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Efficient Uncertainty Quantification Applied to the Aeroelastic Analysis of a Transonic Wing

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The application of a Point-Collocation Non-Intrusive Polynomial Chaos method to the uncertainty quantification of a stochastic transonic aeroelastic wing problem has been demonstrated. The variation in the transient response of the first aeroelastic mode of a three-dimensional wing in transonic flow due to the uncertainty in free-stream Mach number and angle of attack was studied. A curve-fitting procedure was used to obtain time-independent parameterization of the transient aeroelastic responses. Among the uncertain parameters that characterize the time-dependent transients, the damping factor was chosen for uncertainty quantification, since this parameter can be thought as an indicator for flutter. Along with the mean and the standard deviation of the damping factor, the probability of having flutter for the given uncertainty in the Mach number and the angle of attack has been also calculated. Besides the Point-Collocation Non-Intrusive Polynomial Chaos method, 1000 Latin Hypercube Monte Carlo simulations were also performed to quantify the uncertainty in the damping factor. The results obtained for various statistics of the damping factor including the flutter probability showed that an 8^{th} degree Point-Collocation Non-Intrusive Polynomial Chaos expansion is capable of estimating the statistics at an accuracy level of 1000 Latin Hypercube Monte Carlo simulation with a significantly lower computational cost. In addition to the uncertainty quantification, the response surface approximation, sensitivity analysis, and reconstruction of the transient response via Non-Intrusive Polynomial Chaos were also demonstrated.

I. Introduction

For the uncertainty modeling and propagation in aeroelastic problems with multiple uncertain input variables and parameters, computational efficiency becomes an important factor in the selection of the stochastic method when high-fidelity computational fluid dynamics (CFD) tools are used to calculate the time-dependent aerodynamic loads. In addition to the computational efficiency, a desired level of accuracy (confidence) is also sought in the solution of the stochastic fluid-structure interaction problems. Recent studies[1,2] on stochastic CFD problems with multiple uncertain input variables have shown that a Point-Collocation Non-Intrusive Polynomial Chaos (NIPC) approach is a computationally efficient and accurate method for uncertainty modeling and propagation compared to some commonly used stochastic methods such as the crude or Latin Hypercube Monte Carlo (MC). This paper will demonstrate the application of this Point-Collocation NIPC method to the quantification of uncertainty in a stochastic transonic aeroelastic wing problem. The variation in the transient aeroelastic response of a three-dimensional wing in transonic flow due to the uncertainty in free-stream Mach number and angle of attack will be investigated.

The Polynomial chaos (PC) method for the propagation of uncertainty in computational simulations involves the substitution of uncertain variables and parameters in the governing equations with the polynomial expansions. In general, an intrusive approach will calculate the unknown polynomial coefficients by projecting the resulting equations onto basis functions (orthogonal polynomials) for different modes. As its

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name suggests, the intrusive approach requires the modification of the deterministic code and this may be difficult, expensive, and time consuming for many computational problems such as the one considered in this paper. To overcome the inconveniences associated with the intrusive approach, collocation-based NIPC methods have been developed for uncertainty modeling and propagation, such as the Point-Collocation[1,2] approach to be used in the current study and the Probabilistic Collocation approach developed by Loeven *et al.*[3] The other NIPC approaches in the literature are based on *sampling* (Debusschere et al.,[4] Reagan et. al.,[5] and Isukapalli[6]) or *quadrature* methods (Debusschere et al.[4] and Mathelin et al.[7]) to determine the projected polynomial coefficients.

Polynomial Chaos has been used for uncertainty quantification in many previous unsteady stochastic fluid-structure interaction studies.[8–10] One difficulty encountered during the use of PC methods in time-dependent stochastic problems is the degrading accuracy of the approximation in time due to the growing non-linearity of the response. In this paper, a curve-fitting procedure is used to obtain time-independent parameterization of the transient aeroelastic responses. With this approach, the coefficients that characterize the transients are modeled as uncertain response variables. Since they are independent of time, the accuracy in their approximation should not be affected from the growing non-linearity of the response in time. A similar approach was implemented by Witteveen *et. al.*[11] in the time-independent characterization of limit cycle oscillations. In our study, among the uncertain parameters that characterize the time-dependent transients, the damping factor is chosen for uncertainty quantification, since this parameter can be thought as an indicator for flutter. In addition to the Point-Collocation NIPC method, the quantification of uncertainty in the damping factor is also performed with 1000 Latin Hypercube Monte Carlo simulations. Along with the mean and the standard deviation of the damping factor, the probability of having flutter for the given uncertainty in the Mach number and the angle of attack will be also calculated in the stochastic analysis.

In the following section, a description of the non-intrusive polynomial chaos approaches including the Point-Collocation NIPC method is given. Section III includes the details about the stochastic aeroelastic wing case. Section IV presents the results obtained with the Latin Hypercube Monte Carlo and the Point-Collocation NIPC methods. The conclusions are given in Section V.

II. Non-Intrusive Polynomial Chaos

The polynomial chaos is a stochastic method, which is based on the spectral representation of the uncertainty. An important aspect of spectral representation of uncertainty is that one may decompose a random function (or variable) into separable deterministic and stochastic components. For example, for any random variable (*i.e.*, α^*) such as velocity, density or pressure in a stochastic fluid dynamics problem, we can write,

$$\alpha^*(t,\vec{x},\vec{\xi}) = \sum_{i=0}^{P} \alpha_i(t,\vec{x})\Psi_i(\vec{\xi}) \tag{1}$$

where $\alpha_i(t,\vec{x})$ is the deterministic component and $\Psi_i(\vec{\xi})$ is the random basis function corresponding to the i^{th} mode. In the most general case, α^* can be a function of time t, deterministic independent variable vector $\vec{x}$, and the n-dimensional standard random variable vector $\vec{\xi} = (\xi_1, ..., \xi_n)$ which has a specific probability distribution. The discrete sum is taken over the number of output modes ,

$$P + 1 = \frac{(n+p)!}{n!p!} \tag{2}$$

which is a function of the order of polynomial chaos (p) and the number of random dimensions (n). The basis function ideally takes the form of multi-dimensional Hermite Polynomial to span the n-dimensional random space when the input uncertainty is Gaussian (unbounded), which was first used by Wiener[12,13] in his original work of polynomial chaos. Legendre (Jacobi) and Laguerre polynomials are optimal basis functions for bounded (uniform) and semi-bounded (exponential) input uncertainty distributions respectively in terms of the convergence of the statistics. Different basis functions can be used with different input uncertainty distributions (See Xiu and Karniadakis[14] for a detailed description), however the convergence may be affected depending on the basis function used.[15] In the stochastic aeroelasticity problem studied in this paper, we model the input uncertainties as uniform random variables that have bounded probability distributions. Therefore, in our Point-Collocation NIPC method described below we use multi-dimensional Legendre polynomials that are orthogonal in the interval [-1,1] for each random dimension. The detailed information about polynomial chaos expansions can be found in Walters and Huyse[16] and Hosder et al.[1]

