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Polarization-operator approach to electron-positron pair production in combined laser and Coulomb fields

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The optical theorem is applied to the process of electron-positron pair creation in the superposition of a nuclear Coulomb and a strong laser field. We derive representations for the total production rate as twofold integrals, both for circular laser polarization and for the general case of elliptic polarization. Our approach allows us to obtain, by analytical means, the asymptotic behavior of the pair creation rate for various limits of interest. In particular, we consider pair production by two-photon absorption and show that, close to the energetic threshold of this process, the rate obeys a power law in the laser frequency with different exponents for linear and circular laser polarization. With the help of the upcoming x-ray laser sources, our results could be tested experimentally.

I. INTRODUCTION

Studies of electron-positron pair production in strong external fields are an important tool to reveal the structure of the QED vacuum. Dynamical pair creation by two Coulomb fields in relativistic heavy ion collisions has been analyzed in great detail, both theoretically and experimentally (see, e.g., [1] for a topical review). In recent years, because of the sustained progress of laser technology, pair creation in strong laser fields is encountering growing interest. In several theoretical papers, pair production in the standing wave formed by two counterpropagating laser beams has been considered [2]. In most cases, a laser field strength of order or above the critical value $E_c = 1.3 \times 10^{16}$ V/cm is required in order to observe the process in experiment [3], which still is by four orders of magnitude larger than the highest laser field strengths achievable today [4]. More promising, in view of its experimental realization, is pair production in the combination of a laser and a Coulomb field. Here, one can exploit the possibility to let a nucleus or a highly charged ion collide at large value of the relativistic Lorentz factor γ with the laser beam. Then, in the nuclear rest frame, the laser's field strength E and frequency ω are enhanced by a factor $\approx 2\gamma$. Observable pair creation rates should result for $E \sim E_c$ or $\hbar\omega \sim mc^2$ in this frame (here, $\hbar$, c, and m are Planck's constant, light velocity, and electron mass, respectively). With the help of the upcoming laser [5] and accelerator [6] facilities, these conditions can be met. The Large Hadron Collider, that is presently under construction at CERN (Geneva, Switzerland), will accelerate protons to an energy of 7 TeV ($\gamma \approx 7000$) [6]. When such protons are brought into collision with the strongest laser beams available today ($E \approx 10^{-4} E_c$ at $\hbar\omega \sim 1$ eV), then the laser field strength in the proton's rest frame approaches or even exceeds the critical value. On the other hand, the x-ray free-electron laser (XFEL) facilities presently being developed at DESY (Hamburg, Germany) and SLAC (Stanford, California) are proposed to provide spatially coherent and highly brilliant beams of synchrotron radiation with single-photon energies of up to 8–12 keV at maximum field strengths of $E \sim 10^{-6} E_c$ [5]. When combined with a moderately relativistic ion beam, the laser photon energy in the projectile frame can reach the electron's rest energy.

We should mention that a few years ago nonlinear electron-positron pair creation in the collision of an ultrarelativistic electron beam ($\gamma \approx 10^5$) and an intense optical laser pulse was observed at SLAC [7]. In this situation, there are two channels for pair production: A direct channel, via laser-photon absorption in the Coulomb field of the electron, and an indirect two-step channel (so-called Breit-Wheeler process), where first, a high-energy photon is generated by Compton backscattering, which afterward creates the pair via absorption of laser photons [8]. In the experiment, the contribution of the indirect channel was shown to be dominant. In the present paper, however, the indirect production mechanism will not be addressed. It is of importance only in the case of a light projectile beam. In fact, when the incoming electrons were replaced by a beam of heavy particles (e.g., protons), the indirect production channel would be strongly suppressed and direct nonlinear pair production (by a virtual photon from the nuclear Coulomb field) could be investigated. The latter process is the subject of this paper.

The first calculation of pair creation by a Coulomb and a strong laser field is due to Yakovlev [9]. He treated the case of circular laser polarization and derived asymptotic formulas for the production rate in certain parameter regimes. Particularly, the total cross sections in the perturbative regime ($\xi \ll 1$, with the intensity parameter ξ given in Eq. (3)) as well as in the overcritical-field limit ($\xi \gg 1$ and $E \gg E_c$) were obtained. Twenty years later, Mittleman [10] considered the process in a linearly polarized laser wave. His calculation of the leading term for the total cross section in the perturbative multiphoton regime ($\xi \ll 1$ and $\hbar\omega \ll mc^2$) led him to the conclusion that, for the laser intensities and frequencies available at that time, this cross section was the smallest one on record. Despite this pessimistic result, another 20 years later the great headway in laser technology that was following the invention of chirped-pulse amplification [11] triggered newly

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revived interest in the process. Dietz and Pröbsting [12] investigated the influence of a highly charged ion of charge $Z \sim 100$ within a nonperturbative numerical approach. However, to make their coupled-channel calculations feasible they treated the laser field in dipole approximation. Whereas in the earlier work the nucleus was always assumed at rest, Müller *et al.* explored, in detail, the improved opportunities arising from laser-nucleus collisions. In their papers [13,14], they performed numerical calculations for tunneling pair production by an ultrarelativistic nucleus colliding with a superintense near-infrared laser beam and, for multiphoton pair creation, by a moderately relativistic ion beam in combination with an intense x-ray laser wave. Moreover, they considered the variant of bound-free pair production where the electron is created in a bound state of the projectile nucleus [15]. The latter process was also calculated by Matveev *et al.* [16] for an ultrashort laser pulse. Free pair creation in the collision of a relativistic nucleus with an ultrastrong x-ray laser of circular polarization was also treated by Avetissian *et al.* in Ref. [17], where analytical formulas in the high-intensity limit and some numerical results on the energy spectrum of the created particles in the nonperturbative multiphoton regime ($\xi \sim 1$ and $\hbar\omega \sim mc^2$) are given. In a very recent paper by Krajewska *et al.* [18], results of numerical calculations for pair production by ion impact on a linearly polarized, powerful laser beam are presented.

Most of the treatments mentioned above fully account for the influence of the external laser field by using the Volkov solutions to the Dirac equation as basis states in a perturbative calculation of first order with respect to the nuclear Coulomb field. Within this framework, the fully differential pair creation rate can be expressed as a Fourier series over the number of absorbed laser photons, the coefficients of which are essentially given by ordinary or generalized Bessel functions for circular or linear laser polarization, respectively. Because of the large photon numbers involved, taking this sum is one of the main obstacles in this kind of approach, in particular, for a linearly polarized laser wave. As a consequence, it is difficult to get usable expressions for the total production rate.

