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Optimization of a linear controller using dynamic back-propagation

Optimización de un controlador lineal empleando dynamic back-propagation

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Abstract: This paper presents the optimization of a linear controller for a DC motor using Dynamic Back-Propagation algorithm. This algorithm is commonly employed for neural networks training; however, it can be used for optimization of a linear controller. The results show a satisfactory controller optimization.

Keywords: Controller, DC motor, Dynamic, Optimization.

Resumen: En este documento se presenta la optimización de un controlador lineal para un motor DC mediante el algoritmo ‘‘*Dynamic Back-Propagation*’’. Este algoritmo es comúnmente utilizado para el entrenamiento de redes neuronales, sin embargo, puede ser empleado para la optimización de un controlador lineal. Los resultados muestran que la optimización del controlador es satisfactoria.

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Palabras clave: Controlador, Motor DC, Dinámico, Optimización.

1. Introduction

In general, many applications like robotics, servomechanisms, control, and automation normally use motors with Direct Current DC; for instance, an application for implementing a solar tracker with a DC motor can be seen is [1] and [2]. On the other hand, the Dynamic Back-Propagation (DBP) is used for training of neural networks aiming the incorporation the same characteristics as those of dynamic systems, [3] and [4]. Such an algorithm is suitable in the application of the chain rule of the descending gradient method characterized for having a fast convergence [4].

About works carried out using the DBP algorithm, [5] presents an adaptive processing system consisting of a digital filter based on a neural network. This work focuses on the online training algorithms to achieve an association between the characteristics of the input signal of the neural network, and the dynamic responses of the digital filter. For this, a Dynamic Back-Propagation algorithm is developed to train the closed loop network between the output of the digital filter and the inputs to the neural network.

Meanwhile, reference [6] describes a single input-output adaptive neuronal network controller scheme and the training algorithm. For the implementation of the system, a modification of the traditional Back-Propagation algorithm is developed. The proposal is made considering applications for time-variant systems.

A paper that considers the dynamic stability of the neural network (according to Lyapunov) during the training process can be seen in [7]. In this work, to avoid unstable phenomena during the learning process, multiplier and restricted rate learning schemes are proposed. In the

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multiplier method, explicit stability conditions are introduced in the iterative error index and the update equations contain a set of inequality constraints. In the restricted learning rate algorithm, such rate is updated at each iterative instant by an equation derived from the stability conditions. According to [8], adaptive control with reference model is commonly used in the design of controllers based on traditional neural networks, where it often requires a plant emulator when the neural controller is connected to the plant. In this work, the authors propose a plant emulator using a neuro-fuzzy system and a variation of the DBP algorithm to train the neuronal controller. The system is employed to the control of a DC to DC converter.

On the other hand, [9] presents a general framework for dynamic neural networks reviewing two general algorithms for the calculation of Gradients and Jacobians for these dynamic networks: Backward Propagation Through Time (BPTT) and Real Time Recurrent Learning (RTRL). The results show that the BPTT algorithm is more efficient for Gradient calculations, while the RTRL algorithm is more efficient for Jacobian calculations.

Finally, for a general background, in [10] a review is made on different techniques for the training of neural networks, particularly the Dynamic Back-Propagation algorithm for recurrent neural networks.

This paper presents linear controller optimization for a DC motor through the Dynamic Back-Propagation algorithm. A model of the plant is obtained as a transfer function that later is transformed into a discreet time to implementing the optimization algorithm to determine the result of the process, where the simulation shows a satisfactory controller optimization.

The document is organized as follows: first, the dynamic model to a DC motor and the Dynamic Back-Propagation algorithm theory are revised; second, the architecture of the implemented

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controller is also checked. The following aspect is related to the model for the motor in discreet time which is used for implementing the Dynamic Back-Propagation algorithm. The results and conclusions are finally displayed.

2. DC Motor Model

The model of the motor mainly consists of a mechanical and an electrical part [11], as shown in Figure 1.

