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Modified Dispersion Relations and $\pi\pi$ Scattering*

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The $\pi\pi$ S-wave scattering-length predictions of Weinberg have been tested by using dispersion sum rules for the infinite-energy cross section. Reasonable agreement is obtained with the infinite-energy cross section ($\approx$15 mb) estimated from the factorization theorem for the Pomeranchon Regge residues. Experimental phase-shift data of Gutay *et al.*, Walker *et al.*, and Baton *et al.* are used in estimating the dispersion integrals. The analysis seems to rule out an $I=0$ S-wave scattering length $\gtrsim 0.4\mu^{-1}$.

THE question of S-wave $\pi\pi$ scattering lengths is of considerable theoretical interest in view of recent current-algebra[1–4] and dispersion-relation[5] calculations. We have tested the predictions of Weinberg[1] by using modified dispersion relations to calculate infinite-energy cross sections in various isospin channels. By demanding consistency among the equations and by using factorization of Regge residues,[6,7] we can put some limits on possible values for the scattering lengths. Although the experimental data on $\pi\pi$ scattering are rather uncertain,[8–12] our general conclusions are not very sensitive to the precise values for the phase shifts.

We normalize the forward $\pi\pi$ scattering amplitudes as follows:

$$\begin{aligned} \mathrm{Im}T_{00}(\omega) &= q\sigma_{\pi^0\pi^0}(\omega)\,,\\ \mathrm{Im}T_{+0}(\omega) &= q\sigma_{\pi^+\pi^0}(\omega)\,, \end{aligned} \tag{1}$$

where ω, q are, respectively, the laboratory energy and momentum; μ is the pion mass and the $\sigma(\omega)$'s are the total cross sections. In terms of isospin components,

$$\begin{aligned} I_{00} &= \tfrac{1}{3}T_0+\tfrac{2}{3}T_2\,,\\ T_{+0} &= \tfrac{1}{2}T_1+\tfrac{1}{2}T_2\,. \end{aligned} \tag{2}$$

By writing a Gilbert dispersion relation[13] and taking the limit $\omega\to\infty$, we easily derive

$$\sigma(\infty)=\sigma(0)+\frac{2}{\pi}\int_0^\infty \frac{\mathrm{Re}T(q)-\mathrm{Re}T(0)}{q^2}dq\,. \tag{3}$$

If we assume that $\mathrm{Re}T(q)$ for large q is dominated by the P' (f^0 meson) Regge trajectory, $\mathrm{Re}T(q)\sim q^{\alpha_{P'}}$ with $\alpha_{P'}<1$ and the integral in (3) is well defined.

We next evaluate (3) using Weinberg's scattering lengths[1] and the experimental data on phase shifts of Walker *et al.*[9] and of Baton *et al.*[11].

In terms of the S- and P-wave phase shifts, we have near threshold

$$\mathrm{Re}T_I(q)\approx(16\pi E/2\mu p)\cos\delta_I{}^0\sin\delta_I{}^0 \quad I=0,2 \tag{4}$$

and

$$\mathrm{Re}T_1(q)\approx(16\pi E/2\mu p)3\cos\delta_1{}^1\sin\delta_1{}^1\,, \tag{5}$$

where $\frac{1}{2}E$ and p are, respectively, the pion c.m. energy and momentum.

For c.m. energies below 625 MeV, we use the following expansions:

$$\begin{aligned} p\cos\delta_2{}^0 &= a_2{}^{-1}+\tfrac{1}{2}r_2p^2\,,\\ p\cot\delta_0{}^0 &= a_0{}^{-1}+\tfrac{1}{2}r_0p^2+bp^4+cp^6\,,\\ p^3\cot\delta_1{}^1 &= a_1{}^{-1}\,. \end{aligned} \tag{6}$$

The reason we use a different form of expansion for each phase shift is that we want to take the minimum number of parameters necessary to obtain a smooth fit to the experimental data.

