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N. Hemati

J. S. Thorp

Ming-Chuan Leu
Missouri University of Science and Technology, mleu@mst.edu

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ROBUST NONLINEAR CONTROL OF BRUSHLESS DC MOTORS IN THE PRESENCE OF MAGNETIC SATURATION

N. Hemati, Assistant Professor, Drexel University
J.S. Thorp, Professor, Cornell University
M.C. Leu, Professor, New Jersey Inst. of Tech.

ABSTRACT

A robust control law is derived and examined for a direct-drive robot arm driven by a Brushless DC Motor (BLDCM). The complete dynamics of the motor and its interaction with the robot arm are accounted for. This is important, since in a direct-drive servo system the torque generated by the motor is directly transmitted to the load. Effects of magnetic saturation as well as reluctance variations are accounted for, in order to ensure accuracy. The effectiveness of the method is examined through computer simulations. The computational complexity of the overall control scheme is such that it can be readily used for real-time control.

I. INTRODUCTION

In this paper we address the robust control of a direct drive robot arm which is actuated by a Brushless DC Motor (BLDCM). BLDCM has been an attractive choice for direct-drive applications, e.g. [1], mainly because of its large torque producing capabilities suitable for high acceleration and deceleration rates. In a direct-drive servo system the load, e.g. robot arm, is directly coupled to the motor and therefore, the torque generated by the motor is directly transmitted to the load. As a result, in order to ensure high performance of the system, the dynamics of the motor and its interaction with the load must be taken into account.

BLDCM constitutes a coupled nonlinear dynamic system. Accordingly, the tracking control associated with it is addressed here as a nonlinear control problem. The control problem is formulated in the framework of the transformation of nonlinear systems[7]. The control law presented in this paper is composed of a part which guarantees good performance in the absence of modeling and measurement errors[6]. Additionally, to ensure good performance in the presence of uncertainties, a saturating robust control term is derived and included in the overall control structure. Computer simulations are used to examine the effectiveness of the control law.

II. MATHEMATICAL MODEL

A one-degree-of-freedom robot arm actuated by a BLDCM is considered; see figure 1. The motor is to generate a prescribed torque profile such that the payload and the arm are guided along a given trajectory specified in terms of the time histories of position, velocity, and acceleration. For the payload to track the prescribed trajectory, appropriate control commands (voltages/ currents) must be supplied to the windings of the motor. The motor used to drive the arm (figure 1) is a BLDCM with eight Samarium Cobolt permanent magnet poles (i.e. 4 pole pairs), and 3-phase Y-connected stator windings. The BLDCM consists of a means to provide three-phase signals to the phase windings. The signals from the signal generator are synchronized with the output of the position sensor (ususally a resolver) to provide electronic commutation. The differential equations governing the dynamic characteristics of the combined BLDCM and robot arm system can be written as

$$\underline{V}(t) = \underline{R}\ \underline{I}(t) + \frac{d\underline{\Lambda}(\underline{I},\theta)}{dt} \tag{1}$$

$$J\frac{d\omega}{dt} = T - T_L \tag{2}$$

$$T_L = Mgl\cos(\theta) + Ml^2\frac{d^2\theta}{dt^2} \tag{3}$$

$$\frac{d\theta}{dt} = \omega \tag{4}$$

where θ is the position variable, $\underline{V}=[v_1,v_2,v_3]^T$ and $\underline{I}=[i_1,i_2,i_3]^T$ are the phase voltage input and current vectors, respectively. $\underline{R}$ is the resistance matrix, and the flux linkage vector is defined by

$$\underline{\Lambda}(\underline{I},\theta) = \underline{L}(\theta)\ \underline{I} + \underline{\Lambda}_m(\theta) \tag{5}$$

where the inductance matrix, $\underline{L}(\theta)$, is a 3 by 3 symmetric, positive definite matrix whose diagonal elements are the self inductances and the off-diagonal elements are the mutual inductances of the windings. The vector $\underline{\Lambda}_m(\theta)$ defines the flux linkages associated with the permanent magnet rotor. The torque generated by the motor, $T=T(\underline{I},\theta)$, is a function of the phase currents as well as rotor displacement. For brevity purposes, the lengthy equation associated with the torque funtion is not given here, and the interested reader should refer to [5] and [6]. The dynamics associated with the arm and the payload are modelled as given in equation (3). Equations (1)-(4) represent a set of differential equations with time varying coefficients, and are quite complex for analytical and control design purposes. However, it is well known [2][9][11] that for the type of BLDCM under study, a Floquet type transformation (commonly refered to as Park's transformation), $K(\theta)$, exists which transforms equation (1) to a set of nonlinear differential equations with constant coefficients. The transformed governing equations are as follows:

