

**Program Title:**

Meta-didactical Slippages in a Ninth Grade Mathematics Classroom: A Paradox of Teaching

**Abstract:**

This paper examines (a) the nature of meta-didactical slippages that occurred in a ninth grade predominantly African American mathematics classroom; and (b) how these meta-didactical slippages affect students' conceptual understanding on a unit of ninth grade mathematics. A qualitative case study that employed ethnographic techniques of data collection and analysis was conducted. The theory of didactical situations in mathematics (Brousseau, 1997) served as the lens that grounded the interpretation of the data. The study found four themes, which illustrated the nature meta-didactical slippages: (a) over-teaching, (b) situational bypass, (c) language and symbolic representation, and (d) the design of didactical situations.

**Descriptors:**

Mathematics Education, Didactical Situations, Slippages, Qualitative Case Study

**Objective/Purpose**

Mathematics teaching is a complex practice, because teachers have to balance multiple goals and constraints as they decide "how to respond to students' questions, how to represent a given mathematical idea, how long to pursue discussion of a problem, or how to make use of available technologies to develop the richness of an investigation" (Martin & Herrera, 2007, p. 18). Mathematics teachers are also responsible for developing students' mathematical reasoning skills. Mathematical reasoning or learning occurs within a context that is determined by a set of implicit and explicit rules, circumstances, and interactions among several systems such as the teacher system, the student system, and the milieu (Brousseau, 1997). Despite these complexities, a significant responsibility is placed on the teacher to ensure that students are able to do mathematics. Thus mathematics teachers have to create meaningful didactical situations, in order to facilitate the process of doing mathematics. Nevertheless, it is in the didactical situations that complexities and inherent difficulties of the teaching and learning process occur.

According to National Council of Teachers of Mathematics (NCTM) (Mathematics, 2000), "a significant challenge to realizing the vision portrayed in *Principles and Standards* is disengagement" (p. 371). Moreover, disengagement is often reinforced in both overt and subtle ways by the attitudes and actions of adults who have influence over students. For instance, in a study on the influence of classroom practice on the development of subject-matter understanding, Schoenfeld (1988) argued that "despite the fact that the class was well taught, and the students did well on relevant performance measures, the students learned some inappropriate and counterproductive conceptions about the nature of mathematics, as a direct result of the instruction" (p. 146). Research carried out on the teaching of rational numbers during the period 1970-1980, uncovered several phenomena connected with the teaching and learning of mathematics (Brousseau, 1997). These phenomena which occur from the interplay of relationships and constraints

between the teacher, students, and mathematical content, may produce certain unwanted effects. Although these effects are inappropriate for the learning, they are often inevitable, and sometimes unknown (Brousseau, 2008; Schoenfeld, 1988). In this study, I argue that in order to improve students' performance in problem solving situations, we need a better understanding of the didactical situations in the mathematics classroom.

### **Purpose and Research Questions**

The purpose of this study was to (a) understand the nature of meta-didactical slippages and how meta-didactical slippages occur in a ninth grade predominantly African American mathematics classroom; and (b) describe the consequence of meta-didactical slippage on a unit of study of ninth grade mathematics.

The following questions guided the study:

1. What is the nature of meta-didactical slippages that emerge in the practice of teaching mathematics?
2. How did the teacher perceive these meta-didactical slippages affect teaching and learning of a unit of analysis of ninth grade mathematics?

### **Theoretical Perspective**

The philosophical stance of this study was rooted in the perspective of cultural anthropology. This perspective views culture as a set of cognitive structures that children learn as they grow up in a particular community, and that they use to make decisions about their own behaviors and that of the people around them. This stance spawned the theory of didactical situations in mathematics (Brousseau, 1997), which is the theoretical frame used in this study. Meanings are constructed in a social situation, and the meanings change from culture to culture and from individual to individual. The theory of didactical situations in mathematics helped me to isolate particular meanings that teachers and students construct in didactical situations in the mathematics classroom (see chapter 1 for a more detailed discussion).