To model the uncertainty propagation in computational simulations via polynomial chaos with the intrusive approach, all dependent variables and random parameters in the governing equations are replaced with their polynomial chaos expansions. Taking the inner product of the equations, (or projecting each equation onto k^{th} basis) yield $P+1$ times the number of deterministic equations which can be solved by the same numerical methods applied to the original deterministic system. Although straightforward in theory, an intrusive formulation for complex problems can be relatively difficult, expensive, and time consuming to implement. To overcome such inconveniences associated with the intrusive approach, non-intrusive polynomial chaos formulations have been considered for uncertainty propagation.

The objective of the non-intrusive polynomial chaos methods is to obtain approximations of the polynomial coefficients without making any modifications to the deterministic code. Main approaches for non-intrusive polynomial chaos are sampling based, collocation based, and quadrature methods. To find the polynomial coefficients $\alpha_k = \alpha_k(t, \vec{x})$, $(k = 0, 1, .., P)$ in Equation 1 using sampling based and quadrature methods, the equation is projected onto k^{th} basis:

$$\left\langle \alpha^*(t, \vec{x}, \vec{\xi}), \Psi_k(\vec{\xi}) \right\rangle = \left\langle \sum_{i=0}^{P} \alpha_i \Psi_i(\vec{\xi}), \Psi_k(\vec{\xi}) \right\rangle \tag{3}$$

where the inner product of two functions $f(\vec{\xi})$ and $g(\vec{\xi})$ is defined by

$$\left\langle f(\vec{\xi}) g(\vec{\xi}) \right\rangle = \int_R f(\vec{\xi}) g(\vec{\xi}) p_N(\vec{\xi}) d\vec{\xi} \tag{4}$$

Here the weight function $p_N(\vec{\xi})$ is the probability density function of $\vec{\xi}$ and the integral is evaluated on the support (R) region of this weight function. Using the orthogonality of the basis functions,

$$\left\langle \alpha^*(t, \vec{x}, \vec{\xi}), \Psi_k(\vec{\xi}) \right\rangle = \alpha_k \left\langle \Psi_k^2(\vec{\xi}) \right\rangle \tag{5}$$

we can obtain

$$\alpha_k = \frac{\left\langle \alpha^*(t, \vec{x}, \vec{\xi}), \Psi_k(\vec{\xi}) \right\rangle}{\left\langle \Psi_k^2(\vec{\xi}) \right\rangle} \tag{6}$$

In sampling based methods, the main strategy is to compute $\alpha^*(t, \vec{x}, \vec{\xi}) \Psi_k(\vec{\xi})$ for a number of samples ($\vec{\xi_i}$ values) and perform averaging to determine the estimate of the inner product $\left\langle \alpha^*(t, \vec{x}, \vec{\xi}), \Psi_k(\vec{\xi}) \right\rangle$. Quadrature methods calculate the same term, which is an integral over the support of the weight function $p_N(\vec{\xi})$, using numerical quadrature. Once this term is evaluated, both methods (sampling based and quadrature) use Equation 6 to estimate the projected polynomial coefficients for each mode.

A. Point-Collocation Non-Intrusive Polynomial Chaos

The collocation based NIPC approach starts with replacing the uncertain variables of interest with their polynomial expansions given by Equation 1. Then, $P+1$ vectors $(\vec{\xi_i} = \{\xi_1, \xi_2, ..., \xi_n\}_i,\ i = 0, 1, 2, ..., P)$ are chosen in random space for a given PC expansion with $P+1$ modes and the deterministic code is evaluated at these points. With the left hand side of Equation (1) known from the solutions of deterministic evaluations at the chosen random points, a linear system of equations can be obtained:

$$\begin{pmatrix} \Psi_0(\vec{\xi_0}) & \Psi_1(\vec{\xi_0}) & \cdots & \Psi_P(\vec{\xi_0}) \\ \Psi_0(\vec{\xi_1}) & \Psi_1(\vec{\xi_1}) & \cdots & \Psi_P(\vec{\xi_1}) \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_0(\vec{\xi_P}) & \Psi_1(\vec{\xi_P}) & \cdots & \Psi_P(\vec{\xi_P}) \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_P \end{pmatrix} = \begin{pmatrix} \alpha^*(\vec{\xi_0}) \\ \alpha^*(\vec{\xi_1}) \\ \vdots \\ \alpha^*(\vec{\xi_P}) \end{pmatrix} \tag{7}$$

The spectral modes (α_k) of the random variable, α^*, are obtained by solving the linear system of equations given above. Using these, mean (μ_{α^*}) and the variance $(\sigma^2_{\alpha^*})$ of the solution can be obtained by

$$\mu_{\alpha^*} = \alpha_0 \tag{8}$$

$$\sigma^2_{\alpha^*} = \sum_{i=1}^{P} \alpha_i^2 \left\langle \Psi_i^2(\vec{\xi}) \right\rangle \tag{9}$$

The Point-Collocation NIPC was first introduced by Walters[17] to approximate the polynomial chaos coefficients of the metric terms which are required as input stochastic variables for the intrusive polynomial chaos representation of a stochastic heat transfer problem with geometric uncertainty. In 2006, Hosder et. al.[1] applied this Point-Collocation NIPC method to stochastic fluid dynamics problems with geometric uncertainty. They demonstrated the efficiency and the accuracy of the NIPC method in terms of modelling and propagation of an input uncertainty and the quantification of the variation in an output variable. That study included a single random input variable, and the collocation locations were equally spaced in the random space. Following that work, Hosder et. al.[2] applied the Point-Collocation NIPC to model stochastic problems with multiple uncertain input variables having uniform probability distributions and investigated different sampling techniques (Random, Latin Hypercube, and Hammersley) to select the optimum collocation points. The results of the stochastic model problems showed that all three sampling methods exhibit a similar performance in terms of the accuracy and the computational efficiency of the chaos expansions. However, the convergence of the Point-Collocation NIPC statistics obtained with Hammersley and Latin Hypercube sampling exhibit a much smoother (monotonic) convergence compared to the cases obtained with random sampling.