The $\sigma(0)$ are related to the scattering lengths as follows:

$$\begin{aligned} \sigma_{00}(0) &= \tfrac{1}{3}(8\pi a_0{}^2+16\pi a_2{}^2)\,,\\ \sigma_{+0}(0) &= 4\pi a_2{}^2\,. \end{aligned} \tag{7}$$

Phase shifts given by the expansions in (6) and corresponding experimental phase shifts are displayed in Figs. 1 and 2. The Weinberg scattering lengths[1] and physical effective ranges[14] are used.

$$\begin{aligned} a_0 &= 0.2\mu^{-1}\,, \quad r_0=0.5\mu^{-1}\,.\\ a_2 &= -0.06\mu^{-1}\,, \quad r_0=0.5\mu^{-1}\,. \end{aligned} \tag{8}$$

The value of a_1 is taken as $0.054\mu^{-3}$ in order to give a smooth fit to the data of Baton *et al.*[11]

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[1] S. Weinberg, Phys. Rev. Letters 17, 616 (1966).
[2] N. Khuri, Phys. Rev. 153, 1477 (1967).
[3] J. Schwinger, Phys. Letters 24B, 473 (1967).
[4] For discussion of unitary corrections to the S-wave scattering lengths, see J. Sucher and C. H. Woo, Phys. Rev. Letters 18, 723 (1967); F. T. Meiere, Phys. Rev. 159, 1462 (1967); T. Akiba and K. Kang, Phys. Letters 25B, 35 (1967); J. Iliopoulis, Nuovo Cimento 52A, 192 (1967).
[5] F. T. Meiere and M. Sugawara, Phys. Rev. 153, 1702 (1967).
[6] M. Gell-Mann, Phys. Rev. Letters 8, 263 (1912).
[7] V. N. Gribov and I. Ya Pomeranchuk, Phys. Rev. Letters 8, 343 (1962); 8, 412 (1962).
[8] L. Gutay, P. B. Johnson, F. J. Loeffler, R. L. McIlwain, D. H. Miller, R. B. Willmann, and P. L. Csonka, Phys. Rev. Letters 18, 142 (1967).
[9] W. D. Walker, J. Carroll, A. Garfinkel, and B. Y. Oh, Phys. Rev. Letters 18, 630 (1967).
[10] P. B. Johnson, L. J. Gutay, R. L. Eisner, P. R. Klein, R. E. Peters, R. J. Sahni, W. L. Yen, and G. W. Tautfest, Phys. Rev. 163, 1497 (1967).
[11] J. P. Baton, G. Laurens, and J. Reignier, Phys. Letters 25B, 419 (1967).
[12] E. Malamud and P. E. Schlein, Phys. Rev. Letters 19, 1056 (1967).
[13] W. Gilbert, Phys. Rev. 108, 1078 (1957).

[14] For a physical discussion of this choice of effective range, see, e.g., the Appendix of Ref. 5.

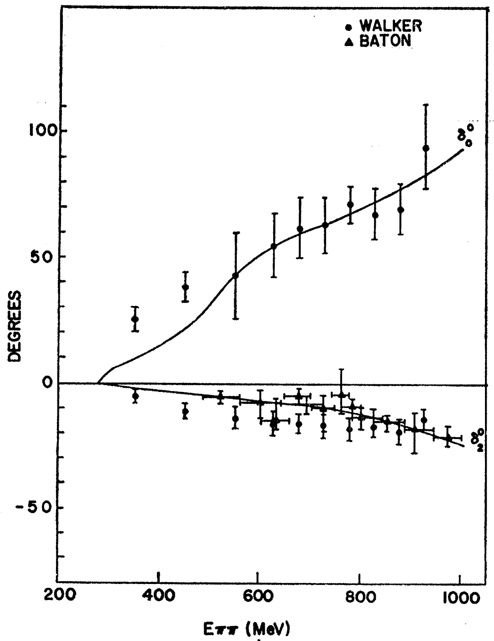

FIG. 1. The δ_0^0 and δ_2^0 phase shifts as a function of the total c.m. energy. The fit to these phase shifts is discussed in the text.

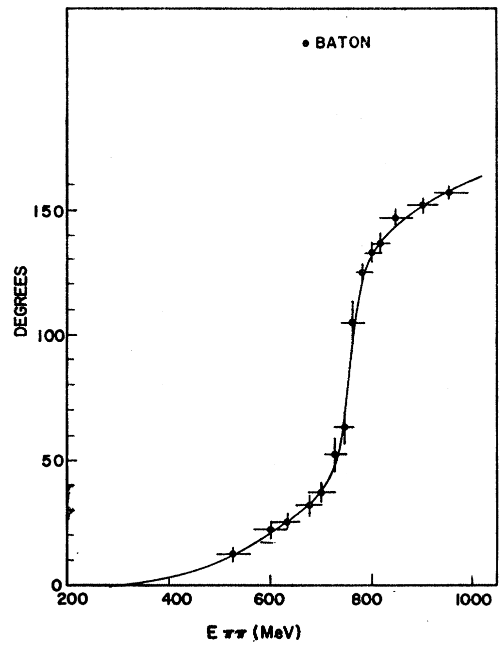

FIG. 2. The δ_1^1 phase shift. The fit to the phase shift is discussed in the text.