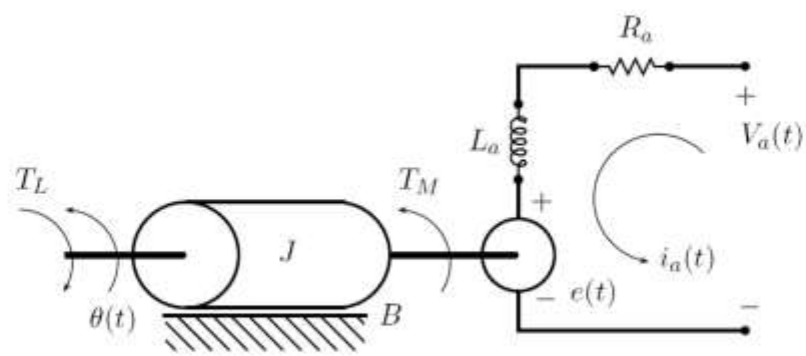

Figure 1. DC Motor scheme. Source: adapted of [11].

For the model in Figure 1, the motor torque is proportional to the armature current as presented in equation (1).

$$T_M = K_T i_a(t) \tag{1}$$

The counter-electromotive force is proportional to the motor angular velocity, according to equation (2).

$$e(t) = K_e \omega(t) = K_e \frac{d\theta}{dt} \tag{2}$$

Equation (3) represents the electrical part.

$$V_a(t)\text{-}e(t) \;=\; L_a \frac{di_a(t)}{dt} + R_a i_a(t) \tag{3}$$

The mechanical part is given by equation (4).

$$T_M(t)\text{-}T_L(t) \;=\; J \frac{d\theta^2(t)}{dt^2} + B \frac{d\theta(t)}{dt} \tag{4}$$

A complete plant dynamic model may be impractical in some applications given the severity to establishing the parameters. An alternative consists of a simplified model via plant parameter identification [12].

Equation (5) describes the plant simplified transfer function corresponding to a first-order system with an integrator where the variable s is associated with the Laplace transform.

$$G(s) = \frac{K}{s(\tau \cdot s + 1)} \tag{5}$$

The parameter identification to this plant is performed considering time and velocity average values as shown in [1] and [12].

In equation (5), K corresponds to the input-output relation in stable state that for the specific case corresponds to $K = \Delta\omega / \Delta V$ where ω is the angular velocity; with a 12 voltage, the average time taken by the system to rotate an angle of $\pi / 2$ is 0.06 rad/sec; thus, the corresponding K parameter value is 0.005 rad/(Vsec) [12].

The parameter τ can be determined as a quarter part of the time to obtain a steady-state output, which is 1.53 seconds; thus, for τ 0.38 seconds are taken [12]. This way, the model of the plant is given by equation (6).

$$G(s) = \frac{0.005}{s(0.38s + 1)} \tag{6}$$

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3. *Dynamic Back-Propagation* Algorithm

This algorithm is used in neural networks for identification and control of dynamic systems; specifically, when having a parallel-type scheme [13]. Figure (2) provides a representation of this architecture for identification process.

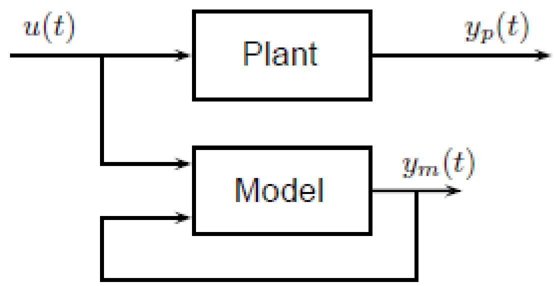

Figure 2. Parallel identification scheme. Source: adapted of [13].

The parallel identification scheme uses plant inputs and the output feedback from the network itself.

In control applications, one of the schemes of neural networks uses one network to model the plant and another to the controller. This approach allows first the plant identification and later the controller training is performed aiming the output of the system to follow the reference [3]. Figure (3) displays the two neural networks used in this process.

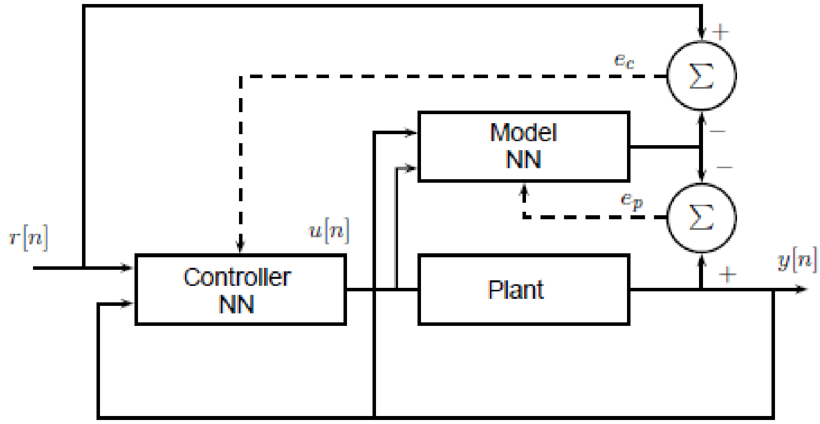

Figure 3. Control scheme using neural networks. Source: adapted of [3].