In the present paper, a different approach based on the optical theorem is used to calculate the total rate for pair creation in combined laser and Coulomb fields. This approach employs the explicit form of the polarization operator of a photon in a laser field found by Baier *et al.* [19] by means of an operator technique. An alternative form of this polarization operator was derived independently by Becker and Mitter [20] by means of another method. Both results are in agreement with each other. Similar to Refs. [9,10,13,14,17,18], the interaction of the leptons with the laser field is taken into account to all orders, while the effect of the Coulomb field is treated in first order (cf. Fig. 1). The main advantage of our method is the possibility to derive, independent of the laser's polarization state, in an analytical manner compact formulas for the total production rate that only involve low-dimensional integrals and no additional summations. Based on these representations, one can rather easily find all different kinds of asymptotics. In this way, we will confirm and extend previous results for a circularly polarized laser wave and give corresponding expressions for
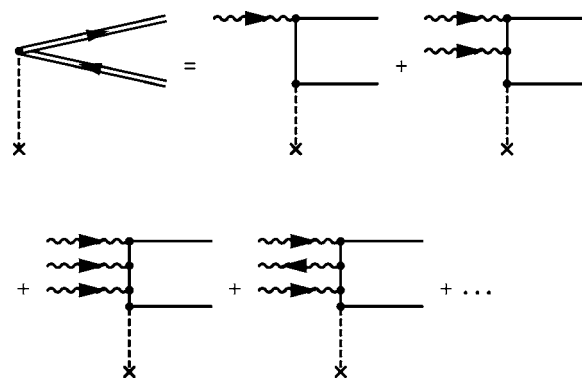

FIG. 1. Pictorial equation in terms of Feynman diagrams describing e^+e^- pair production in combined laser and Coulomb fields. In our approach, the Coulomb field is treated within the first order of perturbation theory, while the effect of the laser wave is nonperturbatively taken into account to all orders. This is expressed by the Feynman graph on the left-hand side, where the double lines represent the exact lepton wave functions in the laser field (Volkov states) and the dashed line stands for their interaction with the Coulomb field. Expanding the Volkov states with respect to the lepton-laser coupling results in a perturbation series, some typical low-order terms of which are shown on the right-hand side. The wavy lines symbolize the laser photons, and the arrows indicate whether the respective laser photon is emitted or absorbed during the process. The first diagram on the right-hand side, e.g., describes the leading order of pair production by the net-absorption of one laser photon (with $\hbar\omega > 2mc^2$). For later reference, we note that the interference between the first and the fourth diagrams leads to a ξ^2 correction to the probability for one-photon pair creation [cf. Eq. (34)].

elliptic laser polarization. As a main result, we show that there are essential differences between the general case of an elliptically polarized laser field and the special case of circular polarization. Furthermore, because of its interest for near-future experiments, particular emphasis is placed on the nonlinear process of pair creation by the simultaneous absorption of two photons from the laser wave in a situation, where the energy of a single photon is not sufficient to produce a pair and where the contribution from higher photon orders is negligibly small. We note that, in general, contributions stemming from different net numbers of absorbed laser photons can be distinguished via the momentum spectra of the produced particles. In addition, the strong-field regimes of pair production shall be examined.

The paper is organized as follows. In Sec. II, we describe how the polarization operator is related to the process of pair creation and how it can be used to calculate the total pair production rate in the superposition of a Coulomb and a laser field. General formulas are given in terms of twofold integrals for both circular and elliptic laser polarization. In Sec. III, we apply these expressions to various parameter regions and determine the corresponding asymptotic behavior of the pair production rate. Special attention is paid to the nonlinear process of pair production by two-photon absorption in the limit of low laser intensity, for which, in particular, the behavior close to the energetic threshold is investigated. Moreover, the pair-creation cross section in the high-frequency

limit ($\hbar\omega \gg mc^2$) is analyzed, where we calculate the next-to-leading-order correction to one-photon pair production at low intensity and analyze the limiting case of high laser intensity. Finally, we consider the overcritical-field regime as well as the quasiclassical limit of high intensity and low frequency, where the rate shows an exponential tunneling behavior. We finish with a conclusion where the significance of our results in view of possible experimental investigations is discussed.

II. THEORETICAL FRAMEWORK

A. Polarization operator and pair production

The polarization operator $\Pi^{\mu\nu}(k)$ describes the propagation of a photon of four-momentum k^μ in a background field (e.g., the QED vacuum), including self-energy corrections. It is related to the corresponding exact photon propagator $\mathcal{D}^{\mu\nu}(k)$ via Dyson's equation [21]

$$\mathcal{D}_{\mu\nu}(k) = D_{\mu\nu}(k) + D_{\mu\sigma}(k)\frac{\Pi^{\sigma\lambda}(k)}{4\pi}\mathcal{D}_{\lambda\nu}(k), \qquad (1)$$

where $D^{\mu\nu}(k)$ denotes the free photon propagator. In Refs. [19,20], the (properly renormalized) polarization operator was evaluated in the combined fields of the QED vacuum and a classical electromagnetic plane wave. The interaction with the vacuum was taken into account to first order in the fine-structure constant $\alpha = e^2$, with e being the electron charge, while the interaction with the plane-wave field was included to all orders. In the following, the shape of the electromagnetic wave will be taken as

$$A_\mu^{(L)}(x) = a_{1\mu}\cos(\kappa x) + a_{2\mu}\sin(\kappa x) \qquad (2)$$

with $\kappa^2 = \kappa a_{1,2} = a_1 a_2 = 0$ and the dimensionless intensity parameters

$$\xi_{1,2}^2 = -\frac{e^2 a_{1,2}^2}{m^2}. \qquad (3)$$

Here and henceforth, we use relativistic units with $\hbar = c = 1$ and write $ab = a^0 b^0 - \mathbf{ab}$ for the product of two four-vectors. The respective result on the polarization operator of a photon in a constant electric field instead of a plane electromagnetic wave can be found in Ref. [22]. Apart from its theoretical significance, the knowledge of the polarization operator allows for a number of applications. For example, the total probability for e^+e^- pair production by a photon of momentum k in an external field (e.g., a plane laser wave) is related to the imaginary part of the corresponding polarization operator via [21]

$$W = \frac{\epsilon_\mu \epsilon_\nu^*}{k^0}\,\mathrm{Im}\,\Pi^{\mu\nu}(k), \qquad (4)$$

where ϵ_μ denotes the photon's polarization four-vector. In Fig. 2, we give an intuitive, pictorial explanation of this formula.

