

# Proceedings of the Interdisciplinary STEM Teaching and Learning Conference

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# Volume 2. Proceedings of the Interdisciplinary STEM Teaching and Learning Conference

## **Abstract**

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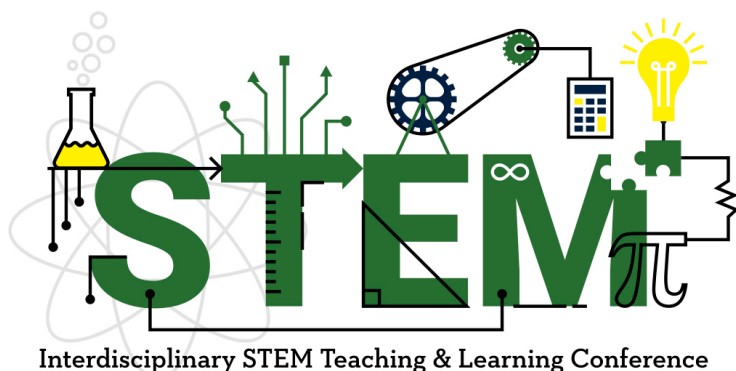
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## Conference Proceedings

2018, Volume 2



### **Editor's Note**

I want to start off by thanking all the authors for their time, interest, and energy they channeled into this second volume of the Proceedings of the Interdisciplinary STEM Teaching and Learning Conference. This volume, without intention, emphasizes the role of mathematics within interdisciplinary STEM teaching and learning. This unexpected focus sings the praises of an important piece of the STEM puzzle that often is overlooked and underrepresented. The Proceedings of the Interdisciplinary STEM Teaching and Learning Conference second volume is proud to highlight mathematics.

My second thank you goes out to Lisa Stueve and Kania Greer, our wonderful reviewers, and the conference planning team at Georgia Southern University for their help along the way. This volume speaks to our growth for both the conference and the Proceedings.

Cheers to round two of the Proceedings of the Interdisciplinary STEM Teaching and Learning Conference!

Best,  
Lisa Millsaps



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## **Augmented Reality Chemistry: Transforming 2-D Molecular Representations into Interactive 3-D Structures**

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### **Abstract**

Spatial reasoning is defined as the ability to generate, retain, and manipulate abstract visual images. In chemistry, spatial reasoning skills are typically taught using 2-D paper-based models, 3-D handheld models, and computerized models. These models are designed to aid student learning by integrating information from the macroscopic, microscopic, and symbolic domains of chemistry. Research has shown that increased spatial reasoning abilities translate directly to improved content knowledge. The recent explosion in the popularity of smartphones and the development of augmented reality apps for them provide, a yet to be explored, way of teaching spatial reasoning skills to chemistry students. Augmented reality apps can use the camera on a smartphone to turn 2-D paper-based molecular models into 3-D models the user can manipulate. This paper will discuss the development, implementation, and assessment of an augmented reality app that transforms 2-D molecular representations into interactive 3-D structures.

### **Keywords**

augmented reality, molecular representations, molecular visualizations, app development, molecular modeling

### **Recommended Citation**

Behmke, Derek; Kerven, David; Lutz, Robert; Paredes, Julia; Pennington, Richard; Brannock, Evelyn; Deiters, Michael; Rose, John; and Stevens, Kevin (2018) "Augmented Reality Chemistry: Transforming 2-D Molecular Representations into Interactive 3-D Structures," *Proceedings of the Interdisciplinary STEM Teaching and Learning Conference: Vol. 2* , Article 3.

## Introduction

STEM students continue to consider chemistry one of the most difficult subjects they must complete. Johnstone states that students find chemistry difficult to grasp because they must integrate information from the macroscopic, microscopic, and symbolic domains of the discipline (Johnstone, 1991, 2000). For students to be successful, instructors must teach them to develop mental models of the microscopic interactions between atoms and molecules that explain their macroscopic observations. Chemistry instructors typically use 2-D drawings, 3-D handheld models, and computer models to ease the cognitive load associated with developing mental models (Barak, 2013; Suits & Sanger, 2013). The ability to seamlessly transition between physical models and mental models is important to student success. This ability is most commonly referred to as spatial ability or reasoning (Coleman & Gotch, 1998; Harle & Towns, 2011; Suits & Sanger, 2013).

Chemical education literature contains numerous studies that demonstrate the importance of providing students with some type of molecular model when they are carrying out tasks that require the use of spatial reasoning skills (Barak, 2013; Booth et al., 2005; Suits & Sanger, 2013; Williamson et al., 2012). Springer demonstrated that organic chemistry students who watched an instructor properly manipulate computer models outperformed their peers who did not witness the manipulation (Springer, 2014). Abraham et al. randomly assigned students to one of three treatment conditions (2-D drawings, handheld models, and computer models) to see if there was a difference in their performance on stereochemistry assessments when compared to a reference group that did not use any models (Abraham, Varghese, & Tang, 2010). Students using computer models scored 15% higher than the other treatment groups, and 37% higher than the reference group on subsequent stereochemistry assessments (Abraham et al., 2010). Kuo et al. administered a stereochemistry exam broken into subtests (Kuo, Jones, Pulos, Hyslop, & Nan, 2004). During each subtest, students used a different model type (2-D drawings, dash wedge drawings, handheld models, and computer models) (Kuo et al., 2004). Scores were significantly higher on subtests where students used handheld and computer models (Kuo et al., 2004).

Despite the overwhelming evidence in favor of using models to learn chemistry concepts, it is still difficult to convince students to use molecular models to learn concepts unless the models are provided by the instructor. The explosion in popularity and availability of smartphones and augmented reality technology may provide a means to bridge this gap.

Augmented reality is a technology that virtually overlays information or interactive elements on top of a mediated view of the user's physical environment. A computing device with a camera and a screen, typically a smartphone or tablet, provides the mediation. The user points the device's camera at an object that acts as a trigger in the physical environment, and virtual elements are added to, or over, the object on the device's display. The virtual elements are, in their simplest form, an active element such as a video. In more complicated forms, the trigger can be overlaid with user interface elements and/or 3-D constructs. One example of such a construct is a 3-D molecular model a user can manipulate. (Delello, McWhorter, & Camp, 2015; Dunleavy & Dede, 2014; Ke & Hsu, 2015)

A number of augmented reality platforms have been developed and used in the educational arena; however, this field is not particularly mature (Dunleavy & Dede, 2014; Figueiredo, Gomes, Gomes, & Lopes, 2014). A number of investigators have used various augmented reality platforms to research artifact creation in college classes, including the areas of teacher education, business, and marketing (Delello et al., 2015; Ke & Hsu, 2015). Though augmented reality may already have a role in some chemistry classrooms, no literature could be found documenting its use. Can an augmented reality based molecule viewing app be developed that can provide students with a mobile, technology based solution to enhance their understanding of chemistry?

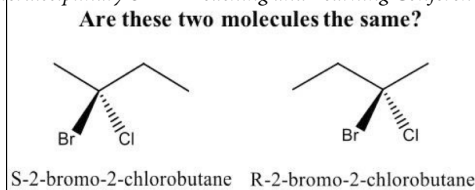
### App Development

For an initial proof of concept of the augmented reality molecule viewing app, the platform Aurasma was selected. Aurasma is one of many augmented reality platforms that has been previously used in educational settings. Non- commercial use of this platform is free, and it provides apps for both Android and iOS mobile devices. It has operated for a number of years and provides a reasonable degree of stability. It also provides a straightforward web-based authoring environment for creating augmented reality artifacts, referred to as auras. (Figueiredo et al., 2014) Given these characteristics, the majority of the development work focused on creating a process to generate and view 3-D molecular representations using this platform.

The process of generating a 3-D molecular aura that a user could manipulate involved a series of steps; first, generating a 2-D drawing of the molecule that was to be displayed. This drawing ultimately served as the trigger that later initiated the augmentation when the mobile device's camera driven by the app was pointed at it. A .mol file of the molecular structure was then generated using a molecular drawing program such as ChemDraw. The .mol file was then converted to a protein database (pdb) file, which was subsequently converted to a 3-D molecular structure using the free, open source 3-D creation suite Blender. The 3-D structure file was then transferred to Aurasma Studio and linked as an overlay to the 2-D drawing to create the aura. At this point additional overlays, such as buttons to allow molecular rotation, were added to the aura to enhance the user experience. Finally, the aura had to be shared so that end users could access the experience. In order to access the augmented reality based auras, users would download the Aurasma app and follow the channel where the auras were shared. Once the user followed the appropriate channel, they simply opened the app and pointed the camera on their mobile device at the appropriate 2-D drawing trigger.

### Implementation and Assessment

Stereochemistry has long been recognized as one of the more difficult topics in organic chemistry courses, because distinguishing between pairs of 2-D molecular drawings without an algorithmic approach requires a great deal of spatial ability. The first augmented reality based activity that was developed, and Institutional Review Board (IRB) approved, sought to enhance student learning in this area. The activity, which was designed to take no more than ten minutes to complete, is shown in Figure 1.



**Figure 1:** The first stereochemistry augmented reality based activity developed.  
(Note: Molecule names were omitted in student version)

The activity was implemented in several sections of organic chemistry I during four different terms (spring 2016, summer 2016, fall 2016, and spring 2017). During each term, there were an equal number of randomly assigned experimental and control sections, which resulted in total of 238 student participants (control N = 116, experimental N = 122). Instructors in both the experimental and control sections were asked to incorporate the activity near the conclusion of their normal instruction on stereochemistry, and students were permitted to discuss their answers with each other. In control sections, the activity was completed as a paper and pencil exercise that concluded with the instructor discussing the correct answer. Students in control sections were permitted to use model kits if they had them during the activity, but no student elected to build models. In the experimental sections students first downloaded, installed, and setup the Aurasma app, and then they were told that the image in Figure 1 above, with the molecule names removed, was a trigger to begin an augmented reality experience. The experience allowed students to view side-by-side 3-D representations of the drawings in Figure 1 which they could rotate by pushing a button on their screen. Additionally, they could push a button and see a video explaining the correct answer. At the conclusion of the activity both experimental and control sections completed the same assessment.

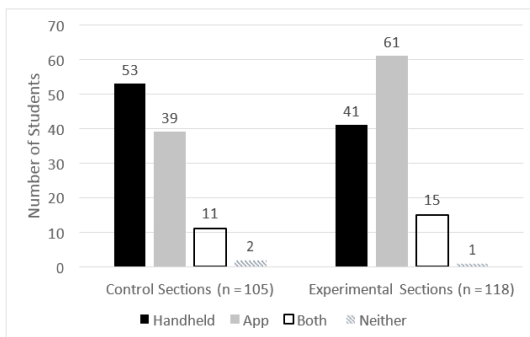
The assessment asked students to indicate whether pairs of molecules were the same (identical) or different (enantiomers or diastereomers). Experimental section students were not permitted to use the app on the assessment. All students were permitted to use their own model kits on the assessment, but none did. During the assessment students were also asked which type of model (handheld, computer/app, both, or neither) would have assisted them in better learning the stereochemistry material, and they were asked to briefly explain their choice.

## Results and Discussion

Students in the experimental sections had an average score of 68.0% on the assessment, compared to 63.3% for students in the control sections. A t-test showed that these results were not significantly different ( $p = 0.12$ ). These results are not surprising considering students only participated in one ten-minute activity prior to completing the assessment. These findings, however, suggest that the inclusion of the augmented reality activity did not negatively impact student learning. These results, combined with the enthusiastic response of the students as they interacted with the Aurasma app, indicate that augmented reality based molecular models have a future in the chemistry classroom.

The positive student performance results are further validated when their opinions

of model types are examined. Figure 2 summarizes student opinions of useful model types to learn stereochemistry material broken down by experimental and control sections.



**Figure 2:** Student opinions of useful model types to learn stereochemistry material broken down by experimental and control sections.

Students in the control section preferred handheld models ( $N = 53$ ) over computer or app based models ( $N = 39$ ). Student explanations indicated that they did not believe computer models were tactile enough, and that they did not believe computer based models could accurately depict 3-D structure. Students in the experimental sections preferred computer based models ( $N = 61$ ) over handheld models ( $N = 41$ ). Students preferred the computer based models because they accurately portrayed 3-D structure, it was easier and faster to access them compared to constructing handheld models, and they did not have to carry around a physical model kit. Students in the experimental section who still preferred handheld models cited the lack of tactile interaction as the app's number one weakness. Notably, all students in each section are required to have physical model kits, but no student in either section used these, even though they are highly encouraged to use them for the same reasons noted by student feedback.

When the performance data is combined with student opinions regarding the app, it becomes clear that augmented reality based molecular models have a role in the chemistry classroom. In order to provide a more definitive answer regarding their impact on student performance, a more detailed and rigorous set of augmented reality based activities will need to be developed, implemented, and assessed. Additionally, modifications will need to be made to the app experience to enhance tactile interactions the user has with the models. Work by McCollum et al. validates the need for more tactile interactions by demonstrating that users equipped with an iPad's touch screen to interact with models showed superior representational competence compared to their peers who used 2-D paper drawings (McCollum, Regier, Leong, Simpson, & Sterner, 2014).

## Conclusions

An augmented reality based molecule viewing app that can provide students with a mobile, technology based solution to enhance their understanding of chemistry was developed. The process for developing 3-D, augmented reality based molecular representations from 2-D, paper-based drawings is time and developer intensive.

Nevertheless, students using augmented reality models perform at least as well as those using no models. Students who have used augmented reality models find them more

### Current and Future Work

As work progressed, it became increasingly apparent that the selection of Aurasma as the platform for the app came with some serious limitations. The freely available, non-commercial package necessitated a cumbersome and time intensive process for generating the augmented reality based molecules. Given the large number of molecules end users would want to use, this is a significant hurdle. Additionally, the platform provided no seamless way to increase the tactile interactions with the models, which both the initial pilot testing of this app and the work by McCollum et al.(McCollum et al., 2014) suggests would be desirable and beneficial to the user experience. In order to alleviate these deficiencies, it became apparent that a standalone augmented reality app developed specifically for the intended use in a chemistry classroom was needed.

The development of such an augmented reality molecule viewing app is well underway. The Android and iOS app is currently in beta testing. The app uses 2-D drawings of molecules as triggers, much like the Aurasma app. The 3-D molecular structure is retrieved from the PubChem Database and is dynamically converted to a 3-D model that is presented to the user to generate an augmented reality experience. This automated process only requires the development of a trigger for each molecule. Users can rotate molecules using their fingers, and they can use pinch gestures to zoom in and out. Both of these features increase the tactile interactions users have with the models. The app also provides a color key, which clearly identifies the type of atom(s) in each substance. Assuming successful pilot testing, the app should become more widely available in the near future.

One of the more recent technologies to burst onto the scene are mixed reality headsets, such as Microsoft's HoloLens. These headsets have a wider viewing angle. This will allow the user to view larger numbers of more complex molecules in greater detail. We have designed a prototype molecule viewing app for the HoloLens. This app does not use a trigger but rather allows the user to directly select the molecule(s) he/she wants to view from a pre-populated list. The 3-D molecular structure is retrieved from the PubChem Database and automatically converted to a 3-D model. Users can zoom in and out and rotate molecules with both hand gestures and speech. The HoloLens app will enter beta testing soon.

### Acknowledgements

This project was supported by funds from a USG-STEM Initiative II Mini- Grant and a Georgia Gwinnett College SEED Grant. Special thanks to the faculty of the chemistry and information technology disciplines at Georgia Gwinnett College who volunteered their expertise and class time to help with the project.

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## **Selecting, Sequencing, and Connecting: Using Technology to Support Area Measurement through Tasks, Strategies, and Discussion**

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### **Abstract**

This paper supports grades 3-5 mathematics teachers and considers how technology in the classroom can be used to support “low threshold, high ceiling” tasks and productive discussion. We present a description of a card-sorting task to support the “5 Practices of Productive Mathematics Discussions” focused on an online task designed to: be open to multiple levels of strategies, reveal misconceptions, and support students in developing more sophisticated conceptual understandings of area measurement. We present a sampling of strategies created by teachers (who were pretending to be elementary students) in past activities. We discuss approaches to connecting strategies for deeper understanding of area measurement.

### **Keywords**

Area measurement, Productive discussion, Open tasks, Technology

### **Recommended Citation**

Stehr, Eryn Michelle; He, Jia; and Nguyen, Ha (2018) “Selecting, Sequencing, and Connecting: Using Technology to Support Area Measurement through Tasks, Strategies, and Discussion,” *Proceedings of the Interdisciplinary STEM Teaching and Learning Conference*: Vol. 2 , Article 4.

Teachers at all levels are expected ever more frequently to integrate use of emergent digital technologies (e.g., calculators, software, online tools, device applets) in mathematics teaching and learning. Too often technology is used for technology's sake, rather than in intentional ways to support mathematical reasoning, sense-making, and understanding. Online offerings of mathematical tools, tasks, and experiences span pedagogical spectra similar to those recognized in traditional curricula: procedural/conceptual, simple/rich, disconnected/connected, and so on. Applying long-tested educational quality frameworks to online tasks may help teachers choose and implement technology in ways that help students' reasoning, sense-making, and understanding. One such set of strategies is the *Five Practices for Facilitating Productive Mathematical Discussion*.

In this paper we describe how teachers can be supported in planning their use of the "Five Practices" productively. We use the context of an open, online area measurement task. That is, we describe two tasks: (a) an online area measurement task and (b) a sorting card task to be used to support teachers in developing strategies for using the Five Practices to support learning goals. The sorting task was developed based on preservice and practicing teachers' strategies and discussion after encountering the area task. The area task was designed by a mathematics education research group focused on spatial measurement in K-8 curricula. The area task asks students to measure the area of an irregular shape using rectangles. The team designed the area task to support multiple grade level learning goals, to allow multiple student strategies, to allow access from a wide range of levels of sophistication, and to potentially reveal student misconceptions about important ideas surrounding area measurement. Because of its openness, the area task can be used to support productive discussion. The sorting task was developed, based on the area task, to support future or practicing teachers to discuss the affordances and limitations of different selections and sequences of student responses. The goal of the sorting task is to support teachers in thinking through a practical application of the Five Practices *without* the chaos of a live classroom to support their classroom implementation and decision-making.

Through our discussion of these tasks, we hope teacher educators and teachers will be supported in developing practical and critical considerations for integrating open, online tasks and productive mathematics discussions into their pedagogies.

### Literature Review

In this review of relevant literature, we describe what has been said in the field of mathematics education about how beginning teachers and practicing teachers can be supported to use technology to support mathematical reasoning and sense-making. We propose that not all online tasks are designed for such mathematical practice. Hence, we describe perspectives on mathematical tasks from mathematics education literature that may help teachers and educators recognize particular characteristics of online tasks that are more likely to support these practices. Providing opportunities to students to interact with such tasks does not reliably result in deep understanding. Stein and Smith (2011) have developed and tested a set of strategies that teachers can use to support students in productive discussion (i.e., discussion that supports students in actively reflecting, analyzing strategies, and making mathematical connections).

In the following paragraphs, we describe recommendations for use of technology to support mathematical reasoning and sense-making. First, we explain what is meant by mathematical reasoning and sense-making and how digital technologies can allow students to encounter mathematical consequences of their actions. Next, we give examples of how online applets have helped students engage in these tasks.

The Association of Mathematics Teacher Educators (AMTE) developed a set of recommendations and standards for preparing beginning K-12 mathematics teachers. AMTE standards recommend that teachers are able to use technology in ways that support “mathematical reasoning and sense-making” and allow students to encounter mathematical consequences of actions (p. 11). Beginning teachers may have such opportunities in teacher preparation programs, but practicing teachers may have fewer opportunities to search for, choose, and implement such tools in their classrooms. In searching for online mathematical tasks and activities, teachers may find a wide variety of types of tasks. Some tasks are targeted to support teachers’ evaluation of isolated skills, others provide interactions with procedures meant to support conceptual understanding, and still others are developed based on recognized needs for students to engage in rich, messy, or open mathematical tasks.

To provide background on these tasks, we provide some examples here. Many online applets and tasks have been developed by for-profit companies (e.g., IXL Math [ixl.com/math/grade-8/graph-a-line-using-slope](https://www.ixl.com/math/grade-8/graph-a-line-using-slope)) to connect directly with content standards from the *Common Core State Standards in Mathematics (CCSSM, 2010)*. The mathematical tasks in such applets are intended as targeted assessments that provide feedback to teachers and students about students’ performance. Many other online applets and tasks have been developed by educational researchers and mathematics educators (e.g., [teacher.desmos.com/waterline](https://teacher.desmos.com/waterline)). These applets and tasks are designed to allow students to encounter and experiment with mathematical consequences of their actions, to support mathematical thinking and conceptual understanding. Along with other types, these two types of applets and tasks--assessment/feedback and exploration/consequences--can be used in balance to support student growth. When student work is confined to only assessment/feedback tasks, however, they lose opportunities to develop deeper mathematical understanding and to gain expertise in using online tools and technologies to support their reasoning.

In this paper, we propose building on research and professional development opportunities about use of open, rich tasks to support productive discussion of mathematics to support teachers in similar use of open, rich online tasks.

## **Open and Rich Mathematical Tasks**

Open, rich tasks are described from multiple perspectives. In this paper, we focus on five of these perspectives, although there are others that could be included. We focus on complex instruction and group-worthy tasks, cognitive demand of tasks, tasks that support mathematical connections, open tasks with multiple entry points, and authentic and relevant tasks. We briefly discuss these perspectives here to support later discussion about the use of these perspectives in categorizing online mathematical tasks.

One perspective is on complex instruction and group-worthy tasks (e.g., Featherstone et al., 2011). Group-worthy tasks include complex problems open to and [https://digitalcommons.georgiasouthern.edu/stem\\_proceedings/vol2/iss1/1](https://digitalcommons.georgiasouthern.edu/stem_proceedings/vol2/iss1/1)

requiring multiple smartnesses. In engaging with such tasks, each student is a valued member of the team and students can learn from each other's strengths, especially when those strengths are not typically privileged.

A second perspective is on high cognitive demand tasks and tasks that require critical thinking (e.g., Stein, Grover, & Henningsen, 1996). Such tasks include problems that make students think and develop their own strategies and solutions. One goal of such tasks is to support students in developing and owning the ideas and strategies that emerge. High cognitive demand tasks are defined in a framework with lower cognitive demand tasks (e.g., memorization, procedures without connections) compared to higher cognitive demand tasks (e.g., procedures with connections, doing mathematics through complex, non-procedural thinking).

Mathematical tasks that support integrating funds of knowledge, past knowledge or understanding, and connecting to different types of knowledge (e.g., mathematical, other subject areas, lived experiences) are a third perspective (e.g., Aguirre et al., 2013). With a problem that integrates multiple topics, strategies, and mathematical understandings, students continue to make connections between classroom mathematics and previously learned knowledge from mathematics or other subject areas and from their lived experiences outside of school.

A fourth perspective focuses on multiple access or entry points (e.g., Boaler, 1998; Turner et al., 2012). Problems that allow students access at their own level of mathematical sophistication, bring challenges and growth to all students whether they are normally identified as struggling, average, or advanced. This type of task may be called "low threshold/high ceiling." Then, through discussion, students learn from each other's strategies and consider the mathematical ideas at a higher level, asking which strategies (and in which situations) are more efficient, more straightforward, or more valid.

A fifth perspective focuses on authentic and relevant tasks with meaningful contexts (e.g., Aguirre et al., 2013, Turner et al., 2012). With a problem that connects to students' real-world experiences in a way that is interesting and motivating, they can bring their prior knowledge, experiences, intuition, and previously developed problem-solving strategies into the classroom.

## **Five Practices for Productive Discussion of Mathematics**

Stein and Smith (2011) described how teachers can build on tasks that meet the five criteria in the previous section; they explained that the tasks alone are not as effective in supporting learning as incorporating discussion that helps students put together their ideas and develop more sophisticated strategies. They proposed five practices for supporting productive mathematics discussions: (1) anticipating likely student responses, (2) monitoring students' actual responses, (3) selecting student strategies to be shared in the discussion, (4) sequencing shared strategies to support a learning goal, and (5) connecting mathematical ideas across strategies and to larger mathematical concepts. We describe each practice in more depth in the next paragraph.

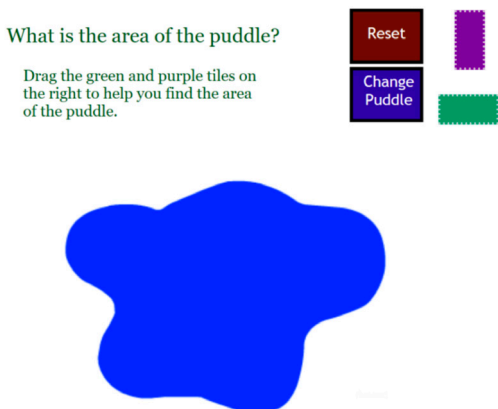
A rich, open task should allow students to develop individual strategies, resulting in many student solution strategies emerging in one classroom (Stein & Smith, 2011). The student solution strategies will not all contribute to productive discussion; indeed, using more than a few strategies may be overwhelming to both teacher and students (Stein & Smith). Hence, the authors explain that teachers must develop their expertise in

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 selecting and sequencing strategies that support a particular learning goal. The strategies and discussion can result in very different stories depending on which strategies are selected and how they are sequenced (Stein & Smith). A teacher must plan a task by generating possible strategies that she might see and focusing on a handful that she hopes to see (Stein & Smith). This step is *anticipating student strategies* (Stein & Smith). During the implementation of the task, the teacher must engage in everyday classroom management along with *monitoring actual strategies*; she asks students questions, looks at their work, and marks down on her planning sheet to track which student groups have developed which strategies (Stein & Smith). As she monitors, she draws on what she had planned for the selecting and sequencing (Stein & Smith). She asks particular groups to share their strategies in the discussion (Stein & Smith). Finally, she orchestrates the discussion by asking students to share their strategies, and asking their classmates questions to keep them involved in analyzing the strategies (Stein & Smith). Each step is non-trivial in terms of effort and necessity.

### Open Online Area Task

The *Strengthening Tomorrow's Education in Measurement (STEM)*, a multi-stage project funded by *National Science Foundation* with principal investigator Dr. John P. Smith, III, found that elementary mathematics curricula lack tasks that target student development of conceptual understanding of area measurement (Smith et al., 2008). As a response to this finding, the *STEM* project developed and adapted measurement tasks that could be accessed through physical manipulatives or online applets. One such area task is the “area of a puddle” task (see <http://tinyurl.com/STEM-puddle>) shown in Figure 1.

**Figure 1.** Open online area measurement task: Measuring the area of a puddle.



In the puddle task, the user is asked to drag green and purple tiles to the puddle (a blue, irregular shape) as units to measure the area of the puddle. The purple tiles and green tiles have the same measurements, but the purple has a vertical orientation and the green has horizontal. For each, one length is twice the measure of the other length. Neither type of tile exists in sufficient quantity to entirely cover the irregular shape. The designers chose elements of the manipulative and task deliberately to allow students

to experiment with different strategies, to make mistakes, and reveal misconceptions. For example, because puddle is an irregular shape, and because the measuring units are rectangles, it is not possible for a student to find one valid solution. Rather, students must devise strategies to using the tools to estimate. The measuring units are rectangles rather than squares to support student thinking about filling an area, the meaning of area units, and why square units can be useful. The rectangles can be placed anywhere, so students might overlap the units or leave gaps. There are not enough rectangular units to cover the puddle with only one orientation, so students must use both orientations or develop other strategies for measuring. The designers intended to push students to use two orientations to support discussion about the meaning of the area formula; that the units are rectangles rather than squares supports useful discussion, even if the rectangles are all one direction.

Based on examination of the *Georgia Standards of Excellence in Mathematics: K-5* (an adaptation for Georgia students of *Common Core State Standards in Mathematics*), we created six potential learning goals. That is, by the end of a lesson, students should show ability to do one of the following: (a) give reasons to not leave gaps/overlaps (MGSE3.MD.5b, MGSE3.MD.6), (b) divide a whole area into equal area parts (MGSE3.G.2), (c) find strategies to partition a shape and add areas (MGSE3.MD.7c/ MGSE4.MD.8), (d) building to area formula: Find strategies to count number of rows & columns (MGSE3.MD.7a), (e) reason about the meaning of the area formula for rectangles, specifically: describe reasoning about whether or not the area formula changes if rectangular units are used to measure a rectangle instead of square units or describe reasoning about whether or not area formula changes if rectangle units are two different orientations (MGSE3.MD.5a /MGSE5.NF.4), or (f) develop strategies that use over- and under- estimates to approximate a more accurate measure (MP5). Even though elements of each of the learning goals above could emerge through a productive discussion about a task, focusing the selection and sequence of strategies on one learning goal may make the discussion more manageable for both teachers and students.

