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Recommended Citation

J. Shen and S. N. Balakrishnan, "A Class of Modified Hopfield Networks for Control of Linear and Nonlinear Systems," *Proceedings of the 1998 American Control Conference, 1998*, Institute of Electrical and Electronics Engineers (IEEE), Jan 1998.

The definitive version is available at <https://doi.org/10.1109/ACC.1998.703552>

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A Class of Modified Hopfield Networks for Control of Linear and Nonlinear Systems

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Abstract

This paper presents a class of modified Hopfield neural networks (MHNN) and their use in solving linear and nonlinear control problems. This class of networks consists of parallel recurrent networks which have variable dimensions that can be changed to fit the problems under consideration. It has a structure to implement an inverse transformation that is essential for embedding optimal control gain sequences. Equilibrium solutions are discussed. Numerical results for a motivating aircraft control problem (linear) are presented. Furthermore, we formulate the state-dependent Riccati equation method (SDRE) for a class of nonlinear dynamical system and show how MHNN provides the solution. Two examples that illustrate the potential of this network for the SDRE method are also presented.

1 Introduction

There has been a spurt of activities in the area of artificial neural networks (ANN) during the last ten years. For a survey of ANN work done in the areas of identification and control, see bibliography. There are two types of networks used in almost all ANN applications, the feedforward network and the recurrent network. The former where data flow is unidirectional are essentially static; the later, on the other hand, are based on feedback connections. Due to the feedback connections, the recurrent networks are better suited for control problems which are based on closed-loop solutions.

In this paper, a variation of the Hopfield network is proposed. Similar to the classic Hopfield network, it keeps the characteristic of energy mini-

mization. However, based on the equilibrium analysis, these networks can perform an inverse transformation on matrices and other auxiliary mathematical operations. This feature allows the networks to produce optimal control gain sequences. Unlike any other existing neural networks, inputs to the networks are the parameters of system dynamics and control matrices. In addition, this class of networks has more degrees of freedom than the classic Hopfield networks. match the problems at hand.

There has also been an increasing interest in nonlinear control. For quite a long time, LQR and its variations have been widely used. Other methods based on linear systems theory such as dynamic inversion, feedback linearization, and sliding mode control are becoming more popular due to the inherent limitation of the linear regulator. A recent development in the area of nonlinear regulation is SDRE[4-5]. This method converts the nonlinear structure of a class of nonlinear problems to a linear structure. For a quadratic cost function, the resultant Riccati equation is state-dependent (hence the name SDRE method) though its structure is the same as that for the linear cases. The formulation and development of MHNN is such that it helps solve the SDRE-based nonlinear problems.

2 Modified Hopfield Neural Networks(MHNN)

2.1 Stability

MHNN is a variant of the classical Hopfield network. Fig (1) shows its basic features.

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We will demonstrate its stability by analyzing its dynamics and using an energy function.

$$C_i \frac{du_i}{dt} = -a_i - G_i u_i - \sum_{j=1}^m w_{ji} f\left(\sum_{k=1}^n w_{kj} v_k - b_j\right) \quad (i = 1, 2, \dots, n) \quad (1)$$

If defining the following *Lyapunov* function as an energy function E for MHNN

$$E(v) \triangleq \sum_{k=1}^n a_k v_k + \sum_{j=1}^m F\left(\sum_{k=1}^n w_{kj} v_k - b_j\right) + \sum_{i=1}^n G_i \int_0^{v_i} g^{-1}(v) dv, \quad (2)$$

we can find the time derivative of the energy function as

$$\begin{aligned} \frac{dE}{dt} &= \sum_{i=1}^n \frac{dv_i}{dt} \left(a_i + u_i G_i + \sum_{j=1}^m w_{ji} \right. \\ &\quad \left. \cdot f\left(\sum_{k=1}^n w_{kj} v_k - b_j\right) \right) \\ &= - \sum_{i=1}^n C_i g^{-1'}(v_i) \left(\frac{dv_i}{dt} \right)^2 \end{aligned} \quad (3)$$

Since $C_i > 0$, and $g^{-1}(v_i)$ is a monotonically increasing function, the sum on the right side of (3) is nonnegative, and therefore we have $dE/dt \leq 0$, unless $dv_i/dt = 0$, in which case $dE/dt = 0$. This means that the evolution of dynamic system (1) in state space always seeks the minima of the energy surface E . and (2) shows that the outputs v_j do follow gradient descent paths on the E surface.