Thus, for the plant model, there is an input $u[n]$ and output $y[n]$ signal in a way that the result is a structure given by equation (7).

$$y[n] = f_p(y[n-1], y[n-2],..., y[n-p], u[n], u[n-1],..., u[n-q]) \quad (7)$$

Meanwhile, the inputs for the controller are given by the reference $r[n]$ and the process signal measured $y[n]$, while the output is given by the control action $u[n]$ as shown in equation (8), where p, q and m corresponds to the number of outputs, inputs and reference delays. Typically, the number of delays increases according to the order in the plant [3].

$$u[n] = f_c(y[n],..., y[n-p], r[n],..., r[n-m], u[n],..., u[n-q]) \quad (8)$$

4. Controller architecture

Figure (4) represents the considered control scheme; both the plant and the controller are models in discrete-time.

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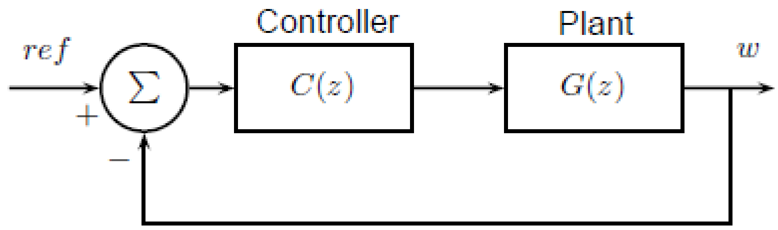

Figure 4. System simplified architecture. Source: own.

In discrete-time, the transfer function of the controller is given by equation (9).

$$C(z)=\frac{U(z)}{E(z)}=\frac{B_0+B_1z^{-1}+B_2z^{-2}}{1+A_1z^{-1}+A_2z^{-2}} \tag{9}$$

Thereby, the controller difference equation is (10). Under this approach the general action of the controller corresponds to the equation (11).

$$u[n]=B_0e[n]+B_1e[n-1]+B_2e[n-2]-A_1u[n-1]-A_2u[n-2] \tag{10}$$

$$u[n]=f_c(e[n],...,e[n-N_e],u[n-1],...,u[n-N_u],\mathbf{H}_c) \tag{11}$$

Similarly, the output for the controller training is given by equation (12).

$$y[n]=f_p(y[n-1],...,y[n-N_y],u[n],...,u[n-N_u],\mathbf{H}_p) \tag{12}$$

Where N_y corresponds to the total number of output samples, N_u the number of samples in the inputs, and $\mathbf{H}_p$ the parameters vector of the plant model. Meanwhile, the controller parameters set is given by equation (13).

$$\mathbf{H}_c=[B_0,B_1,B_2,A_1,A_2] \tag{13}$$

The adaptation (optimization) of parameters in the controller is made with equation (14) using a learning rate η.

$$\mathbf{H}_c(k+1) = \mathbf{H}_c(k) - \eta \frac{\partial J(k)}{\partial \mathbf{H}_c(k)} \tag{14}$$

The adjustment function J used in equation (14) is defined by (15). The variation of J with respect to the controller parameters can be calculated using equation (16).

$$J = \frac{1}{2} P\left(r[n] - y[n]\right)^2 + \frac{1}{2} Q\left(u[n]\right)^2 \tag{15}$$

$$\frac{\partial J}{\partial \mathbf{H}_c[n]} = \frac{\partial J}{\partial y[n]} \frac{\partial y[n]}{\partial \mathbf{H}_c[n]} \tag{16}$$

5. Plant implementation in discreet time

The transfer function of the plant is described by equation (17).

$$G(s) = \frac{0.005}{s(0.38s+1)} \tag{17}$$

Thus, transforming this transfer function to discreet time considering a sampling time of $T_s = 0.1s$, and using the method of bilinear transformation is obtained the equation (18). In general, this transfer function can be represented using equation (19).