### Strategies Card Sorting Task

The sorting task was originally developed in a mathematics methods course to support senior-level preservice teachers who were preparing to design and teach their first lesson. As a part of the lesson, the preservice teachers would use the Five Practices to lead a productive discussion after the students worked on a high-level task. Future teachers were divided in two groups: one group to engage with the task (as themselves first and then pretending to be third grade students) and the other group to monitor, select, and sequence strategies to support discussion. Based on their strategies, we created 21 strategy cards shown below, with brief descriptions. In the following sections we show selected strategies to illustrate their use in supporting productive discussions.

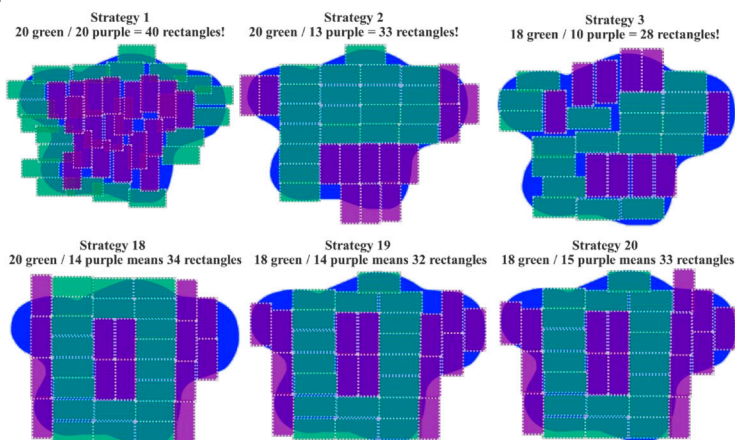
In Figure 2, six sorting strategies are shown. The strategies are chosen to illustrate *covering the space*, a method of measuring area that has a lower level of sophistication. In Strategy 1, the student has use 40 rectangles in two orientations to cover the space. The student left gaps and overlaps which reveals potential misconceptions about the meaning of area and the need for tessellation to ensure consistent results. Strategy 2 illustrates covering the space in a more systematic way. The rectangles are tessellated. Some rectangles hang off the irregular shape, while other parts of the shape are left uncovered.



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 Strategy 3 is more systematic than Strategy 1: there are no overlapping rectangles, and there are only a few overhanging rectangles, but there are many gaps left across the shape. Because each solution is different (40, 33, and 28 rectangles, respectively) these may support productive discussion about the need to avoid gaps and overlaps in order to obtain consistent results.

Strategies 18-20 are very similar and result in measures of 34, 32, and 33, respectively. They are systematically created with no gaps or overlaps in the central portion of the irregular shape. The rectangular units are shifted to different locations to cover as much of the enclosed space as possible, while leaving as little overhang as possible. These strategies could be used to discuss consistent results, in comparison with Strategies 1-3. They could also be used to discuss precision and limitations of measuring tools.

**Figure 2.** Low sophistication strategies: covering through tessellation or leaving gaps and overlaps.



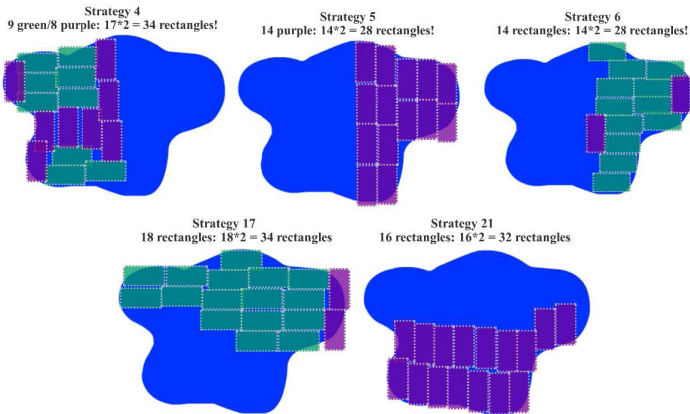
In Figure 3, Strategies 4-6 are shown with Strategies 17 and 21. These strategies show a slightly higher level of sophistication because the student covers only half of the irregular shape and then multiplies by two. The students seem to have attempted to tessellate the rectangles, and have different strategies for covering the space that lead to different solutions in Strategy 4, compared to the other two. Strategies 5 and 6 may be used to compare the same strategy with the same solution, but differently oriented rectangles. Strategies 17 and 21 also use the strategy of covering half and multiplying by two, but the lower and upper halves are covered rather than the left and right. The right and left side are less clearly different sizes, while the bottom side is clearly smaller than the upper side. In Strategy 17, some attempt is made to address this inequality by cutting the half along a diagonal rather than straight across. In Strategy 21, the rectangles trespass slightly into the upper portion to address the inequality between upper and lower sides. The solutions are similar, despite using different orientations of rectangles and measuring different portions of the irregular shape. Considering the five strategies together could support good discussion about the meaning of half of an irregular shape as well as the ways to estimate measures of half. Students can discuss whether the orientation of the rectangles matters in these estimates and how the two

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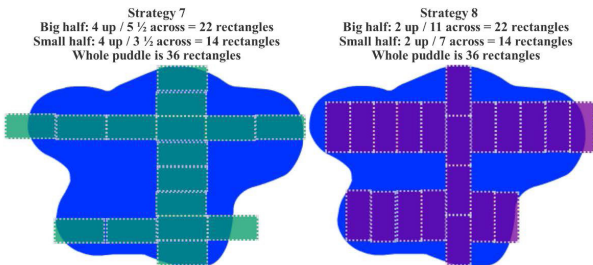
et al.: Volume 2, Proceedings of the Interdisciplinary STEM Teaching and orientations may be used strategically for better estimates. For example, in Strategy 6, two purple rectangles seem to be used to fill the space precisely.

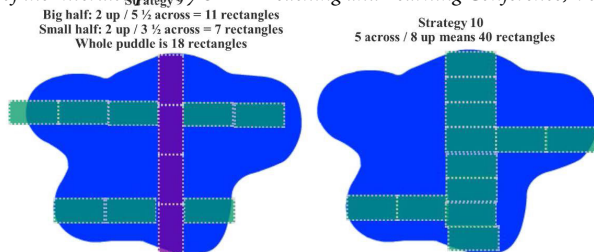
**Figure 3.** Slightly higher sophistication strategies: covering half and multiplying by two.



In Figure 4, we show Strategies 7-10. These strategies illustrate the use of the area formula for rectangles. They build on the strategy of dividing the irregular shape into two “rectangles.” Rather than simply measuring and multiplying by two, however, they measure two regions and add to find the overall area. Adding the measures of two areas in this way seem to create a reasonably accurate estimate of the overall measure. Strategies 7 and 8 can be compared because the measure is the same for both, despite different orientations of rectangles. Students may discuss the meaning of multiplying to find area when rectangular units are used rather than square units. The area formula for rectangles can be used to find the number of rectangular units that cover a larger rectangle, because it is simply counting the number of objects in an array (number of rows multiplied by number of columns). Strategy 9 reveals an important misconception about the meaning of the area formula and its validity. In Strategy 9, the number of rows and columns loses meaning because two orientations of rectangles are used. This mismatch may result in questions about the imagined array: Are rectangles in the horizontal or vertical orientation in its rows and columns? Can it be both, or must it be only one? The solution is much lower than other solutions which indicates the strategy is invalid.

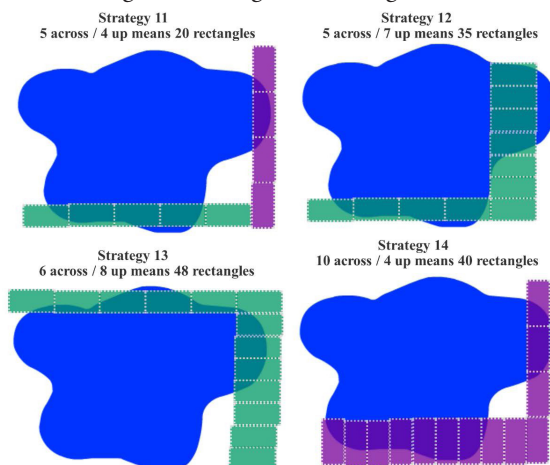
**Figure 4.** Sophisticated strategies: using tiles to find heights and lengths; subdividing the irregular shape in different ways for more accurate estimates.





Strategies 11-14 are shown in Figure 5. These strategies illustrate a higher sophistication. Similar to Strategy 9 above, Strategy 11 can be used to question the meaning of the area formula when it is used with rectangular objects in arrays rather than square units found by multiplying lengths and widths. The resulting solution is similar to Strategy 9, and solutions from Strategies 9 and 11 are quite a bit lower than other solutions which may indicate to students that something is amiss. Strategies 12 and 13 further illustrate under-estimating and over-estimating the area. Students may visualize creating a box based on the placement of the green rectangles. Strategy 14 has a solution almost midway between those of Strategies 12 and 13. Students may discuss the accuracy of each estimate.

**Figure 5.** Sophisticated strategies involving the meaning of area and estimates.



### Sorting the Cards to Tell a Story

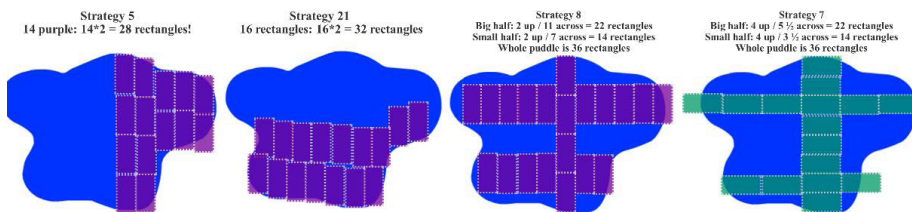
There are many valid ways of selecting and sequencing student strategies when using the Five Practices for productive mathematics discussion (Stein & Smith, 2011). Teachers may choose student strategies to ensure that all students participate. At times, tracking participation and ensuring all students have a chance to show their strategies can be overwhelming for the teacher and the students. Another method is selecting students' strategies that illustrate particular conceptions and misconceptions to support student thinking. Our method presented here is to choose strategies to tell a story that supports the lesson learning goal; that is, to select and sequence student strategies allow comparison and analysis and that build on each other along levels of sophistication or complexity toward a natural conclusion, the lesson learning goal.

Selecting and sequencing to tell a story to students through student-centered discussion is challenging for teachers to do, with many complexities in implementation. This card-sorting task can help teachers think through possibilities without the pressure of the classroom. We provide two examples of selecting and sequencing the cards to tell a story through discussion to support a learning goal. In the first example, we present a potential selection and sequence to support two of the learning goals listed above: (c) students are able to find strategies to partition a shape and add areas (MGSE3.MD.7c/ MGSE4.MD.8) and (d) students build to area formula and are able to find strategies to count number of rows & columns (MGSE3.MD.7a).

### Strategies for Partitioning Shapes and Adding Areas

Several of the student strategies might support discussion about partitioning shapes and adding areas. As one example, we select strategies 5, 21, 8, and 7 (shown and described in more detail above). We show their sequencing in Figure 6 below.

**Figure 6.** Strategies selected and sequenced for (c) strategies to partition a shape and add areas.



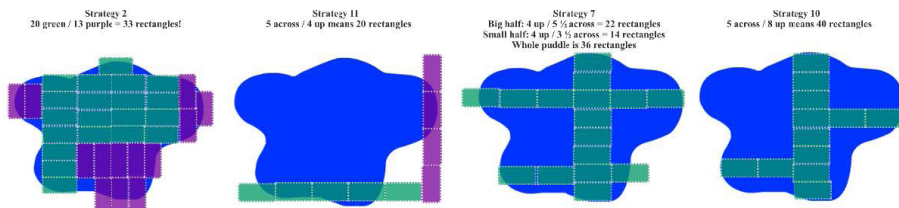
We selected strategies 5 and 21 to support students in thinking about two ways that the irregular shape (the puddle) can be partitioned into two parts. In these strategies, discussion might focus on the meaning of *half* for an irregular shape; that is, that dividing the shape into two equal parts is difficult in this situation. Students can discuss why dividing the shape into two parts vertically results in a fairly different solution than dividing the shape in half horizontally. We chose the two strategies because they both use the same orientation of rectangles. Moving from Strategies 5 and 21 to Strategy 8, may help students connect to a more sophisticated approach. In Strategy 8, the student divided the irregular shape into two parts, but noticed that the two parts are different in size. The student used the area formula to measure the area of each part and then to add the areas. This strategy leads to Strategy 7 where the two parts are measured using the area formula, but with rectangles in a different orientation. Although the strategies are different, the solutions in Strategies 8 and 7 are the same which could be surprising and might support discussion about the way the area formula works when rectangles are used rather than squares.

This sequence of strategies then can lead discussion that focuses on making connections between strategies for partitioning shapes using the covering method to using area formulas. The discussion may support analysis of strategies for efficiency and validity. That is, students can discuss efficiency of strategies: as shapes grow larger, time and materials become more important; covering the space uses more time and more materials than measuring the length and width. Students can also discuss validity of strategies: covering one part of an irregular shape leads to less precision than measuring

### Strategies for Building to the Area Formula

In the previous example, the strategies could support thinking about the meaning of the area formula. In this example, we select strategies to specifically target this learning goal. As in the previous example, many strategies might be chosen; we choose Strategies 2, 11, 10, and 7 shown in Figure 7 that could support this story. (These strategies are shown and described in more detail above.)

**Figure 7.** Strategies to build to area formula and to count number of rows & columns.



We selected these strategies to tell a story, moving from (2) covering the entire irregular shape with rectangles (both orientations must be used because the students run out of rectangles if they try to use only one orientation), to (11) measuring length and width with two orientations (which is problematic for the area formula and results in a much smaller measure), to (10) using one orientation and measuring the longest length and longest width, to (7) using one orientation, taking two measures of width, and using fractional parts of a rectangle in the solution.

Similar to the previous example, strategies can be analyzed for efficiency of time and materials, validity, and precision. In Strategy 11, some discussion can explore the meaning of the area formula. Students may notice that this measure is much smaller than measures resulting from other strategies (including their own strategy and comparing to Strategy 2). Students may discuss the consequences of multiplying rectangles in two orientations. Students could discuss the differences in units: rectangles, green rectangles, purple rectangles, squares. One consequence can be shown by comparing the results when adding all of the green and purple rectangles and then multiplying the height and width; this comparison could support a discussion about the result of multiplying green and purple rectangles (does the multiplication result in rectangular units or square units?). A second consequence that can be discussed is that this measure actually can be accurate and meaningful if the units are considered as squares rather than rectangles. In moving from Strategy 2 to Strategies 10 and 7, students can discuss the precision of the area formula when estimating the area of an irregular shape.

### Conclusion

Teachers need support in developing expertise over time and through community and collaboration with other teachers. In the first year they use a particular task, they may not know what strategies will emerge from their own students. Sharing strategies from other teachers' classrooms can help them anticipate. Over time, as they gather

strategies from students through particular tasks, they can develop their use of those strategies to support more rich and complex discussions. In this paper, we discuss one open task and how strategies from the task might support six learning goals across third, fourth, and fifth grade standards.

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## **The Effects of Integrating LEGO Robotics Into a Mathematics Curriculum to Promote the Development of Proportional Reasoning**

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### **Abstract**

A mixed methods, action research case study was conducted to investigate the effects of incorporating LEGO robotics into a seventh-grade mathematics curriculum focused on the development of proportional reasoning through the lens of Social Constructivist Theory. This study applied students' prior knowledge of the distance, rate, and time formula as they used LEGO EV3 robots to calculate the rate of a robot. The information gained was applied to different iterations, and structures, of the formula to support the development of proportional reasoning skills. The purposefully designed lessons were integral to the development of the students' understanding of the proportionality existing among the variables. The quantitative analysis reflects the acquisition of understanding of proportional relationships with the greatest increase being from low- performing students. The qualitative analysis provides an in-depth look at how students used their understanding of the distance, rate, and time relationship to develop proportional reasoning skills. Overall, the inclusion of robotics was productive for learning; however, future studies should be completed, on larger student populations, as a means to validate the quantitative findings and continue to improve the curriculum.

### **Keywords**

LEGO robotics, robotics, proportional reasoning, mathematics, distance

### **Recommended Citation**

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As a middle school mathematics teacher, I was always looking for ways to engage my students in authentic learning tasks that were engaging, hands-on, and, most importantly, fun for the students. However, I did not want the fun and engaging part to replace the learning and understanding aspects of the curriculum. Furthermore, knowing technological advances have increased the demand on educational programs to create students who are thinkers and doers; I wanted my students to be able to apply their knowledge while working in collaborative environments. I knew the “drill and kill” solution to learning mathematics was no longer a viable solution to advancing through mathematics education – students should be able to develop a solution, and apply that solution, when given a problem. With this in mind, mathematics teachers, myself included, need to create avenues to educate students to produce the type of person that is capable of succeeding in today’s technological world.

According to the *Partnership for 21st Century Skills* students require hands-on, engaging activities that promote intrinsic motivation to learn and acquire the skills so sought after in this 21st Century (2014). Carbonaro, Rex, and Chambers (2004) believe education needs to shift; instead of learning from technology (i.e., computer programs) students should learn with technology (i.e., robotics). Therefore, future research needs to be gathered with this thought in mind, which brings us to the research being reported in this paper. I sought to engage my students in authentic tasks by the integration of technology, namely LEGO robotics, into the educational environment to promote and enhance learning. When robotics are appropriately integrated into the mathematics classroom through specific tasks and challenges, students can “develop more sophisticated solutions and understandings of those solutions” (Silk, Higashi, Shoop, & Schunn, 2010, p. 21). If students are given the opportunity to learn mathematics through the use of LEGO robotics they would be provided with hands-on, engaging activities that promote learning.

A specific area of interest, that I felt could be supported by LEGO robotics technology, is how students develop proportional reasoning skills. Proportional reasoning has been a focus of research for over fifty years and has once again come to the forefront with the onset of the Common Core State Standards in Mathematics (CCSSM). Although CCSSM has recently become a topic of concern for educators, proportional reasoning has been a topic of much importance for mathematics educators since the 1970’s due to its influence on student success in higher-level mathematics (Jitendra, Star, Dupuis, & Rodriguez, 2013).

Langrall and Swafford (2000) claim a student’s ability to reason proportionally is imperative to aid their mathematical understanding at higher levels of mathematics and therefore it must be developed and strengthened during the middle school years. For the purposes of this research, I defined proportional reasoning as one’s ability to determine the multiplicative relationship between two quantities and to apply that knowledge to predict how the quantities will be affected when one of the quantities is changed.

Previous research studies incorporating LEGO robotics have reported positive results. Martinez Ortiz (2015) investigated the effects of robotics on students’ proportional reasoning skills through a one-week, extra-curricular intervention. The findings of his research showed that although there was not a statistically significant difference in student achievement at the end of the intervention period for either the



Intra-Prop or Extra-Prop questions, there did exist a moderate difference in student understanding on the final assessment given ten weeks after the completion of the intervention for the experimental group; a significant difference was found for the both the end of intervention and ten-week assessment for the Engin-Prop questions with the experimental group (Martinez Ortiz, 2015).

Ardito, Mosley, and Scollins (2014) integrated robotics into a sixth-grade mathematics class and found the highest level of success achieved by the students was in the areas most reflective of problem solving and critical thinking skills – Algebra; Measurement; and Statistics and Probability. Williams, Igel, Poveda, Kapila, and Iskander (2012) investigated the effects of integrating robotics into mathematics and science curricula classes and found the students' mathematics understanding improved by 25%, their science understanding improved by 47% and student surveys showed that students preferred the hands-on learning afforded by robotics.

The portion of my research project being reported in this paper investigated how the application of the distance, rate, and time relationship through the use of LEGO Robotics influenced the development of proportional reasoning skills among seventh grade students. More specifically, this portion of the research study sought to explore how students' solution strategies to distance, rate, and time problems supported the growth of developing, and applying, proportional reasoning skills. The research questions guiding this research were:

- (1) How does the incorporation of LEGO robotics into a unit on ratios and proportions influence students' proportional reasoning?
- (2) In what ways do students reason about distance, rate, and time while using the LEGO robots?

My research study investigated the four main types of proportional reasoning problems: part-part-whole, associated sets, well-known measures, and growth<sup>1</sup>. Part-part-whole problems relate two subsets (e.g., lions or tigers) to one another or one of the subsets to the whole (e.g., number of tigers as compared to the whole population of zoo animals). Associated sets are proportional relationships with quantities that are not regularly associated with one another (e.g., ounces of juice and students). Well-known measures involve quantities that are regularly associated together (e.g., miles per hour is equal to speed). Growth problems deal with the dilation or shrinking of objects (e.g., a photo is enlarged from 3x5 to 4.5x7.5) and are considered to be the most difficult types of problems for students to master (Langrall & Swafford, 2000; Lamon, 1993).

### **Theoretical Framework**

This research was guided by the Social Constructivist Theory as explained through the work of Lev Vygotsky (Moll & Whitmore, 1993; Cobb, Wood, & Yackel, 1993; Hatano, 1993; Vygotsky, 1978; Wertsch, 1985). Vygotsky's (1978) Social Constructivist Theory was based on his belief that learning was a result of social activity which allowed children to construct knowledge and understanding by playing and conversing with other children and adults. This theory was the foundation for the development of the

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<sup>1</sup> The results of the growth problems will be presented in a separate paper as they were investigated separate from the distance, rate, and time formula  
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et al.: Volume 2, Proceedings of the Interdisciplinary STEM Teaching and Curriculum and every investigation and activity was designed to focus on the social aspect of LEGO robotics. I was careful to incorporate discussion and *play* into the curriculum as students used the robots for learning. As the students worked through structured tasks, the LEGO robots required the “children [to] solve practical tasks with the help of their speech, as well as their eyes and hands” (Vygotsky, 1978, p. 26).

As the research was analyzed, another framework, primarily applied to problem-based learning (PBL), evolved. Carbonaro, Rex, & Chambers (2004) found when working in PBL environments that technology integration must involve five stages in order to be effective. The stages are engagement – teams are formed, the challenge explained, and questions are asked; exploration – perform specific tasks to acquire knowledge and skills; investigation – make predictions, plan experiment, and test; creation – design, test and modify as needed; and evaluation – present findings to peers and formal/informal assessment of knowledge gained (Carbonaro, Rex, & Chambers, 2004). As I analyzed the data, these stages were very pronounced and became an important piece of the coding scheme. Since this framework relates closely with Social Constructivist Theory, it was used to analyze the research data.

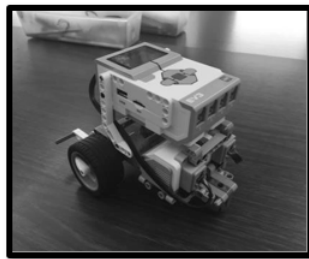
### Methodology

The mixed methods format utilized for this action research allowed me to assess the students’ growth of understanding, document student engagement, and allowed for student feedback to become part of the data collection. The participants studied were six (6) students in my seventh-grade mathematics class who attended a small, progressive, independent school. The research was comprised of a pre- and post-test, eight purposefully designed lessons/investigations (see Appendix A to view a lesson), and three activities (given at specific intervals throughout the intervention). <sup>2</sup>The activity completed after investigation 4 is shown in Appendix B.

This research integrated the use of the LEGO Mindstorms EV3 Robots (see Figures 1 and 2) programmed with a basic movement block (see Figure 3) that was relatively easy for students to understand and manipulate. The students were purposefully grouped into heterogeneous pairs to complete the investigations. The data collected consisted of pre- and post-tests, classroom observations, student interviews, field notes, student journals, and student work artifacts. The four investigations addressing the concept of distance, rate, and time were specifically designed for this research and allowed students to change the values of time and speed in the programming block as required in each investigation.



**Figure 1.** Right Side View of Driving Base



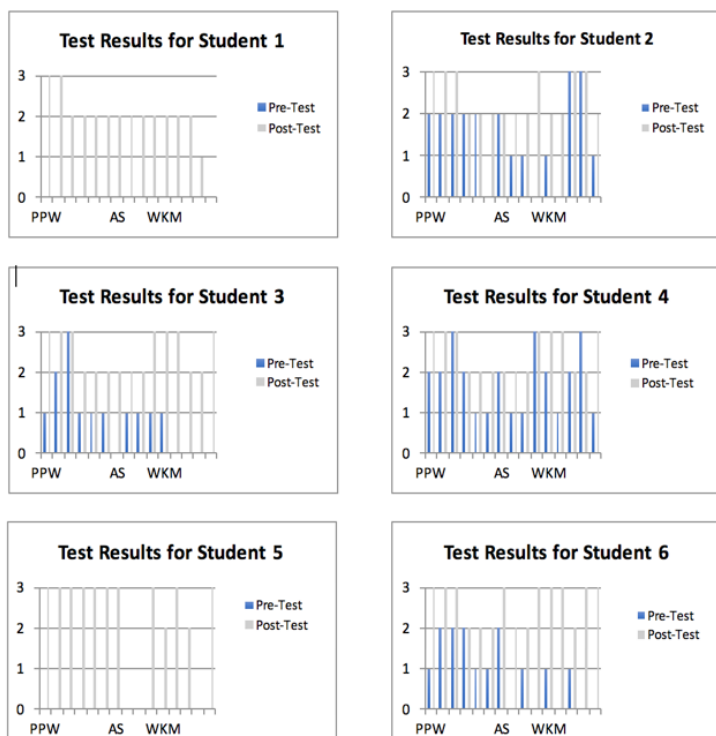
**Figure 2.** Left Side View of Driving Base

<sup>2</sup>The research reported in this paper only involves the first four lessons/investigations and one activity.  
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In addition to the overall test scores, each question on the pre- and post- tests were analyzed according to Langrall and Swafford's Proportional Reasoning Rubric (2000). The rubric allows for classification of students' proportional reasoning among four levels - non-proportional reasoning, informal reasoning about proportional situations, quantitative reasoning, and formal proportional reasoning. At the non-proportional reasoning stage students are likely to make guesses or randomly choose numbers. At the informal reasoning stage students may draw pictures to represent their understanding. Students at the quantitative reasoning stage have begun the transition from additive to relative thinking and begin to understand and use scale factors. At the formal proportional reasoning stage students understand how to set up and solve proportions (Langrall & Swafford, 2000).

The results for each student are shown in Figure 5 below. The figure depicts the development of proportional reasoning skills by each of the six students who participated in the research study. Each of the six students developed and/or improved proportional reasoning skills. Students 1 and 5, who are low-performing students, reflected the most growth in their proportional reasoning skills. The low-performing students demonstrated informal proportional reasoning skills (level 0) on the pre-test, but developed quantitative and formal proportional reasoning (levels 2 and 3), as demonstrated on the post- test. Students 3 and 6, average-performing students, exhibited growth by improving to consistently reflect quantitative and formal proportional reasoning skills on the post-test. Finally, students 2 and 4, high-performing students, demonstrated improved understanding of proportional reasoning as shown by their growth from the pre-test to the post-test.



**Figure 5.** Results of pre- and post-test by question type.<sup>5</sup>

<sup>5</sup>The abbreviations in the table refer to the problem types: part-part-whole (PPW), associated sets (AS), and well-known measures (WKM).

I analyzed students' discussions as the students solved problems involving distance, rate, and time, to identify the students' application of the five stages of technology integration (Carbonaro, Rex, & Chambers, 2004) and determine how this integration guided the development of understanding.

Carbonaro, Rex, and Chambers (2004) reported the students appeared to progress through the stages in a linear fashion in the PBL environment, however, in my research the students' movement among the stages was more fluid. Engagement was an overarching stage, present at each of the other levels, and students progressed through the stages *as needed*. For instance, students may have read a question, explored a solution, created a solution, evaluated the findings, and, if wrong investigated why it was wrong, which may have required the creation of a new solution. It was the progression among these stages in which the students' understanding of proportional reasoning was developed, improved, and applied.

