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Simon J. E. Radford

Paul E. Boynton

Bobby L. Ulich

Bruce Partridge
Haverford College, bpartrid@haverford.edu

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A SEARCH FOR THE SUNYAEV-ZEL'DOVICH EFFECT AT $\lambda = 3$ MILLIMETERS

SIMON J. E. RADFORD,[1,2] PAUL E. BOYNTON,[2] BOBBY L. ULICH,[3] R. BRUCE PARTRIDGE,[4] ROBERT A. SCHOMMER,[5] ANTONY A. STARK,[6] ROBERT W. WILSON,[6] AND STEPHEN S. MURRAY[7]
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ABSTRACT

Using both drift scan and source tracking observations, we have searched for the Sunyaev-Zel'dovich effect in seven clusters of galaxies at a wavelength of 3 mm using the National Radio Astronomy Observatory 12 m (formerly 11 m) telescope on Kitt Peak and the AT&T Bell Laboratories 7 m telescope at Crawford Hill. This wavelength is advantageous because flux from sources in the clusters is negligible and the Sunyaev-Zel'dovich signal is near maximum amplitude. Our measurements set upper limits on the magnitude of the Sunyaev-Zel'dovich effect that are consistent with all previous, longer wavelength, searches. We introduce a method for comparison of observations that is independent of the details of the measurement and only weakly dependent on the intracluster gas model. The consistency of existing observations and the prospects for future measurements are discussed.

Subject headings: galaxies: clustering — radiation mechanisms — radio sources: general

I. INTRODUCTION

In the dozen years since Sunyaev and Zel'dovich (1972) predicted that inverse Compton scattering of microwave photons by the hot gas in clusters of galaxies would affect the spectrum of the cosmic background radiation in the directions of the clusters, many observers have attempted to measure this effect (Pariiskii 1973; Rudnick 1978; Perrenod and Lada 1979; Lake and Partridge 1980; Birkinshaw, Gull, and Northover 1981; Schallwich 1982; Andernach *et al.* 1983; Lasenby and Davies 1983; Meyer, Jeffries, and Weiss 1983; Birkinshaw and Gull 1984; Birkinshaw, Gull and Hardebeck 1984). Heating of the microwave radiation by the hot intracluster medium leads to a photon-conserving shift of the background spectrum toward higher frequency, giving rise to a characteristic signature of the Sunyaev-Zel'dovich (SZ) effect: a decrease in radiation intensity at wavelengths in the Rayleigh-Jeans region of the background spectrum ($\lambda > 1$ mm). To date long-wavelength ($\lambda = 3$ cm) measurements have offered the highest sensitivity but have been limited by the presence of discrete sources, whose flux must be measured and subtracted. Observations at higher frequencies (30–100 GHz), although offering relief from the problem of contamination by these sources, have been plagued by instrumental difficulties and atmospheric effects. Nevertheless, the 90 GHz atmospheric window is attractive for these measurements because it is near the frequency at which the effect is maximum.

Detailed X-ray imaging data from the *Einstein Observatory* is available for many clusters. The images of A2218, for example, display a smooth, centrally condensed X-ray brightness distribution with no significant indication of clumping of the emitting gas. Simple polytropic models can account for the radial distribution of both the galaxies and the gas in this cluster and indicate the fractional decrease $\Delta I/I_0$ in intensity at the cluster center due to the SZ effect should be approximately 5×10^{-4} in the Rayleigh-Jeans limit (Boynton *et al.* 1982).

This paper describes attempts to detect the Sunyaev-Zel'dovich effect in 1981 with the AT&T Bell Laboratories 7 m telescope at Crawford Hill and in 1982 and 1983 with the NRAO[8] 12 m (formerly 11 m) telescope on Kitt Peak. The observations are discussed in §§ II, III, and IV. Section V presents a brief development of a method for consistent comparison of measurements made with different equipment. In § VI we discuss the application of this method to our work and previous studies, and we review our results in § VII.

II. KITT PEAK DRIFT SCAN OBSERVATIONS (1982)

Two groups have attempted measurements of the SZ effect at a wavelength of 9 mm with the NRAO telescope at Kitt Peak (Perrenod and Lada 1979; Lake and Partridge 1980). These observations were made in the traditional on-off beam-switching mode while tracking the source. To help eliminate instrumental asymmetries, the source is placed alternately in the two beams. This measurement mode is attractive because the source is observed fully 50% of the time, and has proved quite successful for observations of strong continuum sources. When applied to weak continuum sources, however, this scheme has a major drawback, as the side and back lobe power from the local environment changes as the telescope moves, giving rise to a large offset which varies with antenna position (Boynton and Partridge 1973; Lake and Partridge 1980). Systematic offsets probably of this origin and an order of magnitude larger than the expected SZ signal were observed by both groups at $\lambda = 9$ mm. The offset amplitudes measured by Perrenod and Lada (1979) were approximately 10 mK antenna temperature. Attempts by those observers to subtract these offsets by measuring them in nearby blank sky regions met with limited success. While Perrenod and Lada (1979) reported a decrement in A2218, Lake and Partridge (1980) reported an equally significant positive increment for the same cluster.

[1] Battelle Observatory, Richland, Washington.
[2] University of Washington, Seattle.
[3] Multiple Mirror Telescope Observatory.
[4] Haverford College.
[5] Rutgers University
[6] AT&T Bell Laboratories.
[7] Harvard-Smithsonian Center for Astrophysics.