$$v_q(t) = Ri_q + n\lambda_d(t)\frac{d\theta}{dt} + \frac{d\lambda_q(t)}{dt} \tag{6}$$

$$v_d(t) = Ri_d - n\lambda_q(t)\frac{d\theta}{dt} + \frac{d\lambda_d(t)}{dt} \tag{7}$$

where

$$\lambda_q(t) = L_q i_q \tag{8}$$

$$\lambda_d(t) = L_d i_d + K_e \tag{9}$$

$L_q = (\frac{3}{2})(L_a - L_g)$, and $L_d = (\frac{3}{2})(L_a + L_g)$, where L_a and L_g are inductance parameters. The number of permanent magnet pole pairs is denoted by n. The torque expression in terms of the new variables is

$$T(i_q,i_d) = (\frac{3n}{2})\{K_e i_q + (L_d - L_q)\, i_q i_d\} \tag{10}$$

Equation (2) remains unchanged, except that now the torque generated by the motor is no longer an explicit function of the position variable, θ. In our case, this has very important implications, since it is now possible to address the control problem independently of the commutation function involved. Furthermore, the transformed model (i.e. equations (2), (6), (7), and (10)) is much more attractive for computational, and as a result, for real-time control purposes.

So far, in the development of the model the assumption has been made that the torques generated by the motor lie within the range where the magnetic structure of the motor retains its linearity. It turns out that for applications, such as direct-drive robotics, where large torques are needed, this becomes an unrealistic assumption. A motor from the class of motors under investigation has been experimentally tested and it has been found that magnetic saturation plays an important role; see [5]. To account for the presence of magnetic saturation, the nonlinear characteristics associated with the motor have been modelled by a set of multi-dimensional surfaces which accurately describe the flux linkages and the torque function associated with the saturated motor. However, the application of Floquet transformation to the original equation, i.e. eqn. (1), would result in a more simplified representation only if the flux linkage vector, equation (5), is a linear function of the current vector. To alleviate this problem, the nonlinear functions representing the flux linkages are divided into intervals within each of which they are closely approximated by linear functions. This approach, in turn, allows us to retain the general formulation obtained in equations (6)-(10), with the exception that now the parameters L_a, L_g, and K_e are piecewise constant functions of the phase currents. It is important to note that the interval widths within which linearity is assumed can be made arbitrarily small for better accuracy.

III. NONLINEAR CONTROL OF BLDCM

The mathematical model, equations (2), and (6)-(7), represents a coupled nonlinear system. The tracking control of the system is approached by considering the full combined dynamics of the system. In this section, a nonlinear control law is presented to compensate for the nonlinearities in the system and to give good tracking performance. The goodness of the control law, however, will be based on the assumption that an accurate description of the system and that accurate measurements are available. In section IV, this assumption will be relaxed by supplementing a robust contoller to the control law of this section.

To proceed, it is appropriate to rewrite the governing equations of motion in the following form:

$$\frac{dx}{dt} = f(x) + G(x)\,u(t) \qquad (11)$$

$$e(t) = \theta_d - \theta \qquad (12)$$

where

$$x(t) = \begin{bmatrix} \int\theta\, dt \\ \theta \\ \omega \\ i_q \\ i_d \end{bmatrix};\; f(x) = \begin{bmatrix} x_2 \\ x_3 \\ k_1x_4+k_2x_4x_5 \\ -k_4x_3-k_3x_4-k_5x_3x_5 \\ k_6x_5+k_7x_3x_4 \end{bmatrix}$$

$$G(x) = \begin{bmatrix} 0 & 0 & 0 & q_1 & 0 \\ 0 & 0 & 0 & 0 & q_2 \end{bmatrix}^T;\; u(t) = \begin{bmatrix} v_q \\ v_d \end{bmatrix}$$

where $k_1=3nK_e/2J$, $k_2=3n(L_d-L_q)/2J$, $k_3=-R/L_q$, $k_4=-K_e/L_q$, $k_5=-L_d/L_q$, $k_6=-R/L_d$, $k_7=L_q/L_d$, $q_1=1/L_q$, $q_2=1/L_d$.