B. Pair creation in combined laser and Coulomb fields

The calculation in Ref. [19] was performed for an arbitrary photon momentum k^μ, including the case $k^2 \neq 0$. There-
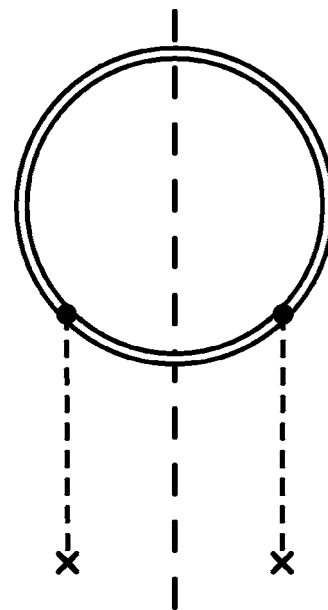

FIG. 2. Lowest-order (in α) Feynman graph for the polarization operator of a photon in an external laser field. In our case the photon is a virtual one stemming from a Coulomb field (outer dashed lines). The diagram can be viewed as describing the elastic forward scattering of the photon through an intermediate laser-dressed e^+e^- state (double line). By performing, as indicated, a cut along the central dashed line, it is seen that the graph represents the product of the amplitude for the production of a laser-dressed e^+e^- pair by the virtual photon and its complex conjugate, including a sum over all electron states. Application of the optical theorem (i.e., the unitarity of the scattering matrix) therefore relates the imaginary part of the polarization operator with the total probability for pair creation, as expressed by Eqs. (4) and (5).

fore, the polarization operator found there not only applies to the combination "photon+laser wave." Instead, the photon may be replaced by any additional external field $A_\mu^{(ext)}(k)$, e.g., a nuclear Coulomb field. Since a Coulomb field can transfer momentum but not energy, here one has $k^\mu = (0, \mathbf{q})$. To get, at a given value of $\mathbf{q}$, the differential probability (per unit time) for e^+e^- pair creation one has to replace the photon wave function $A_\mu^{(ph)}(k) = \sqrt{4\pi/2k^0}\,\epsilon_\mu$ in Eq. (4) by the Fourier transform of the Coulomb field $A_\mu^{(C)}(\mathbf{q}) = (4\pi Ze/\mathbf{q}^2)\delta_{\mu 0}$. Here, Z denotes the nuclear charge number. The total production rate then is found by integration over all possible momenta

$$\dot{W} = \frac{(4\pi Ze)^2}{4\pi}\int\frac{d^3q}{(2\pi)^3}\frac{\mathrm{Im}\,\Pi^{00}}{q^4}. \qquad (5)$$

Note that in Eq. (5) the additional factor of $1/2$ compensates for the double counting of the contributions from the momenta $\mathbf{q}$ and $-\mathbf{q}$. For the case of a constant homogeneous electromagnetic field, the relation (5) was used in Ref. [23], where the probability for pair production was found within the quasiclassical approximation for the polarization operator. According to the results of Ref. [19], the required component of the polarization operator in Eq. (5) can be written as

$$\Pi^{00} = -\frac{\alpha}{\pi} m^2 \int_0^\infty \frac{d\rho}{\rho} \int_{-1}^{+1} dv$$

$$\times \exp\left\{\frac{-2i\rho}{|\lambda|(1-v^2)}\left[1 + \frac{q^2(1-v^2)}{4m^2} + A(\xi_1^2 + \xi_2^2)\right]\right\}$$

$$\times \frac{1}{\cos^2\theta}[\sin^2\theta(d_3\cos^2\phi + d_4\sin^2\phi) - d_5], \qquad (6)$$

where we introduced the angles $\theta = \angle(\mathbf{q}, \boldsymbol{\kappa})$ and $\phi = \angle(\mathbf{q}, \mathbf{a}_1)$, and write $\lambda = -\omega q\cos\theta/2m^2$. The coefficients d_j $(j=3,4,5)$ in Eq. (6) are given by Eq. (2.32) in Ref. [19]; they read

$$d_3 = \left(A_1\xi_1^2 - \sin^2\rho\frac{\xi_1^2 - \xi_2^2}{1-v^2}\right)[J_0(\zeta) + iJ_1(\zeta)]$$

$$+ \xi_1^2\sin^2\rho\frac{1+v^2}{1-v^2}J_0(\zeta)$$

$$+ \frac{1}{4}\left[\frac{q^2}{m^2} - i\frac{|\lambda|(1-v^2)}{\rho}\right][J_0(\zeta) - e^{iy}]$$

$$d_4 = d_1(\xi_1 \leftrightarrow \xi_2)$$

$$d_5 = \frac{q^2}{4m^2}(1-v^2)[J_0(\zeta) - e^{iy}], \qquad (7)$$

with the abbreviations

$$A = \frac{1}{2}\left(1 - \frac{\sin^2\rho}{\rho^2}\right), \quad A_0 = \frac{1}{2}\left(\frac{\sin^2\rho}{\rho^2} - \frac{\sin 2\rho}{2\rho}\right),$$

$$A_1 = A + 2A_0,$$

$$\zeta = \frac{2\rho A_0(\xi_1^2 - \xi_2^2)}{|\lambda|(1-v^2)}, \quad y = \frac{2\rho A(\xi_1^2 + \xi_2^2)}{|\lambda|(1-v^2)}. \qquad (8)$$

For simplicity, we first consider the case of a circularly polarized laser wave ($\xi_1^2 = \xi_2^2 \equiv \xi^2$). Then, the coefficients in Eq. (6) can be simplified to read [19]

$$d_{3,4} = \xi^2\sin^2\rho\frac{1+v^2}{1-v^2} - \frac{1}{2}\left[1 - \frac{q^2}{4m^2}(1+v^2)\right](1 - e^{iy})$$

$$d_5 = \frac{q^2}{4m^2}(1-v^2)(1 - e^{iy}), \qquad (9)$$

with

$$y = \frac{2\xi^2\rho}{|\lambda|(1-v^2)}\left(1 - \frac{\sin^2\rho}{\rho^2}\right).$$

Employing spherical coordinates such that $d^3q = q^2 dq d\cos\theta d\phi$, the integration over ϕ in Eq. (5) is trivial and the total rate becomes

$$\dot{W} = -\frac{2}{\pi^2}(Z\alpha)^2 m \operatorname{Im} \int_0^1 dv \int_0^\infty \frac{dQ}{Q^2} \int_0^1 \frac{dt}{t^2} \int_0^\infty \frac{d\rho}{\rho}$$

$$\times \exp\left[-i\frac{a\rho}{Qt(1-v^2)}[1 + Q^2(1-v^2)]\right]$$

$$\times \left\{(1-t^2)\xi^2\sin^2\rho\frac{1+v^2}{1-v^2}e^{-iy}\right.$$

$$\left. + \left[Q^2(1-v^2) + \frac{1-t^2}{2}[1 - Q^2(1+v^2)]\right](1 - e^{-iy})\right\}, \qquad (10)$$

where we have introduced the dimensionless quantities $Q = q/2m$, $t = \cos\theta$, and $a = 2m/\omega$. Going over to the variables $Q \to Q/\sqrt{1-v^2}$, $t \to t/\sqrt{1-v^2}$ considerably simplifies the v dependence such that also this integral can be taken

$$\dot{W} = -\frac{2}{\pi^2}(Z\alpha)^2 m \operatorname{Im} \int_0^\infty \frac{dQ}{Q^2} \int_0^1 dt \int_0^\infty \frac{d\rho}{\rho}$$