The solution of linear problem given by Equation 7 requires $P+1$ deterministic function evaluations. If more than $P+1$ samples are chosen, then the over-determined system of equations can be solved using the Least Squares method. Hosder et. al.[2] investigated this option by increasing the number of collocation points in a systematic way through the introduction of a parameter n_p defined as

$$n_p = \frac{number\ of\ samples}{P+1}. \tag{10}$$

In the solution of stochastic model problems with multiple uncertain variables, they have used $n_p = 1, 2, 3,$ and 4 to study the effect of the number of collocation points (samples) on the accuracy of the polynomial chaos expansions. Their results showed that using a number of collocation points that is twice more than the minimum number required ($n_p = 2$) gives a better approximation to the statistics at each polynomial degree. This improvement can be related to the increase of the accuracy of the polynomial coefficients due to the use of more information (collocation points) in their calculation. The results of the stochastic model problems also indicated that for problems with multiple random variables, improving the accuracy of polynomial chaos coefficients in NIPC approaches may reduce the computational expense by achieving the same accuracy level with a lower order polynomial expansion.

III. Stochastic Aeroelastic Transonic Wing Problem

A. Geometry

The wing geometry involved in our study is the AGARD 445.6 Aeroelastic Wing,[18] which has been extensively used in experiments to validate computational aeroelasticity tools[19–21] especially in the determination of flutter boundary at various transonic Mach numbers. The wing has a quarter-chord sweep angle of 45 deg., a panel aspect ratio of 1.65, a taper ratio of 0.6576, and a NACA 65A004 airfoil section. Figure 1 shows the planform view of the AGARD 445.6 Wing. The shapes of the first four structural modes for the AGARD 445.6 Wing and the corresponding model frequencies[18] in vacuo are presented in Figure 2. The modes are first bending (Mode 1), first torsion (Mode 2), second bending (Mode 3), and second torsion (Mode 4). The details about the wing (various structural versions, experiments, *etc.*) can be found in the report by Yates.[18]

B. Computational Methods

For the numerical aeroleastic analysis of the AGARD 445.6 wing in transonic flow, CFL3Dv6.0 code has been used with the aeroelasticity option enabled to couple the unsteady flow field computation with the dynamic deformation of the wing. CFL3Dv6.0 is a three-dimensional, structured, finite-volume Navier-Stokes code capable of solving steady or time-dependent aerodynamic flows ranging from subsonic to supersonic speeds.[22] With the assumption of small deformations for the wing geometry which has a small thickness, a linear structural model has been considered for the modal aeroelastic analysis built in the CFL3Dv6.0 code. First four mode shapes of the wing described in the previous section have been used in the model analysis and the transient response for each mode has been calculated as output. For the uncertainty analysis, we have focused on the transient response of the first mode (bending) shape. For the structural modeling of the wing, experimental model data from Yates[18] has been used. The structural damping parameter has been set to zero for all aeroelastic runs.

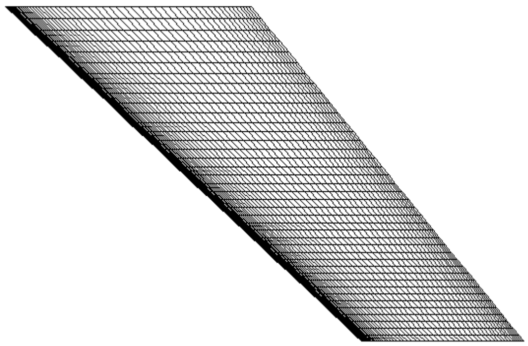


Figure 1. AGARD 445.6 Wing used in aeroelastic calculations and the surface grid

To model the the time-dependent transonic aerodynamics over the deforming wing, three-dimensional, unsteady Euler Equations were solved with CFL3D on a computational grid having a C-H topology with $193 \times 65 \times 42$ grid points (Figure 1). The flux difference splitting method of Roe was employed in the calculation of the fluxes at the cell faces and and the primitive variables on the cell faces were obtained using a third-order upwind-biased spatial differencing scheme with the MinMod flux limiter. To march the solution in time, a second-order backward time-differencing scheme with sub iterations was implemented. A non-dimensional time step of 0.3 with 5 sub-iterations per time step was used in the calculations. At each time step, a multigrid method was employed for convergence increase and error reduction.

C. Stochastic Problem

For the stochastic aeroelasticity problem, the free-stream Mach number (M_∞) and the angle of attack (α) are treated as uncertain variables. The Mach number is modeled as a uniform random variable between $M_\infty = 0.8$ and 1.1 (mean Mach number of 0.95 and a coefficient of variation of 9.1%). The angle of attack is defined to have a uniform distribution between $\alpha = -2.0$ and 2.0 degrees (mean angle of 0.0 degrees and a standard deviation of 1.155). The Mach number variation is chosen such that the uncertainty interval includes transonic regime ranging from high subsonic to low supersonic Mach numbers. In the formulation of the stochastic problem the free-stream pressure and the density were kept constant, so the uncertainty in the Mach number can be also interpreted as the uncertainty due to the variation in the free-stream velocity or the dynamic pressure. Both parameters are chosen to study the variation in aeroelastic characteristics due to the uncertainty in two common aerodynamic variables . Future studies will involve random variables coming from both aerodynamics and structural dynamics to extend the uncertainty quantification approach discussed in this paper to a representative multidisciplinary problem.