Above 1-BeV c.m. energy (3.45-BeV lab energy) we assume that $\mathrm{Re}T(\omega)$ is given by the P' exchange

$$\mathrm{Re}T(\omega)=(\gamma_{P'}/\tan\tfrac{1}{2}\pi\alpha_{P'})(\omega/\omega_0)^{\alpha_{P'}}. \tag{9}$$

In order to estimate $\gamma_{P'}$ we use the factorization theorem[6,7]

$$\gamma_{P'}{}^{\pi\pi}=(\gamma_{P'}{}^{\pi N})^2/\gamma_{P'}{}^{NN}. \tag{10}$$

Taking the usual value of $\alpha_{P'}\approx 0.6$ and $\omega_0=1$ BeV, we find[15]

$$\gamma_{P'}{}^{\pi N}=10.4 \text{ mb BeV},\quad \gamma_{P'}{}^{NN}\approx 62 \text{ mb BeV}, \tag{11}$$

yielding

$$\gamma_{P'}{}^{\pi\pi}\approx 1.7 \text{ mb BeV}. \tag{12}$$

With these assumptions we have calculated $\sigma(\infty)$ and the results are displayed in Table I. It can be seen that with these assumptions we obtain values for $\sigma(\infty)$ in reasonably good agreement with the prediction following from the factorization theorem for the Pomeranchon Regge residues,[6,7]

$$\sigma_{\pi\pi}(\infty)=\sigma_{\pi N}{}^2(\infty)/\sigma_{NN}(\infty)\approx 15 \text{ mb}. \tag{13}$$

TABLE I. Calculated contributions to $\sigma(\infty)$.

Contribution	$\sigma_{00}(\infty)$ (mb)	$\sigma_{+0}(\infty)$ (mb)
$\sigma(0)$	+ 7.9	+ 0.9
Integral, $2\mu<E<625$ MeV	+12.9	+17.0
Integral, 625 MeV$<E<1$ BeV	− 2.4	− 2.8
Integral, 1 BeV$<E<\infty$	− 0.1	+ 1.4
Total	18.3	16.5

[15] See, e.g., W. Rarita, R. J. Riddell Jr., C. Chiu, and R. J. Phillips, Phys. Rev. **165**, 1615 (1968).

The important point of this calculation is that most of the contribution to the integral comes from below 625 MeV and so depends strongly on the scattering lengths used. It is not very sensitive to the effective ranges used however; it was found that varying the effective ranges over the interval $0.25\mu^{-1}<r_{0,2}<0.75\mu^{-1}$ changed the value of $\sigma(\infty)$ by less than 0.5 mb.

We also calculated $\sigma_{\pi^0\pi^0}(\infty)$, using the data of Gutay *et al.*[8,10] for δ_0^0, adjusting b and c in (6) to fit their δ_0^0. We found

$$\sigma_{\pi^0\pi^0}(\infty)=12.1 \text{ mb}. \tag{14}$$

By inverting the calculation, we can obtain rough limits on a_0 and a_2. We assume a value for $\sigma(\infty)$ and then determine the values of a_0 and a_2 which are consistent with it, the effective-range parameters and a_1 being held constant.

With $\sigma_{\pi^+\pi^0}(\infty)=15\pm5$ mb, $a_1=0.054\mu^{-3}$, $r_2=0.5\mu^{-1}$, we find

$$a_2=(-0.1\pm0.08)\mu^{-1}. \tag{15}$$

A second value of $-0.5\mu^{-1}$ can be eliminated on the basis of inconsistency with the experimental δ_2^0 phase shifts.[11]

The 5-mb uncertainty in $\sigma_{\pi^+\pi^0}(\infty)$ is introduced so as to reflect uncertainties in a_1, r_2, the validity and application of factorization theorem and the experimental phase shifts.