$$\times \exp\left[-i\frac{a\rho}{Qt}(1 + Q^2)\right]\left\{\xi^2\sin^2\rho\left[\frac{2}{3}(2+t^2)\frac{\sqrt{1-t^2}}{t^2}\right.\right.$$

$$\left. - \ln\left(\frac{1+\sqrt{1-t^2}}{1-\sqrt{1-t^2}}\right)\right]e^{-iy} + \left[\frac{1}{3}\frac{(1-t^2)^{3/2}}{t^2}\right.$$

$$\left.\left. + \frac{Q^2}{2}\ln\left(\frac{1+\sqrt{1-t^2}}{1-\sqrt{1-t^2}}\right)\right](1 - e^{-iy})\right\}, \qquad (11)$$

where now $y = (a\rho\xi^2/Qt)(1 - \sin^2\rho/\rho^2)$. Alternatively, the Q integration can be expressed by Neumann functions Y_n

$$\dot{W} = -\frac{(Z\alpha)^2}{\pi} m \int_0^1 dt \int_0^\infty \frac{d\rho}{\rho}\left\{\frac{2\xi^2\sin^2\rho}{z}Y_1(\beta z)\right.$$

$$\times \left[\frac{2}{3}(2+t^2)\frac{\sqrt{1-t^2}}{t^2} - \ln\left(\frac{1+\sqrt{1-t^2}}{1-\sqrt{1-t^2}}\right)\right]$$

$$+ \frac{2}{3}\left[Y_1(\beta) - \frac{1}{z}Y_1(\beta z)\right]\frac{(1-t^2)^{3/2}}{t^2} + [Y_1(\beta)$$

$$\left. - zY_1(\beta z)]\ln\left(\frac{1+\sqrt{1-t^2}}{1-\sqrt{1-t^2}}\right)\right\} \qquad (12)$$

with $\beta = 2a\rho/t$ and $z = [1 + \xi^2(1 - \sin^2\rho/\rho^2)]^{1/2}$. Equation (12) is a very compact, general expression for the total pair production rate in a circularly polarized laser field. For applications, both Eqs. (11) and (12) will prove to be useful.

In the general case of an elliptically polarized laser wave, the integrations over ϕ and v can be taken in the same manner as before yielding

$$\dot{W} = -\frac{(Z\alpha)^2}{\pi^2} m \operatorname{Im} \int_0^\infty \frac{dQ}{Q^2} \int_0^1 \frac{dt}{t^2} \int_0^\infty \frac{d\rho}{\rho}\exp\left[-i\frac{a\rho}{Qt}(1\right.$$

$$\left. + Q^2)\right]e^{-iy}\left((\xi_1^2 + \xi_2^2)J_0(\zeta)\left\{\frac{2}{3}A_1(1-t^2)^{3/2} + \sin^2\rho\left[\frac{2}{3}(2\right.\right.\right.$$

$$\left.\left.\left. + t^2)\sqrt{1-t^2} - t^2\ln\left(\frac{1+\sqrt{1-t^2}}{1-\sqrt{1-t^2}}\right)\right]\right\}\right]$$

$$+ i(\xi_1^2 - \xi_2^2) J_1(\zeta) \left\{ \frac{2}{3} A_1 (1-t^2)^{3/2} - \sin^2 \rho \left[2\sqrt{1-t^2} \right. \right.$$

$$\left. \left. - t^2 \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right) \right] \right\} + \left[J_0(\zeta) - e^{iy} \right] \left[\frac{2}{3} (1-t^2)^{3/2} \left(Q^2 \right. \right.$$

$$\left. \left. - i \frac{Qt}{a\rho} \right) - Q^2 t^2 \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right) \right] \right). \tag{13}$$

The integral over Q can again be taken analytically

$$\dot{W} = \frac{(Z\alpha)^2}{4\pi} \omega \int_0^1 \frac{dt}{t} \int_0^\infty \frac{d\rho}{\rho^2} \left(\frac{\xi_1^2 + \xi_2^2}{\sqrt{g_0^2 - g_1^2}} [\varphi_- J_1(\varphi_-) Y_0(\varphi_+) \right.$$

$$- \varphi_+ J_0(\varphi_-) Y_1(\varphi_+)] \left\{ \frac{2}{3} A_1 (1-t^2)^{3/2} \right.$$

$$\left. + \sin^2 \rho \left[\frac{2}{3} (2+t^2)\sqrt{1-t^2} - t^2 \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right) \right] \right\}$$

$$- \frac{\xi_1^2 - \xi_2^2}{\sqrt{g_0^2 - g_1^2}} [\varphi_- J_1'(\varphi_-) Y_1(\varphi_+) - \varphi_+ J_1(\varphi_-) Y_1'(\varphi_+)]$$

$$\times \operatorname{sgn}(g_1) \left\{ \frac{2}{3} A_1 (1-t^2)^{3/2} - \sin^2 \rho \left[2\sqrt{1-t^2} \right. \right.$$

$$\left. \left. - t^2 \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right) \right] \right\} - [\varphi_- J_1(\varphi_-) Y_0(\varphi_+)$$

$$+ \varphi_+ J_0(\varphi_-) Y_1(\varphi_+) - \beta Y_1(\beta)] \left[\frac{2}{3} (1-t^2)^{3/2} \right.$$

$$\left. - t^2 \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right) \right]$$

$$- \frac{4}{3} [J_0(\varphi_-) Y_0(\varphi_+) - Y_0(\beta)] (1-t^2)^{3/2} \right) \tag{14}$$

with $g_0 = 1 + (\xi_1^2 + \xi_2^2)A$, $g_1 = (\xi_1^2 - \xi_2^2)A_0$, and $\varphi_\pm = (g_0 \pm \sqrt{g_0^2 - g_1^2})^{1/2} \beta / \sqrt{2}$.

The total production rate can be converted into a cross section according to the relation

$$\sigma = \frac{\dot{W}}{j}, \quad j = \frac{\omega m^2}{8\pi\alpha} (\xi_1^2 + \xi_2^2) \tag{15}$$

with j being the photon flux.