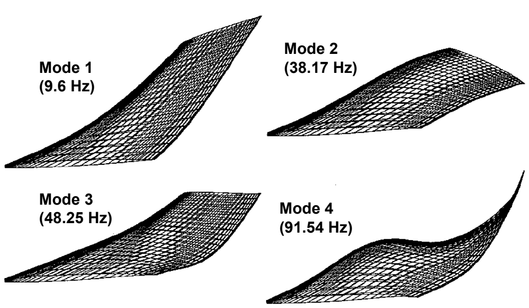


Figure 2. The first four natural vibrational mode shapes and frequencies for the AGARD 445.6 Wing

Since the dynamic aeroleasticity problems, which may also include flutter are inherently unsteady, various output quantities or the response of the system will be also time-dependent. To quantify the uncertainty in the time dependent output of the AGARD 445.6 wing in transonic flow, we parameterize the transient response of a particular aeroleastic mode with damped Sine waves (or complex exponential functions) using the equation

$$x(t) = a_0 + e^{-\eta t}\left[aCos(\omega t) + bSin(\omega t)\right]. \tag{11}$$

In this equation, $x(t)$ corresponds to a time-dependent response variable such as the generalized displacement or the force, which is obtained from a deterministic numerical aeroelastic simulation described in the previous section. The LHS of the equation includes the parameters a_0 (the offset or the static value), η (the damping factor), a (the coefficient of the Cosine term), b (the coefficient of the Sine term), and ω (the angular frequency), which are determined with curve-fitting to the simulation data using the method described by Bennett and Desmarais.[23] All variables and coefficients in Equation 11 including time t are non-dimensional quantities obtained with the scaling convention of CFL3Dv6.0 [22] code. In the current work, the curve-fitting procedure

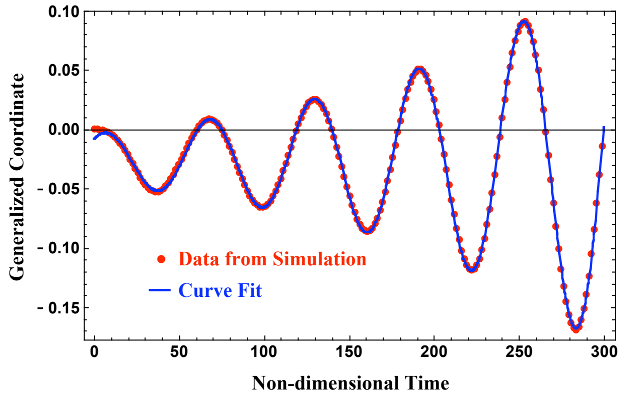


Figure 3. The curve-fit to the generalized displacement response of the first mode shape (bending) for the AGARD 445.6 wing at $Mach = 0.977$ and $\alpha = -0.709^o$ with the parameters $a_0 = -0.02495$, $\eta = -0.006772$, $a = 0.01698$, $b = 0.01232$, and $\omega = 0.1021$ (See Equation 11).

was implemented in Mathematica© version 6 using the CFL3Dv6.0 aeroelastic output data. Figure 3 gives an example result of the curve-fitting procedure, which shows the good representation of the transient response of the first mode shape (bending) with the parameterized curve for Mach $= 0.977$ and $\alpha = -0.709^o$. For this case, the damping factor η is obtained as -0.00677 which indicates that the wing experiences flutter for the given Mach number and the angle of attack. In general, $\eta > 0$ will correspond to a stable (damped) transient, $\eta = 0$ a neutral condition, and $\eta < 0$ a flutter solution. One can think of the curve-fitting procedure as a time-independent parameterization of the transient aeroelastic response. This type of an approach is particularly useful in the stochastic analysis of the time-dependent output with Polynomial Chaos, since the accuracy of the approximation should not be affected from the growing non-linearity of the response in time.

In the stochastic problem, the parameters that characterize the transient will be uncertain due the variation in the free-stream Mach number and the angle of attack. Each parameter will posses a probability distribution. In particular, we will focus on the quantification of uncertainty in the damping factor, since this parameter is an indicator of flutter. Along with the mean and the standard deviation, we will also investigate the flutter probability using the probability distribution of the damping factor. For the given uncertainty in the Mach number and the angle of attack, the probability of flutter will be determined from the ratio of samples (or area) with $\eta < 0$ to the total number of samples (area) in the damping factor distribution.

The propagation and the quantification of uncertainty in stochastic aeroelastic wing problem are studied with two approaches: Latin Hypercube Monte Carlo with 1000 samples and Point-Collocation NIPC with different polynomial degrees. Based on the observations from the stochastic model problems,[2] Hammersley Sampling with a number of collocation points that is twice more than the minimum number required ($n_p = 2$) has been used for the selection of collocation points for the NIPC approach. Various statistics obtained with the Point-Collocation NIPC are compared to the Latin Hypercube Monte Carlo results.

IV. Results

The initial stochastic results with the AGARD 445.6 wing were obtained with the rigid wing assumption in transonic flow and the results were reported in a recent paper by Hosder *et al.*[2] That work, which included the same geometry and the same uniform uncertain input variables (Mach number and the angle of attack) with the same limits can be considered as a preliminary investigation before the application of the Point-Collocation NIPC method to the current stochastic computational aeroelasticity problem. In that study, Hosder *et al.* quantified the uncertainty in various output quantities of interest such as the lift coefficient, drag coefficient, and the surface pressure distributions. Figure 4 shows the uncertainty bars for the pressure coefficient (C_p) distributions at three spanwise stations obtained using the Point-Collocation NIPC approach for the rigid transonic wing case. The results showed that a 5^{th} degree Point-Collocation NIPC expansion obtained with Hammersley sampling was capable of estimating the statistics at an accuracy level of 1000 Latin Hypercube Monte Carlo simulations with a significantly lower computational cost. The following two