$$G(z) = 10^{-5} \frac{2.907z^2 + 5.814z + 2.907}{z^2 - 1.767z + 0.7674} \tag{18}$$

$$G(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{19}$$

Finally, expression (20) represents the difference equation of the plant, which is utilized to establishing the parameter expressions to perform the controller training.

$$y[n] = b_0 u[n] + b_1 u[n-1] + b_2 u[n-2] - a_1 y[n-1] - a_2 y[n-2] \tag{20}$$

6. Equations in discrete-time to optimization algorithm implementation

Implementing the Dynamic Back-Propagation algorithm requires the difference equations of each parameter to be used in equations (14), (15), and (16). In [14] the application of this algorithm for the training of neural networks can be seen.

First, using (20) the error equation is (21).

$$e[n] = r[n] + a_1 r[n-1] + a_2 r[n-2] - a_1 e[n-1] - a_2 e[n-2] - b_0 u[n] - b_1 u[n-1] - b_2 u[n-2] \quad (21)$$

Meanwhile, the controller difference equation is (22).

$$u[n] = -A_1 u[n-1] - A_2 u[n-2] + B_0 e[n] + B_1 e[n-1] + B_2 e[n-2] \quad (22)$$

Consequently, the training equations of the parameter A_1 are (23).

$$\begin{gathered} \frac{du}{dA_1}[n] = -u[n-1] - A_1 \frac{du}{dA_1}[n-1] - A_2 \frac{du}{dA_1}[n-2] + B_0 \frac{de}{dA_1}[n] + B_1 \frac{de}{dA_1}[n-1] + B_2 \frac{de}{dA_1}[n-2] \\ \frac{de}{dA_1}[n] = -a_1 \frac{de}{dA_1}[n-1] - a_2 \frac{de}{dA_1}[n-2] - b_0 \frac{du}{dA_1}[n] - b_1 \frac{du}{dA_1}[n-1] - b_2 \frac{du}{dA_1}[n-2] \end{gathered} \quad (23)$$

For parameter A_2 is given by (24).

$$\begin{gathered} \frac{du}{dA_2}[n] = -u[n-2] - A_1 \frac{du}{dA_2}[n-1] - A_2 \frac{du}{dA_2}[n-2] + B_0 \frac{de}{dA_2}[n] + B_1 \frac{de}{dA_2}[n-1] + B_2 \frac{de}{dA_2}[n-2] \\ \frac{de}{dA_2}[n] = -a_1 \frac{de}{dA_2}[n-1] - a_2 \frac{de}{dA_2}[n-2] - b_0 \frac{du}{dA_2}[n] - b_1 \frac{du}{dA_2}[n-1] - b_2 \frac{du}{dA_2}[n-2] \end{gathered} \quad (24)$$

For parameter B_0 the training equations are (25).

$$\begin{gathered} \frac{du}{dB_0}[n] = e[n] - A_1 \frac{du}{dB_0}[n-1] - A_2 \frac{du}{dB_0}[n-2] + B_0 \frac{de}{dB_0}[n] + B_1 \frac{de}{dB_0}[n-1] + B_2 \frac{de}{dB_0}[n-2] \\ \frac{de}{dB_0}[n] = -a_1 \frac{de}{dB_0}[n-1] - a_2 \frac{de}{dB_0}[n-2] - b_0 \frac{du}{dB_0}[n] - b_1 \frac{du}{dB_0}[n-1] - b_2 \frac{du}{dB_0}[n-2] \end{gathered} \quad (25)$$

Similarly, for parameter B_1 is (26).

$$\frac{du}{dB_1}[n] = e[n-1] - A_1\frac{du}{dB_1}[n-1] - A_2\frac{du}{dB_1}[n-2] + B_0\frac{de}{dB_1}[n] + B_1\frac{de}{dB_1}[n-1] + B_2\frac{de}{dB_1}[n-2] \quad (26)$$

$$\frac{de}{dB_1}[n] = -a_1\frac{de}{dB_1}[n-1] - a_2\frac{de}{dB_1}[n-2] - b_0\frac{du}{dB_1}[n] - b_1\frac{du}{dB_1}[n-1] - b_2\frac{du}{dB_1}[n-2]$$

Finally, the training equations of the parameter B_2 are (27).