III. SPECIAL CASES

A. One-photon limit

Let us consider the case of small intensity ($\xi \ll 1$) and frequency $\omega > 2m$. The leading term in the ξ^2 expansion corresponds to pair creation by a single photon in a Coulomb field. Under these circumstances, the total production probability is independent of the laser's polarization state and it is convenient to derive the asymptotic behavior by using the formula (11) for circular laser polarization. Assuming $\xi \ll 1$, we can expand the exponentials to lowest order with respect

to y. Afterward, we perform the integration over ρ with the result

$$\dot{W} = \frac{(Z\alpha)^2}{2\pi} m\xi^2 \int_0^\infty \frac{dQ}{Q^2} \int_0^1 dt\,\vartheta(2-\gamma) \left\{ \frac{2}{3} (2+t^2) \frac{\sqrt{1-t^2}}{t^2} \right.$$

$$- \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right) + \frac{a}{Qt} \left[\frac{2}{3} \frac{(1-t^2)^{3/2}}{t^2} \right.$$

$$\left. \left. + Q^2 \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right) \right] \left(1 - \frac{\gamma}{2} \right) \right\}, \tag{16}$$

where $\gamma = a(1+Q^2)/Qt$ and $\vartheta(x)$ is the step function, which restricts the integration to the kinematically allowed region $a \leq t \leq 1$ and $Q_- \leq Q \leq Q_+$ with $Q_\pm = t/a \pm \sqrt{(t/a)^2 - 1}$. Integrating over Q gives

$$\dot{W} = \frac{(Z\alpha)^2}{2\pi} m\xi^2 \int_a^1 dt \left\{ \frac{2}{3} \sqrt{\frac{t^2}{a^2} - 1} \frac{\sqrt{1-t^2}}{t^2} \left[\frac{4}{3} t^2 + \frac{14}{3} + \frac{2}{3} a^2 \right. \right.$$

$$\left. - \frac{2}{3} \frac{a^2}{t^2} \right] - 2 \left(1 + \frac{a^2}{t^2} \right) \sqrt{\frac{t^2}{a^2} - 1} \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right)$$

$$\left. + \frac{2a}{t} \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right) \ln\left(\frac{t}{a} + \sqrt{\frac{t^2}{a^2} - 1} \right) \right\}. \tag{17}$$

The corresponding expression for the cross section [see Eq. (15)] agrees with the well-known result of Bethe and Heitler [24]. Accordingly, in the ultrarelativistic limit ($\omega \gg m$) we find

$$\sigma = \frac{28}{9} Z^2 \alpha r_e^2 \left[\ln\left(\frac{2\omega}{m} \right) - \frac{109}{42} \right] \tag{18}$$

with the classical electron radius $r_e = \alpha/m$, and in the nonrelativistic limit ($\omega - 2m \ll m$)

$$\sigma = \frac{\pi}{12} Z^2 \alpha r_e^2 \left(\frac{\omega - 2m}{m} \right)^3. \tag{19}$$

B. Two-photon pair creation

It is interesting to consider the cross section for two-photon pair production at $\xi \ll 1$, which represents a nonlinear strong-field process of lowest possible order in the number of absorbed photons. We assume that $m < \omega < 2m$, which excludes the possibility of one-photon pair creation. Going one step further in the y expansion of Eq. (11) and performing the ρ integration, we obtain for the circular polarization case

$$\dot{W} = \frac{(Z\alpha)^2}{2\pi} m\xi^4 \int_0^\infty \frac{dQ}{Q^2} \int_0^1 dt\,\vartheta(4-\gamma) \left\{ \left(1 - \frac{\gamma}{4} \right) \frac{a}{Qt} \left[\frac{2}{3} (2 \right. \right.$$

$$+ t^2) \frac{\sqrt{1-t^2}}{t^2} - \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right) \right] + \frac{1}{2} \left(1 - \frac{\gamma}{4} \right)^2 \frac{a^2}{Q^2 t^2}$$

$$\left. \times \left[\frac{2}{3} \frac{(1-t^2)^{3/2}}{t^2} + Q^2 \ln\left(\frac{1 + \sqrt{1-t^2}}{1 - \sqrt{1-t^2}} \right) \right] \right\}. \tag{20}$$

Proceeding as before, we get

$$\dot{W} = \frac{(Z\alpha)^2}{2\pi} m\xi^4 \int_{m/\omega}^{1} dt \left\{ \frac{(4t^2 - a^2)^{3/2}}{3at^2} \left[\frac{2}{3}(2 + t^2) \frac{\sqrt{1 - t^2}}{t^2} \right. \right.$$
$$\left. - \ln\left(\frac{1 + \sqrt{1 - t^2}}{1 - \sqrt{1 - t^2}} \right) \right] + \frac{(4t^2 - a^2)^{5/2}}{45at^4} \frac{(1 - t^2)^{3/2}}{t^2} + \frac{a}{6t^4}$$
$$\times \ln\left(\frac{1 + \sqrt{1 - t^2}}{1 - \sqrt{1 - t^2}} \right) \left[(2t^2 + a^2)\sqrt{4t^2 - a^2} \right.$$
$$\left. \left. - 3a^2 t \ln\left(\frac{2t}{a} + \sqrt{\frac{4t^2}{a^2} - 1} \right) \right] \right\}. \tag{21}$$

After conversion into a cross section, Eq. (21) reads

$$\sigma = \alpha Z^2 \xi^2 r_e^2 F\left(\frac{\omega}{m} \right) \tag{22}$$

with

$$F(x) = \frac{8}{3x} \int_{1/x}^{1} dt \left\{ \frac{[(xt)^2 - 1]^{3/2}}{(xt)^2} \left[\frac{2}{3}(2 + t^2) \frac{\sqrt{1 - t^2}}{t^2} \right. \right.$$
$$\left. - \ln\left(\frac{1 + \sqrt{1 - t^2}}{1 - \sqrt{1 - t^2}} \right) \right] + \frac{4}{15} \frac{[(xt)^2 - 1]^{5/2}}{(xt)^4} \frac{(1 - t^2)^{3/2}}{t^2}$$
$$+ \frac{1}{(xt)^4} \ln\left(\frac{1 + \sqrt{1 - t^2}}{1 - \sqrt{1 - t^2}} \right) [(2 + (xt)^2) \sqrt{(xt)^2 - 1}$$
$$\left. - 3xt \ln(xt + \sqrt{(xt)^2 - 1})] \right\}. \tag{23}$$

In the limit $0 < x - 1 \ll 1$, Eq. (23) becomes

$$F(x) = \frac{128}{45}(x - 1)^4 \int_{0}^{1} ds \sqrt{s}(1 - s)^{3/2}(6 - s) = \pi(x - 1)^4. \tag{24}$$

Hence, the threshold behavior of the cross section for two-photon pair production in a circularly polarized laser field reads

$$\sigma = \pi \alpha Z^2 \xi^2 r_e^2 \left(\frac{\omega - m}{m} \right)^4. \tag{25}$$

We note that, based on the results of their numerical calculations, the authors of Ref. [14] found that the rate for two-photon pair creation within the whole frequency range $m < \omega < 2m$ approximately scales as $(\omega - m)^3$. However, Eq. (25) now shows that the exact scaling in the threshold region ($m \lesssim \omega$ and $\omega - m \ll 2m$) is given by $(\omega - m)^4$. Furthermore, in Ref. [14] a total production rate of 190 s^{-1} (in the nuclear rest frame) was reported for $Z = 1$, $\omega = 900$ keV, and $\xi = 7.5 \times 10^{-4}$. Applying Eq. (21) to these parameters, perfectly reproduces this result.