This study was grounded in a social constructionist epistemology. Mathematics teachers and students construct meanings from both the situation and from the act of teaching and learning mathematics. Moreover, according to Crotty (1998), because of the essential relationship that human experience bears to its object, no object can be adequately described in isolation from its conscious being experiencing it, nor can any experience be adequately described in isolation from its object.

Constructionist epistemology consists of at least two schools of thought. These schools of thought are sometimes called empirically oriented constructivism and radically oriented constructivism. The former holds that knowledge is anchored in the external environment and exists independently of the learner. The latter maintains that knowledge resides in the constructions of the subject. In this study, I follow the latter.

### **Study Design**

This study was a descriptive, qualitative, case study conducted in one, ninth grade mathematics classroom over a 15-week period. The case study was selected because it is well suited to take into account the complexity of didactical interactions between teacher, student, and the content of mathematics. These interactions are intangible, yet the impact of these interactions on the student, the teacher, the institution, and hence the society at large, are very tangible. The case study is grounded in the lived reality and can help us to understand complex inter-relationships (Hays, 2004). Furthermore, the case study

according to Hays (2004) seeks to answer focused questions by producing in-depth descriptions and interpretations over a short period of time. Thus, in order to probe beneath the surface of the didactical situations, and to get a better understanding of meta-didactical slippages, the qualitative case study methodology is well suited.

Qualitative researchers use multiple methods. The use of multiple methods reflects the researchers' aim to secure an in-depth understanding of the phenomenon in question. This process of using multiple methods is referred to as triangulation in the literature (Berg, 2009; Cohen, Manion, & Morrison, 2013; DeMarrais & Lapan, 2004; Denzin & Lincoln, 2005; Hays, 2004; Yin, 2002). According to Denzin and Lincoln (2005), this combination of multiple methodological practices, empirical materials, perspectives, and observers is understood as a strategy that adds depth, rigor, breadth, complexity, and richness to an inquiry. Since didactical situations in the mathematics classroom are examples of a complex situation that involves multiple representations, the qualitative case study is an ideal research methodology.

The case study is one of the many strategies of inquiry that the qualitative bricoleur can use to conduct research. According to Yin (2009) a rationale for selecting the case study is when the researcher is studying contemporary events but "the relevant behaviors cannot be manipulated" (p. 11). Additionally, the research questions, that is to say the substance (what the research is about) and form ("who", "what", "where", or "how", questions) of the research questions, provide a good rationale for choosing the case study. The case study recognizes and accepts that there can be many factors operating in a single case. Accordingly, many types of data can be incorporated into a case study such as interviews, participant observations, documents, and quantitative data to provide rich and vivid descriptions of events relevant to the case.

By studying didactical situations in the mathematics classroom, my aim was to describe the real-life, complex dynamic unfolding interaction of the phenomena in its natural occurring environment. Consequently, the qualitative case study is well suited to study didactical situations in mathematics classrooms.

The descriptive case study design required that the researcher presents a priori, a descriptive theory, which served as a framework for the study (Yin, 2009). In this study the theory of didactical situations in mathematics served as the descriptive theoretical framework that guided the study (Brousseau, 1997). Once the study is grounded in a theoretical framework, Yin (2009) identified five components of the research design: (1) the studies questions; (2) the studies propositions, if any; (3) the study's unit(s) of analysis; (4) the logic linking the data to the propositions; and (5) the criteria for interpreting the findings. In this study no propositions was formulated, because the primary goal was to describe the phenomenon as it occurred in its natural environment.

**Data collection and analysis.** To answer the research questions proposed, I used four data collection techniques: (a) collection of document artifacts, which included student work samples and teacher lesson plans; (b) direct observation (c) open ended interviews, conducted with the teacher; and (d) researcher introspection. Data collection instruments include the interview protocol, the observation log, and the documents artifacts.

In order to achieve a more fine grained analysis two analytic techniques were used: ethnographic analysis using Spradley's (1998) model and discourse analysis using Gee's (2011) model. Episodes from the classroom were coded using the theory of

didactical situations in mathematics to guide the construction of codes. A summary of my analytic procedure is shown in Figure 4. In order to maintain focus throughout the analysis, I asked the following questions of the data: (a) what is the genesis of these slippages? (b) how may this slippage be identified? (c) what are their attributes? (d) what are their affordances? (e) can they be predicted? and (f) how can they be controlled if possible? I used a combination of hand coding and computer qualitative software coding. I used the ATLAS.ti qualitative software to manage the data files and to retrieve codes quickly. A summary of the analytic procedure is shown in Figure 1.