Student discussion was an important aspect of each investigation within, and among, each of the groups and was an important factor in how students applied their knowledge about distance, rate, and time to create, and analyze, proportions related to their given tasks; decisions made within all three groups were made by both group members and not by one individual. Students were applying the DRT formula in each of its three forms ( $d = rt$ ,  $r = d/t$ , or  $t = d/r$ ), in order to respond to the tasks presented in each investigation. It was through the understanding of these formulas that students were able to make sense of, and create, proportions. For example, when students were working with the same programming speed, say 50, they knew their robot's rate was approximately 24 cm/s (from previous tasks). After determining the time required to travel a specific distance at this rate, they would be able to predict the time needed to travel a different distance by applying the following proportion:

$$\frac{\text{known distance}}{\text{known time}} = \frac{\text{new distance}}{x}$$

The students were able to substitute the known numbers, calculate the predicted time, input the information into the program software, and test their prediction. Once students obtained the results, they were required to justify their answer if they were correct or determine possible causes of error if they were incorrect. It was through these actions, and the conversations occurring as these actions were completed, the students' understanding was developed. It became clear, while analyzing the conversations, this was how the students were developing proportional reasoning skills. An example would be the following conversation when students were attempting to determine the rate at programming speed 25 when they knew the rate at programming speed 50:

*Casey:* ...the speed of 50.

*Bailey:* That means you do half the rate.

*Casey:* Half the rate?

*Bailey:* Or double the rate, I'm not sure.

*Casey:* No, half the rate because if we double the rate then we're going too fast.

Another consistency among the groups was the ability of the students to *make sense* of the data. For example, Dakota stating, “if it went that far with 5 seconds, maybe we should try some smaller numbers” or Bailey saying, “That doesn’t make sense, what did I do wrong?”

The investigation and activities designed for this portion of the research were developed in a manner to support student’s development of proportional reasoning skills by applying their knowledge of distance, rate, and time through the tasks presented. The format required the students to work together to predict, program, test, and evaluate their data; each of these tasks required the students to perform an activity (e.g., calculate numbers, measure a distance), thus applying the DRT formula while developing and/or improving proportional reasoning skills.

## Discussion

### Implications of Research

My research has provided evidence to support the inclusion of robotics as a means to apply student understanding of the distance, rate, and time relationship to improve students’ development of proportional reasoning. The inclusion of robotics promoted discussion within, and among, student groups as they worked through the investigations and activities. In this day and age when so much attention is given to purposeful technology integration, units such as the one I developed for this research is beneficial – it provides an example of how technology integration can support the learning of mathematics. This type of technology integration allows students to learn *with* technology rather than *from* technology (Carbonaro, Rex, & Chambers, 2004).

Throughout education students have been developing proportional reasoning skills in mathematics classrooms through many different methods (e.g., lecture or manipulatives) long before the introduction of robotics. The inclusion of robotics to promote the development of proportional reasoning skills may not be a unique method for promoting understanding, but it is a meaningful method.

LEGO Robots allows students to *see* proportionality as they progress through the activities. Students echoed this statement through their responses to the interview question, “How do you feel about using the robots in math class? Do they help you learn better?” Each of the four interviewed students<sup>6</sup> replied with similar responses:

Jordan: “I feel like they can actually really help with the ratios and proportions because the way, or the things that we’ve been doing so far have helped me better understand, I think, rather than using a book. Cause [sic] with a book sometimes you can’t really understand what you’re doing, but with the robots you can actually see what’s happening and calculate further.”

Dakota: “Yeah, because its more hands on than just like, here’s a worksheet fill out the answers... cause in life if you have...a math problem integrated in life you’re not going to be handed a worksheet. You have to analyze it and then figure out from that. That’s sorta [sic] what we’re doing with the robots.”

<sup>6</sup>The gender-neutral student names are pseudonyms to ensure student anonymity.  
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Casey: "I think it's good, like, I think it's fun and you learn things, like, like, uh, rates and times and distances. I like using the robots better than just doing math on paper...it's more fun with the robot...it's, like, more interactive so you're doing something and then you're learning math, not just looking at a workbook, reading the question, and writing whatever's down on it."

Harley: "I like it a lot.... because it's, like, you can see what you're doing. If you program something and you turn on the robot and it goes however long it goes, you can see what you're doing and if it messes up you can always improve instead of, like, writing on a sheet of paper... you messed this up and you have to, like, redo it, but...you can see what you did..."

The LEGO robots bring another dimension to the learning, a sense of play that tends to mask the learning, in my experience. I have witnessed students struggle to arrive at the "correct" answer and give up when working out of a book, with a worksheet, or with manipulatives. However, when students are learning collaboratively with robots they tend to have much more perseverance – they continue to talk through the issues and try different numbers in the program until they arrive at the answer – the robots create a "can-do" environment. My experience as a mathematics teacher has allowed me to witness that low- performing students tend to "give up" more quickly than average- or high- performing students. However, it was the low-performing students that achieved the greatest growth in my research, which, I believe, is due to the positive environment generated through the playfulness of the robots. I argue LEGO robotics provides students the opportunity to develop proportional reasoning skills in a manner more effective than other learning methods due to the playful aspect and positive environment created by the robots.

### **Limitations of Research**

The results of the quantitative data show the students developed proportional reasoning skills, as evident in the change in the levels of proportional reasoning from the pre-test to the post-test and overall improvement in test grades, but since the class consisted of only six students the data is not generalizable to larger populations. The breadth and depth of the qualitative analysis was limited as well. The breadth of the data analysis was limited as with a small class size there is a lack of multiple occurrences of comments and/or actions. The depth is limited because although I was able to find evidence of the benefit of robotics, it is insufficient verification due to having only six students.

### **Proposed Changes for Future Research**

This research provided evidence for the positive effects of incorporating LEGO robotics into a mathematics curriculum focusing on the development of proportional reasoning. However, after conducting the research and analyzing the data, I have found areas I would like to improve to produce stronger, more convincing evidence for the power of robotics inclusion in future studies. In addition to researching a larger sample of students, future studies will include at least one additional investigation to focus more clearly on ratios (separate from proportions), will include different types of daily journal

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questions (more objective to better assess student understanding), and will investigate, in more depth, the playful nature of the robotics.

## Conclusion

The findings show students reason about distance, rate, and time through discussion as they transition through the five stages of technology integration (Carbonaro, Rex, & Chambers, 2004). It is through this process the students develop, improve, and apply proportional reasoning skills. The students reported the benefit of incorporating robotics into the unit as it allowed them to learn in a visual manner and more easily determine accuracy – they could see if they were right or wrong. In addition, the creative and playful aspect of the robotics appeared to create a natural engaging environment for student learning. When students are given the opportunity to learn mathematics through the use of LEGO robotics they are provided with hands-on, engaging activities that assist in, and promote, learning.

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## Appendix A

### Rates and Proportions - Investigation 2

#### How much time do I need?

In Investigation #1, “What is my rate?” you determined the rate at which the robot travels at programming speed 50. In this investigation, you will use your knowledge of the robot’s rate to determine different times that are needed to travel a specific distance.

*This lesson will allow students to continue to develop their ability to reason proportionally. The objective of this lesson is for students to begin to reason proportionally as they predict how the rates of the robots will change from a programming speed of 50 to a programming speed of 25, or 100.*

#### Class Discussion:

- 1) How can I use a known speed to determine how much time is needed to travel a specific distance?
- 2) What variables could affect your predictions and results?

#### Group Work:

For each question below, you will first need to predict the time required, program the time using the software, and test your prediction. If your prediction is inaccurate, you will need to continue to test until you find the correct time.

In Investigation #1 you determined your robot’s average rate at programming speed 50.

What was your robot’s average rate? \_\_\_\_\_cm/s

- 1) How much time is needed for your robot to travel at programming speed 50 for 15 cm?  
Was your prediction correct? If not, what was the time needed?  
Why do you think your calculations were incorrect?
- 2) How much time is needed for your robot to travel at programming speed 100 for 25 cm?



- a. What do you predict the robot's rate will be at programming speed 100? Why?
  - b. Was your prediction correct? If not, what was the time needed? Why do you think your calculations were incorrect?
- 3) How much time is needed for your robot to travel at programming speed 25 for 50 cm?
- a. What do you predict the robot's rate will be at programming speed 25? Why?
  - b. Was your prediction correct? If not, what was the time needed? Why do you think your calculations were incorrect?
- 4) Develop your own speed rate and distance, make the prediction and test your results. Make sure to record your speed, distance, time prediction and results.

## Appendix B

### Rates and Proportions – Check-Up Activity Sheet #1

I would like you to answer each of the following questions. You may work in your groups to complete these problems. You must show all of your work and answer each question completely. Please add any comments you feel are necessary to explain your thinking.

All of these problems were taken from *Connected Mathematics 2* “Comparing and Scaling: Ratio, Proportion, and Percent.” (Lappan, Fey, Fitzgerald, Friel, & Defanis Phillips, Comparing and scaling: Ratio, proportion, and percent, 2006, p. 7)

*This activity will be given to students during class upon the completion of the first four investigations. The objective of this activity is to document the students' ability to transfer their new knowledge to problems requiring proportional reasoning skills to determine a solution.*

- 1) Students at Neilson Middle school are asked if they prefer watching television or listening to the radio. Of 150 students, 100 prefer television and 50 prefer radio.
  - a. Determine if each statement accurately reports the results of the Neilson Middle School survey by answering true or false. Please justify your answer in detail.
    - i. At Neilson Middle School,  $\frac{1}{3}$  of the students prefer radio to television.
    - ii. Students prefer television to radio by a ratio of 2 to 1.
    - iii. The ratio of students who prefer radio to television is 1 to 2.
    - iv. The number of students who prefer television is 50 more than the number of students who prefer radio.
    - v. The number of students who prefer television is two times the number who prefer radio.
    - vi. 50% of the students prefer radio to television.

## **Integrating LEGO Robotics Into a 5th Grade Cross Curricular Unit to Promote the Development of Narrative Writing Skills**

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### **Abstract**

This paper describes a unit designed to promote the development of narrative writing skills among 5th grade students through the use of LEGO robotics. Over the course of four, two and one-half hour sessions (one day per week for four consecutive weeks), the students learned how to construct and program robots, write and present a proposal to complete a mission, and connected the learning to their personal experiences with Hurricane Irma. The students began the activity with prior knowledge of World War II and Hiroshima. After learning the basics of building and programming robots, they were presented with a scenario similar to the impact of the bomb drop in Hiroshima – a city in ruins with survivors in need of supplies. Students took the role of engineers to work in pairs to create a proposal, which stated the problem and defined, and justified, a solution to pick up, and deliver, supplies through a specially-designed course using their robots. After the proposals were presented, and accepted, students programmed their robots according to their proposed solution; the activity required students to apply mathematical skills to measure distances in order to traverse parts of the course. Writing reflections were collected to determine individual student understanding and to include an additional element of writing. A final culminating activity required the students to write a narrative piece to relate the events of Hiroshima to their personal experiences with Hurricane Irma.

### **Keywords**

LEGO robotics, cross-curricular, mathematics, writing, social studies

### **Recommended Citation**

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Writing has been at the forefront of education since the onset of formal educational settings and I argue it may be more important now than ever before. The increase in technology has allowed more and more students to have access to texting and social media – avenues where proper grammar and formal writing take a back seat. In addition, educational policies of the 21st Century (e.g., No Child Left Behind and Race to the Top) have increased the demand on teachers to improve students' reading and writing abilities. The issue at hand then becomes how to teach writing, or apply the concepts learned, in an engaging and meaningful way for students. Furthermore, with such time constraints placed on classroom teachers, finding a way to incorporate more learning in less time would help to alleviate some of the stresses felt by educators. I posit the time constraints can be reduced, and learning can become engaging and meaningful, through the implementation of cross-curricular units.

I have experience incorporating LEGO robotics into mathematics curricula and found the students to be more engaged in the learning when the robots were implemented. In addition, studies have reported positive learning outcomes in mathematics classrooms (i.e., Ardito, Mosley, & Scollins, 2014; McDonald & Howell, 2012; Martinez Ortiz, 2015) and in science classrooms (i.e., Williams, Igel, Poveda, Kapila, & Iskander, 2012) when learning and understanding were gained through the incorporation of robotics into the curricula. We, the teachers and faculty involved in this intervention, were interested in stretching the boundaries of previous robotics interventions to determine if positive results can be achieved in the domain of writing.

### **The Problem**

In rural Georgia, the writing deficits of elementary students are documented through annual Georgia Milestones exams. These deficits may not be more severe than other rural areas around the nation, however, the number of English Language Learners (ELs) and students living in poverty substantially impacts this area; any additional support to meet the needs of all learners would be welcomed by faculty and administration. One local elementary school reached out to the university at the start of the 2016-2017 school year to inquire about possible programs to support their students. After school stakeholders participated in a meeting with university faculty it was decided LEGO robotics could be a possible solution. School faculty and administration decided to bring a sampling of their students to campus for a robotics intervention intended to promote narrative writing skills. The school faculty and administration felt the robotics writing intervention showed positive results and early in the fall of 2017 the school reached out to the university to determine if it would be feasible for their students to participate in another robotics program, but with a new group of students. After meeting to determine the school's expectations for the program the following unit was developed.

### **Methods**

The school's focus was to integrate Social Studies and narrative writing, with priority given to students performing below Georgia Milestones proficiency levels. Since the University's Innovation Studio is supplied with eight LEGO Robotics kits, it was agreed a total of 32 students would participate in two separate programs of instruction.

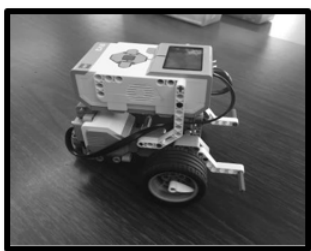
Each program supported 16 students and lasted four weeks. The students met for four

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consecutive weeks, one morning per week, for 2.5 hours each morning – a total of 10 hours of instruction; one group participated in November, 2017 and the second group participated in January/February, 2018.

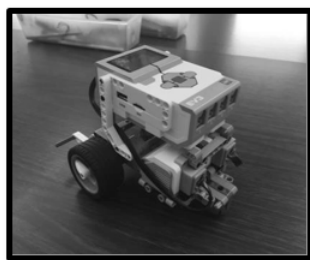
A unit was designed to integrate Social Studies, narrative writing, mathematics, and technology through the use of LEGO Robotics. Since the students were studying World War II in Social Studies, one of the teachers suggested the students read an article about the bombing of Hiroshima (Hiroshima, 2012). The article was read in school prior to the students' first robotics session and the topic became the overarching focus of the intervention. The plan was to have students learn about robots, build and program a robot, and be presented with a final challenge – helping survivors of Hiroshima obtain necessary supplies. The final challenge would require them to work in pairs to complete a proposal, program their robots to complete the final challenge, and the culminating activity would require the students to write a narrative piece connecting their understanding of Hiroshima to their experiences with Hurricane Irma.

## The Unit

The 4-day unit was designed to incorporate robotics education, building, programming, collaboration, problem-solving, planning, and design. The first class began with an introduction to robots through a PowerPoint presentation and class discussion – what they are and what they do. The students were provided with a LEGO Mindstorms EV3 instruction manual and robot parts – they were required to work in purposefully chosen pairs to build the basic driving base (see Figures 1 and 2). In addition, students were instructed to add the color sensor to their build for use in the final challenge. Once the robots were built (which utilized a significant part of the class time) students were introduced to basic programming blocks used with the Mindstorms software. To end the class, students were asked to participate in a closing reflection activity which required them to write a response to two questions: (1) What did you like about today? Why? and (2) What is a robot and what does it do?



**Figure 1.** Driving base viewed from the right.



**Figure 2.** Driving base viewed from the left.

The second class began with a quick review initiated by the question “What is a robot and what does it do?” The discussion concluded by determining the different methods for turning the robot – drag turns, pivot turns, and point turns – and scenarios in which one may be favored over the others. Drag turns involve both wheels turning, but at different speeds. Pivot turns require one wheel to be stopped while the other wheel moves. Point turns are tight turns in which both wheels move at the same

speed, but in different directions. Once the discussion ended students were required to program their robot to move in a square. This task required basic programming blocks, but allowed students to apply their knowledge of straight movement and turning. As student pairs completed this task, they were asked to create different polygon shapes (e.g., triangles, pentagons) as a form of differentiation and to allow other student pairs time to complete the task.

A major focus of the final challenge was the requirement of the robots to follow a line, therefore the next challenge required students to program their robots to follow a colored line and to stop on a specific color. This introduced students to programming logic (if/then statements) known as switches and loops. Differentiation was implemented in this activity by allowing early finishers to add “song and dance” to their robots after completing the designated challenge, which was done by adding sound blocks in the programming and creating unique movements with the programming blocks.

Once all student pairs had completed these tasks they were presented with the final challenge – to deliver supplies to the survivors of Hiroshima. The students were shown the course specifically designed for this unit (see Figure 3), which required the robots to pick up supplies and deliver them to the survivors of Hiroshima. It was at this point the students were asked to write a proposal to present to the “Disaster Relief Administration” with their plan to complete the mission. The students were presented an outline with the following required information for the proposal:

- What is the problem?
  - o Write 1-2 sentences to describe the problem to be solved.
- What is your solution?
  - o Write 4-5 sentences to describe your solution plan.
- How do you plan to implement your solution? What will you need to do?
  - o Write 5-10 sentences to describe how your team plans solve the problem. Describe the programming blocks you will use.

Students were asked to collaborate to complete this task and informed they would present their solutions to the entire class and teachers at the next session. The remaining class time was devoted to writing (approximately 30 minutes) and time to complete the proposal was provided during their regularly scheduled support time at school (teachers reported an additional 30-45 minutes was provided).



**Figure 3.** View of the Final Challenge course.

blocks – moving straight, turning, loops, switches. After the discussion, the students presented their proposals. Upon “acceptance” of the proposals students were tasked with programming their robots to complete the final challenge. The final challenge required the robots to follow a line to reach the supplies (the robots were programmed to stop when the color sensor “saw” the red tape), attach the supplies to their robots, travel a specified distance to another line following section, and then follow the line to the survivors to deliver the supplies (the robots were programmed to stop when the color sensor “saw” the green tape). In order to travel the specified distance between the supplies and second line, the students applied mathematical understanding to measure the distance and determine how far their robot traveled in one rotation of the wheels in order to determine how many rotations would be required to travel to the next line following segment.

The students worked diligently for the entire class period and were all actively engaged throughout the time period. It was rewarding to witness their perseverance to work through the issues at each stage – they did not give up. After working through several trials to obtain success it was common for students to be “jumping up and down” in excitement and “high-fiving” one another; they experienced success due to working hard and not giving up. One pair of students completed the challenge fairly quickly and were asked to build a basket to transport the supplies that could easily be attached and unattached from everyone’s robot during the challenge – they appreciated the opportunity to complete the additional project and collaborated well to design a solid, lightweight structure. By the end of the session, most student groups had completed the challenge, and only needed to complete some fine-tuning at the next, and final, session. However, there were two groups who only partially completed the challenge at this point and it was decided additional support would be provided at the final session to ensure their success. This class ended with students completing a written reflection to address the following questions: (1) What challenges did you encounter today as you programmed your robot to complete the final challenge? And (2) How did you and your partner successfully conquer the challenge? Be specific.

The final session began with a quick discussion reviewing the functions of a robot and the functions of the different programming blocks. Once the discussion ended, time was devoted to completing the programming of the final challenge. Those teams who successfully completed the challenge were given the opportunity to program a celebratory song and dance upon reaching the survivors as the instructors worked with the two teams in need of support.

After all teams had successfully programmed their robots to traverse the course, each student pair was asked to present their solution by having their robot complete the challenge while the rest of the class watched, which allowed each team of students to witness similarities and differences among the movements of each team’s robot. After all of the student pairs had completed their presentation, a final discussion was conducted for students to share the hurdles they encountered and strategies for overcoming those hurdles in order to achieve success in the final challenge. This discussion afforded students the opportunity to understand how similar issues can be approached from different perspectives, thus creating multiple solutions to similar problems.

The culminating activity was introduced to the students – write a narrative document comparing their understanding of the events of Hiroshima to their recent experiences with Hurricane Irma. The remaining class time was devoted to student writing (approximately 30 minutes) and support was provided to students as needed. All of the students required additional writing time, which was provided upon their return to school.<sup>1</sup>

## Results

An important result of this intervention, although subjective, was the improvement in the students' ability to write; when students were asked to write a reflection at the end of the first session there was a lot of hesitation and their final products barely answered the questions posed, many writing samples reflected incomplete sentence structure. This response was mirrored in the second session when students were asked to write a proposal. The students struggled to get started, even with the prompts given. They were questioning the teachers and instructors about what to write and required much scaffolding to put words on the paper even though they were provided with an outline to guide their thinking. By the third session, their reflections seemed easier to complete, which I speculate was due to their active engagement during the session.

When the culminating activity was announced, although the students were not excited about the writing aspect, they were able to begin writing rather quickly. It was impressive to see 15 of the 16 students feverishly writing - some even asking for additional sheets of paper! One student, who the teachers reported had a history of struggling to organize their<sup>2</sup> thoughts in order to complete writing assignments, was facing the same struggle with this activity. When I sat down with the student I asked specific questions to guide their thinking to support the facilitation of the writing assignment. Some of the questions I posed were:

"What did you do during the hurricane - did you stay home or evacuate?"

"Where did you go?"

"Why did your family choose to leave the area?"

"Did the people of Hiroshima have the opportunity to evacuate?"

"What happened during the bombing of Hiroshima?"

"How is this similar to, or different from, the hurricane?"

After asking each question, I gave ample wait time for the student to respond and make notes on their paper. Once I completed the questioning I told the student "you have everything you need to tell your story now" and allowed them to begin writing. To everyone's surprise, this student had completed an entire page of writing by the end of the session.

## Discussion

Although I do not have statistics or student artifacts to provide concrete data on the effects of this unit on students' narrative writing skills, it has provided a solid foundation from which I can move forward with a formal research plan in the future. I can also conclude this unit provided students with a fun, engaging way to apply their understanding of the events of Hiroshima. One teacher reported "The kids couldn't

<sup>1</sup> The information reported in this paper is based upon the November 2017 session; at the time of this writing, the January/February 2018 session had not begun.

<sup>2</sup> The word, "their," is chosen to maintain anonymity in regard to the students' gender.

stop talking about it on the way home. Yay!" (v. Woodrum, personal communication, November 16, 2017), which provides further evidence of the students' positive response to the intervention. Additionally, I have found robotics can move past the STEM disciplines to support learning in other academic areas.

This intervention has provided some basic, subjective findings to promote continued investigation into the benefits of this type of curriculum to promote narrative writing skills. I intend to continue with type of intervention and would like to make this unit and/or sessions available to more students at more schools, however, funding and logistics will need to be investigated in depth in order to make this feasible.

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## **Incorporating the Dragging Feature of Dynamic Geometry Environments in Teaching and Learning College Geometry**

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### **Abstract**

In a study about pre-service secondary mathematics teachers' (PSMTs) understanding about the nature of theorems in geometry, the researcher noticed that it was challenging for the PSMTs to visualize and draw counterexamples to disprove the given mathematical statements. Meanwhile, the use of the dragging feature of dynamic geometry environments (DGEs), such as the Geometer's Sketchpad and GeoGebra, in teaching and learning proof and reasoning has been widely discussed and become an ongoing research trend. In this paper, the researcher and her colleague will present a research design aimed at investigating PSMTs' conceptions of counterexamples in geometric reasoning when using the dragging feature of DGEs. Expected results of the study are potentially beneficial to pre-service/in-service secondary math teachers as well as teacher educators.

### **Keywords**

teacher education, dynamic geometry environments, proof and reasoning, geometry, dragging tools

### **Recommended Citation**

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Proof and theorems form part of the core content of secondary geometry curriculum and should be well grasped by secondary math teachers and their students (NCTM, 2000, 2003, 2012). Studies show that both secondary teachers and students have encountered challenges in teaching and learning proofs (Cirillo, 2009; Knuth, 2002; McCrone & Martin, 2004; National Center for Education Statistics (NCES), 1998; Senk, 1985). In order to develop knowledge about pre-service secondary mathematics teachers' (PSMTs) conceptions of theorems and provide mathematics educators and researchers with a possible means to unpack their conceptions, the researcher investigated the essential elements of four PSMTs' conceptions of the nature of theorems (NoT) through research-informed task-based interviews in 2016-2017. Findings of the study provided interpretations of PSMTs' conceptions of the NoT, in terms of the ways they claimed the truth of mathematical statements, examined validity of the given proof, disproved the given statement, as well as the role of the task-based interviews in understanding their conceptions (An, 2017).

During the above study, the researcher noticed that it was challenging for the PSMTs to draw and visualize counterexamples using paper and pencil to disprove the given geometrical statements. Both the PSMTs and the researcher felt the need to modify some of the geometry tasks that the researcher developed for the study by incorporating the dragging feature of dynamic geometry environments (DGEs), such as The Geometer's Sketchpad and GeoGebra. Especially, under the background of the rapid development of mobile devices and touch technology, the use of the dragging feature of DGEs in teaching and learning proof and reasoning has been discussed more and more widely and has become an ongoing research trend in mathematics education (Mariotti, 2014; Sinclair et al., 2016). Furthermore, introducing the dragging feature of some easily accessible DGEs to my secondary geometry content classes can provide PSMTs with a handy tool to explore the meaning and applications of counterexamples in writing proofs. In this paper, the researcher and her colleague propose a research design intending to answer the research question: *What are PSMTs' conceptions of counterexamples in geometrical reasoning when using the dragging feature of DGEs?*

## Literature Review

### Challenges in Learning and Teaching Proof and Theorems

The relationship between proof and theorems can be viewed as the relationship between a process and products in the world of mathematics (Farrell & Farmer, 1980). Being able to understand and apply theorems is considered as a relatively high level of proof and reasoning ability. For example, understanding of the axiomatic system of Euclidean geometry is ranked as higher level geometric thinking by the van Hiele levels (van Hiele, 1959). Despite the important role of proof and theorems in school mathematics, many secondary students have difficulty in writing valid geometry proofs (McCrone & Martin, 2004; NCES, 1998; Senk, 1985). Their difficulties may relate to incomplete conceptions or confusion about proof and theorems, such as accepting empirical evidence as formal proofs, questioning the generalizability of deductive reasoning, not accepting counterexamples as refutation, and overemphasizing the forms without logical coherence in proofs (Chazan, 1993; McCrone & Martin, 2004; Schoenfeld, 1994; Weber, 2001). Studies also show that teachers' conceptions, [https://digitalcommons.georgiasouthern.edu/stem\\_proceedings/vol2/iss1/1](https://digitalcommons.georgiasouthern.edu/stem_proceedings/vol2/iss1/1)

knowledge, and prior learning experiences of proof and theorems can have significant impact on their teaching of proof and theorems and thus affect their students' understanding and achievement in learning proof and theorems (Cirillo, 2009; Knuth, 2002; Lacourly & Varas, 2009; Oehrtman & Lawson, 2008; Rozner, Noblet, & Soto-Johnson, 2010). The extensive focus on what teachers do not know for teaching proof and theorems has driven the researchers' interest in examining what they do learn about proof and theorems in their undergraduate programs.

### Conceptual Framework for Task Design

As mentioned earlier in this paper, the proposed study is an extension of an earlier study in which the researcher focused on unpacking PSMTs' conceptions of geometric theorems. The researcher created a set of principles of theorems which served as the conceptual framework for the development of the data collection instrument: task-based interviews. The principles were developed by incorporating Cirillo's (2014) conceptual model of mathematical proof tools, Dreyfus and Hadas' (1987) six logical principles of geometry theorems and proofs with additional revisions by McCrone and Martin (2004), and Duval's (2007) indicators of misunderstandings in proof writing. The principles of theorems included three aspects: (a) nature of theorems (NoT), (b) logic of theorems (LoT), and (c) application of theorems (AoT). The current study only focuses on the two tasks built on two subcategories (NoT 1 and NoT 3) of the principles of the NoT (Table 1), as these tasks reflected PSMTs' need of utilizing draggable figures to explore possibilities of counterexamples.

**Table 1.** Principles of the NoT

Nature of Theorems (NoT)	<ol style="list-style-type: none"> <li>1. A mathematical statement is not a theorem until it has been proved (using axioms, definitions, postulates, previously proved theorems, lemmas, and propositions) (NoT 1)</li> <li>2. A theorem has no exceptions               <ol style="list-style-type: none"> <li>a. A general statement is true implies that the statement is true for all specific instances. Therefore, a proof must be general to be valid for proving a statement (i.e., non-exhaustive proofs, empirical evidence, and checking a few specific cases are invalid proofs) (NoT 2)</li> <li>b. A general statement is not true in one specific instance, proves that the statement is false (i.e., one counter example is sufficient to disprove a statement) (NoT 3)</li> </ol> </li> </ol>
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### Use of Dragging Feature of DGEs in Geometry Education

Dynamic geometry environments (DGEs) refer to geometry software that supports the “continuous real-time transformation often called ‘dragging’” (Goldenberg & Cuoco, 1998, p. 351). The dragging feature allows the user to change certain elements (e.g., a point) in a constructed geometric figure and observe the change of the corresponding geometric relationships in the figure. The constructed figures are referred as “draggable” or “moving” figures, which can provide the user with opportunities to experience “motion dependency” and further explore “logical consequence between properties within the geometrical context” (Mariotti, 2014, p. 159).