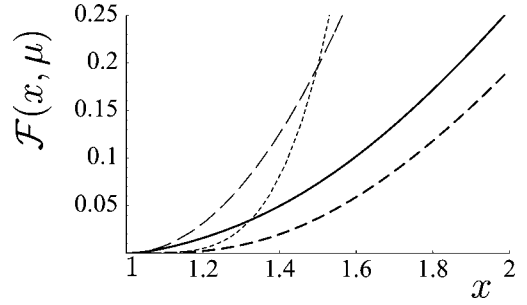In the general case of elliptic polarization, the corresponding asymptotic cross section reads

FIG. 3. The function $\mathcal{F}(x, \mu)$ in the interval $1 \leq x \leq 2$ for $\mu = 1$ (linear polarization, solid line) and $\mu = 0$ (circular polarization, short-dashed line). The long-dashed and dotted lines show the respective threshold laws for $x \approx 1$ according to Eqs. (24) and (28).

$$\sigma = \alpha Z^2 \frac{\xi_1^2 + \xi_2^2}{2} r_e^2 \mathcal{F}\left(\frac{\omega}{m}, \mu \right), \tag{26}$$

where $\mathcal{F}(x, \mu) = F(x) + \mu G(x)$ with the function $F(x)$ from Eq. (23) and

$$G(x) = \frac{1}{3x^2} \int_{1/x}^{1} \frac{dt}{t^3} \left\{ \frac{2}{5} \frac{(1 - t^2)^{3/2}}{(xt)^3} \left[((xt)^4 - 7(xt)^2 \right. \right.$$
$$\left. + 16) \sqrt{(xt)^2 - 1} - 10xt \ln(xt + \sqrt{(xt)^2 - 1}) \right] - [(xt)^2$$
$$- 4] \frac{\sqrt{(xt)^2 - 1}}{xt} \left[2\sqrt{1 - t^2} - t^2 \ln\left(\frac{1 + \sqrt{1 - t^2}}{1 - \sqrt{1 - t^2}} \right) \right]$$
$$+ \frac{1}{2(xt)^3} [((xt)^2 - 16)\sqrt{(xt)^2 - 1} + 12xt \ln(xt$$
$$\left. + \sqrt{(xt)^2 - 1})] \left[\frac{2}{3}(1 - t^2)^{3/2} - t^2 \ln\left(\frac{1 + \sqrt{1 - t^2}}{1 - \sqrt{1 - t^2}} \right) \right] \right\}$$
$$\tag{27}$$

and the ellipticity parameter $\mu = [(\xi_1^2 - \xi_2^2)/(\xi_1^2 + \xi_2^2)]^2$. This time the threshold behavior ($\omega \approx m$) is found to be

$$\mathcal{F}(x, \mu) = \frac{\pi}{4} \mu(x - 1)^2. \tag{28}$$

Hence, in the general case, the cross section for two-photon pair creation close to threshold raises like the second power of the excess energy $\omega - m$. Only in the special case of circular polarization ($\mu = 0$) does the threshold cross section scale with the fourth power as given in Eq. (25). We note, however, that our first-order treatment of the Coulomb field requires the parameter $Z\alpha/v_\pm$ to be small, where $v_\pm$ denotes the velocities of the created particles. For pair production very close to the threshold, Coulomb corrections are expected to be important. A summary of our results on two-photon pair production for linear and circular polarization including various approximations is displayed in Fig. 3.

When applying our general formula (26) to the same parameters as above but for a linearly polarized laser wave ($\mu = 1$, $\xi_1/\sqrt{2} = 7.5 \times 10^{-4}$, $\xi_2 = 0$, $\omega = 900$ keV, and $Z = 1$), we get a total production rate of 290 s^{-1}, which is in rather good agreement with the corresponding value of 240 s^{-1} found in

Ref. [14]. The difference is most likely due to small inaccuracies in the five-dimensional integration along with the numerical evaluation of the generalized Bessel functions in Ref. [14]. As is seen from Fig. 3, the pair production rate for a linearly polarized laser wave is noticeably larger than the corresponding rate for circular laser polarization within the whole frequency range $m < \omega < 2m$.

C. High-frequency limit

In this section, we analyze the high-energy behavior of strong-field pair creation, i.e., we shall assume $\omega \gg m$ but allow for an arbitrary value of ξ. Let us first consider the circular polarization case and start from Eq. (12). We exchange the order of integration and first integrate over t. Since $\omega \gg m$, we can introduce a splitting parameter ϵ with $a \ll \epsilon \ll 1$, and divide the t integration into ranges from 0 to ϵ and from ϵ to 1. In the first region, we have $t \ll 1$. Substituting $s = 1/t$, we get

$$\int_0^{\epsilon} dt\{\cdots\} = \int_{1/\epsilon}^{\infty} ds \left\{ \frac{8\xi^2 \sin^2 \rho}{3z} Y_1(2a\rho z s) + \frac{2}{3}\left[Y_1(2a\rho s) - \frac{1}{z} Y_1(2a\rho z s) \right] \right\}.$$ (29)

With the help of the identity $Y_1(x) = -\frac{d}{dx}Y_0(x)$ this is readily integrated to give

$$\int_0^{\epsilon} dt\{\cdots\} = \frac{2\xi^2}{3\pi a \rho z^2} \left\{ \left[4\sin^2 \rho + \left(1 - \frac{\sin^2 \rho}{\rho^2}\right) \right] \times \left(\ln \frac{a\rho}{\epsilon} + C \right) + \left(4\sin^2 \rho - \frac{1}{\xi^2}\right) \ln z \right\},$$ (30)

where the relation $Y_0(x) \approx (2/\pi)[\ln(x/2) + C]$ was used, which is valid for $x \ll 1$; C denotes Euler's constant. In the second range, we have $a \ll t$ and can exploit the small-argument behavior of the Neumann function: $Y_1(x) \approx -2/(\pi x)$. Then the integration is easily performed

$$\int_{\epsilon}^{1} dt\{\cdots\} = -\frac{2\xi^2}{3\pi a \rho z^2} \left\{ \ln \frac{2}{\epsilon} \left[4\sin^2 \rho + \left(1 - \frac{\sin^2 \rho}{\rho^2}\right) \right] - \frac{19}{3}\sin^2 \rho - \frac{4}{3}\left(1 - \frac{\sin^2 \rho}{\rho^2}\right) \right\}.$$ (31)

Putting both parts together, ϵ drops out as it should and we arrive at

$$\dot{W} = \frac{2}{3\pi^2}(Z\alpha)^2 \frac{m\xi^2}{a} \int_0^{\infty} \frac{d\rho}{\rho^2 z^2} \left\{ \left[4\sin^2 \rho + \left(1 - \frac{\sin^2 \rho}{\rho^2}\right) \right] \times \left(\ln \frac{2}{a\rho z} - C \right) + \frac{z^2}{\xi^2} \ln z - \frac{19}{3}\sin^2 \rho - \frac{4}{3}\left(1 - \frac{\sin^2 \rho}{\rho^2}\right) \right\}.$$ (32)