2.2 Solution

In order to get the analytic expression for the converged value of the networks, we assume small signals and that they work in the linear region of the amplifier. difference if we denote the connection matrices in the left and right adjoint subnets separately. These connection matrices are nothing but the weights w_{ij} . Let the right connection matrix be D_1 , and the left connection matrix be D_2 , Under these mild assumptions, and with *Kirchhoff's* law, we can have a relation in a matrix form as

$$C \frac{dU}{dt} = -a - GU - D_1^T Q \quad (4)$$

$$\begin{aligned} Q &= K_2(D_2 V - b) \\ &= K_2(K_1 D_2 U - b) \end{aligned} \quad (5)$$

Substitute Equation (5) into Equation (4) to get

$$C \frac{dU}{dt} = -a - GU - D_1^T K_2(K_1 D_2 U - b) \quad (6)$$

$$\begin{aligned} &= -(G + K_2 K_1 D_1^T D_2) U \\ &\quad + K_2 D_1^T b - a \end{aligned} \quad (7)$$

When the networks reach equilibrium, $dU/dt = 0$, and

$$V = \left(D_1^T D_2 + \frac{G}{K_1 K_2} \right)^{-1} \left(D_1^T b - \frac{a}{K_1} \right) \quad (8)$$

2.3 Discussion

Equation (8) gives the general solution for MHNN. Compared with the classical Hopfield networks, an obvious feature is that this network involves more parameters. We may find some applications in which these parameters can be taken advantage of. Also some of them can be nulled out depending upon the desired objective.

Note we get two factors involved in the inverse operation. As a result, the structure of this kind of recurrent networks is quite flexible. While the classical Hopfield is self-recurrent, that is, it feeds back its own output; the variation is mutually recurrent, that is, it feeds back the outputs of its two-adjoint parts. This architecture can be expanded further with ease to three or four subnets or several layers as needed. The dimensions of parameters a , b , D_1 and D_2 depend on the applications. K_1 and K_2 also can be designed to provide appropriate magnitudes.

3 Linear Control Application

3.1 Problem Formulation

Let the plant to be controlled be described by the linear equation

$$x_{k+1} = A_k x_k + B_k u_k \quad (9)$$

with $x_k \in \mathbf{R}^n$ and $u_k \in \mathbf{R}^m$. The associated performance index is the quadratic function

$$J_i = \frac{1}{2} x_N^T S_N x_N + \frac{1}{2} \sum_{k=i}^{N-1} \left(x_k^T Q_k x_k + u_k^T R_k u_k \right) \quad (10)$$

defined over the time interval of interest $[i, N]$. matrices can be time-varying. The initial plant state is given as x_i . We assume that Q_k , R_k and S_N are symmetric positive semidefinite matrices, and in addition that $|R_k| \neq 0$ for all k .

The Hamiltonian procedure leads to the control,

$$u_k = -K_k x_k, \quad k < N$$

where the Kalman gain K_k is given by

$$K_k = \left(B_k^T S_{k+1} B_k + R_k \right)^{-1} B_k^T S_{k+1} A_k \quad (11)$$

In terms of the Riccati variable S_k , now

$$S_k = A_k^T S_{k+1} (A_k - B_k K_k) + Q_k \quad (12)$$

3.2 Missile Roll Control Problem

We consider the synthesis of an optimal missile roll autopilot in this section. The performance index in this application is an infinite-time quadratic cost function. The minimizing control is expected to maintain the roll orientation Φ close to zero, while the roll rate p does not exceed the maximum p_{max} and the aileron deflection δ_a does not exceed the limit δ_{max} .

The elements of the state space \mathbf{x} are

$$\mathbf{x} = [\phi \quad p]^T \quad (13)$$

The matrix A represents the dynamic stability derivatives and is given by

$$A = \begin{bmatrix} 0 & 1 \\ 0 & L_p \end{bmatrix}$$

The matrix B represents the control derivatives and is given by

$$B = [0 \quad L_\delta]^T$$

The control variable u represents aileron deflection δ .

The performance index, J , is formulated so as to keep ϕ , p and δ low and penalize if they exceed the prespecified maximum values. i.e.

$$J = \int_0^\infty \left[\left(\frac{\Phi}{\Phi_{max}} \right)^2 + \left(\frac{p}{p_{max}} \right)^2 + \left(\frac{\delta}{\delta_{max}} \right)^2 \right] dt \quad (14)$$

This performance index can be easily transformed into an optimal control problem with

$$J = \int_0^\infty \left(\mathbf{x}^T Q' \mathbf{x} + u^T R' u \right) dt$$

The values of the parameters are $L_p = -2$ rad/sec, $L_\delta = 9000/\text{sec}^2$, $\Phi_{max} = 10$ deg, $p_{max} = 300$ deg/sec and $\delta_{max} = 0.524$ rad.