Research has shown that using the dragging feature of DGEs can promote student understanding and reasoning ability in learning geometry. By letting junior high students work on a series of DGE tasks designed for constructing draggable quadrilaterals, Jones (2000) found that students were able to make a transition from DGE-based arguments to formal mathematical arguments, which indicated their development of “formal-geometric conceptualizations” (p. 877). By studying the use of a particular dragging modality in dynamic geometry with pairs of high school students, maintaining dragging (MD) – “dragging a base- point of the dynamic figure on the screen trying to maintain some geometrical property of the figure” (p. 110), Baccaglini-Frank’s (2011) study implied the potential of MD to foster “a greater cognitive unity” (p. 117) – generating of conjectures that can lead to proofs, when internalized as a psychological tool by the learners.

As Battista (2007) pointed out as one of the future research issues, both qualitative (to understand the nature) and quantitative research (to determine the effectiveness) is much needed on the use of DGEs in teaching and learning geometry, and it serves the best to integrate the two types of studies. Therefore, the proposed study adopts a qualitative case study design to develop knowledge about PSMTs’ conceptions of counterexamples when using the dragging feature of DGEs. The result of study will be compared with the result of the previous study in which non-DGE tasks were used. If the comparison indicates the dragging feature of DGEs as an effective instructional tool, a quantitative study will be conducted to examine its effectiveness.

## Methodology

### Overall Design

Since the purpose of the study is to provide descriptive accounts targeted at understanding PSMTs’ conceptions of geometric counterexamples, the nature of the study is determined as a basic interpretive case study. Basic interpretive studies aim at “understanding a phenomenon, a process, or a particular point of view from the perspective of those involved” (Ary, Jacobs, & Sorensen, 2010, p. 453). Case studies are appropriate when the objective is an “in-depth data collection involving multiple sources of information rich in context” (Creswell, 1998, p. 61). In this study, a case is defined as each individual PSMT’s work on the two tasks designed for the task-based interview session. Task-based interviews are used as the data collection method, because they provide a structured and somewhat controlled mathematical environment for the researcher to focus directly on the subjects’ processes of addressing the tasks, rather than just on the correctness of the answers (Goldin, 2000).

### Site and Participants

Four to six participants will be recruited from the department of mathematical science at a research-oriented public university in the South. The recruitment criteria include: (a) candidates committed themselves to secondary mathematics teaching in the future, namely, they are PSMTs; and (b) candidates have taken a university-level geometry course to ensure sufficient prior knowledge of geometric theorems and proofs. A recruitment letter will be sent to the students through department email lists, introducing the goal of the study, recruitment criteria, and involvement in the study. Students’ participation is completely voluntary and irrelevant to their course grades.

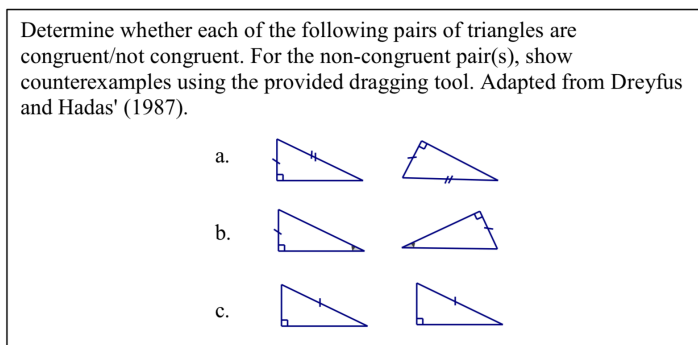
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Each PSMT will take part in an individual task-based interview session, lasting approximately 60 minutes. The interview session focuses on unpacking PSMTs' conceptions of the NoT, including the subcategories *theorem has to be proved* (NoT 1) and *one counterexample is sufficient to disprove* (NoT 3) (see Table 1). The researchers will demonstrate how to use the dragging feature of DGEs and let the participant practice on a few geometric constructions in the first 20 minutes of each session. During the interview sessions, PSMTs will be asked to think aloud while working on the tasks. The researcher will ask probing questions following a pre-designed interview protocol in order to understand their thinking process through the tasks. Because the goal of the study is not to assess participants' memorization skills, a list of Euclidean geometry definitions, postulates, and theorems will be provided to the PSMTs. Interviews will be video and audio recorded. The researchers will collect interview notes and PSMTs' worksheets as supplementary materials. A pilot interview will be conducted before the official data collection to test out the tasks and the interview protocol.

Goldin (2000) summarized the exploration process of task-based interviews as a four-stage process: (1) posing the question (free problem solving), (2) minimal heuristic suggestions (if no spontaneous responses), (3) the guided use of heuristic suggestions (if again no expected spontaneous responses), and (4) exploratory, metacognitive questions. This sequence of interview questions for each task is consistent with the "hard to easy" order suggested by Tzur (2007) in terms of minimizing the influence of prompts on students' conception development during the task exploration process. Since the main goal of this study is to unpack PSMTs' current conceptions rather than foster new conceptions, minimal heuristic suggestions will be provided when a PSMT is not able to provide any responses to the task questions.

## Tasks

The two tasks used in the interview session are designed based on the Principles of the Nature of Theorems in Geometry framework (see Table 1), and targeted PSMTs' conceptions of NoT 1, and NoT 3, respectively. Task 1 (NoT 1) shown in Figure 1 is designed to unpack the ways in which PSMTs can show that a mathematical statement is true or not true, which indicates their conceptions that theorems are true mathematical statements proved through deductive reasoning using axioms, definitions, postulates, etc.



**Figure 1.** Task 1 (NoT 1) – Theorem must be proved  
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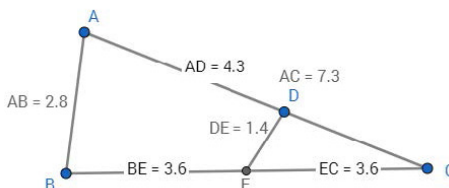
can disprove a statement, which indicates their conceptions about the meaning and use of counterexamples.

Disprove the following statement using the provided dragging tool.

In  $\triangle ABC$ , if D and E are points on AC and BC respectively,  
 $BC=2BE$  and  $DE=1/2 AB$ , then  $AC=2AD$ ,  $DE\parallel AB$ .

**Figure 2.** Task 2 (NoT 3) - One counterexample is sufficient to disprove

The dragging tool (GeoGebra) will be provided to the participant using an iPad during the interview. Figure 3 below shows a possible counterexample of Task 2 that can be created with the dragging feature of GeoGebra. As mentioned earlier, each interview will start with a 20-minute tutorial session about the use of the tool.



**Figure 3.** A counterexample of Task 2 created by GeoGebra

## Data Analysis

Since the data are collected through task-based interviews with a semi-structured interview protocol, the data analysis process will follow steps suggested by the typological analysis method that “data analysis starts by dividing the overall data set into categories or groups based on predetermined typologies” (Hatch, 2002, p. 152). The typologies of the study are PSMTs’ *goals* of the given task, their available *goal-directed activities* (GA), and what they believe are the *effects* of their goal-directed activities, which are the essential components of their conceptions (Simon et al., 2004). PSMTs’ goal of a task refers to their desired status of problem solving, namely, what they want to or intend to do. The goal structures what they can notice, compare, and abstract, but it is not necessarily conscious (Simon et al., 2004; Tzur & Simon, 2004). PSMTs’ GA of a task refers to their mental activity that is related to their goal of the task. GAs include the observable actions and the corresponding mental process that generated these actions (Simon et al., 2004; Tzur & Simon, 2004). An effect refers to what the PSMT identifies as the outcome of his/her GA (Simon et al., 2004; Tzur, 2007; Tzur & Simon, 2004), which, in this study, includes his/her conception of whether the goal of the GA is met (or about to be met) and the decision about the next step of actions. PSMTs’ goals, GAs, and effects of GA are structured and governed by their current conceptions. In other words, their conceptions are embodied by their goals, GAs, and effects of GAs. Taking a close look at these elements can help the researchers gain a deeper understanding of PSMTs’ current conceptions of theorems.

## Timeline and Potential Impact

In terms of the timeline of the research, a pilot study will be conducted in Spring 2018 to collect feedback on the tasks and interview protocol. The institutional review board (IRB) application will be submitted by the end of Spring 2018. The participant

et al. Volume 2, Proceedings of the Interdisciplinary STEM Teaching and recruitment and official data collection will be completed in Fall 2018. Data analysis and dissemination will start in late Fall 2018 and will continue until publications are generated.

This research project could potentially benefit all PSMTs at the university in which the researchers are teaching. Once the research results indicate the effectiveness of incorporating the dragging feature of DGEs in geometry learning, the researchers plan to introduce this tool to students in her college geometry classes. If the students can gain the knowledge of how to use the dragging feature to better understand the concept of counterexample, they will have a tool for more effective learning of geometric proof and reasoning. In turn, their new knowledge will be able to be applied in their future teaching roles and increase the quality of secondary geometry education in Georgia.

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## **Using Active Learning Strategies in Calculus to Improve Student Learning and Influence Mathematics Department Cultural Change**

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### **Abstract**

An interdisciplinary team of physics, education, math and chemistry faculty developed MATH-GAINS (Growing as Adaptive INstructors) creating an ecosystem where mathematics faculty persistently and sustainably apply active learning strategies in their teaching of calculus courses. As a result of implementation, MATH-GAINS proposed to positively affect the wide-spread adaptation of active learning strategies by department faculty as well as student learning, retention and graduation of over 900 students annually. The objective of this paper is to provide details on how the project was conceived and implemented; instruments, research methodologies and active learning strategies used; and examples of faculty projects and preliminary results of the study. Results of the study add to the growing body of knowledge of how research-based instructional strategies designed in other STEM disciplines work in math courses, as well as an understanding of the critical factors that influence math faculty's teaching practices.

### **Keywords**

calculus, active learning, teacher change

### **Recommended Citation**

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Nationally, less than 40% of students who begin college in pursuit of a science, technology, engineering or mathematics (STEM) degree complete in one of these disciplines (PCAST, 2012). Furthermore, almost a quarter of the students who leave STEM fields cite low grades in the early years of study as a factor for their decision to leave. These claims are supported by earlier work from Seymour and Hewitt (1997) who identified seven reasons students leave these disciplines, including feeling overwhelmed by the rigor of courses and dissatisfaction with instruction or the climate found within the discipline. Their findings of poor science, math and engineering teaching and lack of student preparation for the mathematics and sciences support the need for identifying not just more educational innovations, but specifically those having a *significant impact* on student learning (ASEE, 2012). Charged with the call from PCAST (2012) to study the attributes of successful math courses for dissemination of best practices, researchers developed the MATH-GAINS project to enhance math learning environments. Using active learning strategies proven successful in other STEM disciplines, researchers aimed to arm faculty with the necessary tools to enrich instruction and improve student learning within the calculus sequence.

The goal of this research – MATH-GAINS (Growing as Adaptive Instructors) – was to create an ecosystem where an influential number of the Department of Mathematics faculty at a large metropolitan university persistently and sustainably applied evidence-based practices in their teaching of calculus courses, the courses considered as a *gateway to success in STEM disciplines* (PCAST, 2012). As a result of improved instruction, MATH-GAINS proposed to positively affect student learning, retention and graduation within STEM. By the end of the MATH-GAINS effort six faculty and ten math graduate students (many of whom will teach post-secondary math upon graduation) were provided training and support to apply evidence-based practices in their math courses. These faculty members and graduate teaching assistants (GTAs) *annually affected over 900 students*. Data collected during this project continues to add to the growing body of knowledge of how research-based instructional strategies designed in other STEM disciplines work in math courses, as well as the community's understanding of the critical factors that influence math faculty's teaching practices.

The MATH-GAINS project was grounded in the recommendation from the 2012 PCAST Undergraduate STEM Education report, to identify and broaden implementation of research-based instructional strategies and address the problem of excessively high failure rates in introductory mathematics courses at the undergraduate level, in order to open pathways to more advanced STEM courses. Project activities were designed in such a way to ensure the Mathematics Department could sustain a culture of using evidence-based teaching practices in math courses with a plan to use state and national existing partnerships to disseminate best practices. The objective of this paper is to provide details on how the project was conceived and implemented; instruments, research methodologies and active learning strategies used; and examples of faculty projects and preliminary results of the study.

### Project Overview

The researchers desire to provide a thorough understanding of the MATH-GAINS project with the goal of allowing replication across other institutions. To assist, this

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paper outlines the detailed objectives of the project, why the interdisciplinary team was formed, how each phase of the project was implemented broken down by faculty and student components and the instrumentation and methodology used in the research.

## Objectives

Several objectives were identified to guide the research with the role of MATH-GAINS faculty increasing in responsibility through the sequential path of project activities.

*Objective 1 – Develop and Retrain.* Two Learning Communities (one in Year One and one in Year Two of the project), consisting of three faculty and five GTAs, participated in a year-long project with on-going training.

*Objective 2 – Implement and Reinforce.* Each year, the learning community participants implemented self-selected evidence-based practices during both fall and spring semesters.

*Objective 3 – Disseminate.* Faculty participants from each year's learning community disseminated their projects to (a) other on-campus faculty (local),  
(b) other state institutions (regional) and (c) faculty from institutions in other states (national) through existing consortia and partnerships.

These objectives allowed researchers to meet the goal of creating an ecosystem of mathematics faculty persistently and sustainably applying evidence-based practices in their teaching of calculus courses. Objective 1 which provided for faculty development with consistent reinforcement of the strategies used in the classroom was considered the most critical for the success of the MATH-GAINS effort. For this reason, the supporting activities will be deliberately detailed in the faculty section of this paper.

## Implementation

A very thoughtful process went in to selecting the right mix of faculty partners to develop and implement the MATH-GAINS project. An interdisciplinary team from physics, education, math, the Faculty Center for Teaching and Learning (FCTL) and the Center for Initiatives in STEM (iSTEM) were hand-picked with the necessary expertise for the project's success. Ensuring the project had the proper support within mathematics at all levels, the chair and an associate professor from the department led the project. Faculty from education, physics and FCTL were chosen to provide appropriate training and professional development to the faculty participants, assess the level of implementation of evidence-based practices, prompt faculty reflection and suggest avenues for improvement. The physics, education and FCTL faculty members had experience personally implementing evidence-based teaching practices and designing evidence-based curriculum for use by other faculty and GTAs. Additionally, they had expertise in assessing student learning, using protocols to observe instruction, and interviewing faculty about their teaching practices. The iSTEM Executive Director tracked the progress of the students in the target cohort [https://digitalcommons.georgiasouthern.edu/stem\\_proceedings/vol2/iss1/1](https://digitalcommons.georgiasouthern.edu/stem_proceedings/vol2/iss1/1)

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 and made the necessary arrangements to ensure registration for MATH-GAINS' calculus courses, student group coding and data collection.

Prior to commencement of the project six mathematics faculty were selected to participate over the course of the project – three in year one and three in year two. Each faculty member was assigned to teach a section in the calculus sequence (Calculus with Analytical Geometry 1, Calculus with Analytical Geometry 2 and Calculus with Analytical Geometry 3) for both the fall and spring terms of the same academic year. Faculty formed a learning community and attended personalized professional development training. After being immersed in the research literature, they were provided with a menu of evidence- based teaching practices to implement in the classroom, from which they selected one or more practices to implement in the subsequent two semesters. Non-project faculty experienced in implementing evidence-based teaching practices and designing evidence-based curriculum served as mentors for the Year One faculty. Year One faculty then served as mentors to Year Two faculty participants. Five graduate teaching assistants (GTAs) were selected to assist the faculty each year, for a total of 10 GTAs. Both faculty and GTAs participated in professional development activities in support of the MATH- GAINS experience.

The mathematics courses included in MATH-GAINS (MG) contained no specific designation that would assist students in identifying which sections were included in the project. This allowed registration for the courses to be random. All students meeting the appropriate prerequisites for the calculus sequence had an equal opportunity to register for a MG course. After the first term of participation in a MG course, students were invited to continue into the next MG course in the Calculus sequence if they desired. Once current MG participants were registered, the remaining seats in the section were opened to the general population. Table 1 includes the number of students registered in MATH-GAINS for each term.

**Table 1.** MATH-GAINS student enrollments by course, term and year of the project

Course	Year One		Year Two		Total (course)
	Fall	Spring	Fall	Spring	
Calculus 1	392	271	439	414	1516
Calculus 2	49	49	48	49	195
Calculus 3	49	49	49	50	197
Total (term)	490	369	536	513	1908

Student demographic and performance data were collected using university system databases and faculty course records. All student perception and concept knowledge data were collected in the various calculus courses each term. The Characteristics of Successful Programs in College Calculus (CSPCC) gauged student attitudes and efficacy about learning mathematics. The instrument was administered using Qualtrics survey software in a pre and post-test format during the first and last weeks of each term. The Calculus Concepts Inventory (CCI) measured the understanding of Calculus concepts. The CCI was administered as a paper survey utilizing Scantron forms for easy data collection and was also offered in a pre and post-test format at the beginning and end of the term. The student consent process occurred in class during the first week of the semester. Students were informed that participating in the research was voluntary and had no bearing on their course grades. Students provided consent on the Undergraduate Explanation of Research online via Qualtrics.

methods. To measure demonstrated positive change in attitudes and beliefs about the efficacy of evidence-based teaching practices in the identified courses, two survey instruments were used in a pre and post-test format: Culture, Cognition, and Evaluation of STEM Higher Education Reform (CCHER) (Hora, 2011) and a calculus teaching efficacy questionnaire modified from Gill, Ashton and Algina (2014). Data were collected electronically using Qualtrics survey software. To provide further validity of belief change beyond simple self-report, two additional measures were used: (a) ratings of teacher scenarios (Bullough, 2015; Gill, Ashton & Algina, 2014) and (b) examinations of faculty rationales for their instructional decisions during interviews and training sessions (Gill & Hoffman, 2009). Classroom observations using the Reformatted Teaching Observation Protocol (RTOP) (Piburn, Sawada, Falconer, Turley, Benford, & Bloom, 2000) were used to gauge the extent to which faculty implemented the evidence-based practices. RTOP was chosen for MATH-GAINS due to the focus on the extent of the implementation of evidence-based teaching practices. Pre, mid and post-implementation observations were conducted by an external observer prepared in utilizing the protocol. In addition to being the instrument used by the external observers during MATH-GAINS' assessment, the RTOP was used as a formative assessment tool through peer observations conducted by other participants in the learning community and by the assigned mentors.

In addition to professional development workshops, every semester the GTAs used a mixed reality teaching simulator to engage in virtual practice. Aimed at helping GTAs to acquire and refine their skills through the use of TeachLivE technology (Andreasen and Haciomeroglu, 2009), the virtual practice sessions integrated immediate feedback and reflection in between short virtual teaching sessions. With the TeachLivE technology, GTAs focused on implementing strategies to facilitate group discussion including – providing specific feedback, asking higher-level questioning and practicing wait time. Each simulator experience consisted of two 7-minute interactive sessions in a classroom with five virtual avatars with a ten minute break for feedback and reflection in between. GTAs were also expected to watch at least one of their peer's sessions to get more familiar with different implementations of the strategies. After the sessions, GTAs were prompted to explain how they were going to use the techniques they practiced in their own classes.

## Research on Faculty

### Selection

MATH-GAINS faculty participants were selected according to three main criteria. First, it was important to have faculty representing various ranks. Over the course of the study, there was one Professor, one Associate Professor, one Assistant Professor, one Lecturer, one Associate Instructor, and one Instructor. Second, the faculty participants needed to have interest and potential to truly implement evidence-based practices, which they had not used in their courses previously. Third, it was important to select faculty that had potential to influence other faculty and/or department policy. To this end, the faculty participants possessed at least one of the following characteristics:

- Taught calculus courses regularly
- Served as course coordinator for Calculus II or III

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- Chaired or served on committees that effect course changes
- Displayed prior participation in education related grants or research
- Held the rank of tenure, which may indicate an influential voice with other mathematics researchers in the department.

In year one, there was one female and two male faculty with ranks of instructor, associate instructor and professor. All three participants in year two were female with ranks of lecturer, assistant professor and associate professor.

## Training

Research (Henderson, Beach & Finkelstein, 2011) shows that short workshops do not facilitate institutional change. Instead, prolonged, consistent, intervention with reflection incorporated into the process leads to systemic change. MATH-GAINS was a one-year cognitive apprenticeship embedded within a vertical learning community of faculty and GTAs where faculty had the autonomy to select for themselves and implement on their terms evidence-based practice(s) in the Calculus classroom. Motivation theory suggests that providing autonomy for teachers leads to better outcomes for students (Roth, Assor, Kanat-Maymon & Kaplan, 2007).

Teams consisting of three faculty (one for each course – Calculus 1, 2 and 3) and five GTAs comprised the Learning Community (LC) for each year. Faculty LCs were designed to be a forum for exchange of information regarding evidence-based teaching strategies and the environment that nurtures support for the implementation of these practices. MATH-GAINS participants focused on developing mathematical understanding utilizing strategies centered on active engagement, effective use of technology and classroom assessment techniques. Faculty selected from a menu of evidence-based practices and developed learning materials that incorporated these practices in math courses over a two-semester period.

The totality of the professional development experience is summarized here, chronologically, and captured more succinctly in Table 2.

1. Faculty LC participated in a summer workshop, led by a faculty member from the Faculty Center for Teaching and Learning (FCTL) whose background was in chemistry. Programming included an introduction to STEM education research and the theories guiding effective practice. Through this context, the menu of MATH-GAINS' evidence based practices was introduced
2. Throughout the summer, the faculty LC participants brainstormed, discussed, reflected and developed curriculum and materials for their upcoming courses
3. Projects were implemented in fall semester and, through direct observations by trained mentors, the LC participants received formative feedback. LC faculty also visited each other's classes for support
4. GTAs made use of the teaching simulator once each semester
5. LC participants met monthly to debrief on their project, seek group support, and share ideas for success
6. LC participants attended a one-day winter workshop to discuss common experiences and "tweak" the evidence- based practices for the spring semester
7. Adjusted projects were implemented in the spring semester and underwent



- 8. LC participants disseminated lessons-learned, best practices and materials developed to a faculty audience at the university's summer faculty development conference. Findings were also shared with populations in the math department including the Year Two MATH-GAINS LC cohort
- 9. The cycle continued in the second year

Faculty and GTA teaching efficacy and beliefs were measured at the beginning and end of the year.

**Measures**

A variety of evidence was used to measure the effect that the professional development had on the instructors. Each faculty member was observed by an external observer who used the Reformed Teaching Observation Protocol (RTOP) to document the extent to which their lessons were reformed (according to the national science and mathematics standards for K-20 classrooms). Observations pre, during, and at the end of the faculty's participation in the program were analyzed. Faculty generated implementation plans, reflections and exit interviews were used to gain a better understanding of what the faculty were trying to do in their classroom and where they felt they had barriers. Researchers also surveyed all math faculty using the Culture, Cognition, and Evaluation of STEM Higher Education Reform questionnaire (CCHER; Hora, 2011) to ascertain faculty members' degree of acceptance of active learning classrooms. Two additional measures help provide insight into MATH-GAINS' faculty's beliefs about what constitutes good instruction in calculus (Calculus Teaching Scenarios; modified from Gill, Ashton & Algina, 2014), and their confidence in teaching calculus effectively (Calculus Teaching Efficacy Scales, modified from Gill, Ashton & Algina, 2014).

**Table 2.** MATH-GAINS training timeline by activity and participant type

Training Component	Training Category	Term			Participant	
		Su	Fa	Sp	Fac	GTA
Training Workshops	Professional development	X			X	X
TeachLive simulator	Support & feedback		X	X		X
Winter meeting	Professional development		X		X	X
Observation & mentor meeting	Support & feedback		X	X	X	
Monthly meetings	Professional development		X	X	X	X
Implementation (initial)	Intervention		X		X	X
Implementation (revised)	Intervention			X	X	X
Faculty conference	Sharing experience	X			X	

**Outcomes**

Based on Roth, Assor, Kanat-Maymon & Kaplan (2007), autonomy in letting the faculty member decide which strategies to use and how they were going to use them was



a large part of the project's theoretical framework. Because of this, the implementation for each faculty member varied. One faculty member, for example, decided to include an active learning activity, suited to the day's objectives, into every lesson taught. Another faculty member mostly focused on modifying the discussions sections of the course led by GTAs. In this case, the faculty member designed student-centered lesson plans and assisted in mentoring the GTAs to lead an active discussion section once a week. Still another faculty member decided to flip the course and use the majority of face-to-face time for active student-centered learning.

When looking at efficacy and attitudes, analyses showed that faculty held more positive views of reform instruction (using evidence-based practices), and more negative views of traditional instruction following the intervention in Year One. RTOP analyses revealed that changes in instructor practice varied across instructors. One instructor showed a strong change in practice, which continued across the second year of the study. Multiple faculty showed moderate change in practice continued across the second year. It should be noted that there were a couple of faculty whose practice did not show noticeable change despite a change in efficacy and attitudes. Further investigation into factors that indicate readiness of faculty to change is warranted.

The TeachLivE simulator data of the GTAs is still being analyzed. Interview data with the GTAs indicates they thought that the avatars responded similarly to the way their students responded in class. They felt like the simulator helped them learn how to work with small groups of students, particularly when trying to lead students through a series of questions as opposed to direct instruction. The GTAs also felt that a limitation of the technology was that they did not feel like the practices they focused on would scale up to their larger classes. Most of the GTAs expressed difficulty in translating the skills they practiced in the simulator to their actual classes.

Research on results of the faculty interviews and beliefs measures is ongoing. One presentation has resulted from early analyses to date (Gill, James, Saitta, Moore, Dagley, Philips & Chini, 2017, August).

## Research on Students

### Selection

Student participation within the MATH-GAINS (MG) project included enrollment in one of the designated calculus sequence sections in the fall or spring over the two year project period, Fall 2015 to Spring 2017. As previously indicated, the calculus courses contained no specific designation that would assist students in identifying which sections were included in the project. This allowed registration for the courses to be random. All students meeting the appropriate prerequisites for the calculus sequence had an equal opportunity to register for a project course. Calculus 1 was random enrollment each term. Once a student participated in a MG course, he/she was invited to continue into the next MG course in the calculus sequence if they desired. This was allowed until the course reached capacity or the end of the primary registration period. Once current MG participants were registered, the remaining seats in the section were opened to the general population. For example, in MATH- GAINS' first semester, Fall 2015, all participants in the project randomly enrolled into the MG sections on their own. Before registration opened for Spring 2016, a member of the research team visited each MG section and invited the current students to register for the MG section

Those who expressed an interest were enrolled in the MG section of the appropriate course for spring. The same thing occurred each fall and spring term until the end of the project. This meant that some students enrolled in all three calculus courses with MATH- GAINS, some only enrolled in one and others chose to enroll in two courses.

Opening the remaining seats to the general population provided a comparison group built within each class. Future analyses will use this group to compare learning differences between those students who took multiple classes and those who experienced only one of the MG courses and to investigate any cumulative effect of experiencing multiple sections in an active learning environment. One potential factor that must be considered is whether academic differences in students in the comparison group affected outcome results. Students who register later for classes may be unsure of their performance in the class or future in a major, and thus differ significantly from early registrants. For this reason, a comparison holding constant for past academic performance or standardized test scores may be necessary to ensure there is no bias.

Coding

In order to be able to assess the MATH-GAINS effort, researchers had to appropriately define the cohorts for each course, term and year of the project. Once determined, each student enrolled in MATH-GAINS courses was coded in the university database using one of these definitions. Using only four characters as allowed by the parameters of the database, researchers determined ME## would be the best format. The first character “M” was chosen to designate the project “M”ATH- GAINS. The second character “E” represented the experimental group. This was important for future studies when specific control groups would be established and could use the designation of “C”. The third character and first number corresponded to the number of the course in the calculus sequence: 1 = Calculus 1, 2 = Calculus 2 and 3 = Calculus 3. The final character and second number corresponded to the term in chronological sequence of the project. For example, Fall 2015 was designated as 1 for term one of the project, Spring 2016 designated as 2 for term two of the project, Fall 2016 as 3 and so on. Table 3 includes each of the term definitions.