If we now use the value in (15) for a_2 together with $r_0=r_2=0.5\mu^{-1}$ and $\sigma_{\pi^0\pi^0}(\infty)=15\pm5$ mb, we obtain, for the data of Walker *et al.*,[9]

$$a_0=(0.18\pm0.08)\mu^{-1}$$

and

$$a_0=(0.33\pm0.07)\mu^{-1} \tag{16}$$

for the data of Gutay *et al.*[8,10]

Thus if our estimates of $\sigma_{\pi\pi}(\infty)$ and the effective ranges are valid, it would seem unlikely that $a_0 \gtrsim 0.4$.[16]

We wish to thank Professor Laszlo Gutay for several interesting discussions concerning $\pi\pi$ phase shifts.

[16] For a discussion of the possibility of $a_0>0.4\mu^{-1}$, see, e.g., J. R. Fulco and D. Y. Wong, Phys. Rev. Letters **19**, 1399 (1967).

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Current Algebra and Photoproduction of Pions

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A low-energy theorem from current-algebra techniques is employed to derive an expression for the pion photoproduction amplitude which is shown to be gauge invariant. The low-energy predictions for the differential cross sections and the multipole amplitudes are analyzed extensively and compared with the experimental data. The predictions for the production of charged pions are in as good agreement with experimental data as those of dispersion theory are. However, the current-algebra predictions for $\gamma p\to\pi^0 p$ characteristically differ from other theoretical models, especially at threshold. Accurate low-energy experimental data for this process should therefore provide a test of the validity of the current-algebra approach.

1. INTRODUCTION

The algebra of vector and axial-vector currents[1] together with an assumption of the pole dominance of the divergence of the axial-vector current[2] (PDDAC) have been employed in the past to derive low-energy theorems for scattering amplitudes. Such theorems lead to definite predictions which turn out to be reliable for those simple cases where the coupling constants involved are well known and when the assumption of PDDAC is a justifiable approximation. The predictions of the s- and p-wave scattering lengths for πN scattering[3] are typical examples for which there exists good agreement with experiment. It is then of considerable interest to apply this method to the photoproduction of single pions. This approach has been used by Fubini, Furlan, and Rossetti to derive sum rules by making use of unsubtracted dispersion relations. We shall instead use it to infer the local properties of the photoproduction amplitude in the low-energy region near threshold.

A recent analysis[4] of the available experimental data reveals that the early Born-approximation calculations[5] as well as the dispersion-theory calculations[6] which take account of final-state interaction via the $N_{33}^*(1238)$ resonance do not provide satisfactory agreement with experiment in the low-energy region for all the observed photoproduction reactions. It is the purpose of this paper to apply current-algebra techniques to the photoproduction process.

There are two problems which arise. One is that the amplitude does not satisfy the gauge constraint when the pion is off its mass shell.[7] Secondly, the extrapolation to the physical amplitude when the pion is on-shell has to satisfy some criterion of smoothness since the approximation by PDDAC will otherwise be meaningless. A recent investigation[8] of pion photoproduction, using current algebra, restricts it to the production by isoscalar photons in view of the gauge-invariance problem. In the same work, the smoothness of the extrapolation is guaranteed by resorting to a method which makes use of a power-series expansion.

Here we shall use the full electromagnetic interaction with isoscalar as well as isovector photons and work with the off-mass-shell amplitude which does not satisfy the gauge constraint. After using current algebra and imposing PDDAC, we pass to the physical amplitude, which is shown to be explicitly gauge-invariant. We shall not attempt any Taylor expansion in order to justify the smoothness of the extrapolation, but we shall

[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[2] J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento **17**, 757 (1960); M. Gell-Mann and M. Lévy, *ibid.* **16**, 705 (1960).
[3] For example, A. P. Balachandran, M. G. Gundzik, and F. Nicodemi, Nuovo Cimento **44**, 1257 (1966); S. Weinberg, Phys. Rev. Letters **17**, 616 (1966); K. Raman and E. C. G. Sudarshan, Phys. Letters **21**, 450 (1966).
[4] G. Hohler and W. Schmidt, Ann. Phys. (N. Y.) **28**, 34 (1964).
[5] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957), referred to as CGLN.
[6] For example, J. S. Ball, Phys. Rev. **124**, 2014 (1961). See also discussion in Ref. 4.
[7] M. Nauenberg, Phys. Letters **22**, 201 (1966).
[8] A. P. Balachandran, M. G. Gundzik, P. Narayanaswamy, and F. Nicodemi, Ann. Phys. (N. Y.) **45**, 339 (1967).