Applying an integration by parts to one of the terms, this can be rewritten as

$$\dot{W} = \frac{2}{3\pi^2}(Z\alpha)^2 \frac{\xi^2}{a} m \int_0^{\infty} \frac{d\rho}{\rho^2 z^2} \left\{ \left[4\sin^2 \rho + \left(1 - \frac{\sin^2 \rho}{\rho^2}\right) \right] \times \left(\ln \frac{2}{a\rho z} - C \right) + \left(1 - \frac{\sin 2\rho}{2\rho}\right) - \frac{19}{3}\sin^2 \rho - \frac{7}{3}\left(1 - \frac{\sin^2 \rho}{\rho^2}\right) \right\}.$$ (33)

At $\xi \ll 1$, the cross section resulting from Eq. (33) in the leading order in an expansion with respect to ξ^2 coincides with Eq. (18). The next-to-leading-order term for the cross section reads

$$\Delta\sigma = -\frac{52}{45}Z^2\alpha\xi^2 r_e^2 \left[\ln\left(\frac{2\omega}{m}\right) - \frac{22}{13}\ln 2 - \frac{124}{195} \right].$$ (34)

The correction $\Delta\sigma$ is negative. This is because it not only contains the process of two-photon pair production but also includes the ξ^2 corrections to one-photon pair creation (cf. Fig. 1). The latter is negative and has a larger absolute value than the contribution from two-photon pair creation.

Now we consider the case $\xi \gg 1$, which corresponds to a supercritical laser field strength $E \gg E_c$. Note that, for the very high frequencies ($\omega \gg m$) assumed, such extremely large field strengths are far beyond present and near-future technical capabilities. In this situation, the main contribution to the integral over ρ in Eq. (33) comes from the region $\rho \sim 1/\xi \ll 1$. Performing the according expansion of the integrand and taking the integral, we obtain

$$\sigma = \frac{26}{3\sqrt{3}} \frac{Z^2\alpha}{\xi} r_e^2 \left[\ln\left(\frac{\omega\xi}{2\sqrt{3}m}\right) - C - \frac{58}{39} \right].$$ (35)

We note that the leading logarithm in Eq. (35) has already been derived by Yakovlev [9]. Furthermore, Eq. (35) agrees with the corresponding asymptotic limits found in Ref. [25] for pair creation by a Coulomb field and a constant crossed field and in Ref. [23] for pair creation in a Coulomb field and a constant homogeneous field. It is worth stressing that the ξ dependence in Eq. (35) cannot simply be expressed in terms of the effective (laser-dressed) electron mass $m_* = m\sqrt{1 + \xi^2}$ [21].

For the general case of an elliptically polarized laser wave, the expression corresponding to Eq. (32) reads

$$\dot{W} = \frac{(Z\alpha)^2\omega}{3\pi^2} \int_0^{\infty} \frac{d\rho}{\rho^2} \left\{ \left[\ln\left(\frac{4}{a\rho\chi}\right) - C \right] \left[\frac{\xi_1^2 + \xi_2^2}{g_2}(A_1 + 2\sin^2 \rho) \right. \right.$$

$$\left. - 2\ln\frac{\chi}{2} + \frac{1}{A_0}\left(\frac{g_0}{g_2} - 1\right)(A_1 - 3\sin^2 \rho) \right]$$

$$+ \frac{\xi_1^2 + \xi_2^2}{g_2}\left[A_1\left(\ln\frac{g_0 + g_2}{2g_2} - \frac{4}{3}\right) + 2\sin^2 \rho\left(\ln\frac{g_0 + g_2}{2g_2}\right.\right.$$

$$\left.\left. - \frac{19}{12}\right) \right] - \ln\frac{\chi}{2}\left(\ln\frac{\chi}{2} - \frac{5}{3}\right) - \frac{1}{2}L\left(\frac{g_2 - g_0}{g_2 + g_0}\right)$$

$$+ \frac{1}{A_0 g_2} \left[A_1 \left(g_0 \ln \frac{g_0 + g_2}{2 g_2} - \frac{4}{3} (g_0 - g_2) \right) \right.$$

$$\left. \left. - 3 \sin^2 \rho \left(g_0 \ln \frac{g_0 + g_2}{2 g_2} - \frac{3}{2} (g_0 - g_2) \right) \right) \right] \right\} \quad (36)$$

with $g_2 = \sqrt{g_0^2 - g_1^2}$, $\chi = \sqrt{2(g_0 + g_2)}$, and Spencer's function

$$L(x) = \int_0^x \frac{dy}{y} \ln(1 + y). \quad (37)$$

In the limit $(\xi_1^2 + \xi_2^2)^{1/2} \gg 1$, we again can expand the integrand for small values of ρ and, within logarithmic accuracy, arrive at the following cross section:

$$\sigma = \frac{26}{3\sqrt{3}} \frac{Z^2 \alpha}{\sqrt{\frac{1}{2}(\xi_1^2 + \xi_2^2)}} r_e^2 {}_2 F_1 \left(-\frac{1}{4}, \frac{1}{4}, 1, \mu \right) \ln \left(\frac{\omega}{m} \sqrt{\xi_1^2 + \xi_2^2} \right).$$

$$(38)$$

Here, ${}_2 F_1$ denotes a hypergeometric function, which smoothly depends on the ellipticity parameter μ and monotonously decreases from unity for $\mu = 0$ (circular polarization) to $2\sqrt{2}/\pi \approx 0.9$ for $\mu = 1$ (linear polarization).

D. Quasiclassical limit

Now we consider the limit $\xi \gg 1$. Note that today's most powerful lasers achieve $\xi \sim 10^2$. In this limit, the main contribution to the integral over ρ in Eqs. (12) and (14) comes again from the region $\rho \sim 1/\xi \ll 1$. Hence, for the case of circular polarization, we can write

$$\dot{W} = -\frac{(Z\alpha)^2}{\pi} m \int_0^1 dt \int_0^\infty \frac{d\rho}{\rho} \left\{ \frac{6\rho^2}{u} Y_1(\beta u) \right.$$

$$\times \left[\frac{2}{3} (2 + t^2) \frac{\sqrt{1 - t^2}}{t^2} - \ln \left(\frac{1 + \sqrt{1 - t^2}}{1 - \sqrt{1 - t^2}} \right) \right]$$

$$+ \frac{2}{3} \left[Y_1(\beta) - \frac{1}{u} Y_1(\beta u) \right] \frac{(1 - t^2)^{3/2}}{t^2}$$