The comparison groups used for analysis in the ongoing studies related to this project were composed of all other non- Honors and non-Learning Community sections of Calculus 1, 2 and 3 offered during fall and spring terms during the same period, Fall 2015 – Spring 2017.

Table 3. MATH-GAINS coding definitions

Code	Coding Definition
ME11	Experimental group Calculus 1 in term 1
ME21	Experimental group Calculus 2 in term 1
ME31	Experimental group Calculus 3 in term 1
ME12	Experimental group Calculus 1 in term 2
ME22	Experimental group Calculus 2 in term 2
ME23	Experimental group Calculus 3 in term 2
ME13	Experimental group Calculus 1 in term 3
ME23	Experimental group Calculus 2 in term 3
ME33	Experimental group Calculus 3 in term 3
ME14	Experimental group Calculus 1 in term 4
ME24	Experimental group Calculus 2 in term 4
ME24	Experimental group Calculus 3 in term 4

Of the 1,908 students who enrolled in a MATH-GAINS course, 17 were eliminated because of admission in a graduate non-degree seeking category. Another 237 were removed from any analysis due to previous participation in a STEM learning community. Of the 1,654 eligible students the vast majority (n=1,329) were enrolled in Calculus 1 with the remaining 163 and 162 students enrolled in Calculus 2 and Calculus 3, respectively. The comparison group consisted of 4,528 students of which 1,456 were enrolled in Calculus 1 with the remaining 1,573 and 1,499 enrolled in Calculus 2 and Calculus 3, respectively.

Other demographic characteristics considered in the study included admission status to the institution, socioeconomic standing during the semester completing the course, gender, ethnicity and classification as a first-generation college attendee. The characteristic details for the MATH-GAINS participants and comparison groups are shown in Table 4.

Outcomes

Course Performance

Overall, there was no statistically significant difference in student performance based on DFW rates – those not successfully completing the course by failure or withdrawal – when comparing those students participating in MATH-GAINS courses and those in the general population courses (comparison group). In general, students in MATH- GAINS sections of Calculus 1 offered in fall had lower DFW rates than the comparison group, but the comparison group outperformed MATH-GAINS students in spring sections. For Calculus 2, the comparison group outperformed MATH-GAINS in every term. Just the opposite, MATH-GAINS students outperformed the comparison group in Calculus 3 in all terms except one where performance was almost identical. Deeper analysis is necessary to determine the reasons behind these differences including individual review by section each term to hold constant for instructor.

Table 4. Demographic characteristics of MATH-GAINS and comparison group students

Variable	MATH-GAINS	Comparison
First Generation	368	954
Low Income	391	1,068
First-Time in College Admit	1,169	2,995
Transfer Admit	433	1,419
Second or Non-Degree Admit	52	114
Female	523	1,072
Male	1,131	3,456
African American	150	338
Hispanic	442	1,136
White	781	2,331
Other or non-specified	281	723

First-time-in-college and transfer student admits in MATH- GAINS had about the same DFW rates, 41% and 42% respectively. However, when analyzed alongside the comparison group, first-time-in-college students performed much better (DFW for MATH-GAINS 41% vs. Comparison 37%) in the general population courses

Comparison 56%) in the MATH-GAINS courses. This could be attributed to the academic maturity of transfer students, having experience in college level courses and leading to the ability to adapt to different types of instructional methodologies whereas, first-time-in-college students may be accustomed to a more traditional style of lecture and are still acclimating to college level rigor. There was no significant difference in performance based on gender though both males and females had slightly lower DFW rates in the general population courses. When looking at ethnicity, African- American students had lower DFW rates in MATH-GAINS while Caucasian students had lower DFW rates in the general population sections, but there were only slight differences for each group. The most significant differences were for Hispanic students who had lower DFW rates in the general population courses compared to MATH-GAINS, 42% and 51% respectively.

A few factors impacted this portion of the student analysis. Limitations include, but may not be limited to:

1. The use of grades in courses which are known to be a less effective and more subjective variable for comparison.
2. Students repeating a course were included in the total counts therefore, student counts were not uniquely identified. Additionally, a poor performing student in one class could be expected to be poor performing in subsequent attempts of the course.
3. Most MATH-GAINS sections contain late enrolled students. Late enrollment occurs when a student postpones registration which is often due to indecision on continuing with a major or expected or actual poor performance in a class. This could mean that a larger percentage of students with a poor performance record enrolled in MATH-GAINS sections.

When looking at only MATH-GAINS participants, 91 students took at least two courses in the calculus sequence with the program. Of this group, 56 passed (61%) and 35 failed (39%) the second course. Only 7 students took all three courses in the sequence with MATH-GAINS. For those with low performance in a MATH-GAINS course, 96 repeated a course in the sequence with MATH-GAINS. Of this group, 37 failed (39%) the second attempt and 59 passed (61%). The number of students taking additional courses in the sequence or repeating courses with the project was limited for two reasons: (1) because MATH-GAINS offered large lecture Calculus 1, but only one section each of small lecture Calculus 2 and 3 only approximately 100 students could move forward each semester and (2) by the time students found out they needed to repeat a course the majority of the MATH-GAINS seats were filled, eliminating the opportunity for many to repeat with the program.

## Persistence

One student outcome associated with the project related to persistence of students in a STEM major. The outcome was divided into two measures, persistence in and graduation from the STEM major. Initial analysis combined the two measures for a single retention rate. Preliminary results were positive.

Because students enrolled in MATH-GAINS courses were not from a single

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entering cohort (i.e., students enrolled were admitted in many different terms and years), a traditional fall- to-fall retention calculation would not accurately reflect retention in STEM as defined by the project. Instead, MATH-GAINS STEM retention was determined by taking all students enrolled in a MATH-GAINS section, reviewing their major upon admission to the institution (STEM vs. non-STEM) and conducting two-year or one-year term-to-term (i.e., fall MATH-GAINS enrollment to fall one and two years out, spring MATH-GAINS enrollment to spring one and two years out) persistence or graduation in STEM. The comparison group for this outcome was the All STEM population inclusive of both first-time-in-college and transfer students. All calculus courses offered in the MATH-GAINS' sequence boasted higher two-year and one-year retention rates in STEM than the All STEM population. Table 5 outlines the one and two-year retention (persistence and graduation) rates for each MATH-GAINS calculus course and the All STEM population.

When examining MATH-GAINS' participants, there were a number of trends that one would expect to see.

1. There was a higher percentage of STEM majors as a total of enrollment in Calculus 2 and 3 than in Calculus 1. This can be credited to actual volume of students in Calculus 1 or that it is the first course in sequence and many students enroll with limited intentions of moving forward in STEM (i.e., students who change a major during the first term of enrollment).
2. One-year and two-year retention rates in Calculus 2 and 3 were significantly higher than Calculus 1 in most terms. By the time students reach these courses, they are further along in their STEM major with more time invested. The majority of students choosing to leave the STEM major early on typically do so after the initial gateway course.
3. Persistence in the STEM major was higher one year out than two years out as shown in Table 4. Retention research (Braxton, Brier & Steele, 2008) shows that though the majority of students who leave do so in the first year, a similar percentage exit during year two. A large percentage attempt to persevere beyond the first gateway course, but make the decision to leave when not performing well during the second or third course in the sequence before investing too much time.

**Table 5.** One and Two-Year Retention Rates for MATH-GAINS and the All STEM Population

Year	MATH-GAINS			All STEM
	Calculus 1 (%)	Calculus 2 (%)	Calculus 3 (%)	(%)
Two-Year	63	76	76	57
One-Year	74	81	86	69

Continued review of the data is warranted. Future analysis will include break downs of retention by gender, ethnicity and even individual STEM majors.

### Sustainability

The purpose of this study was to examine faculty change and its impact on students, as the teacher change literature often does not directly connect changes in

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Proceedings of the Interdisciplinary STEM Teaching and Learning Conference, Vol. 2 [2018], Art. 1 teachers' knowledge, professional development or behaviors with student practice (Buehl & Beck, 2015). Thus, we began with the analyses on student data. Though student data shows initial positive results, it was the research with and development of the faculty that were the primary focus of this project. Advancing student success would not be possible without sustaining the faculty development component. The goal set forth by MATH-GAINS was to create an ecosystem where an influential number of the Department of Mathematics faculty at a large metropolitan university persistently and sustainably applied evidence-based practices in their teaching of Calculus courses. As evaluation of the year two faculty data continues, the researchers believe the project has been very successful in moving faculty towards the use of evidenced-based practices.

Over the course of two years, which was the MATH-GAINS project duration, the Department of Mathematics made many significant changes, each one influenced to some degree by MATH-GAINS. Three of these changes were initiated and accomplished by the principle investigator of MATH-GAINS. First, a regular (semi-weekly) math education seminar series was organized. The seminars showcase teaching practices and results from faculty both inside and outside the department, promoting regular exchange of ideas, and typically boast higher attendance than other regular mathematics research seminars in the department. Second, one mathematics colloquium is devoted to mathematics education each year. These colloquia are generally given by experts from outside the university, and are attended by most of the department. Third, the department hired a tenured professor who has secondary research interests in math education and a tenure-track faculty member, whose primary research interest is math education. As there are no other faculty in the department with the same primary research focus, this denotes a significant change, which is necessarily reshaping the role of mathematics education research within the department.

In addition, further changes in the department have resulted from the actions of faculty that participated in the MATH-GAINS program. To be specific, four MATH-GAINS faculty participants serve on the department's Calculus Committee; one of the four is serving as the committee chair. The committee continues to gather and analyze data in order to better understand failure rates, and they are actively pursuing bold changes to course design, materials and curriculum. Finally, a new Mathematics Education Committee has been created to assess, promote and implement further developments, now that the MATH-GAINS program has officially ended. Though much analysis remains, researchers are encouraged by the progress of cultural change initiated within the Department of Mathematics at all levels of the faculty ranks.

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## **Deepening Place Value Understanding in K-2 Through Explanation and Justification**

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### **Abstract**

This paper seeks to describe the importance of using the context of a candy shop and how focusing on discourse can deepen place value understanding in the base ten number system. Using the language of pieces (ones), rolls (tens), and boxes (hundreds) helps situate place value in a familiar context for K-2 students. Best practice in mathematics instruction is also addressed, including examining the progression of learning for place value concepts, using effective tools to support place value learning, and using explanation and justification to help students deepen their understanding of place value. The authors focus not only on content and pedagogy, but also show how the content connects to the Mathematical Practices and ELA Speaking and Listening standards. The purpose of this paper is to give current teachers something to think about as they plan lessons on place value in their K-2 classrooms. The authors also want to give teachers a reason why discourse can help both them and their students as they progress through content.

### **Keywords**

discourse, place value, explanation, justification, primary grades

### **Recommended Citation**

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Place value is an important concept that crosses multiple grade levels in elementary mathematics, which is why it is critical to build a strong foundation for making sense of place value in primary grades. Discourse plays a key role in helping young children develop a deep understanding of place value concepts. Discourse should be perceived as the process students go through to make sense of mathematics, not as the tool students use to talk about mathematics (Hiebert & Wearne, 1993; Imm & Stylianou, 2011). This is a shift in the role of the student and the role of the teacher, because in the past teachers controlled much of the class conversation (Falle, 2004). Making a shift towards a more student-centered classroom focused on discourse may not be easy in classrooms that are more teacher-centered, where teachers are the only ones expected to give explanations and justifications. Students tend to look at the teacher and expect teachers to give these explanations instead of relying on themselves or their peers (Yackel & Cobb, 1996). However, encouraging students to dig deeper into their solutions and clarify their explanations through oral and written communication can lead students to be more autonomous in the classroom (Kamii, 1985). Thus, place value is the mathematical concept through which discourse will facilitate sense-making among young children.

### **Explanation and Justification**

Communication plays a vital role in mathematics learning, both in terms of developing conceptual understanding among students and helping teachers to develop a deeper understanding of student thinking (Kosko, Rougee, & Herbst, 2014). The National Council for Teachers of Mathematics (NCTM) has long advocated for students to engage in discourse when learning mathematics (NCTM, 1989, 1991, 2000). These expectations for communication position children across the K-12 grade span as active participants in the mathematics classroom where they engage in explanation, justification, questioning, and sense making. In fact, “interacting with classmates helps children construct knowledge, learn other ways to think about ideas, and clarify their own thinking” (NCTM, 1989, p. 26). Studies indicate students should actively construct new information through classroom activities and discussions (Fosnot & Perry, 1996, Nathan, Eilam, & Kim, 2007), and students should ask questions from their peers to dictate the direction of these discussions (Bennett, 2013; Hiebert & Wearne, 1993; Imm & Stylianou, 2011). When students explain and justify different strategies to solve problems and share those strategies with their peers, students have a deeper understanding of the problem and are able to make connections between different strategies, both of which leads to a richer discussion (Nathan et al., 2007) and a deeper understanding of the content.

Currently, a continued focus on discourse as an integral part of mathematics teaching is reflected in both the Common Core State Standards for Mathematics (CCSSM) and NCTM’s *Principles to Actions* (2014). The CCSSM includes the Standards for Mathematical Practice, which focus on the process of doing mathematics and mathematical habits of mind that must be developed in students (NGA Center & CCSSO, 2010b). The need to communicate and engage in discourse is essential to student learning as identified in the Mathematical Practices, including when students “construct viable arguments and critique the reasoning of others” and when students

“attending to precision” as part of sharing their explanations and justifications (NGA Center & CCSSO, 2010b, pp. 6–7). Additionally, NCTM identified eight Mathematics Teaching Practices in *Principles to Actions* that reflect research-based best practices that will ensure deep mathematics learning (2014). One of these practices is focused on discourse: “Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments” (NCTM, 2014, p. 29). To facilitate student learning, teachers must purposefully plan opportunities for students to engage in discourse and the Standards for Mathematical Practice as they explain and justify their understanding of mathematics concepts.

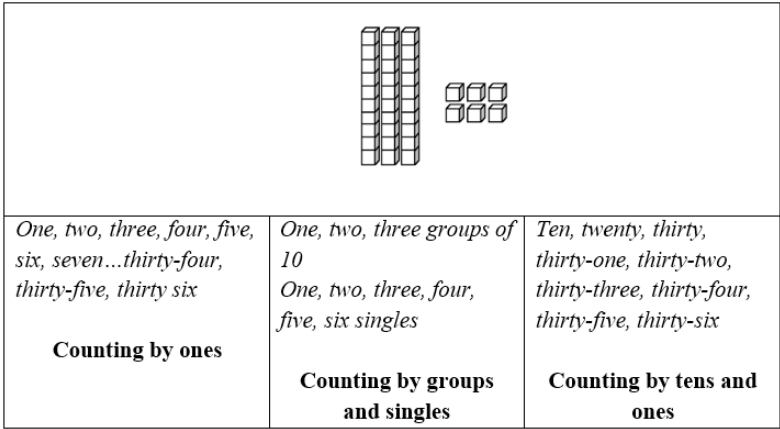
Discourse can encompass both oral and written communication. In primary grades, there is a strong emphasis to build literacy skills across the subject areas. Because young children are at various stages in developing their reading and writing skills, the mathematics classroom is a natural place to emphasize the skills of speaking and listening. The Common Core State Standards for English Language Arts (ELA) have specific standards for Speaking and Listening, which include engaging in discussions with both peers and adults in small and whole group settings (NGA Center & CCSSO, 2010a). Many of these standards in the K-2 grade span are applicable to the mathematics classroom, including engaging in conversations, building on the responses of others, asking questions for clarification, adding drawings or visual representations to provide detail, and expressing ideas clearly ((NGA Center & CCSSO, 2010a, p. 23). As part of creating and sharing explanations and justifications during mathematics instruction, it is necessary for students to share their thinking using words, numbers, objects, and/or drawings, compare their strategies to the strategies of others, and ask questions to clarify meaning. These Speaking and Listening standards link to many of the Standards for Mathematical Practice, including “making sense of problems,” “constructing viable arguments,” and “attending to precision” (NGA Center & CCSSO, 2010b, pp. 6–7). Thus, the nature of mathematics learning with its emphasis on communication supports both mathematics and literacy development in young children.

## Place Value

Developing a strong foundation with place value is a key learning goal in the primary grades. When students first learn about the idea of place value, they make sense of regrouping. This understanding is vital as they move towards number operations and beyond. Because of the mathematical significance of this concept, place value is a critical area in both first and second grades (NGA Center & CCSSO, 2010b). The critical areas in the CCSSM are essential learnings for a given grade level that should be taught for depth and, thus, should have a significant amount of instructional time devoted to them. Place value understanding is scaffolded across the primary grades: (a) In kindergarten, students understand teen numbers as ten ones and some more ones; (b) In first grade, students understand that 10 ones is the same as 1 ten; and (c) In second grade, students extend place value patterns to understand that 10 tens is the same as 1 hundred (NGA Center & CCSSO, 2010b, pp. 12, 14, & 19).

In addition to the instructional progression of place value reflected in the K-2 standards, students move through a defined learning progression as they come to understand place value. As students work within the base ten number system, they

et al. · Volume 2, Proceedings of the Interdisciplinary STEM Teaching and initially count by ones, then they count by groups and singles, and finally they count by tens and ones as illustrated in Figure 1 (Van de Walle, Lovin, Karp, & Bay-Williams, 2014). Children will progress through these stages at different paces, but they will all need multiple and varied experiences to construct these relationships among the place values.



**Figure 1.** Progression of counting within base ten number system

Another element that will support student learning is the choice of mathematical tools or materials that teachers select. It is imperative that initial physical models for place value must be proportional (Van de Walle, et al., 2014). It is best to begin with groupable models where students can construct tens using single items, such as placing counters into ten frames, connecting individual snap cubes into chains of ten, or bundling Popsicle sticks with rubber bands (Van de Walle, et al., 2014). Not only do children need experience with bundling 10 ones into 1 ten, they also need to unbundle 1 ten into 10 ones (Dougherty, Flores, Louis, & Sophian, 2010). It for this reason that groupable models are so important for supporting the development of place value concepts. Too often we begin with or transition to pregrouped models, like Base Ten blocks, before students have a solid foundation of constructing and deconstructing tens and hundreds. Van de Walle et al. (2014) noted, “A significant disadvantage of the pregrouped physical models is the potential for children to use them without reflecting on the ten-to-one relationships” (p. 181). Therefore, students need ample time and multiple experiences with groupable models to construct meaning of our base ten number system.

Not only do students need opportunities to use these hands-on groupable models for sense-making, they also need opportunities to reason about and discuss the action of grouping and ungrouping 10 ones or 10 tens. To promote discourse about place value concepts, teachers could ask questions like: How are 10 ones and 1 ten alike? How are they different? What happened when you put 5 ones and 6 ones together? Why did you make a ten? What does it mean to *make a ten*? Why did you decide to unbundle that ten? All of these questions seek to probe student thinking and help make the mathematics of place value, the ten-to- one relationship, visible. Thus, use of appropriate mathematical tools support primary students’ engagement in discourse as a means for making sense of place value concepts.

In order to develop mathematical reasoning in young students, it is vital to present content in a familiar and understandable context, which applies to the concept of place value. Interestingly, when young children begin exploring our base ten number system, grouping by tens may not be natural for children given that they often group in smaller amounts like twos or fives (Dougherty, et al., 2010). Moreover, because children begin counting by ones, as a teacher “you cannot arbitrarily impose grouping by ten on children” (Van de Walle, et al., 2014, p. 182). Rather, you must provide some context that requires students to make sense of numbers being grouped by tens. This is where we can use the context of a candy shop to help students have a reason to group by tens based on the mathematical situation (Dixon, Nolan, Adams, Brooks, & Howse, 2016; Gregg & Yackel, 2002; Whitenack, Knipping, Novinger, & Underwood, 2001).

Candy is a context that many students can relate to. The following activity was taken from *Making Sense of Mathematics for Teaching Grades K-2* (Dixon et al., 2016). In this activity, students are given the following information: Each snap cube represents one piece of candy. We can combine 10 pieces of candy to make 1 roll. We can combine 10 rolls to make 1 box. The context of a candy shop corresponds to our base ten number system in terms of ones (pieces), tens (rolls), and hundreds (boxes). The language we use with our students (boxes, rolls, pieces) helps put place value into context so students can make sense of what each place value means in terms of packaging the candy, both in terms of physically grouping the ones and tens and explaining what these actions mean in the candy shop. Once they have an understanding of these candy shop terms and what they mean when making sense of how to package an amount, we can move towards the language of ones, tens, and hundreds. Based on the K-2 mathematics standards, packaging 10 pieces into 1 roll would be appropriate for first grade whereas packaging 10 rolls into 1 box would be appropriate for second grade (NGA Center & CCSSO, 2010b, pp. 14, & 19).

Giving students the background of the ten-to-one relationship between boxes and rolls or rolls and pieces allows teachers to then ask the following, “How many pieces are in a box? and “How do you know?” Students might make sense of this task by counting by ones or tens, but ultimately they should arrive at the same conclusion, 100 pieces are in a box, and be able to justify how they arrived at that conclusion. Giving students a set amount, for example 143 snap cubes displayed as 1 box, 4 rolls, and 3 pieces, and asking students to count how many pieces are at their table would be another task teachers can ask students to complete. To facilitate deeper understanding of place value concepts, teachers can then ask students explain how they counted the amount of pieces and justify their strategy. Identifying possible student misconceptions, such as counting a roll as 1 piece instead of 10 pieces, could also support understanding during class discussions where teachers ask students to think about the meaning of the rolls or boxes in relation to place value and discuss justifications as to why or why not this makes sense in their small groups and then again as a whole class. This context of a candy shop connects to the Mathematical Practice “make use of structure” as students are making sense of the ten-to-one relationship in this context (NGA Center & CCSSO, 2010b, p. 8). As students have continued experiences with this context and create explanations and justifications, they are “express[ing] regularity in repeated reasoning” (NGA Center & CCSSO, 2010b, p. 8). Inherent in both of these Mathematical Practices is the need

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for students to engage in discourse by expressing their thinking and reasoning verbally and/or in writing.

## Discussion

Using the context of the candy shop incorporates many elements of mathematics education best practice for K-2 learners. First and foremost, it provides a familiar and relatable context that requires students to group items into tens and/or hundreds. Because young children will not naturally group by tens, the candy shop context provides a purpose for such grouping (Dixon et al., 2016; Dougherty, et al., 2010; Gregg & Yackel, 2002; Van de Walle, et al., 2014; Whitenack et al., 2001). Additionally, in the candy shop, context brings meaning to the language of place value. When young learners first hear the words *ones*, *tens*, or *hundreds*, they may not have an understanding of what those words really mean as they relate to the mathematics of place value. To help give these words meaning and to support students in using the beginnings of place value language in their discourse, the students explore and explain what ones (pieces), tens (rolls), and hundreds (boxes) are in the candy shop. Teachers can use realia to further support meaning for students by bringing in Mentos, Starbursts, Life Savers, or other stacked candies to allow students to see what a piece, roll, or box looks like in real life. Then by giving students snap cubes to represent the pieces of candy, they have an opportunity to make sense of how to group by physically packaging (snapping) 10 pieces (ones) into 1 roll (ten) or unpackaging 1 roll (ten) into 10 pieces (ones) and how to describe the mathematics of packaging or unpackaging through discussions. This same idea can be extended to packaging 10 rolls (tens) into 1 box (hundred) or unpackaging 1 box (hundred) into 10 rolls (tens). Thus, the context of the candy shop provides meaning for the actions and language of bundling and unbundling in our place value system.

The candy shop activity also provides many opportunities to support and develop mathematical communication among young learners. First, the candy shop activity helps students connect contextual language (piece, roll, box) to more formalized vocabulary (ones, tens, hundreds). The use of accurate mathematical vocabulary is part of “attend[ing] to precision,” one of the Standards for Mathematical Practice (NGA Center & CCSSO, 2010b, p. 7). Teachers, however, must facilitate these language connections by pairing the language of the candy shop with the language of place value. Additionally, the candy shop activity provides students with opportunities to engage in explanation and justification. Throughout the activity, the teacher should be challenging students to justify how they know 1 roll is the same as 10 pieces or 10 rolls is the same as 1 box. Likewise, the teacher should be facilitating partner or small group talk among the students where they have to explain how they packaged various amounts of candy and their classmates must agree or disagree with their solution approach and justify why they agree or disagree. Engaging in such discourse via explanation and justification reflects another Standard for Mathematical Practice, where students are “construct[ing] viable arguments and critiqu[ing] the reasoning of others” (NGA Center & CCSSO, 2010b, p. 6). Additionally, students are using the ELA Speaking and Listening skills while engaging in conversations about packaging the candy with their partners and in small and whole group settings ((NGA Center & CCSSO, 2010a, p. 23)

Finally, the candy shop activity supports the progression of place value learning

in K-2 students. The context of the activity gives students a purpose in grouping ones (pieces) into tens (rolls), which moves them beyond counting by ones only (Van de Walle, et al., 2014). Therefore, the candy shop activity is a means by which teachers can challenge students who are only counting by ones to consider a more efficient way of counting by using groups of ten. By allowing students the opportunity to understand our base ten number system in a way that makes sense to them, the focus is on grouping and renaming values. For example, we might start with 14 individual pieces, then move towards describing that same amount as a roll and some pieces, and finally arrive at a more formalized conception of 1 ten and 4 ones. Although this progression through place value can be difficult for students to grasp, pairing the base ten language with the language of the candy shop gives students a context in which they are familiar, to move through this progression. Additionally, the candy shop activity utilizes snap cubes, which are groupable models for place value. Such groupable models support students in making sense of the ten-to-one relationship through physically bundling and unbundling (Dougherty, et al., 2010; Van de Walle, et al., 2014). The use of groupable models is essential for developing a deep understanding of place value concepts in K-2 learners. Therefore, the candy shop activity is developmentally appropriate, and it is a rich task that allows students to make sense of the complexity of our base ten number system through modeling and the use of explanations and justifications.

As suggested in this paper, allowing students to use manipulatives and engage in discourse, all grounded within the context of a candy shop, promotes a deeper understanding of place value among K-2 learners. It is essential that students have multiple experiences with making sense of the ten-to-one relationship. The candy shop activity is one such experience that provides students with opportunities to reason about our base ten number system in a familiar context and to explain and justify their thinking as they work with renaming 10 ones into 1 ten and 10 tens into 1 hundred. Thus, a rich task with an emphasis on mathematical communication, such as the candy shop, is an effective way to ensure K-2 students engage in deep learning about the complexities of place value.

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## **Using Google Forms to Inform Teaching Practices**

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### **Abstract**

Kay and LeSage (2009) conducted a literature review of research on use of student response systems in university courses (typically Science, Technology, Engineering, and Mathematics courses) and categorized benefits into classroom environment, learning, and assessment. The objectives of the proposed session are to discuss how using Google Forms will benefit those three above categories. Examples of Google Forms used to gather data, receive in-the-moment feedback to students and instructors, engage students' learning, and assess their learning will be shared throughout the paper. Limitations of Google Forms will also be discussed. This session can be beneficial to all K-College educators.

### **Keywords**

Google Forms, Student Response System, Technology

### **Recommended Citation**

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To improve student learning, engagement, and feedback, various student response technologies have been developed and used since their introduction in the 1960s (Kay & LeSage, 2009; Cubric & Jefferies, 2015). Although response systems have had many names (e.g., clickers; classroom, student, personal, audience, or audio response systems; electronic voting systems), they have consistently made promises to deepen student learning and engagement and provide in-the-moment feedback to students and instructors (Kay & LeSage). A typical student response device allows students to send responses to software that an instructor can access; the device can be a dedicated keypad (often referred to as a 'clicker') or an app on a student's internet-capable device.

Over the past 20 years, lowered costs, increased availability, and increased ease of set up have contributed to widespread use of student response systems (SRS) (Burgess, Bingley, & Banks, 2016; Kay & LeSage, 2009). Instructors may choose to use SRS in different ways. For example, in mathematics courses, SRS could be used as a tool for engaging in mathematical work and thinking, or as a tool for feedback in the form of in-class content assessments, student self-reflection, course feedback, or peer review. Depending on how the devices are used, benefits and challenges have been identified (Kay & LeSage, 2009; Cubric & Jefferies, 2015).

Kay and LeSage (2009) conducted a literature review of research on use of SRS in university courses (typically Science, Technology, Engineering, and Mathematics courses). They discussed benefits categorized into: classroom environment, learning, and assessment. We briefly describe each of the three categories here, as they provided a theoretical structure for our discussion in this paper.