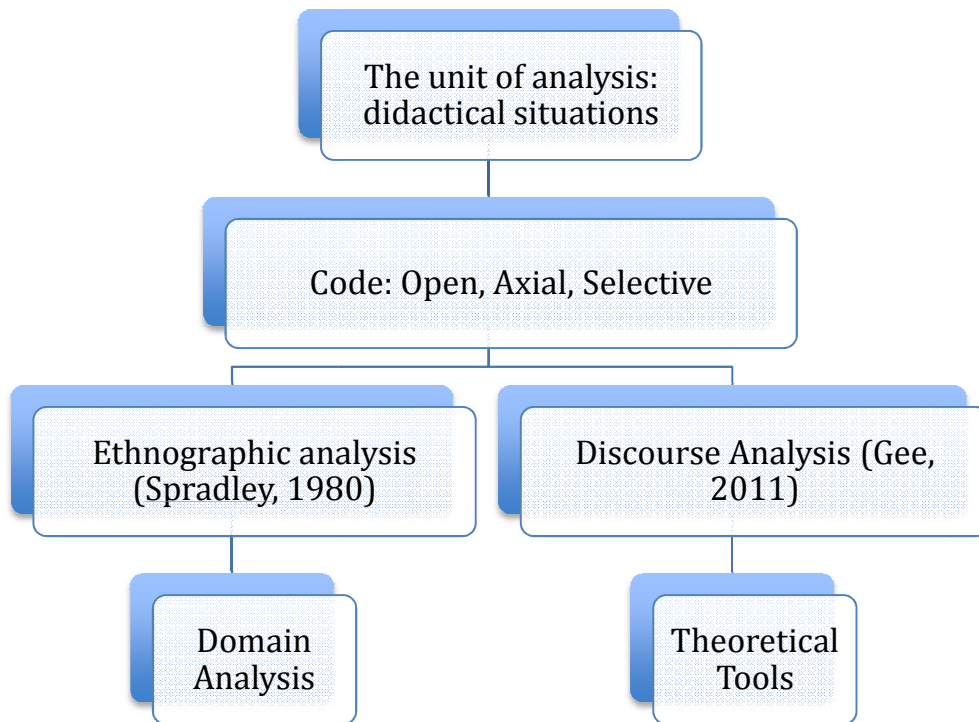


Figure 1. Summary of analytic procedure followed in this study.

## Results

In this study four themes emerged as illustrative of the nature of meta-didactical slippages: (1) over-teaching, (2) situational bypass, (3) language and symbolic representation, and (4) the design of didactical situation. Each theme emerged as an instance of meta-didactical slippage. The findings further supported that meta-didactical slippages manifest as paradox of the teaching endeavor.

## Implications for Actions

Whereas a single case study cannot provide a sound basis for the practice of teaching and learning in the mathematics classroom, this study would suggest that teachers should be more purposive in how and when they intervene in problem situation in the mathematics classroom. This is so that they do not replace an initial mathematical situation that would have permitted an authentic activity on the part of the student, by a study of the mathematical circumstances, or by reducing the cognitive demand of the task.

A second implication of this study is that the results of research on didactical situation be disseminated to mathematics teachers. I recommend that the results of this study (and other studies with similar findings) be included in professional development for mathematics teachers so that they can become aware of the phenomenon of meta-didactical slippages. The findings showed that the mathematics classroom is a very complex and highly nuanced community. Thus the increased awareness of the phenomenon should influence teachers' didactic decisions as they plan and implement mathematical lessons. In this way, the teacher is more sensitive to resist desire to take all mathematics activities as an object of teaching.

Finally, it is recommended for school districts to provide professional development for mathematics teachers to learn how to design didactical situations. Moreover, I recommend continuous discourse among mathematics teachers on the design and implementation of didactical situations.

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