The controls which are calculated by networks, compared with **LQR** results are shown in Fig (3). The states trajectories are shown in Fig (2). We can observe that for various initial conditions, the control and the state histories for the LQR results using MATLAB and MHNN outputs are the same. Note that the solid lines in these plots are LQR histories, and the dashed lines are results using MHNN.

4 SDRE-Based Nonlinear Control

4.1 Overview of the Method

Consider the general infinite-horizon nonlinear regulator problem of the form:

Minimize

$$J = \frac{1}{2} \int_{t_0}^\infty \left(x^T Q(x) x + u^T R(x) u \right) dt \quad (15)$$

with respect to the state x and control u subject to the nonlinear differential constraint

$$\dot{x} = f(x) + g(x)u \quad (16)$$

According to SDRE [4-5], a suboptimal solution of (15)-(16) can be obtained by:

1. Bring the nonlinear dynamics to the form

$$\dot{x} = A(x)x + B(x)u \quad (17)$$

2. Solve the state-dependent Riccati equation

$$A^T(x)S + SA(x) - SB(x)R^{-1}(x)B^T(x)S + Q(x) = 0 \quad (18)$$

3. Construct the nonlinear feedback controller via

$$u = -R^{-1}(x)B^T(x)S(x)x \quad (19)$$

The properties of this approach are discussed in [5].

4.2 Nonlinear Regulator Examples

In this section MHNN approach is applied to two nonlinear regulator problems set in SDRE form.

Example 1

The first example is the scalar problem found in Freeman and Kokotović [6] and illustrates the fact that, in scalar case, the new method produces the optimal solution of the nonlinear regulator problem (15)-(16).

Minimize

$$J = \frac{1}{2} \int_{t_0}^{\infty} (x^2 + u^2) dt \quad (20)$$

with respect to x and u subject to the constraint

$$\dot{x} = x - x^3 + u \quad (21)$$

The optimal control for this problem is given by [6]

$$u_{opt} = -(x - x^3) - x\sqrt{x^4 - 2x^2 + 2} \quad (22)$$

For this problem,

we use a parameterization of $a(x)$ and $b(x)$ in Eqn (17) as

$$a(x) = 1 - x^2 \quad b(x) = 1 \quad (23)$$

The control is given by

$$u(x) = -s(x)x \quad (24)$$

where $s(x)$ is obtained by solving Eqn (18) online with MHNN. Numerical results using Eqs (22) and

(24) are presented in Fig (4). The MHNN-based solution is identical to the exact result.

Example 2

Minimize

$$J = \frac{1}{2} \int_0^{\infty} x^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + u^T \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} u dt \quad (25)$$

subject to the constraints

$$\begin{aligned} \dot{x}_1 &= x_1 - x_1^3 + x_2 + u_1 \\ \dot{x}_2 &= x_1 + x_1^2 x_2 - x_2 + u_2 \end{aligned} \quad (26)$$

For this problem,

we select the parameterization as

$$A(x) = \begin{bmatrix} 1 - x_1^2 & 1 \\ 1 & x_1^2 - 1 \end{bmatrix} \quad (27)$$

The numerical results using an exact method and MHNN approach with SDRE are presented in Fig. (5). We can observe that the SDRE method with MHNN produces the nearly the same trajectories as the exact optimal control. The control histories are also almost overlapping for most of the time. These results show the potential of MHNN with SDRE to be an accurate nonlinear controller.

5 Conclusions

A class of MHNN has been presented to solve optimal control problems raised in linear and nonlinear systems. Similar to the Hopfield network, the stability of MHNN is guaranteed. But they provide more degrees of freedom and flexibility to accommodate different applications. Optimal control is obtained for a four-dimensional aircraft control problem. The results for nonlinear control demonstrate the potential of this method for online applications. Future work on this topic will investigate the robustness of such network controllers and the use of these methods for other relevant applications.

* * * *

ACKNOWLEDGMENT

This study was partially funded by NSF Grant ECS-9634127, the Missouri Department of Economic Development Center Grant R-5-31125 and by Air Force Grant F08630.