Kay and LeSage (2009) identified classroom environment benefits as including improvements in students' attendance, attention, participation (especially avoiding judgment through anonymity of SRS responses), and engagement. Learning benefits described by Kay and LeSage, based on their review of the literature, were: interaction, discussion, contingent teaching, learning performance, and quality of learning. Finally, Kay and LeSage described assessment benefits as allowing feedback, formative assessment, and student comparison of responses (in the moment reflection). Since Google Forms, an online application from Google, is free and easy to use, we propose using Google Forms as a tool to benefit classroom environment, learning, and assessment in any classroom.

### **Using Google Forms to Benefit Classroom Environment**

Google Forms can be used in several ways to increase classroom environment benefits such as surveying students outside of class to learn about them as individuals, engaging them in class by collecting responses in the moment, and collecting their self-reflections after a lesson. The authors of this paper have engaged their students in each activity and they describe them in more detail here.

At the beginning of the semester, the authors create a Google Forms survey to learn about their students' interests, learning styles, and previous experiences, which can be emailed to students prior to the first day of class. By assigning the survey outside of class, it saves class time, decreases the amount of paper needed to be printed, and gives students more time to type their responses. Assigning such a survey also allows instructors to skim the responses before the first class and, potentially, create first day

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experiences tailored to students' interests and backgrounds. Figure 1 displays part of a "Getting to Know You" Survey in Google Forms that were created for the content mathematics courses for pre-service teachers at the authors' university.

**Figure 1.** Part of a "Getting to Know You" Survey in Google Forms

What are some of your interests and activities? \*

Your answer

Name the last math class taken and where (e.g., college algebra at GSU). \*

Your answer

Indicate your math experience as a LEARNER in \*

	Very negative	Somewhat negative	Neutral	Somewhat positive	Very positive
Grades K-5	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Grades 6-8	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Grades 9-12	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Current courses (in college)	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Why do you want to be a teacher? \*

Your answer

When instructors ask students a question during class, not all students may attempt to answer the question. Not only does the lack of engagement mean students are missing opportunities to learn but the instructor is less likely to be able to respond to students' needs. It is easy for an instructor to incorrectly assume that students do or do not understand the material based on the outspoken students who quickly respond to questions. Instructors may move on from a topic when the majority of the class needs more help, or spend too much time on a topic when the majority of the class actually does understand but is not communicating. The need for accurate in-the-moment feedback from all students can be addressed by using Google Forms as free clickers to involve all students in the learning and feedback process. Additionally, at the end of each lesson, students can use Google Forms to fill out a one-minute reflection to summarize the main point of the class and name one new thing they learned or the "muddiest point" of the lecture, which supports their engagement and the instructor's ability to adapt the lessons to their needs. Google Forms can also be used as an exit ticket or to check student attendance, especially in large lecture hall style classrooms where more than 100 students are present.

**Using Google Form to Benefit Learning**

To promote learning, Google Forms can be used to gauge students' pre-existing knowledge, identify misconceptions, and engage students in discussion. Because quizzes in Google Forms can be graded automatically and a summary of all answers can be viewed instantly under the "responses" tab in Google Forms, instructors can easily spot which questions were missed the most and decide on which concepts to

et al.: Volume 2, Proceedings of the Interdisciplinary STEM Teaching and review for students. For example, students enrolled in a content mathematics class called Foundations of Data and Geometry for pre-service teachers at a southeast university were given a set of problems to select the appropriate measurement units for a given figure, prior to the measurement lesson. They worked individually in class and submitted their answers using their own devices. Figure 2 shows one of the problems.

**Figure 2.** A measurement problem

Question from Carnegie Learning (2011)

Which is the best measurement to describe the area of Figure A? \*

1 point

3 cm

20 cm

Figure A

14 in.

8 in.

Figure B

radius = 3 cm

height = 10 cm

Figure C

2 cm

5 cm

7 cm

Figure D

☐ 60 cubic centimeters
 ☐ 60 square centimeters
 ☐ 60 centimeters

Their responses to this problem revealed that only about half of the students could correctly identify the appropriate units for Figure A (**Fig. 2**). This led to a discussion of dimensions and why units of an area are square but not linear nor cubic.

Moreover, questions in Google Forms are not restricted to multiple-choice but can be created to stimulate deeper student thinking where they need to do more than just a click to answer. For instance, the following problem illustrated in Figure 3 required students to exercise higher-order thinking and make comparisons among area measurements.

**Figure 3.** A measurement problem

Adapted from Beckmann (2017)

Which of the following describe the same area? (Type complete sentences; for example, "g and h describe the same area.") \*

0 points

a) 3 in<sup>2</sup>

b) A 9-inch-by-9-inch square

c) 9 square inches

d) A 3-inch-by-3-inch square

e) 9 in. × 9 in.

f) 9 in<sup>2</sup>

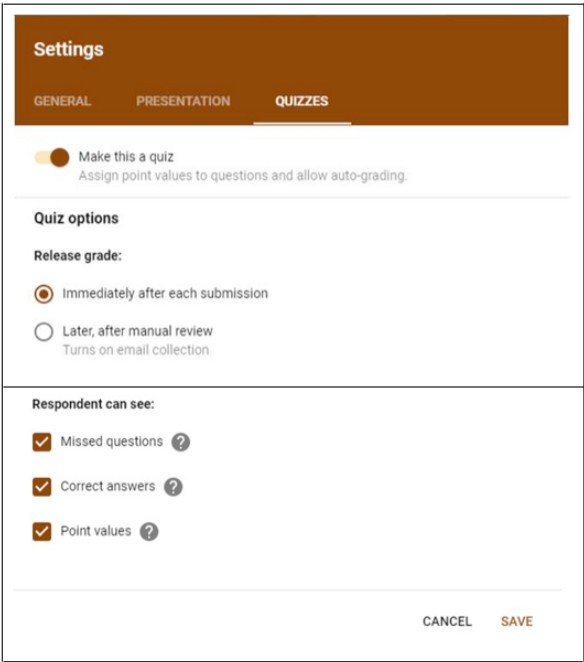
Your answer

The correct answer would be “b) and e) describe the same area and c), d), f) describe the same area.” Of 23 responses, only three students gave a correct answer while the other students listed either the first part or the second part but not both, or completely missed it. From an instructor’s perspective, receiving this feedback was important so she could respond quickly to the students’ confusion. The feedback

**Using Google Forms to Benefit Assessment**

Google Forms can provide in-the-moment feedback to both students and instructors. As shown in Figure 4, quiz settings in Google Forms have options to release grades immediately after each submission and allow students to see their total score and which questions they answered correctly or incorrectly. This immediate feedback allows them to immediately begin questioning their understanding and asking for help. In turn, faculty can assess how well students understand the material. Particularly, a formative assessment can be given to students during class immediately after a concept has been introduced, at the beginning of the next class as a follow-up activity, or at the end of a unit. Student responses give instructors ideas about which concepts need to be revisited or how to adapt follow-up lessons to the students’ needs.

**Figure 4.** Quiz Settings in Google Forms



**Limitation of Google Forms**

While Google Forms have many benefits, there are also limitations. Currently, Google Forms do not allow mathematical symbols or a way to enter anything but the most basic of equations. Also, there are no formatting options such as italicizing, underlining, text, or bold facing. Another possible concern with using Google Forms is that students may get distracted easily when they have their smartphones or laptops in front of them. It may be difficult to bring distracted students back to class discussions and engaging in classwork. Nonetheless, these issues can be addressed; for example, inserting pictures when an equation cannot be typed, capitalizing words to be emphasized, or walking around the classroom to ensure every student stays on task.

As technologies have developed, many changes have taken place in the classroom to support education and help teachers inform their teaching practices. Google Forms is a free online tool that can be used in the classroom to improve students' participation, engage them in their learning, and evaluate their learning. Moreover, it is user-friendly, easy to administer, and helps instructors save paper and time grading assignments.

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## **Graph It Out! Create Graphing Manipulatives to Explore Evolutionary Selection: A Lesson for High School Biology Students**

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### **Abstract**

Students often struggle with the relationship between mathematical graphs and the data they represent. To truly understand types of evolutionary selection, students need to be proficient with several different skills in math, science, and literacy contexts. With math, students must be able to identify variables, design appropriate graphs based on those variables, and convert data to graphical format. With science, students must be able to relate identified variables to scientific classifications and interpret those classifications based on evaluation of the scenarios presented. With literacy, students must be able to comprehend, dissect, and interpret a given passage. This presentation provides a multifaceted approach to teaching about types of evolutionary selection by making and using graph modeling manipulatives. Though the examples provided in this presentation are primarily focused for biology teachers, anyone who teaches students to interpret graph data could find the graphing manipulatives to be a useful tool as well.

### **Keywords**

Graphing Skills, Graphical Interpretation, Modeling, Evolutionary Selection, Secondary Biology

### **Recommended Citation**

Lawrimore, Cassie and Surber, Emily A. (2018) "Graph It Out! Create Graphing Manipulatives to Explore Evolutionary Selection: A Lesson for High School Biology Students," *Proceedings of the Interdisciplinary STEM Teaching and Learning Conference*: Vol. 2 , Article 11.

Students often struggle with the relationship between mathematical graphs and the data they represent (Gültepe, 2017; Roth & Temple, 2014; Tairab & Khalaf Al-Naqbi, 2004). This struggle can carry over into other areas of study that call for an understanding of the correlation between information and representation.

To truly understand types of evolutionary selection, students need to be proficient with several different skills in math, science, and literacy contexts. With math, students must be able to identify variables, design appropriate graphs based on those variables, and convert data to graphical format. With science, students must be able to relate identified variables to scientific classifications and interpret those classifications based on evaluation of the scenarios presented. And with literacy, students must be able to comprehend, dissect, and interpret a given passage.

What makes this following lesson so powerful is that it provides students with a multifaceted approach to learning about types of evolutionary selection by using small groups and graphing manipulatives, in addition to developing each of the skills mentioned previously. This activity is designed to be done after the initial introduction of the concepts and before individual practice.

### **Making the Graphing Manipulatives**

#### ***Necessary materials (for a class size of 30):***

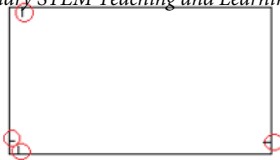
- 1 large foam board (tri-fold presentation size, approximately 36"x48")
- 10 green felt craft squares (8.5"x11" size)
- 1 roll of bright yellow\* yarn (something replaceable)
- 1 roll of purple\* paracord (or strong, waxed cord)
- 1 tube of all-purpose glue
- 1 box cutter or some other razor knife
- 1 black permanent marker
- 1 hot glue gun and glue

*\*The colors listed here are only suggestions because of the contrast. You can use any contrasting colors you prefer.*

#### ***Assembling the manipulatives***

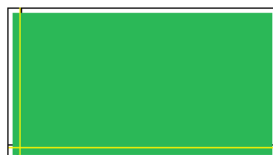
1. Use razor knife to cut foam board into 10 9"x12" size sections.
2. Place 1 felt square on each foam board section and adjust accordingly to make the most efficient cuts; do not allow felt to overlap the notches in foam board too much; this will cause folding in the felt when the yarn is added.
3. Trace around the edges of the felt squares and then place them off to the side.
4. Use the all-purpose glue to affix the felt squares to the newly cut foam board backs. (Reminder: be sure to leave a 1/2" margin of foam board around the edge of the felt sheet)
5. Use the box cutter or razor knife to cut notches into the foam board to secure the yarn. Do not cut through the felt. See Figure 1.

**Figure 1.**



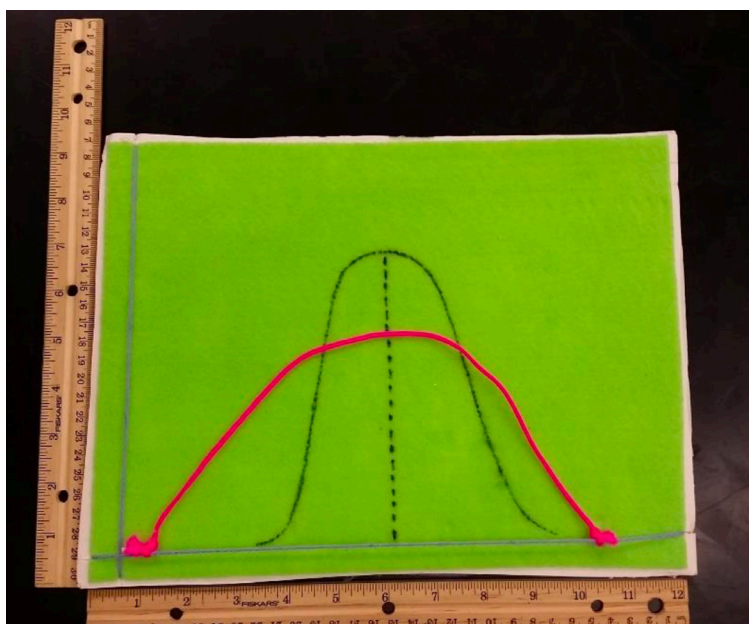
6. Use the yellow yarn to define the x-axis and y-axis of the graph. Tie the ends on the back of the foam board and either tape or hot glue the knots to the foam board.
7. At this point, the manipulative should look similar to Figure 2.

**Figure 2.**



8. Sketch a bell curve on the felt in the center.
9. Cut a length of paracord approximately 10 to 12 inches long; you will need to be able to tie loops in the ends and still have enough length to create a disruptive selection curve.
10. Tie the ends of the paracord into loops around the yarn creating the x-axis. Be sure to leave some space in the loops so the cord can still slide over the yarn.
11. Hot glue the knots to prevent unraveling. Be careful not to hot glue the cord to the yarn.
12. Repeat with the other 9 units. This should result in 10 manipulatives that look similar to the board pictured in Figure 3.

**Figure 3.**



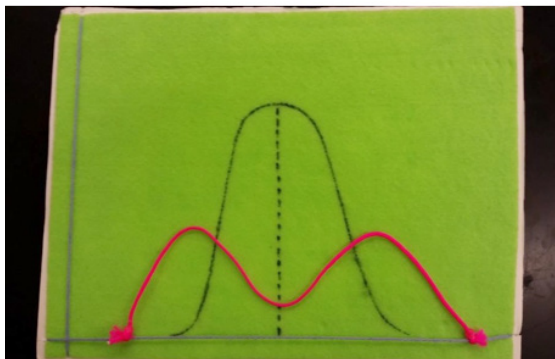


Students of similar skill levels should be assigned to groups of three. This encourages participation by all group members during the lesson. Often when students are placed in a group with a single strong student, the others may tend to not be as open with their ideas. This is especially true with this lesson where students are presented with new information and a novel way of expressing that information.

To begin, the teacher should briefly review the types of selection (stabilizing, selective, and disruptive) with the students and then provide a few possible scenarios. A good starting scenario might be to talk about populations of mice, white, gray, and black living near a volcano. Before a recent eruption, gray mice were selected against. After the eruption, their habitat is now the light gray color of the ash. Both the white and darker mice are easily seen against the light gray volcanic ash, making them more vulnerable to predators. Due to the selection against the white and black mice, the light gray mice have an increase in population because their fur color acts as a camouflage in the ash.

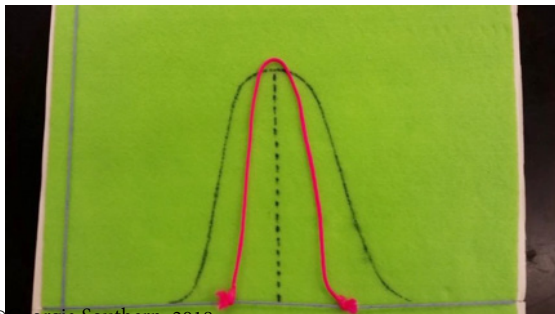
Students should be encouraged to discuss relevant variables within their groups and what population(s) they believe to be present in each scenario to begin with and manipulate their graphs to reflect that initial scenario. In this example, students should start with two different populations at either end of their graph with a dip between the two extremes. (See Figure 4.)

**Figure 4.**



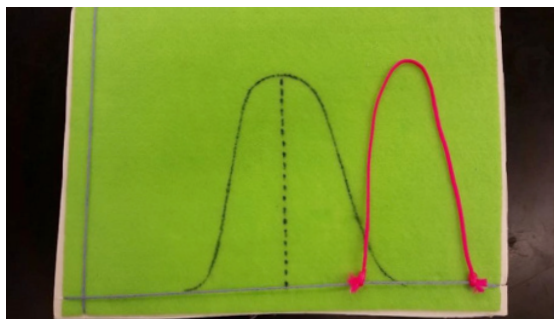
They would then likely want to demonstrate stabilizing selection on their graphs. To do this, they would pull up on the string at the center of the graph to indicate an increasing population with the more moderate trait (i.e. light gray mice). As they pull up on the string, the two beginning population extremes disappear, more clearly illustrating the shift in the population toward more moderate traits over time. (See Figure 5.)

**Figure 5.**



This technique can be used to demonstrate all types of selection. For an example with directional selection, students would pull up on one side of the string on the graph which then causes the height of the original peak to decrease or shift as the new peak is formed (pulled up). (See Figure 6.)

**Figure 6.**



Throughout this activity, the scenarios given are projected onto the screen so that students can re-read the example scenarios as needed. This is helpful because the examples gradually get more and more challenging. At the beginning of this activity, the teacher should read the passage to the class and guide the entire class in identifying the variables together. As the students work through the examples, the teacher should provide progressively less input as to what variables are important for consideration. For the final, more difficult examples, no help is provided other than reading the scenario aloud to the class. To start with, students were generally given two minutes to figure out their response to the scenario and manipulate their graphs accordingly. The time can easily be adjusted based on student need, class length, or speed of advancement through the activity.

Once the timer sounds, each group representative holds up their graph for the class. If there are varying answers from different groups, the teacher should not immediately identify which graph is correct or incorrect. What has typically worked well for this activity is to have a delegate from each group explain the group's rationale. Once the thought processes for all varying answers have been discussed, the class can usually come up with a consensus on what they believe the correct answer to be. Finally, the teacher can verify or provide the correct graph and lead a discussion as to why that particular representation is the most accurate.

### **Conclusion**

This activity has met with considerable success in the classroom as determined with formative and summative assessments. Students enjoy the hands-on aspect of the graphing manipulatives, but also understand the processes of selection more thoroughly than without. Additionally, the majority of students typically are better able to apply their understanding to pictorial graphical representations after engaging in this activity.

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## **Using Cartoons to Make Connections and Enrich Mathematics**

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### **Abstract**

The article discusses the integration of cartoons into a finite mathematics college course. However, cartoon integration is appropriate for any educational level STEM course. Students and the author used an online comic strip creator, MakeBeliefsComix.com, to create cartoons that connected concepts to the real world and history. Following Cho, Osborne, and Sanders (2015), students wrote a paragraph about their cartoon and its mathematics. In addition to connecting mathematics to art and writing and unearthing students' creative side, cartoons helped show the humanistic side of mathematics and promote communication and excitement about mathematics. The author developed a rubric to evaluate students' cartoons. There was evidence that students who did cartoons were better able to explain a concept and give examples of its real-world connection than those who did not. The article has potential to encourage others to brainstorm about cartoon integration in their mathematics courses.

### **Keywords**

writing, finite mathematics, cartoons, teaching

### **Recommended Citation**

St. Clair, Janet (2018) "Using Cartoons to Make Connections and Enrich Mathematics," *Proceedings of the Interdisciplinary STEM Teaching and Learning Conference*: Vol. 2 , Article 12.

The proverb, “A picture says a thousand words,” means that a picture gives as much information as the spoken or written word and maybe, more profoundly. What about a proverb relating cartoons and mathematics? Maybe, “A cartoon *sums* it up.” or “Mathematics; it’s all in the cartoon”. The possibilities about what cartoons can convey about mathematics are limitless. In addition to showing how to problem solve, cartoons in mathematics can convey such things as its beauty, its creativity, its history, its connection with the real world, its continual growth and change, and that its people – mathematicians – are real people who debate and revise. With respect to real people, I think about my favorite cartoon by Sidney Harris on a tee-shirt that I bought for my mother at an American Mathematical Society conference (Figure 1). My interpretation is that this cartoon shows a mathematician as a real person struggling to get the answer.

**Figure 1:** Sidney Harris mathematics cartoon tee-shirt.



Once untouchable in mathematics or any other school discipline for that matter (Cho, 2012; Toh, 2009), cartoons have made a fairly recent appearance in the mathematics world of teaching and learning. In addition to the benefits already listed, cartoons encourage students to express their thinking, motivate students to learn mathematics, reduce mathematical anxiety, help instructors detect students’ misconceptions and adjust instruction accordingly, promote understanding for algebraic symbolism (Cho, 2012; Cho, Osborne, & Sanders, 2014; Toh, 2009), and can be used to convey appropriate mathematical technical language and model how mathematicians really talk about mathematics. Cartoons are appropriate for any level of mathematics. Examples of ways that cartoons can be used in mathematics teaching (Cho, Osborne, & Sanders, 2015) are: (1) use an existing cartoon with mathematical content in a newspaper, for example, and develop activities related to it, (2) use cartoons specifically developed for mathematics (see for e.g., Ashbacher, 2015; Dabell, Keogh, & Naylor, 2008; Gonick, 2011; Gonick & Smith, 1993) and (3) have students create cartoons to convey their mathematical thinking.

The 2015 publication of *The Cartoon Guide to Statistics* by Larry Gonick and Woollscott Smith, sparked my interest in integrating cartoons in my mathematics classes. I decided to have students use an online cartoon maker and first e-mailed several sequential art professors at various universities in the country for their suggestions on a good one. They all said that because they had their students only draw cartoons by hand they were unable to recommend an online cartoon maker. Much googling, led me to decide on using the online comic strip maker at [www.MakeBeliefsComix.com](http://www.MakeBeliefsComix.com) created by Bill Zimmerman, a journalist, book writer, and Pulitzer-prize nominee. The artist, for MakeBeliefsComix.com, Tom Bloom, draws for publications such as *The New*

making through other sources that I found during my googling search (e.g., the annual Michigan State University Comics Forum (<http://comicsforum.msu.edu/>) and the Coursera course (<https://www.coursera.org/>), “How to Make Comic Book (Project-Centered) Course”).

In the sections that follow I start by discussing cartoons that I created and presented to my students to encourage them to realize the relevance of mathematics and that it is more than a collection of facts and skills. Next, I discuss cartoons that students created, showing how they gave me insight about their thinking about mathematical concepts and the nature of mathematics teaching and learning. Finally, I discuss the results of questionnaires that I gave students to determine whether and how mathematics cartooning benefited them and their opinions about it.

## **Integrating Cartoons in Teaching and Learning**

### **Teacher Created Cartoons**

In this section I discuss cartoons that I created to help students think about mathematics in real-world and historical contexts and how I integrated these cartoons into teaching and learning. When using cartoons to introduce objective concepts I encouraged students to interact with the cartoon characters by, for example, verifying cartoon characters’ ideas using a graphing calculator and having them complete Blackboard assignments involving real-world or historical ideas connected with concepts. Cartoon characters actually mention these Blackboard assignments in their dialogue. Other ways that I integrated cartoons that I created into teaching and learning include using them to help students review concepts and to encourage students to make hypotheses about problem solutions.

One way that I used cartoons was to introduce course objectives. Students were surprised when I told them that a Ph.D. dissertation has been written about cartoons in mathematics teaching and learning (Cho 2012). In the cartoons that I created, I made connections to the real world, to historical ideas, and sometimes to stories involving mathematics. Also, I included mathematical ideas and conventions that many students seem to overlook and showed that there are multiple ways to solve a problem. I created follow-up activities connected with the cartoons for further exploration of concepts. The cartoon characters mention these activities. I projected the cartoons on the projector screen and read them aloud, stopping at points to enter the cartoon’s world by expanding on or looking more deeply into a character’s thoughts or adding to or following up on their thoughts. I wrote notes on the board related to this. The students and I often went back and forth between the cartoon and the written remarks on the board related to ideas in the cartoon. Sometimes, I asked students to verify cartoon characters’ ideas by using the graphing calculator or to come to the board to verify characters’ ideas. And, sometimes I used ideas in the cartoon as a springboard for discussing other concepts not specifically addressed in the cartoon. My cartoon characters were Satchel and Tina, two students devoted to thinking and talking about mathematics. One of my students chose the name, Tina. To demonstrate the above ideas, I include cartoons that I created related to the objectives on graphing linear equations and solving systems of linear equations.

The “Graphing Lines” cartoon (Figure 2) starts with a reference to history: the [https://digitalcommons.georgiasouthern.edu/stem\\_proceedings/vol2/iss1/1](https://digitalcommons.georgiasouthern.edu/stem_proceedings/vol2/iss1/1)

ancient Egyptians and Descartes work related to our x-y coordinate system. I created the Blackboard assignment that Satchel mentions to involve students in reading and writing about the ancient Egyptian coordinate system (Lumpkin, 1997) and René Descartes. Students were surprised to learn that the ancient Egyptians had a sense of the rectangular coordinate system. Some questioned why many history of mathematics books do not include this. The cartoon characters discuss various ways to graph the equation,  $2x - 3y = 12$ . While reading the cartoon, students graphed the equation,  $2x - 3y = 12$ , on the graphing calculator to verify cartoon characters' ideas (e.g., the y-intercept, the slope). Satchel points out the connection between Robert Wadlow, the tallest man who ever lived (Jacobs 1994), and graphing lines. The cartoon ends with Satchel and Tina determining the equation of the line that gives the relationship between Wadlow's age and height. Satchel notes ideas that some of my students did not seem to realize: there are an infinite number of points on a line ("Graphing Lines" – Part 2) and the usual standard textbook notation and formula for slope (i.e.,  $m = \frac{y_2 - y_1}{x_2 - x_1}$  for  $(x_1, y_1)$  and  $(x_2, y_2)$  any two points on a line) does not mean that one has to "start with" the second ordered pair in the subtraction ("Graphing Lines" – Part 3). Also, Satchel notes that sometimes other variables instead of  $x$  and  $y$  (e.g.,  $a$  for age and  $h$  for height) are used ("Graphing Lines" – Part 2). We talked about labelling the axes appropriately using  $a$  and  $h$ . With the cartoon characters "setting the stage," I asked the class to follow up on finding the equation for the Wadlow data in the cartoon ("Graphing Lines" – Part 3). We also, discussed the meaning of function using the Wadlow data and how to tell whether the graph of an equation will be a line. Figure 3 shows an excerpt of what I recorded on the board as we read the cartoon.

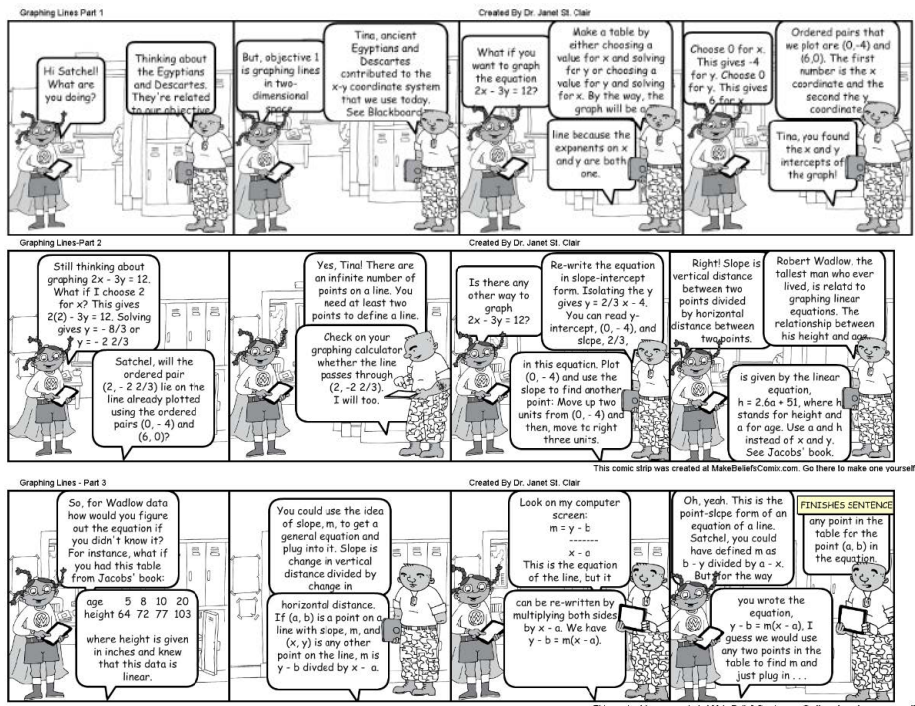


Figure 2: "Graphing Lines" cartoon that I created. Published by Digital Commons@Georgia Southern, 2018



Function - a relationship between two sets of data such that for each member in the first set there is only one member in the second set.