We will be looking for a control law

$$u(t) = \beta^{-1}(x)\,\{\, \upsilon(t) - \sigma(x) \,\} \qquad (13)$$

which will transform the nonlinear system, (11)-(12), through the transformation

$$y(t) = T(x) \qquad (14)$$

to the following linear canonical form

$$\frac{dy}{dt} = Ay(t) + B\,\upsilon(t) \qquad (15)$$

$$e(t) = \theta_d - \theta \qquad (16)$$

where

$$y(t) = \begin{bmatrix} \int(\theta_d-\theta)\, dt & \theta_d-\theta & \omega_d-\omega & \alpha_d-\alpha & i_d \end{bmatrix}^T$$

and the pair (A,B) is in the Brunovsky canonical form with Kronecker indices 4 and 1. θ_d, ω_d, and α_d are the desired position, velocity, and acceleration, respectively.

We will now use the results which have previously appeared in the literature and in particular in [7] to derive conditions under which such a transformation is possible. Note that if it is possible to obtain a control law to transform the governing equations to the linear system equivalent, equations (15)-(16), then to achieve the desired dynamic performance one may simply use an eigen-value assignment/pole placement technique. The necessary and sufficient conditions for the existance of $T(x)$ in our case are as follows: i) The set $C = \{G_1, [f,G_1], (ad^2f,G_1), (ad^3f,G_1), G_2\}$[1] must span a five-dimensional space, where G_1 and G_2 are columns of matrix G; ii) The set $C_1 = \{G_1, [f,G_1], (ad^2f,G_1), (ad^3f,G_1)\}$ and C must be involutive; iii) The span of C_1 must be equal to the span of the intersection of C_1 and C. For space limitations, the derivation of the Lie barackets is not presented and only the main result is given here. Conditions (i)-(iii) are satisfied if

$$k_1 + k_2 x_5 \neq 0 \qquad (17)$$

Consequently, the transformation $T(x)$ exists, provided that the condition in (17) is satisfied. The implications associated with this result are discussed later. For now, suffice it to say that the control law will be designed to ensure the satisfaction of this condition.

A control law of the form given in equation (13) that results in the transformation to the linear canonical form in equations (15)-(16) is given by

$$\beta(x) = \begin{bmatrix} (k_1+k_2x_5)q_1 & q_2k_2x_4 \\ 0 & q_2 \end{bmatrix} \qquad (18)$$

$$\sigma(x) = \begin{bmatrix} (k_1+k_2x_5)(k_3x_4+k_4x_3+k_5x_3x_4)-k_2x_4(k_6x_5+k_7x_3x_4) \\ k_6x_5+k_7x_3x_4 \end{bmatrix} \qquad (19)$$

The existence of $T(x)$, as seen in (18), is equivalent to requiring $\beta(x)$ to be invertible. By substituting for the parameters k_1 and k_2, condition (17) becomes

$$K_e + (L_d - L_q)\, i_d \neq 0 \qquad (20)$$

This condition is likely to always hold, since in general $K_e >> (L_d - L_q)$. However, for further assurance, a stabilizing controller is chosen to drive i_d to zero. To achieve the desirable dynamic response for the overall system, the state feedback control

$$\upsilon(t) = H\, y(t) \qquad (21)$$

is considered, where

$$H = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 & 0 \\ 0 & 0 & 0 & 0 & h_5 \end{bmatrix} \qquad (22)$$

and its elements are appropriately chosen based on a desirable reference model. The characteristic equation of the reference model, for example, may be specified by

$$F(\lambda) = (\lambda+h_5)(\lambda^2+2\xi\omega_1+\omega_1^2)(\lambda^2+2\xi\omega_2+\omega_2^2) \quad (23)$$

Once the feedback control law for the linear system has been specified, the nonlinear control can be computed by

$$u(t) = \beta^{-1}(x)\,\{\, HT(x) - \sigma(x) \,\} \qquad (24)$$

[1] $[f,G_1]$, (ad^2f,G_1), and (ad^3f,G_1) denote the Lie brackets of vector fields and are defined as follows:

$$[f,G_1] = (ad^1f,G_1) = \frac{\partial G_1}{\partial x}f - \frac{\partial f}{\partial x}G_1 \quad \text{and} \quad (ad^kf,G_1) = [f, (ad^{k-1}f,G_1)]$$

As illustrated in the simulation results in section V, the proposed control law of this section behaves well even if the system is subject to significant payload uncertainties and modeling errors, provided that accurate measurements of acceleration are available. If, however, estimated acceleration information are used, which is the case in most practical situations, the performance of the system may be drastically degraded. To account for modeling, payload, and measurement errors, a robust control law will be presented in the following section.

IV. ROBUST CONTROL

In this section a nonlinear robust controller is designed and appended to the control law of the previous section. The overall controller shall be used for position tracking control of the direct-drive arm actuated by a BLDCM. The formulation of the robust controller follows the results of the work pioneered by Gutman[3] and Leitman[10] and later extended by Ha and Gilbert[4]. It will be assumed that the uncertainties in the system are deterministic and bounded.