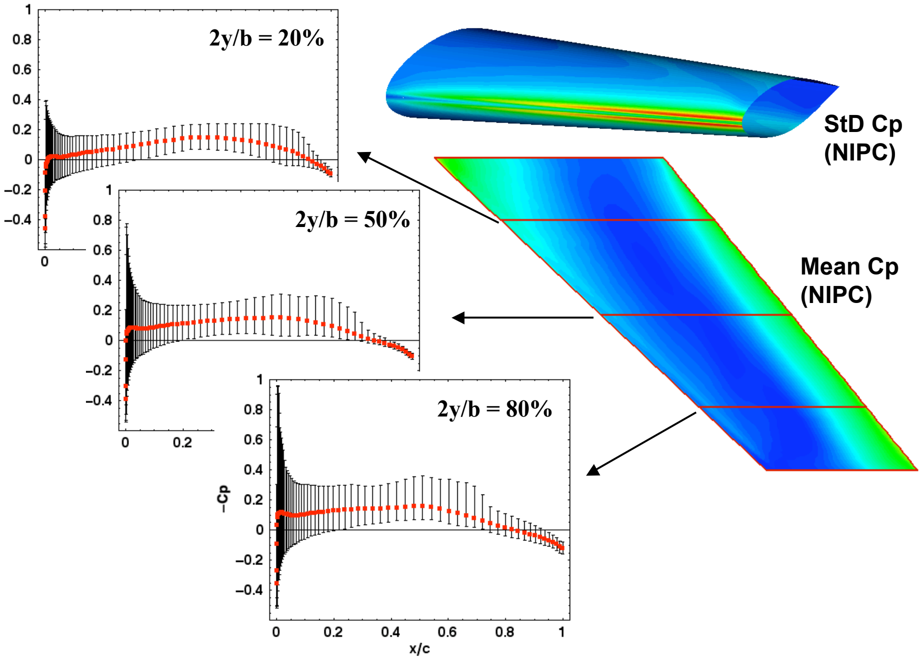


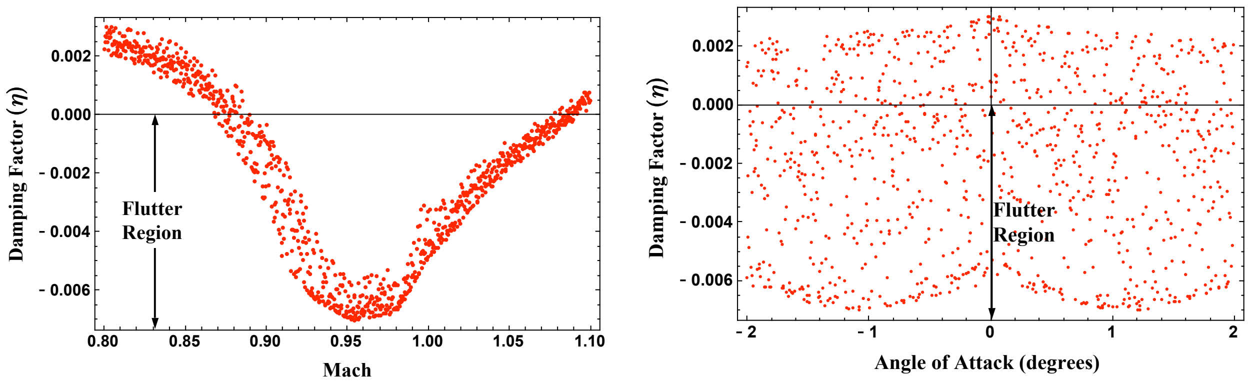
Figure 4. The uncertainty bars for the pressure coefficient (C_p) distributions at three spanwise stations obtained using the Point-Collocation NIPC approach in the rigid stochastic transonic AGARD 445.6 wing study by Hosder *et al.*[2] Each uncertainty bar includes the C_p values within 95% confidence interval.

sections include the results obtained for the current stochastic aeroelastic wing study using Latin Hypercube Monte Carlo and Point-Collocation NIPC methods.

A. Latin Hypercube Monte Carlo Results

Latin Hypercube Monte-Carlo method has been used to calculate various statistics of the curve-fit parameters that characterize the aeroelastic response of the first (bending) mode of the transonic wing. The transient responses were calculated at 1000 Latin Hypercube sample points of Mach number and angle of attack selected from associated uniform probability distributions. Figure 5 shows the scatter of the damping factor η obtained with the Monte Carlo simulations and gives a qualitative description of the dependence of η on the Mach number and the angle of attack within the given range of these random input parameters. As can be seen from Figure 5(a), for all samples that have the Mach number between 0.8 and approximately 0.87, the aeroelastic response is in the stable region ($\eta < 0$) regardless of the variation in the angle of attack. Starting from Mach=0.8, the damping factor starts to decrease with the increase of the Mach number and this trend continues until a Mach number of 0.96 is reached where the minimum value of η is approximately -0.008. Beyond this point, an opposite trend for the damping factor can be observed: with the increase of the Mach number, η value starts to increase, carrying the aeroelastic response towards the stable region. This scatter plot shows that a significant portion of the responses (between Mach$\approx$ 0.87 and Mach$\approx$ 1.07) fall within the flutter region with the given uncertainty in the Mach number and the angle of attack. At a constant Mach number value, the scatter in the data is due to the uncertainty in the angle of attack and the width of this scatter tends to decrease at supersonic Mach numbers with Mach$>$ 1.05. Figure 5(b) shows that the scatter in the damping factor is symmetric with respect to zero degrees angle of attack due to the the symmetry of the AGARD 445.6 wing geometry. At a constant angle of attack, the scatter is due the uncertainty in Mach number and the width of this scatter (variation) is much larger compared to the variation observed at a constant Mach number, which implies that the uncertainty in Mach number has more influence on the value of the damping factor compared to the effect of the angle of attack within the given input uncertainty range.

Table 1 gives the Latin Hypercube Monte Carlo statistics for the damping factor η. From the probability distribution of η, the flutter probability (PF) has been calculated as 68.9%. This indicates that for the given



(a) Effect of Mach Number on η

(b) Effect of angle of attack on η

Figure 5. Scatter plots of damping factor (η) obtained with Latin Hypercube Monte Carlo with 1000 samples.

uncertainty in the Mach number and the angle of attack, the chance of having flutter is much more likely than having a stable aeroelastic response. Table 1 also gives the 95% confidence interval for each Monte Carlo statistics, which was obtained with the Bootstrap Method. The advantage of the Bootstrap Method is that it is not restricted to a specific distribution, e.g. a Gaussian. It is easy and efficient to implement, and can be completely automated to any estimator, such as the mean or the variance. In practice, one takes at least 100 bootstrap samples to obtain a standard error estimate. In our computations, we used 1000 bootstrap samples each consisting of 1000 observations selected randomly from the original Monte Carlo simulations by giving equal probability (1/1000) to each observation.

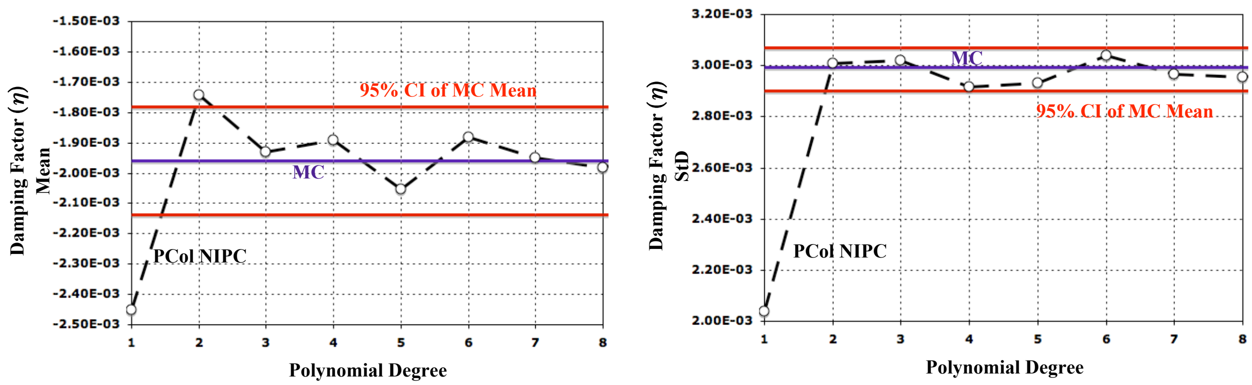
Table 1. The Latin Hypercube Monte Carlo and Point-Collocation NIPC ($p = 8$) statistics for the damping factor η. The 95% confidence intervals for the MC statistics were calculated using the Bootstrap method.