Ways to represent a function: table, graph, equation

Suppose  $(x_1, y_1)$  and  $(x_2, y_2)$  are any two points on a line. (Can use any letters to represent these points, e.g.,  $(m, n)$  and  $(p, q)$ ).

Then,  $m = \frac{y_1 - y_2}{x_1 - x_2}$  or  $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$\frac{m(x_2 - x_1)}{1} = \frac{(y_2 - y_1)(x_2 - x_1)}{x_2 - x_1}$$

$$m(x_2 - x_1) = y_2 - y_1 \text{ or } y_2 - y_1 = m(x_2 - x_1)$$

Drop the subscript, 2:  $y - y_1 = m(x - x_1)$  point-slope form

Compare a (age) h (height in inches) and x y

a (age)	h (height in inches)
5	64
8	72
10	77

Wadlow data

Which table represents a function? Why?

Writing an equation using the Wadlow table:  
Choose any two points:  $(5, 64)$  and  $(8, 72)$

$$m = \frac{64 - 72}{5 - 8} = \frac{-8}{-3} = \frac{8}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 77 = \frac{8}{3}(x - 10)$$

$$y - 77 = \frac{8}{3}x - \frac{80}{3}$$

$$+77 \quad +77 \quad \frac{231}{3}$$

$$y = \frac{8}{3}x + \frac{151}{3}$$

$$y = 2.7x + 50.3 \text{ linear equation}$$

$$h = 2.7a + 50.3$$

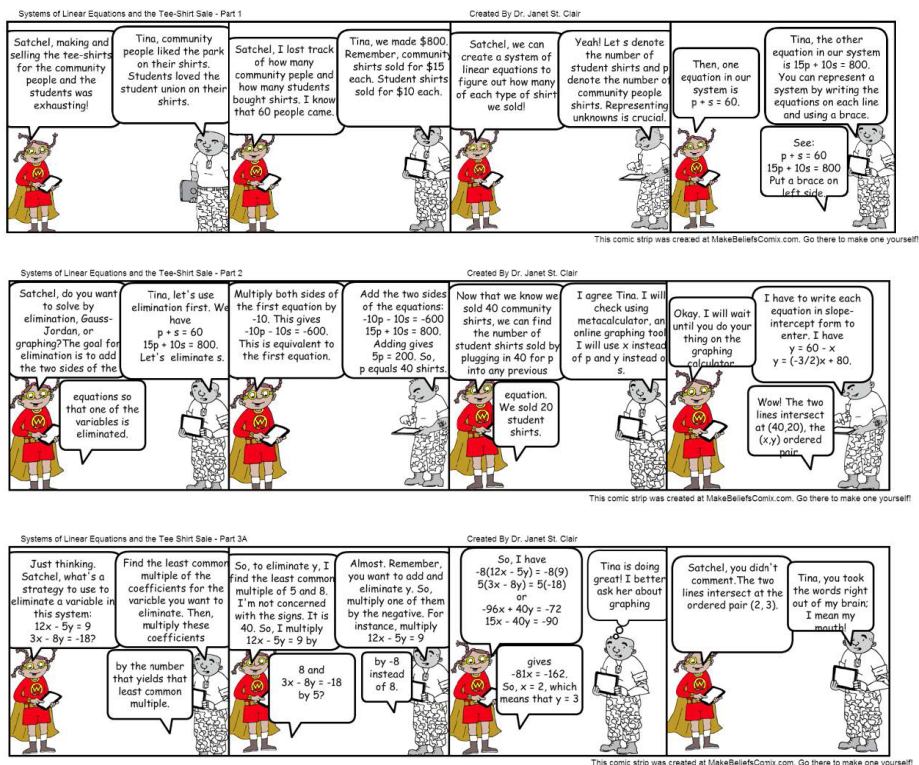
**Figure 3:** Board excerpt related to “Graphing Lines” cartoon (Figure 2) relates to cartoon characters’ discussion of finding the equation of the line for the Robert Wadlow age and height data. The table for age and height is expressed horizontally in the cartoon, but I wrote the table vertically on the board. Satchel and Tina give an idea of the path to take in determining the equation using information in the table, and we follow up on this by picking any two points from the table and using the point-slope form of an equation of a line to determine the equation. The cartoon is used as a springboard for discussing the concept of function. And, the board excerpt also includes a definition of function and a comparison between the Wadlow table, which represents a function, and a table that does not:

x	y
4	3
4	0
4	-4

In the “Systems of Linear Equations and the Tee-Shirt Sale” cartoon that I created (Figure 4) Satchel and Tina are trying to figure out how many student and how many community tee shirts they sold. They agree to create a system of linear equations and solve using the elimination method. Tina mentions a couple of other methods to solve their system. Satchel uses his graphing calculator to check their solution found using elimination, and the class and I viewed the graphical solution on the graphical calculator also. He indicates the appropriate forms for the equations in order to enter them into the graphing calculator. Satchel and Tina go on to talk about strategies for eliminating the x or y variables in another system of linear equations. Satchel doubts whether Tina will use the graphing calculator to check her result. This might show students that the graphing calculator is a useful tool for checking their own solutions rather than wondering or asking whether they are okay. I asked students to also verify the solution graphically as Tina, to Satchel’s surprise, did. Satchel and Tina eventually discuss the merchant problem in the story, The Tutor written in 1884 by Anton Chekhov (<https://www.ibiblio.org/eldritch/ac/tutor.htm>), that can be solved using systems of linear equations (Ochkov & Look, 2015):

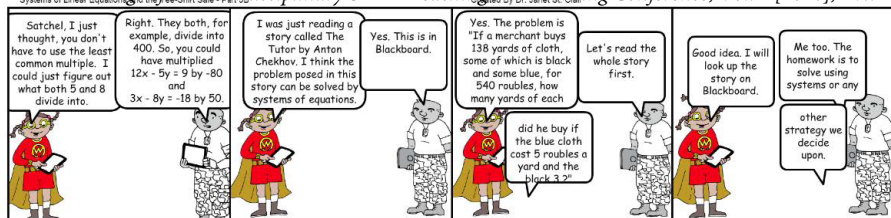
etal.: Volume 2, Proceedings of the Interdisciplinary STEM Teaching and  
 If a merchant buys 158 yards of cloth, some of which is black and some blue,  
 for 540 roubles, how many yards of each did he buy if the blue cloth cost 5  
 roubles a yard and the black cloth 3?  
 (p. 122)

The cartoon ends with Sachel and Tina planning to complete a Blackboard assignment that I prepared for the class. The assignment asked students to solve the merchant problem using a system of linear equations or any other strategy (including a non-algebraic one), write about what they thought about the story and its characters, and to extend the story or write a second part to it. It's interesting that the father in *The Tutor* solves the merchant problem without using algebra, but no details are given about his non-algebraic solution. All of my students solved the problem using a system of linear equations. I contacted Valery Ochkov, a professor at a university in Russia who wrote the article, "A System of Equations: Mathematics Lessons in Classical Literature" (Ochkov & Look, 2015), via e-mail to get his ideas on how the father solved the problem non-algebraically. He was kind enough to send his ideas. Students seemed to be very interested in this communication, and we discussed the non-algebraic solution in class. So, the cartoon was a springboard for communicating with another mathematician about another way to solve the merchant problem in *The Tutor*.



**Figure 4:** "Systems of Linear Equations and the Tee-Shirt Sale" cartoon that I created.

Continued on next page.



This comic strip was created at [kissmycomic.com](http://www.kissmycomic.com). Go there to make one yourself!

A second way that I used cartoons was to review concepts. Sometimes I asked students to read my cartoon creations to review for tests. Once, I revised a student's cartoon from a previous semester to use for review of systems of linear equations. The student had provided an interesting context for systems, a girl buying plants from a man named "Mr. Panda." I made major revisions to the cartoon, naming the girl Inga Schmidt, making historical connections to the ancient Chinese method of solving systems as compared to Gaussian elimination many centuries later, and including Inga's description to Mr. Panda of her sister's trip to China. The trip idea came from my own younger sister's actual trip to China as part of the 2015 Bridge Delegation to China to help educators start or strengthen their institution's Chinese programs and partnerships (<https://www.collegeboard.org/all-access-tags/chinese-bridge-delegation>). I titled the cartoon "Systems, China, and Germany."

A third way that I used cartoons was to have students solve a problem posed in the cartoon before reading the characters' solution either during class or for homework. I noticed that some students became more interested and involved in making hypotheses about solution strategies than they normally were.

## Student Created Cartoons

In this section I discuss students' cartoons, including paragraphs that they wrote to explain their cartoons. This section shows how students' cartoons enabled me to better see their creativity/imagination, their misunderstandings and the need to specifically address problem solving and metacognition in mathematics, and views that they might have about the nature of mathematics and mathematics teaching and learning. One student wrote that creating her cartoon helped her visualize her dream of owning her own business. Another student indicated that he learned from creating his cartoon that integrating life scenarios in cartoons can help learn mathematics.

I gave students several options to choose from to revolve their cartoons around:

- Connection of the concept to the real world
- Connection of the concept to the history of mathematics
- A problem of your choice related to the concept and solved using a method that was a part of the course objectives

I did not include my cartoons in Blackboard so that students will not be tempted to model their cartoons after mine. As I read more research about cartoons in mathematics (e.g., Cho, Osborne, & Sanders, 2015), the second semester that I integrated cartoons in my classes, I asked students to write a paragraph describing their cartoon and its mathematical content.

I developed a rubric using the Rcampus website, an education management system and a Collaborative learning environment (<http://www.rcampus.com/rubricshow.cfm?code=G48C63&sp=true>), as a basis.

The rubric that I developed is as follows:

Your cartoon will be evaluated on number of panels, mathematical relevance, elements, presentation, and creativity. See the maximum 4 points for the maximum requirements. The points will be used for extra credit.

#### 4 points

**Number of Panels** - Cartoon has the required 3 or 4 panels. **Mathematical Relevance** - The cartoon provides a clear picture of the mathematical concept. One would be able to develop understanding of the concept by reading the cartoon. **Elements** - The cartoon includes the required name/title and the author's name. **Presentation** - The cartoon is presented in an attracting way, and the overall appearance is excellent. It includes backgrounds and objects in addition to such items as talk and thought balloons. Characters are scaled to realistic proportions in relation to backgrounds and objects. **Creativity** - The cartoon sparks interest in the mathematical concept. Characters are well-chosen, and wording provides some humor or drama.

#### 3 points

**Number of Panels** - Cartoon has 2 panels. **Mathematical Relevance** - The cartoon gives a vague notion of the mathematical concept. One would have a difficult time developing understanding due to missing ideas. **Elements** - The cartoon does not include either the required name/title or the author's name. **Presentation** - The overall appearance of the cartoon is average. There might be some scaling problems with respect to character and background/object sizes. The cartoon seems to be done haphazardly. **Creativity** - The cartoon generates little interest in the mathematical concepts. Choice of characters is good, but wording provides little humor or drama.

#### 2 points

**Number of Panels** - Cartoon has 1 panel. **Mathematical Relevance** - The cartoon does not provide information that would help one develop understanding about the mathematical concept. **Elements** - The cartoon does not include neither the required name/title nor the author's name. **Presentation** - The overall appearance of the cartoon is poor. Cartoon reflects that little to no thought was put into its plan. **Creativity** - The cartoon does not generate interest in the mathematical concept. Characters are not well-chosen or seem to be unrealistic. Wording provides little to no humor or drama.

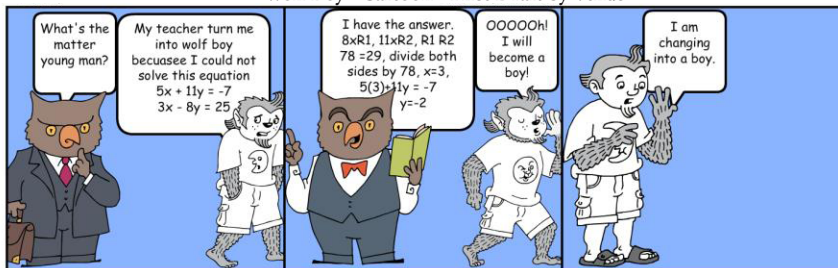
However, some of the follow-up assignments connected with the cartoons were counted toward quiz points. Normally, I don't give extra credit assignments and debated on whether the cartoons should count as extra credit or regular credit. Eventually, as students create a larger number of cartoons during a semester and become more familiar with using MakeBeliefsComix.com, I will probably count cartoons as regular credit.

MakeBeliefsComix.com allows one to both save their cartoon (e.g., on a flash drive) and e-mail it to themselves and someone else. Students e-mailed me their cartoons, and I sent them to campus duplication to be printed in color. It was nice to "get mail." Students did a draft cartoon and then revised using comments that I gave them. Revising conveys the idea of "writing (cartooning) as a process." I typed my reviews of each student's cartoon and gave students a hard copy along with a color copy of their original cartoon sometimes during class and sometimes via e-mail.

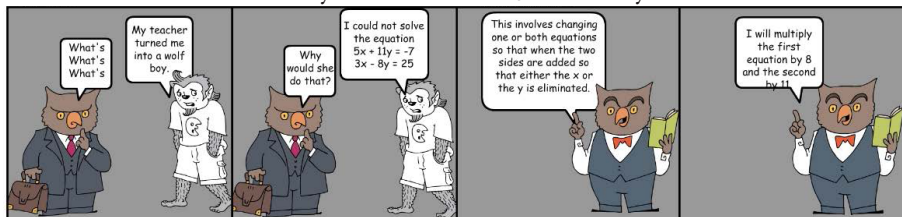
Examples for linear equations and systems of linear equations include small businesses that make head wraps and computers, a pedestrian and a police officer at parking meters, a chemist and his partner creating a new punch, and a grandmother asking her grandson to grocery shop for her.

Figure 5 is an example of a cartoon in which a student is able to integrate suspense, which is often a part of the cartoon genre, and manages the mathematics at the same time. Figure 6 is an example of a cartoon that more or less reflects equal authority between two peers (Cho, Osborne, & Sanders, 2015). Unlike the cartoon in Figure 5, the cartoon in Figure 6 embeds the mathematics in a real-world situation.

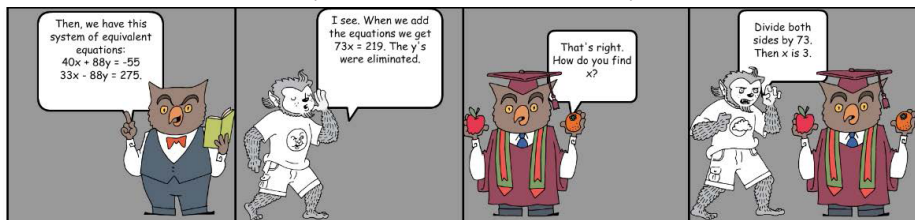
**"Wolf Boy" Cartoon - First Draft by Venus**



**"Wolf Boy" Cartoon - Part 1 of 3 - Revision by Venus**







"Wolf Boy" Cartoon - Part 3 of 3 - Revision by Venus

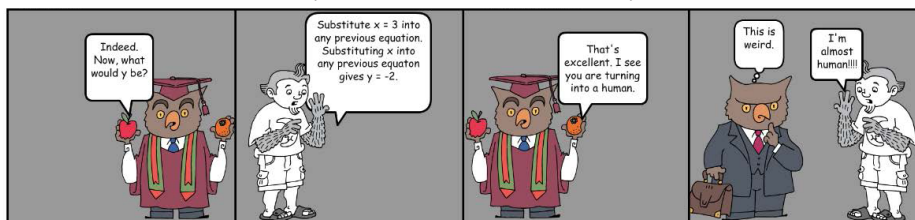
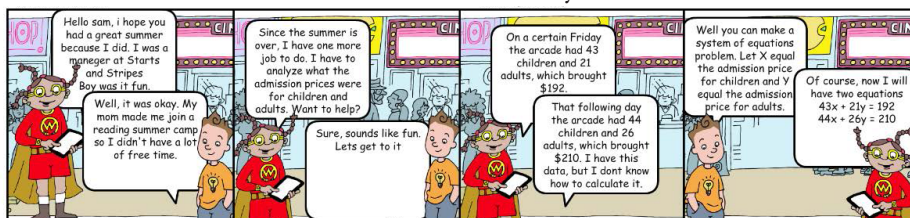


Figure 5: Venus' "Wolf Boy" cartoon. The first cartoon is the draft. The last three cartoons are the three parts of the revised version. In part 2, panel 1 of the revised cartoon, "equivalent" should be "linear."

"A Great Summer" Cartoon – Part 1 of 2 by Hana – No Revisions Done



"A Great Summer" Cartoon – Part 2 of 2 by Hana – No Revisions Done

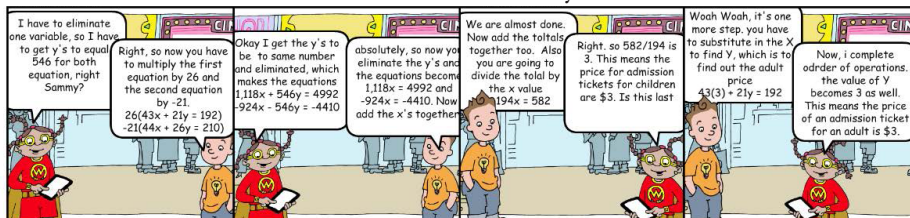


Figure 6: Hana's "A Great Summer" cartoon. She did not do a revision.

The choice of characters in the cartoons reveal students' thoughts about how mathematics interactions occur between people (Cho, Osborne, & Sanders, 2015). The cartoon in Figure 5 shows that the student thinks interactions involve an expert or authoritarian of mathematics knowledge helping a young person figure out the mathematics. In this case, the young person suffered serious consequences for not being able to solve a system of linear equations in the classroom. His teacher turned him into a wolf boy! The student who did this cartoon might view mathematics as an invented,

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 non-changing collection of facts and skills transmitted by the teacher to students (Philipp, 2007; Thompson, 1992). This implies that more work needs to be done in the classroom to challenge some students' views of mathematics. For example, many years ago, Thompson (1992) gave this description of what might be done to help change this type of view:

... more purposeful activities that grow out of problem situations, requiring reasoning and creative thinking, gathering and applying information, discovering, inventing, and communicating ideas, and testing those ideas through critical reflection and argumentation (p. 128).

We want students to view mathematics as being socially constructed by real people and something that is revised and changes over time in the spirit of the characters in Lakatos' (1976) *Proofs and Refutations*.

To illustrate paragraphs that students wrote about their cartoons and to show how cartoons helped me realize students' misconceptions, I will present cartoons done by Evan and Nia (pseudonyms). Evan's cartoon, which he titled "Overthinking at its Finest," relates to our objective on graphing linear equations and finding equations of lines. See Figure 7. It involves a scientist, maybe a "mad" scientist, trying to figure out how the graph of  $y = 5x$  would look. A baby helps the scientist visualize the graph. Evan's



**Figure 7:** Evan's cartoon, "Overthinking at its Finest," relates to graphing linear equations.

paragraph is in Figure 8. He explains his cartoon, indicating that sometimes people make things more difficult than they really are by overthinking and implies that he uses humor to convey that overthinking causes problems. As Cho, Osborne, and Sanders (2015) note, incorporating humor requires an extra layer of thought in addition to the artistic demands and mathematics. Evan's idea of overthinking might also involve the scientist not being able to draw on his metacognitive knowledge or skills. It would have been interesting to weave this into the cartoon. Maybe, the baby could have given the scientist a lesson in metacognition (Schoenfeld, 1987)! Also, George Pólya's ideas about problem solving

I thought of doing this because its funny and people tends to make things more difficult than it really is. Overthinking will do that to you. I picked the scientist because sometimes the smartest people are the ones to over-think something so simple. They might think the answer has to be some extravagant answer. The baby was used to show that he needs to stop overthinking.

**Figure 8:** Evan's paragraph about his cartoon, "Overthinking at its Finest."  
[https://digitalcommons.georgiasouthern.edu/stem\\_proceedings/vol2/iss1/1](https://digitalcommons.georgiasouthern.edu/stem_proceedings/vol2/iss1/1)



come into play (Pólya, 1945). For example, solve a simpler problem and return to the one that one is having trouble with, is a strategy that might have helped the scientist. Unfortunately, Evan did not explain the mathematics in his cartoon as the instructions asked. It is not clear whether the scientist is comparing the graph of  $y = 5x$  with the graph of  $y = x$ . The baby tells the scientist: “. . . The only thing that changed was not there is no  $y$ -intercept. It's still a linear equation.” Based on this, Evan is comparing the graph of  $y = 5x$  with some other graph. Evan might have been thinking about the line,  $y = x$  (which bisects both Quadrants I and III) because the baby mentions a diagonal line. It would have been nice if Evan had included the idea of the scientist using the graph of  $y = x$  to help him visualize the graph of  $y = 5x$ . Obviously, Evan was not aware of metacognition and Pólya's problem-solving principles. This might suggest that time designated for instruction in these would help students. There is a possibility that Evan and other students incorporate in their cartoons the way they would handle solving a problem that they have difficulty with. In this case, Evan might have sought help from another person rather than figuring it out on his own. Another point is that I am not sure whether Evan realized that the  $y$ -intercept of the graph of  $y = 5x$  (as well as the  $x$ -intercept) is  $(0, 0)$ . (The baby's says, “. . . The only thing that changed was not there is *no*  $y$ -intercept . . .”).

I gave Evan typed suggestions for revising his cartoon, including a graph of  $y = 5x$  that I did on the graphing calculator. But, cartoons were optional (worth a maximum of four points extra credit), and Evan did not revise. Here are my suggestions to Evan for revision:

In the first panel, tell what the equation is and maybe, add a little more: For example, let the scientist say, “I can't believe what this equation,  $y = 5x$ , should look like if I plotted it on a graph. Would it be a line? Would it be a parabola? Would it be a hyperbola? Would it touch the  $x$  or  $y$  axes?”

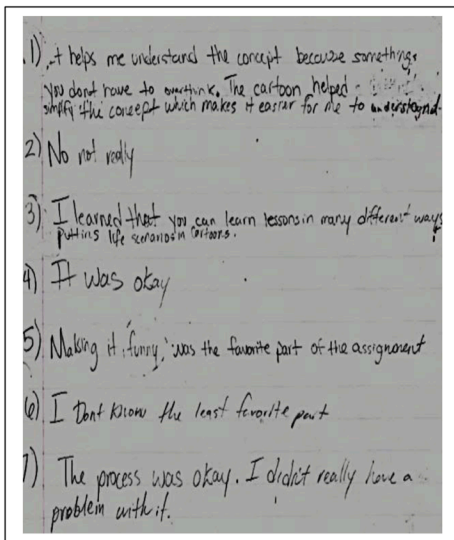
In the third panel, add ideas about the scientist comparing  $y = 5x$  with some other that he knows about, for example  $y = x$ . Evan, graph  $y = x$  and  $y = 5x$  yourself by hand on graph paper or using a graphing calculator. What do you observe about the comparison of these graphs?

In the last panel, re-word the baby's talk balloon to convey this idea: “Dude, it would be a line through the origin. The  $x$ - and  $y$ -intercepts are both  $(0, 0)$ . Another point on the line besides  $(0, 0)$  is  $(1, 5)$ . See,  $(1, 5)$  makes the equation true. You can tell the graph will be a line by looking at the exponents on the variables. When these exponents are 1, the graph is a line. Check out the graph on the online graphing calculator, meta-calculator at [www.metacalculator.com](http://www.metacalculator.com).”

Evan did a decent job of scaling his characters to sizes so that they were in proportion with the background and objects. However, the scientist appears slightly smaller in the first panel than in the last three panels.

Evan's responses to a questionnaire that I gave at the end of the semester to get students' thoughts about creating their cartoons is in Figure 9. Notice that he learned that life scenarios can be integrated with mathematical concepts (question 3) and that the most favorite aspect of creating his cartoon was incorporating humor (question 5).

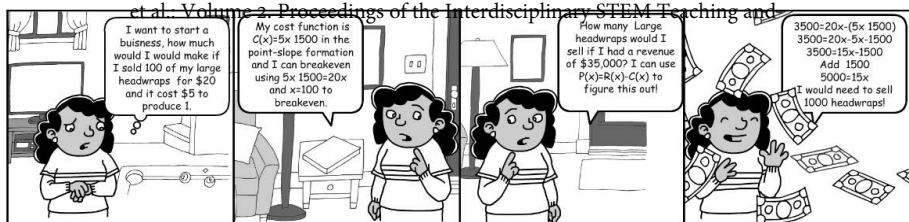
- 1) Please explain whether the cartoon helped you in terms of your understanding of the concept you focused on. If it helped you, clearly explain how.
- 2) Did the cartoon motivate you in any way? If so, how?
- 3) What did you learn from creating your cartoon?
- 4) Did you enjoy the cartoon assignment? Tell why or why not.
- 5) What was your favorite part or thing about the cartoon assignment?
- 6) What was your least favorite part or thing about the cartoon assignment?
- 7) Did you enjoy the process of making the cartoon? Clearly explain why.



**Figure 9:** Questionnaire on students' experiences in creating cartoons given at the end of the semester (left) and Evan's responses to these questions (right).

Notice also that Evan's responses to questions 2 (motivation as a result of creating cartoon), 4 (enjoyment of cartoon assignment), and 7 (enjoyment of process of creating cartoon) would not inspire a person to make cartoons. This might suggest that I try different ways for students to do the cartoons. For example, they could work in pairs and create cartoons and publish their final products on a Website. Also, more time learning how to draw their own cartoons might help. I invited a sequential artist to visit the classroom to give a crash course in drawing cartoons, but due to time constraints, the artist came one time at the end of the semester.

Nia did a cartoon, entitled "Isis's Dream," about a cartoon character named Isis who is planning a head-wrap business and thinks about ideas related to a linear cost function. See Figure 10. Nia's cartoon also related to the objective on graphing lines and writing equations of lines, and it was nice that she thought of a real-world context in which to embed the mathematics. Briefly, the linear cost function,  $C(x)$ , is defined in slope-intercept form as  $C(x) = mx + b$  where  $C(x)$  represents the cost to produce  $x$  items,  $m$  is the marginal cost – the cost to make one item,  $b$  is the fixed cost – the cost that doesn't change (e.g., cost to rent a place to make the product, cost to train workers),  $x$  is the number of items made. The cost equation can be expressed, of course in point-slope form as  $C(x) - C(x_1) = m(x - x_1)$ . Here, the point,  $(x_1, C(x_1))$ , represents the cost,  $C(x_1)$ , make a specific number of items,  $x_1$ . The revenue,  $R(x)$ , made from selling the items made is given by the equation,  $R(x) = px$ , where  $p$  represents the price that an item sells for and  $x$  represents the number of items sold. Break even occurs when the cost to make the items equals the revenue:  $C(x) = R(x)$ . And, profit,  $P(x)$ , is revenue minus cost:  $P(x) = R(x) - C(x)$ .



**Figure 10:** Nia's cartoon, "Isis's Dream," related to the objective on graphing linear equations and finding equations of lines. She leaves out the addition symbol, +, in the equation,  $C(x) = 5x + 1500$ .

Nia typed the following paragraph in Figure 11:

I made "Iris' Dream" because I myself want to have my own business. I used cost function is the only math objective I have completely understood since the start of this class. If I did choose to have my own business I would sell head wraps and I think that having Iris think it through first was a good vision board. I choose the character I chose because she was a black girl trying to start a business which I support personally. This assignment neither helped or stagnated my learning process with math although it was cute in nature and fun as an extra credit it didn't further my success in my continuous struggle with Mathematics.

**Figure 11:** Nia's paragraph that was supposed to explain her cartoon and the mathematics in it.

Nia notes that creating the cartoon was a "vision board" for her because she wants to start a head-wrap business one day. So, the cartoon may have helped Nia "live" her dream of owning her own business. Nia is frank when she points out that creating the cartoon did not help her in her "continuous struggle with mathematics." At the beginning of the course, Nia told me that she would be asking a lot of questions because she felt that she usually has difficulty with mathematics courses. The main misunderstanding shown in Nia's cartoon occurs in the third and fourth (last) panels. Some revision suggestions that I gave to Nia, in typed form, were:

The revenue representation,  $20x$  (where  $x$  is the number of head-wraps sold), in panel 2 implies that the price of each head-wrap is \$20. So, panel 1 could be revised so that Iris says "... I sold 100 of my large head-wraps, making \$2000 ..." instead of "... I sold 100 of my large head-wraps for \$20 ..." (If 100 head-wraps sell for \$20, the price for one head-wrap would only be 20¢. You probably want to sell one head-wrap for more than 20¢ especially because you say that it costs \$5 to make one head-wrap).