$$\left. + \left[Y_1(\beta) - u Y_1(\beta u) \right] \ln \left(\frac{1 + \sqrt{1 - t^2}}{1 - \sqrt{1 - t^2}} \right) \right\}, \quad (39)$$

where now $\beta = 4\sqrt{3} \rho / (t \eta)$ with $\eta = \omega \xi / m = E/E_c$ and $u = \sqrt{1 + \rho^2}$. Concerning the laser field parameters, the total rate in Eq. (39) only depends on the dimensionless ratio η, which is the so-called quasiclassical parameter. In the limit $\eta \gg 1$ we arrive at the production rate

$$\dot{W} = \frac{13}{6\sqrt{3}\pi} (Z\alpha)^2 m \eta \left[\ln \left(\frac{\eta}{2\sqrt{3}} \right) - C - \frac{58}{39} \right], \quad (40)$$

which corresponds to the cross section (35) obtained in Sec. III C. In the opposite limit $\eta \ll 1$, which corresponds to the existing high-power lasers in the optical or infrared frequency range, the integral in Eq. (39) can be evaluated using Laplace's method. The resulting pair creation rate

$$\dot{W} = \frac{(Z\alpha)^2}{2\sqrt{\pi}} m \left(\frac{\eta}{2\sqrt{3}} \right)^{5/2} \exp \left(-\frac{2\sqrt{3}}{\eta} \right) \quad (41)$$

exhibits a tunneling behavior. Equation (41) is in agreement with corresponding results on pair production in a Coulomb field and a constant crossed field [25] or a Coulomb field and a constant homogeneous field [23].

In a similar way we can derive from Eq. (14) the $\eta \ll 1$ limit for an elliptically polarized laser wave. To this end, we assume $\eta_1 > \eta_2$ and $(\eta_1^2 - \eta_2^2)/\eta_1^2 \gg \eta_1$, where $\eta_j = \omega \xi_j / m$ for $j = 1, 2$. The latter condition means that the laser field is considerably different from a circularly polarized wave. Then, we find

$$\dot{W} = \frac{(Z\alpha)^2}{\sqrt{2}\pi} m \sqrt{\frac{\eta_1^2}{\eta_1^2 - \eta_2^2}} \left(\frac{\eta_1}{2\sqrt{3}} \right)^3 \exp \left(-\frac{2\sqrt{3}}{\eta_1} \right). \quad (42)$$

In comparison to the result for circular polarization in Eq. (41), the production rate in Eq. (42) is suppressed by an additional factor of $\sqrt{\eta_1}$. This suppression has the same nature as in the case of $e^+ e^-$ pair production by a real photon in a laser field [26]. The reason for the suppression is that, in general, the modulus of the field strength is truly oscillating, whereas in a circularly polarized wave it has a constant value. We note that Eq. (42) can also be derived by averaging Eq. (41) over one oscillation cycle.

IV. CONCLUSIONS

We have investigated, in detail, the total probability for electron-positron pair creation by a nuclear Coulomb field and an intense laser field. Employing the optical theorem in connection with the known polarization operator of a photon in an electromagnetic plane wave, we have derived compact, general expressions for the total pair production rate and found explicit formulas for various intensity and frequency regimes of interest. We have demonstrated that significant differences exist between the general case of an elliptically polarized laser wave and the special case of circular polarization [see, e.g., Eqs. (28) and (42) vs Eqs. (24) and (41)]. Particular emphasis was placed on the nonlinear process of two-photon pair creation.

Let us estimate the feasibility of $e^+ e^-$ pair production using an XFEL beam [3,5] and a beam of relativistic protons. We will compare the following possible regimes of interaction: (i) the one-photon perturbative regime, where $\xi \ll 1$ and $\omega \gg m$ [see Eq. (18)], (ii) the two-photon perturbative regime, where $\xi \ll 1$ and $m < \omega < 2m$ [see Eq. (26)], (iii) the tunneling regime, where $\xi \gg 1$ and $\xi \ll m/\omega$ [see Eq. (41)], and (iv) the overcritical-field regime, where $\xi \gg 1$ and $\xi \gg m/\omega$ [see Eq. (40)]. We have to take into account that the mentioned formulas are valid in the rest frame of the proton, which moves with a large Lorentz-factor γ. In the rest frame of the proton, the photon frequency is increased with respect to its value in the lab frame: $\omega \approx 2\gamma \omega_L$, while the XFEL pulse duration is decreased: $\tau \approx \tau_L / 2\gamma$. Here, ω_L and τ_L denote the laser frequency and pulse duration in the lab frame, respectively. Note that, nowadays, the proton beam at DESY can be accelerated up to 920 GeV, which corresponds to $\gamma \approx 1000$.

For the following estimates, we will throughout assume an XFEL pulse duration of $\tau_L = 100$ fs. The XFEL beam collides with a bunch of protons moving at $\gamma = 1000$, containing 10^{11} particles, and having a beam radius of 30 μm. In the one-photon perturbative regime, assuming an XFEL photon energy of $\omega_L = 10$ keV, an intensity parameter of $\xi = 10^{-5}$, and a beam radius of $R = 30$ μm, we find that ~ 100 pairs are produced per collision. For a given laser intensity, the number of created particles is independent of the laser's polarization state. In the second regime of two-photon pair creation, employing a linearly polarized XFEL beam is more favorable. Supposing the collision parameters $\omega_L = 0.5$ keV, $\xi_1 = 3 \times 10^{-3}$, and $R = 30$ μm, the process proceeds far above threshold and one pair can be produced per shot. In the overcritical-field regime, circular polarization of the XFEL beam leads to higher production yields. In this regime, a large ξ value is necessary, which can be achieved by focusing the XFEL beam [3]. For $\xi = 3$, $\omega_L = 10$ keV, and $R = 0.4$ nm, one pair can be produced per collision. Here one has to take into account that the XFEL pulse will interact only with a very small fraction of the proton bunch. For the assumed parameters, the latter has a length of 10 cm and contains a particle density of 10^{15} cm^{-3}.

Thus, we see that, in principle, experimental studies of the pair production process are possible by using an XFEL and a relativistic proton beam. Employing a nonfocused XFEL beam (cf. the so-called design and available parameters in Table 1 of Ref. [3]) and an ultrarelativistic proton beam, the perturbative regimes of pair creation can be realized, including nonlinear pair production via two-photon absorption. The realization of the strong-field regimes with $\xi \gg 1$ will require focusing of the XFEL beam (cf. the goal parameters in Table 1 of Ref. [3]). In this case the interaction volume, i.e., the region of beam overlap, is extremely small, which substantially decreases the total production probability. As a result, the pair creation seems unrealistic in the tunneling regime but, at least in principle, feasible in the overcritical-field regime.

In summary, with the future sources of spatially coherent x-rays along with the relativistic proton beams of DESY, the results on pair production presented in this paper could be tested.

ACKNOWLEDGMENTS

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