Statistics	MC	95% Confidence Interval of MC	Point-Collocation NIPC ($p = 8$)
Mean	-0.00195823	[-0.00213605, -0.00178033]	-0.00198330
Standard Deviation	0.0029924	[0.00290271, 0.00307085]	0.00295386
Flutter Probability	68.9%	[66.1%, 71.8%]	70.0%

B. Point-Collocation NIPC Results

The chaos expansions up to a polynomial degree of 8 were obtained to calculate various statistics of the parameters used in the curve-fits representing the transient responses (generalized displacement) of the first (bending) mode. The coefficients in the polynomial expansions were calculated with the Point-Collocation NIPC method using Hammersley samples (with $n_p = 2$) as the collocation points. Table 2 gives the computational cost associated with the Latin Hypercube Monte Carlo and the Point-Collocation NIPC approach. As can be seen from this table, computational time for the the Monte Carlo simulations is approximately 11 times more than the time required for the evaluation of Point-Collocation NIPC with a polynomial degree of 8.

In the NIPC study, we have monitored the convergence of the mean and the standard deviation of the damping factor (Figure 6) along with the convergence of the probability of flutter (Figure 7). As the degree of the polynomial chaos increased, all statistics converged at a polynomial degree of 8. At this polynomial degree, all statistics including the probability of flutter fall within the 95% confidence interval of the Latin Hypercube Monte Carlo simulations. Table 1 gives the statistics obtained with the 8^{th} degree polynomial chaos expansion. The probability of flutter, which was calculated as 68.9% with the Latin Hypercube Monte Carlo takes the value of 70% with the Point-Collocation NIPC method. Figure 8 gives the histograms of the damping factor obtained with the Monte Carlo method and the 8^{th} degree polynomial chaos expansion. Except the peak region on the left hand side, polynomial chaos seems to capture the shape of the Monte



(a) Mean value of η (b) Standard Deviation of η

Figure 6. Mean and standard deviation (StD) of damping factor η obtained with Point-Collocation NIPC and Latin Hypercube MC

Carlo histogram. The shape of histograms also indicate the highly non-linear characteristics of the aeroelastic system considered in this study. Although the input uncertainty is uniform, the output uncertainty exhibit a highly non-conventional distribution, which also explain the relatively high polynomial degree required to estimate the statistics with the NIPC approach.

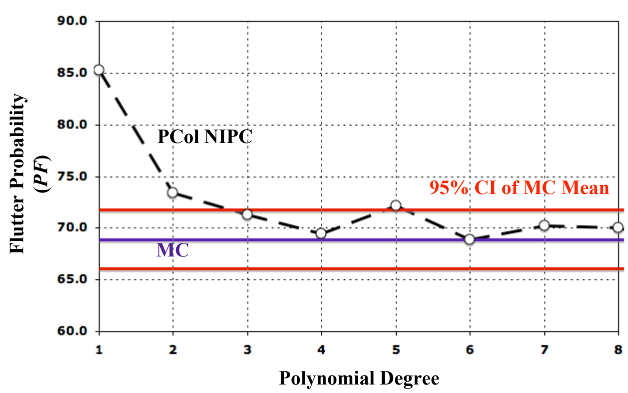
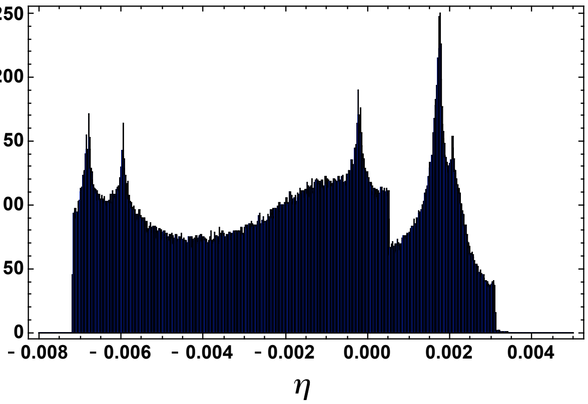


Figure 7. The probability of flutter PF obtained with Point-Collocation NIPC and Latin Hypercube MC

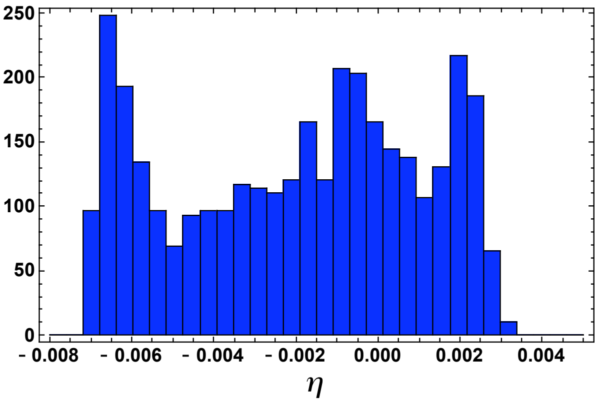
Polynomial Chaos expansions can also be interpreted as stochastic response surfaces, which approximate the output quantities of interest at any location or interval within the region defined by the limits of the uncertain input variables. For the damping factor η, we can write the polynomial chaos expansion as $\eta_{PC} = \eta_{PC}\left[\xi_1(M_\infty), \xi_2(\alpha)\right]$ since the standard uniform input variables ξ_1 and ξ_2 are functions of the free-stream Mach number and the angle of attack, respectively. Once the polynomial coefficients are calculated with the Point-Collocation NIPC, the polynomial representation of η can be used to approximate the response value at any point within the support region of the uncertain variables. As an example, Figure 9(a) shows the 8^{th} degree polynomial chaos representation of the damping factor ($\eta_{PC} = \eta_{PC}\left[\xi_1(M_\infty), \xi_2(\alpha_i)\right]$, $i = 1, 2, 3$) as a function of Mach number at three angle of attack values ($\alpha_1 = 0.0^o$, $\alpha_2 = 0.75^o$, and $\alpha_3 = 1.5^o$) in the interval $0.85 <$Mach< 1.05, whereas Figure 9(b) shows the polynomial chaos representation of the damping factor ($\eta_{PC} = \eta_{PC}\left[\xi_1(M_{\infty i}), \xi_2(\alpha)\right]$, $i = 1, 2, 3$) as a function of angle of attack at three Mach values ($M_{\infty 1} = 0.85$, $M_{\infty 2} = 0.95$, and $M_{\infty 3} = 1.05$) in the interval $-1.5^o < \alpha < 1.5^o$. As can be seen from