In panel 3, instead of Iris asking, "How many large head-wraps would I sell if I had a revenue of \$35,000?" have Iris ask, "How many large head-wraps would I have to sell in order to make a profit of \$35,000?" Then, panel 4 would involve substituting into the profit equation,  $P(x) = R(x) - C(x)$ , as follows:  $35,000 = 20x - (5x + 1500)$ . (Note: Nia, I think, inadvertently substitutes 3500 instead of 35,000 for profit). Solving this for  $x$  gives approximately 2433.3, which should be rounded up to 2434 head-wraps that should be sold to make a profit of \$35,000. Have Iris give some explanation of the various equations that she uses.

In panel 2, with respect to the equation,  $C(x) = 5x + 1500$ , Iris could address why she thinks her fixed cost is \$1500. Note that you could also revise some of the mathematical ideas by having Iris say that it costs \$5 to make one head-wrap and then adding a cost to make a specific number of head-wraps. For example, Iris might find that it costs \$300 to make 50 head-wraps. Then, you could develop the cartoon by having Iris figure out a cost function using this information:

$$C(x) - C(x_1) = m(x - x_1)$$

$$C(x) - 300 = 5(x - 50)$$

$$C(x) - 300 = 5x - 250$$

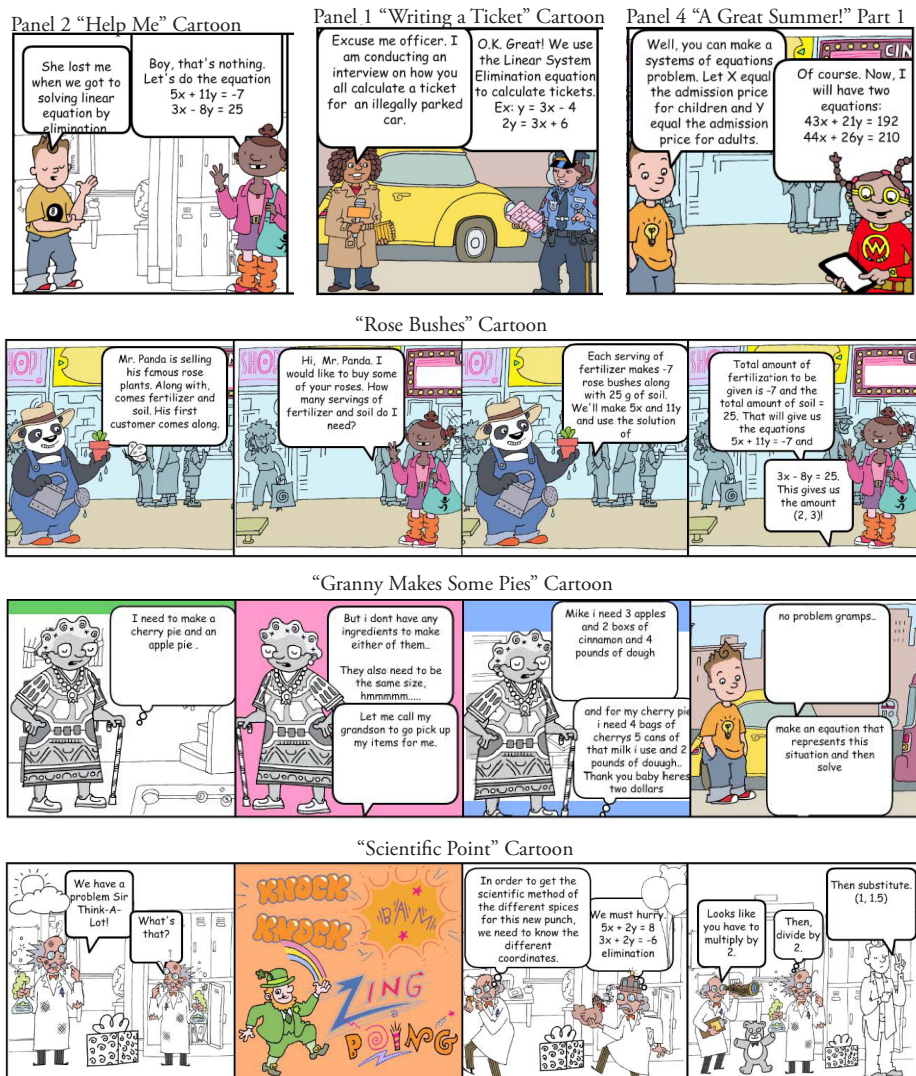
$$C(x) = 5x + 50$$

Like Evan, Nia did not revise her cartoon. She actually ended up dropping the course at the time that I gave her my suggested revisions. Both Evan and Nia provided rich contexts that I and other students could revise to create interesting cartoons related to graphing linear equations and finding equations on lines.

Other misunderstandings that I noticed in students' cartoons included using inappropriate terminology, problems using algebraic notation, and difficulty modeling real-world situations mathematically or omitting mathematics in the cartoon. With respect to inappropriate terminology, I often noticed that some students referred to a system of linear equations as "equations." One student called it a "linear system elimination equation." With respect to algebraic notation, an example is a student who was not consistent is using the same case letters when defining unknowns. The student used X and Y when defining the unknowns and x and y when writing the equations in her system. Finally, with respect to difficulty in modelling, Nia had difficulty with ideas related to modelling linear equations in the context of the linear cost function. Other examples are two other students who had difficulty modelling ideas their cartoons related to systems of linear equations, one who embedded a problem in the context of buying flowers and the other in the context of buying ingredients for a pie. These students thought of rich contexts to embed their mathematics in but were unable to successfully connect their contexts with the mathematics. See Figure 12 for examples of the above misunderstandings. This shows me that I need to think of activities that will target these kinds of misunderstandings. For example, more readings and discussion related to equations, systems of linear equations, and use of variables to represent unknown values might have potential. This could include excerpts of historical readings (e.g., Grcar, 2011; Hart, 2011; Pycior, 1981) that show how these concepts developed over time. With respect to modeling, maybe engaging students in solving more real-world problems in pairs and/or as group projects will be helpful. For example, with respect to algebraic ideas, a function approach that embeds concepts in solving real-world problems using technology has great potential (Laughbaum, 2003; Laughbaum & Crocker, 2004). Another example is the Algebra Project, which involves relating everyday life of students to algebra (Moses, Kamii, Swap, & Howard, 1989; Wilgoren, 2001). Yet another is Realistic Mathematics Education based on Freudenthal's view of mathematics, which embeds mathematics in experiences that students relate to

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(Freudenthal, 1991). Felton (2014) makes an important point (<http://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Mathematics-and-the-Real-World/>). Felton (2014) discusses the value of two approaches to integrating real-world problems: using the real world as a stepping-stone to encourage students to think about mathematical concepts and using authentic real-world problems. The former approach includes problems that are “near” and ones that students will probably not exactly encounter outside of school. The latter includes problems that are open-ended and messy and have multiple ways of solving. This approach would include Realistic Mathematics Education.



**Figure 12 (continued on next page):** Examples of students' misunderstandings including using inappropriate terminology – “Help Me,” “Writing a Ticket,” and “A Great Summer;” problems using algebraic notation – “A Great Summer;” difficulty



### “Off-Call” Cartoon



Figure 12 (Continued)

### Findings Related to Students' Understanding and Thinking

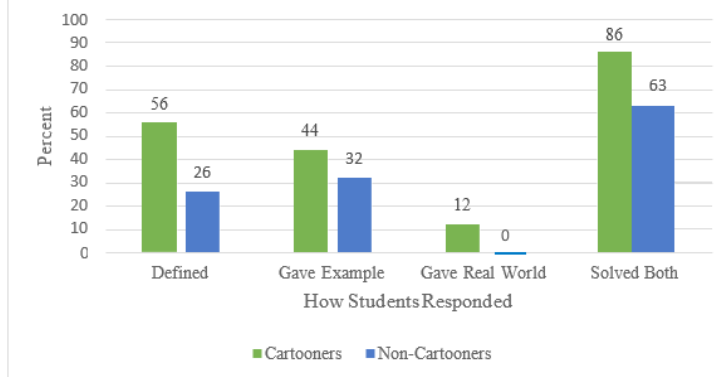
In this section I discuss students' thinking and opinions about cartoons and their understanding of concepts as a result of reading/discussing cartoons that I created and creating their own cartoons. Also, I discuss the results of a pre- and post-questionnaire that measures changes in student motivation, interest, and anxiety given to students in one class. Students tended to have positive opinions about the cartoon experience. There was evidence that students who created cartoons were able to answer questions related to systems of linear equations more successfully than those who did not create cartoons. There was also evidence that cartoons helped students with mathematics anxiety.

The previous section gives examples of misunderstandings that I found as I read students' cartoons. Questionnaires that I created to find out how cartoons influenced students' knowledge and motivation also helped to shed light on students' understanding. I gave a few questions related to each objective before discussing it and again toward the end of the semester and compared students who did cartoons with students who did not. Figure 13 gives results for questions related to systems of linear equations. One question asked students to write everything they knew, including definition, examples, and real-world connections. The other question asked students to solve the following two systems of linear equations using any method:

$$\begin{cases} 2x + y = 9 \\ 3x - y = 16 \end{cases}$$

$$\begin{cases} 4x + 3y = -1 \\ 5x + 4y = 1 \end{cases}$$

Students who did cartoons tended to be able to answer these questions more successfully than those who did not.



**Figure 13:** Results of students' responses to two questions: 1) Write everything you know about systems of linear equations. Examples of things you might include are: definition, example of a system of linear equations, and how a system of linear equations is used to solve real-world problems. 2) Solve the following systems of linear equations:

$$\begin{array}{ll} \text{a) } \begin{cases} 2x + y = 9 \\ 3x - y = 16 \end{cases} & \text{b) } \begin{cases} 4x + 3y = -1 \\ 5x + 4y = 1 \end{cases} \end{array}$$

I also asked students to answer questions about what they thought about the various cartoons that I created and presented in class. All students' responses were positive. Figure 14 gives these questions for the "Graphing Lines" and "Venn Diagram – What's the Fuss" cartoons that I created along with two students' responses. Students indicated such things as appreciating historical information in cartoons, appreciating learning about how concepts related to real life, and appreciating learning more about particular course-related concepts. An education major indicated that she appreciated learning that there are different ways to teach a lesson, i.e., use cartoons.

#### Questions for "Graphing Lines" Cartoon That I Created

- 1) Please explain whether the cartoon helped you in terms of your understanding of graphing and finding equations of lines. If it helped you, clearly explain how.
- 2) Did the graphing lines cartoon motivate you in any way? If so, how?
- 3) What did you learn from the graphing lines cartoon?
- 4) Did you enjoy the graphing lines cartoon? Tell why or why not.
- 5) What was your favorite part or thing about the graphing lines cartoon?
- 6) What was your least favorite part or thing about the graphing lines cartoon?

#### One Student's Responses to Questions for "Graphing Lines" Cartoon

1. The cartoon helped me by explaining in detail how graphing and finding equations of lines and the history of it.
2. It motivated me to do more graphing lines so I can understand them better.
3. I learn more about graphing lines and how it works.
4. Yes, I enjoyed the cartoon it was very informative.
5. My favorite thing about the cartoon was how informative it was.
6. I really did not have a least favorite part in the cartoon I enjoyed it very much.

**Figure 14 (Continued on next page)**



- 1) Please explain whether the cartoon helped you in terms of your understanding of Venn diagrams. If it helped you, clearly explain how.
- 2) Did the Venn diagram cartoon motivate you in any way? If so, how?
- 3) What did you learn from the Venn diagram cartoon?
- 4) Did you enjoy the Venn diagram cartoon? Tell why or why not.
- 5) What was your favorite part or thing about the Venn diagram cartoon?
- 6) What was your least favorite part or thing about the Venn diagram cartoon?

- 1) Yes, I was helped very much. It was helpful to include the paper-plate example in the cartoon, because it became even more alive to me.
- 2) I think the Venn diagram cartoon motivated me to appreciate the many ways to explain and teach a lesson.
- 3) I learned that Charles Dodgson was a mathematician, also John Venn developed Venn diagrams in 1881.
- 4) Yes, I enjoyed the Venn diagram cartoon. I enjoyed the characters. I also enjoyed how everything is explained.
- 5) The names of the characters were very creative.
- 6) I enjoyed reading about the two stories.

**Questions for “Venn Diagrams – What’s the Fuss?” Cartoon That I Created**

**Two Students’ Responses to Questions for “Venn Diagrams – What’s the Fuss?” That I Created**

- 1) Please explain whether the cartoon helped you in terms of your understanding of Venn diagrams. If it helped you, clearly explain how.
- 2) Did the Venn diagram cartoon motivate you in any way? If so, how?
- 3) What did you learn from the Venn diagram cartoon?
- 4) Did you enjoy the Venn diagram cartoon? Tell why or why not.
- 5) What was your favorite part or thing about the Venn diagram cartoon?
- 6) What was your least favorite part or thing about the Venn diagram cartoon?

1. The cartoon did help me because through different problems in math I found that I can use Venn diagrams to make it easier to answer them.
2. No. It didn’t motivate me.
3. I learned that not only are they used for math problems but real life problems as well.
4. The cartoon was okay. I like to see funny cartoons.
5. I don’t have a favorite part.
6. I don’t have a least favorite

**Figure 14 (continued):** Examples of students’ responses to questions about the “Graphing Lines” and “Venn Diagram – What’s the Fuss?” cartoons that I created and presented in class. Note: One student (Evan) e-mailed responses to me, and the last set of responses is a copy of what *he typed* in his e-mail.

In addition to answering questions about cartoons I created and presented, I asked students to respond to similar questions in Figure 14 for the cartoons they created. See Figure 9 in the previous section for an example.

A questionnaire that I gave students in one class at the end of semester showed that students thought the cartoons were helpful. Figure 15 gives questionnaire items along one student’s responses. This student e-mailed me her *typed* responses.

**Figure 15 (on next page):** A student’s answers to questions about their experiences in creating cartoons. The student’s actual *typed, e-mailed* responses are included.  
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1) Please clearly explain how the cartoon helped your understanding of systems of linear equations and how to solve systems of linear equations.

2) Did the cartoons motivate you in any way? If so, how?

3) What did you learn from creating your cartoons?

I like the fact that doing the comic was fun! I had fun taking the time to be creative and learn about the concept of solving linear equations. I never made my own comic before until I started this class and I see that it can be quite challenging. Sometimes it was hard making sure that I used the same characters, backgrounds, term of knowledge, and still be able to teach the concept. This project helped me incorporate my own way of doing a comic without having an actual restriction on what I could do. I really enjoyed doing this project.

Creating the cartoons did motivate me. I never truly was good at math because I felt I was too busy to sit down and actually learn it, but now that I am in college and I know I can't wait on anyone to do my work for me, I decided to learn on my own and teach myself. This cartoon motivated me to work hard at the things I am not so good at, math especially.

I learned from making my own cartoon that creating a comic is not easy. I had to triple check my work to make sure it was done to the best of my ability and I had to make sure that the problem was done correctly. I had to also make sure that people would be able to understand my work without getting confused.

Interestingly, for the first question and also the third question in Figure 15 the student wrote a comment that coincides with a point Cho, Osborne, and Sanders (2015) made: 'They found that their students' cartoons not only presented mathematical concepts, but also showed students' ability to handle the mathematics coupled with a "complex narrative genre" and their thinking about what constitutes mathematical interactions. In Figure 15, my student wrote:

. . . I never made my own comic before . . . it can be quite challenging. Sometimes, it was hard making sure that I use the same characters, backgrounds, term[s] of knowledge and still be able to teach the concept . .

Also interesting is that the student said she was teaching herself as she created the cartoon, noting also that the cartoons encouraged her to persevere with mathematics. Robert A. Heinlein, an American novelist and science fiction writer, expressed the saying between teaching and learning in a nice way: "When one teaches, two learn."

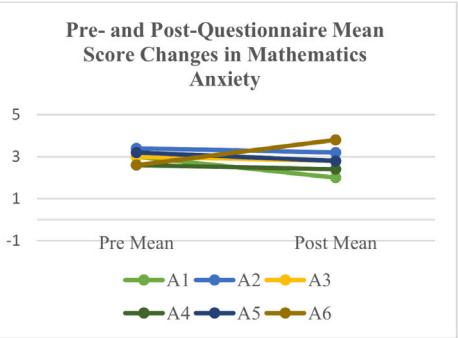
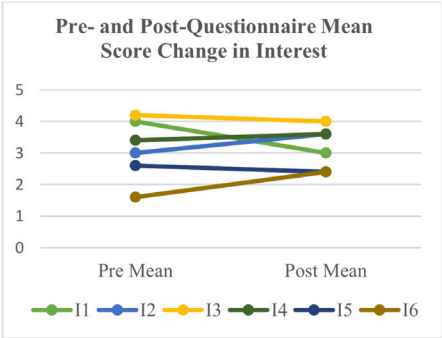
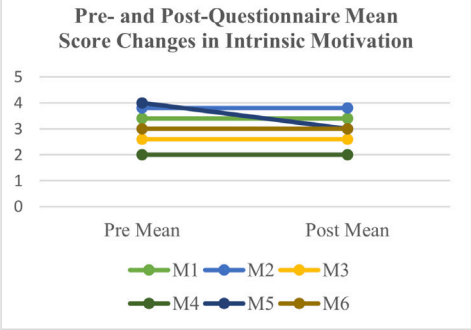
For one of my classes, I gave students a pre- and post-questionnaire entitled Student Motivation, Interest, and Anxiety Changes that Cho (2012) used in his dissertation study. The questionnaire measures changes in student motivation, interest, and anxiety. Table 1 gives the items. Responses to items were on a 5- point Likert scale: "strongly disagree" (1), "disagree" (2), "don't know" (3), "agree" (4), "strongly agree" (5). Five out of eight students (about sixty-two percent) in this particular class chose to do cartoons. Only students who did cartoons completed the pre- and post-questionnaire. Graphs showing changes in mean scores for motivation, interest, and anxiety are in Figure 16. Motivation pre- and post-score means remained the same for all items except item 5 (I don't give up easily when I don't understand a mathematics problem). For this item, the mean score decreased from 4 (agree) to 3 (don't know). Interest pre- and post-score means increased for items 2 (Mathematics is a very interesting subject than other subjects) and 4 (New ideas in mathematics are interesting to me). But, mean scores for interest item 6 (I see mathematics as a subject I will rarely use) went from 1.6 to 2.4. And, for interest item 1 (I am interested in learning mathematics), mean scores went from 4 to 3. Also, there was a small drop in mean scores (0.2 change) for interest items

Type of Item	Item
Motivation (M)	M1. I like mathematics even if I make a lot of mistakes. M2. I enjoy learning new things in mathematics. M3. I enjoy doing an assignment in mathematics. M4. I like difficult problems because I enjoy trying to figure them out. M5. I don't give up easily when I don't understand a mathematics problem. M6. I would like to learn more mathematics in school.
Interest (I)	I1. I am interested in learning mathematics. I2. Mathematics is a very interesting subject than other subjects. I3. I will need mathematics for my future work. I4. New ideas in mathematics are interesting to me. I5. I find that many mathematics problems are interesting. I6. I see mathematics as a subject I will rarely use.
Anxiety (A)	A1. When I hear the word mathematics, I have a feeling of dislike. A2. I have usually worried about being able to solve mathematics problems. A3. Mathematics usually makes me feel uncomfortable and nervous. A4. Mathematics is boring. A5. Mathematics makes me feel uneasy and confused. A6. I usually have been at ease in mathematics classes.

**Table 1:** Questionnaire items related to motivation, interest, and anxiety.

problems are interesting). The most positive evidence was for anxiety. Mean scores for anxiety items 1 (When I hear the word mathematics, I have a feeling of dislike), 2 (I have usually worried about being able to solve mathematics problems), 3 (Mathematics usually makes me feel uncomfortable and nervous), 4 (Mathematics is boring), and 5 (Mathematics makes me feel uneasy and confused) decreased, and those for item 6 (I usually have been at ease in mathematics classes) increased from 2.6 to 3.8. Cho's (2012) motivation and interest results gave a more positive influence of cartoons on students' motivation and interest. I should also point out that I noticed that all students in this class, except one, created cartoons that did not connect mathematics to the real world or to history as the instructions indicated. The one student who did withdrew from the course early and *did not* complete the post- questionnaire.

I also gave the same class that did the motivation/interest/anxiety questionnaire a questionnaire entitled Opinions about Cartoons also used by Cho (2012) in his Ph.D. dissertation study. This questionnaire contains three types of items: enjoyment/interest doing cartoons, value/usefulness of cartoons, and pressure/tension while doing cartoons. Table 2 gives the items. Responses to items were on a 5-point Likert scale: "strongly disagree" (1), "disagree" (2), "don't know" (3), "agree" (4), "strongly agree" (5). As the bar graphs in Figure 17 indicate, students had positive opinions about their experiences doing cartoons.



	M1	M2	M3	M4	M5	M6	I1	I2	I3	I4	I5	I6
Pre Mean	3.4	3.8	2.6	2	4	3	4	3	4.2	3.4	2.6	1.6
Pre Standard Deviation	.8	.4	.8	.63	.63	.63	.7	.89	.4	.8	1	.5
Post Mean	3.4	3.8	2.6	2	3	3	3	3.6	4	3.6	2.4	2.4
Post Standard Deviation	1.2	.75	1.2	.63	.89	.89	.89	.8	.63	1	1	.49

	A1	A2	A3	A4	A5	A6
Pre Mean	3	3.4	3	2.6	3.2	2.6
Pre Standard Deviation	.89	1.4	.89	.49	.98	1.2
Post Mean	2	3.2	2.8	2.4	2.8	3.8
Post Standard Deviation	0	.98	.98	.49	.98	.98

**Figure 16:** Changes in mean scores for motivation, interest, and anxiety.

Type of Opinion Item	Item
Enjoyment/Interest (EI)	EI 1. While I was doing cartoon activities, I enjoyed it. EI2. I thought cartoon activities were a boring activity. EI3. I would describe cartoon activities as enjoyable. EI4. Cartoon activities were fun to do. EI5. I thought cartoon activities were an interesting activity. EI6. I would describe cartoon activities as fun.
Value/Usefulness (VU)	VU 1. I believe that doing cartoon activities could be of some value for me. VU 2. I would be willing to do cartoon activities again because it has some value for me. VU 3. I believe that doing cartoon activities is useful for improved concentration. VU 4. I think cartoon activities are an important activity. VU 5. I am willing to do cartoon activities again because I think it is somewhat useful. 6. VU I believe doing cartoon activities could be somewhat beneficial for me. VU 7. It is possible that cartoon activities could improve my studying habits.
Pressure/Tension (PT)	PT 1. I felt tense while doing cartoon activities. PT 2. I was anxious while doing cartoon activities. PT 3. I felt relaxed while doing cartoon activities. PT 4. I did not feel at all nervous about doing cartoon activities. PT 5. I felt pressured while doing cartoon activities.

Table 2: Questionnaire items related to opinions about cartoons.

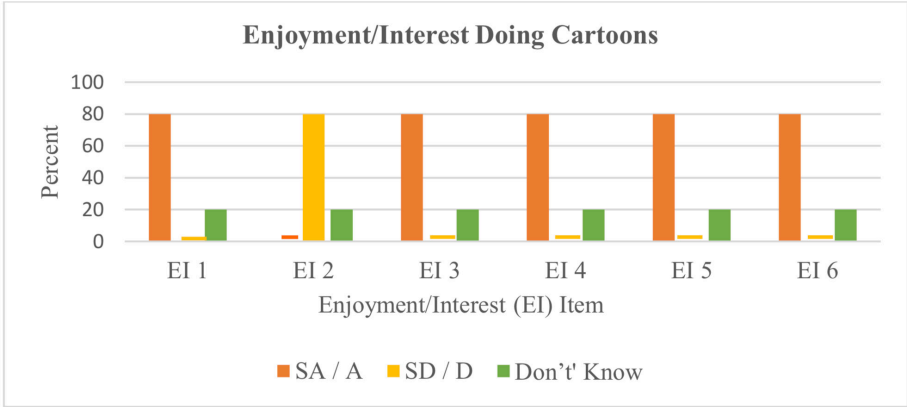
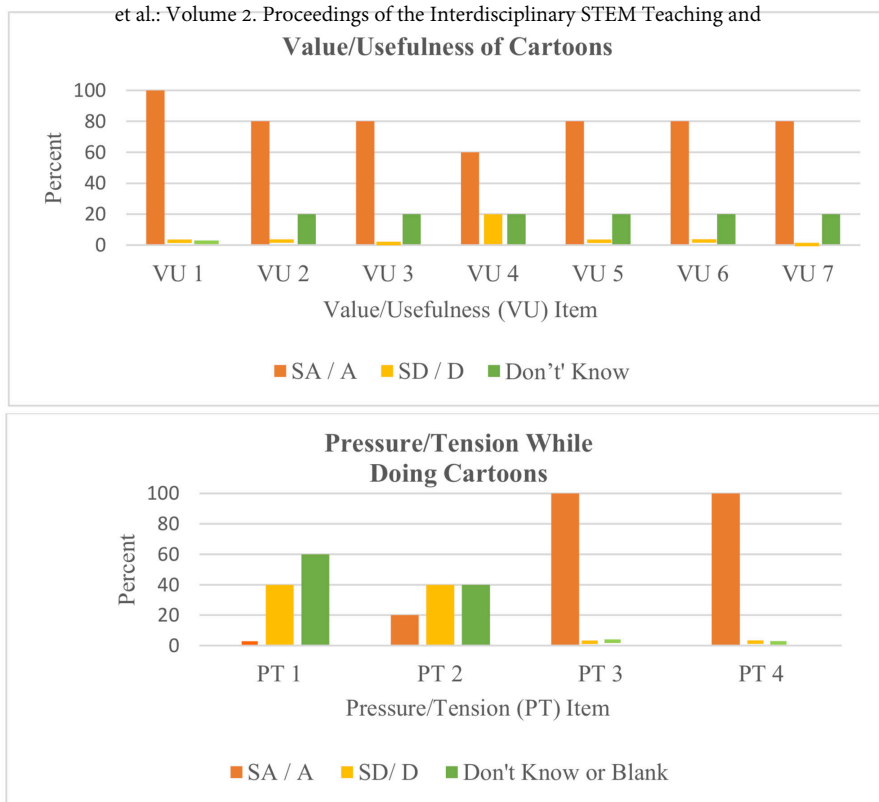


Figure 17 (continued on next page): Students’ responses to opinion questionnaire.

Conclusion

Cartoons lowered students’ mathematics anxiety; unleashed their imagination and creativity; enabled them to draw on their prior knowledge and experiences and in one case, supported their dreams; encouraged them to pose problems; showed them that their ideas are valued; and helped them see that mathematics teaching and learning is not about giving “correct” short answers but involves rich dialogue. With respect to



**Figure 17 (continued):** Students' responses to opinion questionnaire.

the last point, I have noticed that students began to ask more deep-rooted questions involving such ideas as alternative strategies to solve problems and even questions such as "Why do many people not like mathematics?" Further, it encouraged another form of communication (i.e., e-mail). I overheard one student telling another in an excited tone, "I'm going to do my cartoon now!" Also, students seemed to be more apt to give hypotheses about solutions to problems. There was also some evidence that students who did cartoons learned certain concepts more deeply than those who did not. For example, students who did cartoons were able to answer several questions related to systems of linear equations more successfully than those who did not. It is important that there was evidence that most students had positive opinions about doing the cartoons.

Cartoons helped me better understand not only their misunderstandings and ability to use appropriate mathematical language and symbols but also their views about mathematics. This was useful in adjusting instruction. Cartoons also encouraged me to examine concepts more deeply, including their historical roots, their appearance in literature, and new happenings related to them. For example, with respect to Venn diagrams I developed Blackboard assignments that involved students in reading about other ways to show the relationship between sets such as Carroll diagrams and the 11-set Venn diagram done by Khalegh Mamakani and Frank Ruskey at the University of Victoria in British Columbia, Canada (Aron, 2012).

I would like to make them a more integral part of learning and include more authentic real-world problems in the cartoons. This should help give more positive results with respect to motivation and interest similar to Cho's (2012) findings. It would also be useful to get more students' thoughts on how they feel about the cognitive demand of handling both the mathematics and creating a cartoon, which Cho, Osborne, and Sanders (2015) label as a "complex narrative genre".

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