The overall controller is specified to be

$$u(t) = u_n(t) + \Delta u(t) \quad (25)$$

where $u_n(t)$ corresponds to the control law of section III and $\Delta u(t)$ represents the correction term which will make the system robust. In the absence of system uncertainties, the control $u_n(t)$ will provide good dynamic response. $\Delta u(t)$, a saturating function, will guarantee robustness in the presence of uncertainties. Assuming that there are uncertainties in the mathematical model of the system, to distinguish between the actual system and the system model in our previous derivations, the system model is represented by $f^*(x)$ and $G^*(x)$, and the following are defined

$$\Delta f = f - f^* \quad (26)$$

$$\Delta G = G - G^* \quad (27)$$

The derivation of the robust controller proceeds as follows. One of the basic assumptions needed is what is usually known as the matching condition(s). The assumption is that the dynamics of the system are affected by the control input in the same manner as the uncertainties are affected. To enforce this assumption, we will introduce Δf^* and ΔG^* which satisfy the following conditions

$$\frac{\partial T}{\partial x}\Delta f = B\Delta f^* = B\begin{bmatrix} \Delta f_1^* \\ \Delta f_2^* \end{bmatrix} \quad (28)$$

$$\frac{\partial T}{\partial x}\Delta G = B\Delta G^*\beta = B\begin{bmatrix} \Delta G_1^* & \Delta G_2^* \\ \Delta G_3^* & \Delta G_4^* \end{bmatrix}\beta \quad (29)$$

Through the application of the transformation $T(x)$, conditions (28) and (29) become

$$(A-A^*)T(x) = \Delta A T(x) = B\,[\,\Delta f^* - \Delta G^*\alpha\,] \quad (30)$$

$$(B-B^*) = \Delta B = B\Delta G^* \quad (31)$$

By imposing the matching conditions, (30)-(31), two things have been achieved. First, the system uncertainties have now been imbedded in Δf^* and ΔG^* and second, it is shown that the uncertainties are affected by the matrix B the same way as the input to the system. Applying conditions (30) and (31) to the system under consideration, we get

$$\frac{d\Delta y}{dt} = B\,(\Delta G^*\,\beta\,u_n + \Delta f^*) \quad (32)$$

Having assumed bounded uncertainties, we can define ϕ such that

$$\phi \geq \|\,\Delta G^*\,\beta\,u_n + \Delta f^*\,\| \quad (33)$$

to provide a measure for the bound on uncertainties. Assuming that accurate measurements for y_1, y_2, and y_3 are available, a realistic assumption, then

$$\Delta f_1^* = -(k_1 + k_2x_5)\,\Delta f_4 - k_2x_4\,\Delta f_5 \quad (34)$$

$$\Delta f_2^* = \Delta f_4 \quad (35)$$

$$\Delta G_1^* = \delta q_1 \quad (36)$$

$$\Delta G_2^* = k_2x_4(\delta q_1 - \delta q_2) \quad (37)$$

$$\Delta G_3^* = 0 \quad (38)$$

$$\Delta G_4^* = \delta q_2 \quad (39)$$

where $\delta q_1 = \Delta q_1/q_1$ and $\delta q_2 = \Delta q_2/q_2$.

At this point we have developed a set of explicit formulae for the bounds imposed on modeling errors, which can be estimated in quantitative terms. For example, δq_1 and δq_2 express bounds for the percentage errors in inductance values. The correction term, Δu, in the control law which should provide robustness will be defined in terms of the uncertainty bound, ϕ, and a saturating function as follows.

$$\Delta u = -\phi\,\beta^{-1}\,\eta(\zeta) \quad (40)$$

where

$$\eta(\zeta) = \begin{cases} \zeta & \text{if } \|\zeta\| \leq 1 \\ \zeta/\|\zeta\| & \text{if } \|\zeta\| \geq 1 \end{cases} \quad (41)$$

Furthermore,

$$\zeta = \pi\,\phi\,B^T\,P^{-T}\,P^{-1}\,y \quad (42)$$

where P is the matrix whose columns are the eigen-vectors of the matrix (A+BH), and π is a parameter which can be chosen to alter the bound on the tracking error. As will be shown in section V, the controller presented above will provide bounded tracking errors in the presence of uncertainties.