Table 2. The computational cost for evaluation of the Latin Hypercube Monte Carlo simulations and the Point-Collocation NIPC for the stochastic aeroelasticity problem. The CFD runs were performed on a SGI Origin 3800 with 64 processors. (p is the degree of the polynomial chaos expansion).

		NIPC			
	Monte Carlo	$p = 2$	$p = 4$	$p = 6$	$p = 8$
number of CFD runs	1000	12	30	56	90
wall clock time (hours)	446.7	5.36	13.4	25	40.2

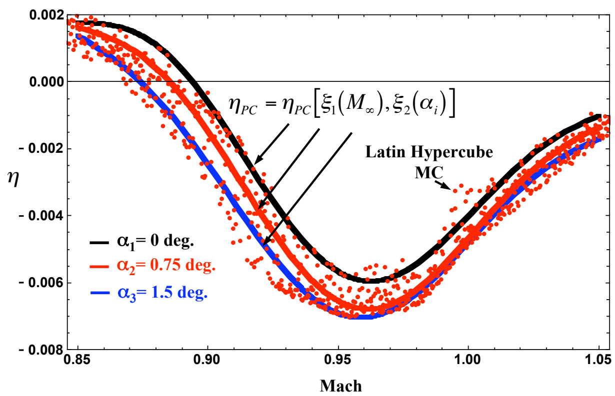


(a) Point-Collocation NIPC

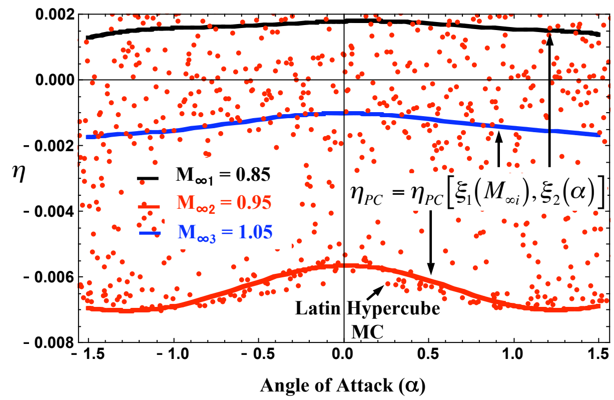


(b) Latin Hypercube Monte Carlo

Figure 8. Histograms of the damping factor η obtained with Point-Collocation NIPC ($p = 8$) and Latin Hypercube MC



(a) Polynomial Chaos representation of η at three angles of attack



(b) Polynomial Chaos representation of η at three Mach numbers

Figure 9. Polynomial Chaos representation (Point-Collacation NIPC with $p = 8$) of damping factor superposed on the Latin Hypercube MC responses

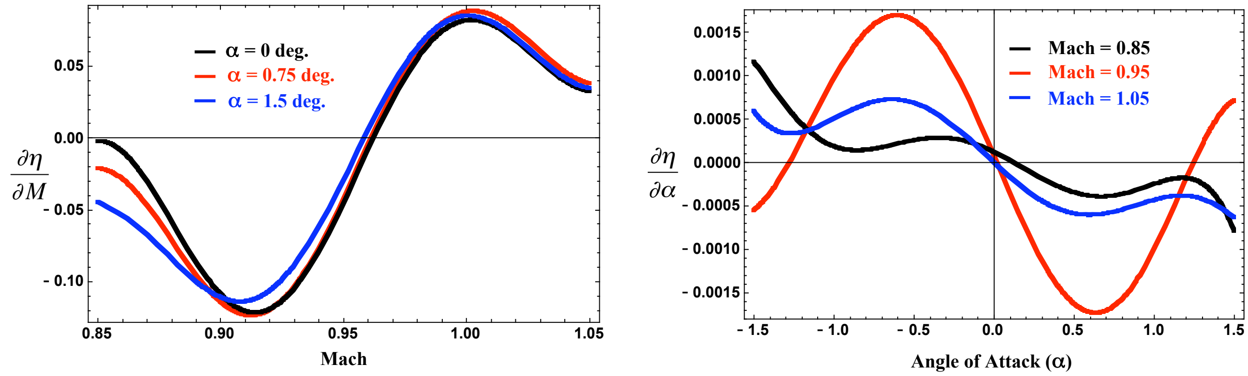
both figures, the polynomial chaos representation of the damping factor follows a trend consistent with the observations made for the Monte Carlo responses. In Figure 9(a), it can also be seen that increasing the angle of attack tends to move the flutter boundary ($\eta = 0$) towards lower Mach numbers in the subsonic region. By using the polynomial expansion, the flutter boundary can be determined to be at Mach=0.894 for $\alpha = 0.0^o$, at a Mach number of 0.885 for $\alpha = 0.75^o$ and at Mach=0.874 for $\alpha = 1.5^o$.

Using the polynomial chaos expansions, one can also calculate the sensitivity of the damping factor to free-stream Mach number and angle of attack with

$$\frac{\partial \eta_{PC}}{\partial M_\infty} = \frac{\partial \eta_{PC}}{\partial \xi_1}\frac{\partial \xi_1}{\partial M_\infty}, \tag{12}$$

$$\frac{\partial \eta_{PC}}{\partial \alpha} = \frac{\partial \eta_{PC}}{\partial \xi_2}\frac{\partial \xi_2}{\partial \alpha}. \tag{13}$$

Figure 10(a) shows the sensitivity of the damping factor to the free-stream Mach number at three angles of attack and Figure 10(b) gives the sensitivity of the damping factor to the angle of attack at three free-stream Mach numbers. The sensitivity information can be calculated at any point within the region defined by the uncertain input variables. As an example, for the flutter boundary obtained at Mach=0.894 and $\alpha = 0.0^o$, the sensitivity derivatives were calculated as $\partial \eta_{PC}/\partial M_\infty = -0.09636$ and $\partial \eta_{PC}/\partial \alpha = 0.00011[deg^{-1}]$, which shows that the damping factor is much more sensitive to small variations in Mach number than the small



(a) Sensitivity of η to Mach number at three angles of attack (b) Sensitivity of η to angle of attack at three Mach numbers

Figure 10. Sensitivity of the damping factor η to Mach number and angle of attack obtained with Point-Collocation NIPC ($p = 8$).

changes in the angle of attack . A positive change in Mach number at this point will onset flutter, which can also be seen from Figure 9(a). In fact, at most regions, the damping factor is more sensitive to the change in Mach number except the locations where $\partial\eta_{PC}/\partial M_\infty \approx 0$ such as at Mach ≈ 0.96.