$$\frac{du}{dB_2}[n] = e[n-2] - A_1\frac{du}{dB_2}[n-1] - A_2\frac{du}{dB_2}[n-2] + B_0\frac{de}{dB_2}[n] + B_1\frac{de}{dB_2}[n-1] + B_2\frac{de}{dB_2}[n-2] \quad (27)$$

$$\frac{de}{dB_2}[n] = -a_1\frac{de}{dB_2}[n-1] - a_2\frac{de}{dB_2}[n-2] - b_0\frac{du}{dB_2}[n] - b_1\frac{du}{dB_2}[n-1] - b_2\frac{du}{dB_2}[n-2]$$

7. Results

The reference of the system is taken for 45° which is an angular position of $\pi/4$ radians (0.7854). The used learning rate is $\eta = 0.01$ and 35 iterations are performed for the training process of the controller parameters. A proportional controller behavior is considered as initial configuration; therefore, its parameters are assigned the zero value excluding B_0 which is taken equal to 10 (under previous experimentation). Initial and final (optimized) values of the controller parameters can be seen in Table 1.

Configuration	B_0	B_1	B_2	A_1	A_2
Initial	10	0	0	0	0
Final	10.0314	0.0306	0.0297	-0.4702	-0.4339

Table 1. Controller parameter values. Source: own.

Figure (5) presents different responses of the control system during the training process. This figure also shows the system output getting closer to the reference. Meanwhile, Figure (6) displays the values obtained of the objective function for each iteration, this allows to observe that the objective function tends towards no variation after iteration 20.

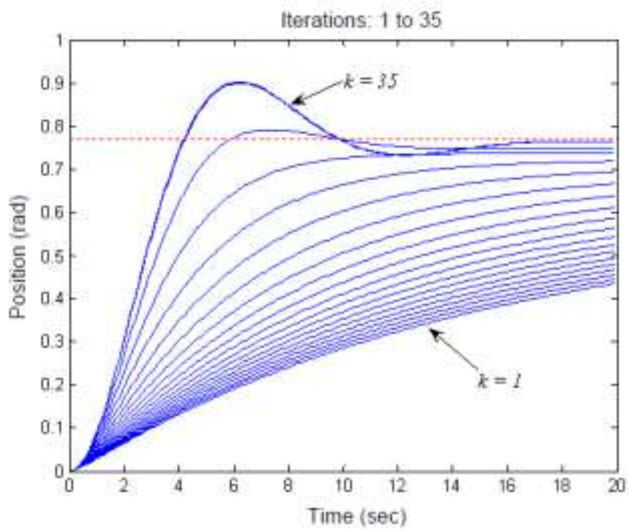

Figure 5. Responses of the system during the training process. Source: own.

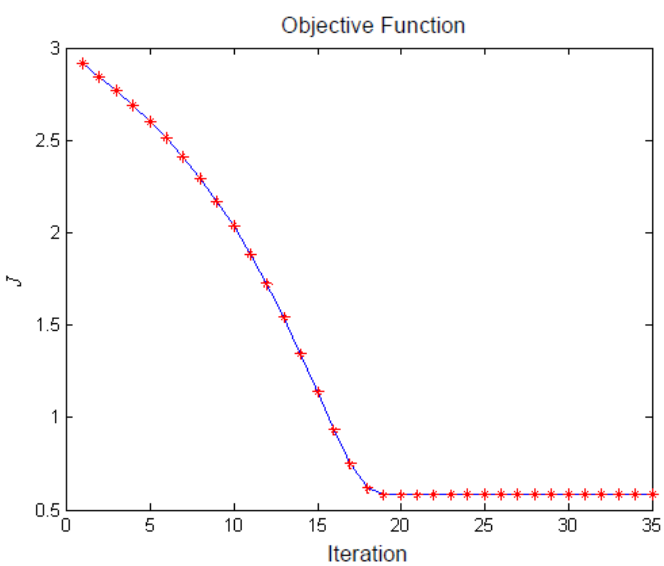

Figure 6. Objective function values during the training process. Source: own.

8. Conclusions

It was visible that the Dynamic Back-Propagation algorithm, which is conventionally used for the training of neural networks, can also be used to optimize a linear controller.

This mechanism offers an alternative for controller tuning in both linear and non-linear controllers and can also be used for schemes of supervised control.

The obtained results are satisfactory showing the response of the system during the training process as well as the evolution of the objective function.

In a further paper, this technique for a neuro-fuzzy type controller training is expected to be implemented.

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