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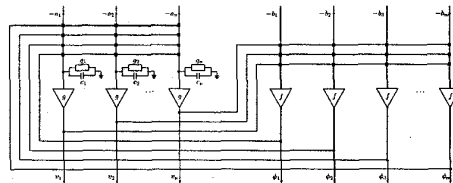


Figure 1: Modified Hopfield Networks

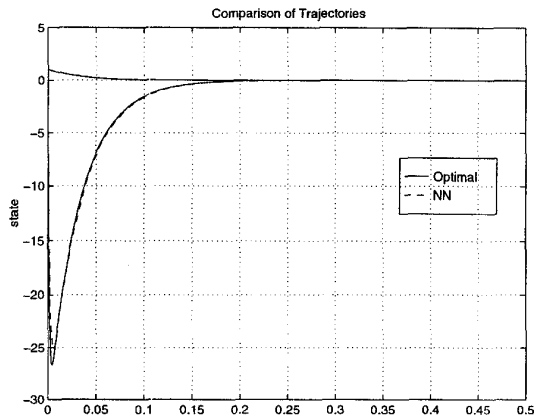


Figure 2: Trajectories of Missile States

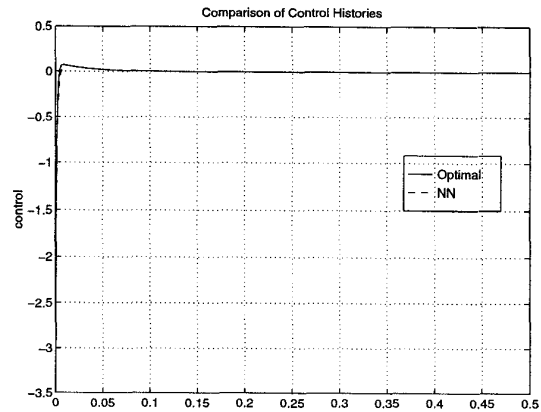


Figure 3: Control History

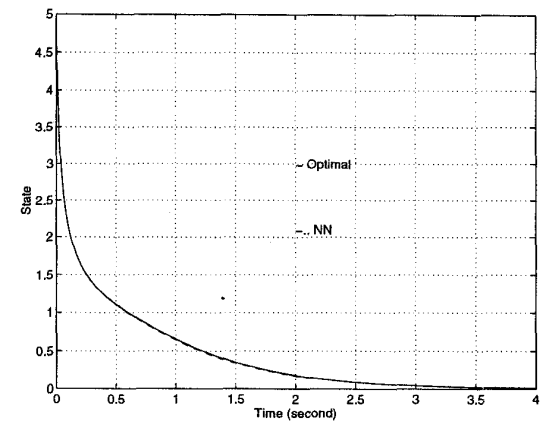
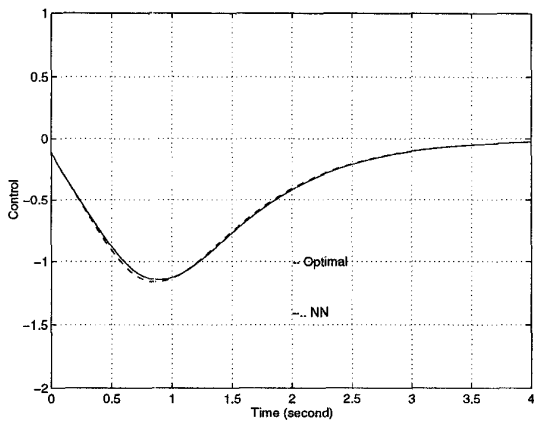


Figure 4: Comparison of Optimal and NN Control and State

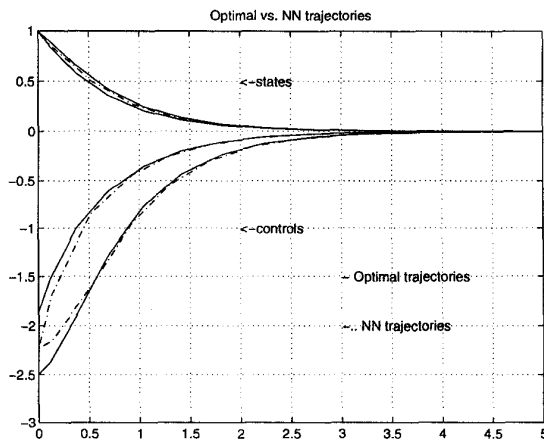


Figure 5: Comparison of Optimal and NN Control and States