The polynomial chaos expansions of the parameters (a_0, η, a, b, and ω) used in the curve-fit to the aeroelastic response (generalized displacement) enable the reconstruction of the time dependent transient for any Mach number and angle of attack within the defined input uncertainty region. To demonstrate this, three random points from the Latin Hypercube Monte Carlo simulations have been selected, which exhibit responses that fall in the flutter region, at the flutter boundary, and in the stable region as seen in Figure 11. The good agreement between the reconstructed transients and the original Monte Carlo responses shows the effectiveness of the time-independent parameterization of the transients and the Point-Collocation NIPC for uncertainty propagation and quantification in stochastic aeroelastic problems.

V. Conclusions

In this paper, we have demonstrated the application of a Point-Collocation Non-Intrusive Polynomial Chaos method to a stochastic transonic aeroelastic wing problem with two uncertain variables. This stochastic computational aeroelasticity problem involved the AGARD 445.6 wing in transonic flow with an uncertain free-stream Mach number and angle of attack, both modeled as uniform random variables. The transient response of the first aeroelastic (bending) mode has been analyzed for uncertainty quantification. A curve-fitting procedure was used to obtain time-independent parameterization of the transient aeroelastic responses. This approach enabled the study of the uncertainty in the aeroelastic response in terms of the variation in the curve-fit parameters that characterize the time-dependent transients. In particular, we have focused on the quantification of uncertainty in the damping factor, since this parameter is an indicator of flutter. Along with the mean and the standard deviation of the damping factor, flutter probability has been also calculated for the specified uncertainty range in the Mach number and the angle of attack. The Point-Collocation NIPC method has been used to quantify the uncertainty in the damping factor. Hammersley Sampling with a number of collocation points that is twice more than the minimum number required ($n_p = 2$) has been used for the selection of collocation points in the NIPC approach. The propagation and the quantification of uncertainty in stochastic aeroelastic wing problem was also studied with 1000 Latin Hypercube Monte Carlo simulations.

The results obtained for various statistics of the damping factor including the flutter probability showed that an 8^{th} degree Point-Collocation NIPC expansion is capable of estimating the statistics at an accuracy level of 1000 Latin Hypercube Monte Carlo simulations. Due to the highly non-linear nature of the transonic aeroelasticity problem, a relatively high polynomial degree was required for the convergence and the accuracy of the statistics obtained with the Point-Collocation NIPC method. However, the computational cost required for the evaluation of NIPC method was still significantly lower than the computational expense of the Monte Carlo simulations. In addition to the uncertainty quantification, the response surface approximation, sensitivity analysis, and reconstruction of the transient response via NIPC were also demonstrated for the transonic stochastic aeroelastic wing problem. Overall, the results show that the Point-Collocation NIPC is a promising method for efficient uncertainty propagation and quantification in stochastic aeroleasticity problems.

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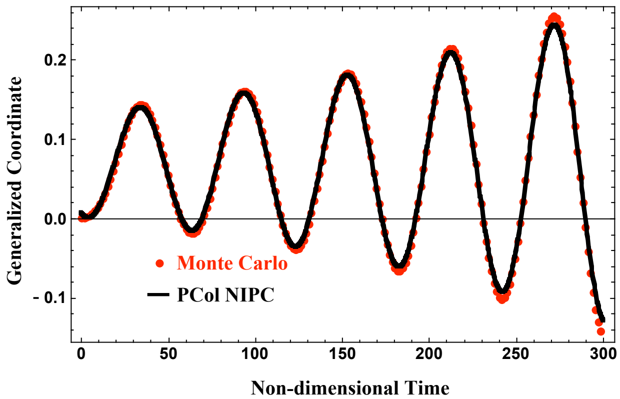
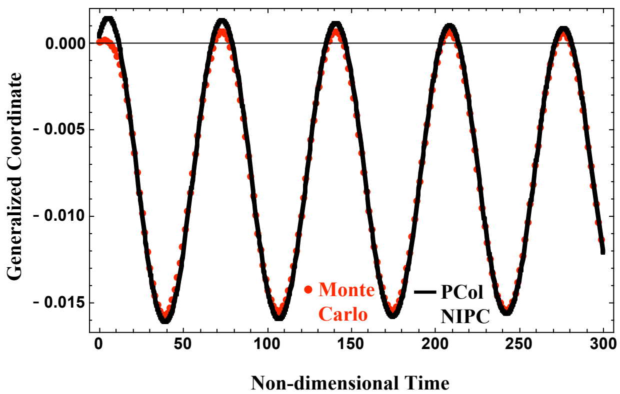
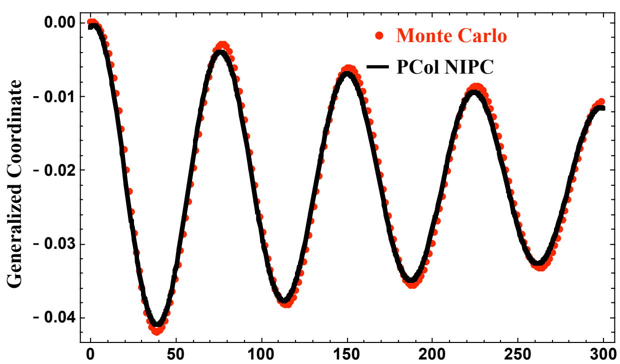


(a) Mach=1.016 and $\alpha = 1.882^o$



(b) Mach=0.889 and $\alpha = -0.277^o$



(c) Mach=0.809 and $\alpha = -0.981^o$

Figure 11. Reconstruction of time dependent aeroelastic responses with Point-Collocation NIPC method