V. SIMULATION RESULTS

In this section the proposed control schemes of sections III and IV are examined through computer simulations. We will start with the assumption that accurate measurements of states, including acceleration measurements, are available. However, it is expected that there will exist payload and modeling uncertainties which the controller has to overcome. Figure 2 shows the time history of position error when the payload is to travel along a cubic trajectory, and when the nonlinear control of section III is used, i.e. the control law does not include any "robust" term. The payload inertia has been considered to have been either underestimated or overestimated. Figure 3 illustrates the performance of the system subject to uncertainties in payload and motor models, when the acceleration information has been estimated based on the approximate system model. Obviously, the performance of the control law has deteriorated since inaccurate acceleration measurements have been used. To alleviate the problem associated with inexact acceleration measurements, the robust control term, derived in section IV, is appended to the control law used in the simulations above. Figure 4 depicts the performance of the robust controller, when $\Delta f_1 = \Delta f_2 = 10^{-5}$, $\delta q_1 = \delta q_2 = 0.35$, and $\pi = 10^{10}$. It is evident from the figure that the tracking error profile has been improved and the error envelope has been significantly reduced.

VI. CONCLUSIONS

We have studied a direct-drive robotic arm system directly coulped to a BLDCM which is capable of producing large torques for high acceleration and deceleration rates. The complete dynamics of the motor and the arm have been combined in investigating the tracking control problem associated with the system. A nonlinear control law was derived which behaves well even when there are significant modeling and payload inertia uncertainties. The behavior of this control law, however, was shown to deteriorate when accurate measurements were not available. To alleviate this problem a correction term was appended to the nonlinear controller to ensure the robustness of the system. It was demonstrated that by appropriately choosing maximum bounds on the uncertainties in the system, favorable results are accomplished. Further investigation through simulation results has indicated that it is possible to create undesirable oscillations in the system if the control law is not properly defined.

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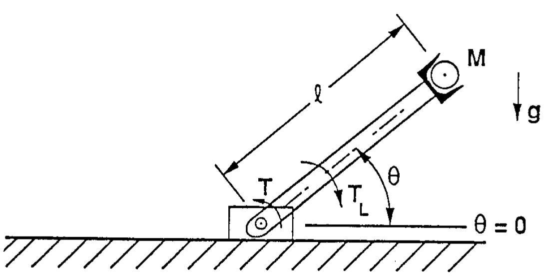

Figure 1: Direct-drive arm actuated by BLDCM, l = 1.0 m, M = 2.0 Kg.

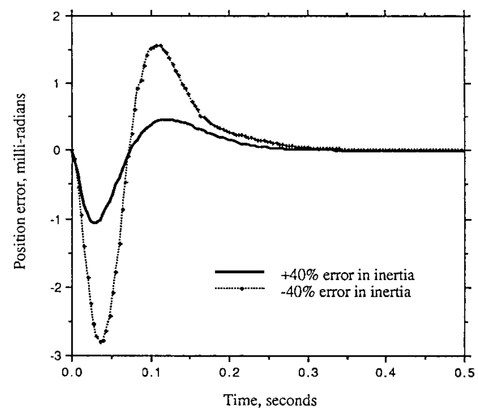

Figure 2: Time history of position error in the presence of payload uncertainties, using accurate acceleration measurements.

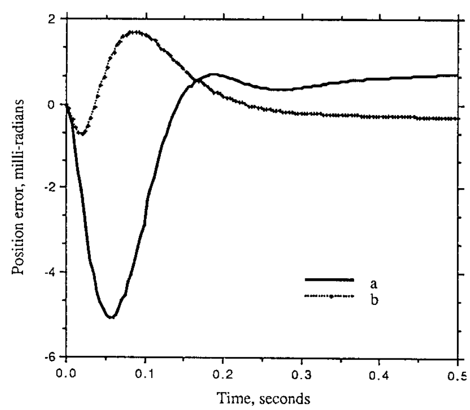

Figure 3: Time history of position error using estimated acceleration measurements; a) -40% error in inertia, +30% error in resistance, b) +40% error in inertia, -30% error in resistance.

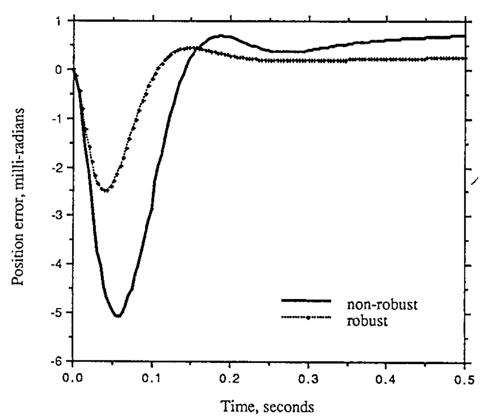

Figure 4: Time history of position error using estimated acceleration in the presence of -40% error in inertia & +30% error in resistance.