[8] NRAO is operated by Associated Universities, Inc., under contract with the National Science Foundation.

To overcome these difficulties, we attempted to observe clusters using short drift scans while beam-switching. In this mode, the telescope is pointed slightly ahead of the source, then held fixed as the source drifts through the beam position. The procedure is then repeated, leapfrogging the source across the sky. The disadvantage of this method is low observing efficiency, since the cluster occupies the beam for only a small fraction of the duration of the drift scan. Indeed, for the expected SZ drift scan profiles for A2218, calculated as discussed in § V, the optimum drift scan length for rejection of a linear trend in the baseline is some eight beam widths or roughly three cluster profile widths. In principle, if the SZ effect were observable in this manner, information about its brightness distribution would be available, which could then be used to further constrain models of the intracluster gas.

In 1982 March, we observed three clusters in this drift scan mode with the 89.6 GHz cooled mixer receiver at the 11 m telescope. The focal plane chopper was used to beam switch by 4′ in azimuth at 70 Hz. The beam width (FWHM) was 79″ ± 1″ and the main beam efficiency, 36% ± 3%. In this operating mode, the rms fluctuation in the antenna temperature of the beam-switched signal was measured to be 14 mK s^{-1}. As a further check on the measurement technique, blank fields separated from each cluster by 1^h in right ascension were observed in the same drift scan mode. Observations of clusters and blank sky regions were alternated each hour and were timed to cover approximately the same ranges of hour angle.

The data were naturally structured into 1 hr blocks of integration time by this process. Column (4) of Table 1 lists the number of blocks obtained for each field in 5 days of telescope time. The drift scans in each block were averaged and then fitted to a linear baseline, which was removed. To obtain a profile of the expected signal, we computed the convolution of the switching beam profile with the modeled profile of the SZ effect as discussed in § V below. To account for the change of orientation of the drift scans on the sky during our observations, we fit each block to a profile calculated for the parallactic angle[9] appropriate to the central scan of that block. From each such fit, the peak amplitude of the SZ signal was obtained. The peak amplitude is independent of the parallactic angle since all scans pass through the cluster center, so the fitted profile amplitudes can be directly combined. The resulting reduced fractional intensity changes (§ IV) are shown in column (6) of Table 1 for each field.

[9] The parallactic angle is the angle between lines of constant declination (the drift scan direction) and lines of constant elevation (beam switching direction; Smart 1956).

An unforeseen difficulty arose, however, when this procedure was followed. The internal estimates of the variance of the profile amplitude obtained within each block of data were consistently lower than the variance between blocks. Both the cluster and blank fields were affected equally by this problem. This excess variance between blocks was identified as an anomalous low-frequency noise component. A power spectrum of the data (Deeter 1983) indicates that only on the longest time scales sampled does the power density of this component become larger than the power density of the receiver noise that is measured at shorter time scales. Thus the spectrum of this low-frequency noise cannot be reliably characterized. Excess variance as large as 5 times the expected white noise of the receiver proved a serious limitation on the sensitivity of these drift scan measurements. The 1 σ error estimates reported in column (6) of Table 1 include this excess contribution. The SZ effect was not significantly detected in any of the three clusters examined.

III. KITT PEAK SOURCE TRACKING OBSERVATIONS (1983)

Further observations were scheduled in 1983 June, to investigate the source of the excess noise. During the intervening year, the telescope primary mirror was replaced with a new 12 m diameter surface. The mounting position of the receiver after this reconfiguration unfortunately precluded use of the fast focal plane chopper used in 1982, so the 70″ FWHM beam was switched 4′.3 at 6 Hz by nutation of the secondary mirror. A main beam efficiency of 49% ± 3% and a noise equivalent antenna temperature of 50 mK s^{-1} were measured with this slow beam-switching arrangement. With this substantial increase in receiver noise over previously experienced levels the integration time necessary either to investigate the excess noise or to improve on our 1982 results by the drift scan technique would have become prohibitively large. Consequently, we examined the performance of the new telescope surface in the tracking mode. These tests proved encouraging; antenna position–dependent offsets were found to be quite small, although slow receiver drifts of a few tenths of mK in antenna temperature required us to include contemporaneous observations of nearby blank fields.

We used a multiple difference scheme to reduce spurious effects. The difference in antenna temperature between the two beam positions, the switched signal, is the fundamental measurement. With the source placed alternately in each beam, pairs of 10 s integrations of the switched signal were recorded and the difference between the two integrations calculated. A sequence of 20 pairs was averaged for a cluster field; then the companion blank field was recorded in an identical manner

TABLE 1

1982 KITT PEAK OBSERVATIONS

Source (1)	R.A. (1950.0) (2)	Decl. (1950.0) (3)	Hours (4)	θ_c (5)	$\Delta I/I\|_0 \times 10^3$ (6)	Reference for θ_c (7)
A478	$4^h10^m36^s$	+10°21′54″	18	2′.0	$+0.76 \pm 1.20^a$	1
Blank	5 10 36	+10 21 54	11	2.0	$+2.99 \pm 1.48$	1
A1413	11 52 48	+23 39 17	6	1.8	-1.01 ± 4.23	1
Blank	10 51 48	+23 39 17	6	1.8	$+1.81 \pm 10.58$	1
A2218	16 35 44	+66 18 00	36	0.97	$+1.41 \pm 0.99$	2
Blank	17 35 44	+66 18 00	16	0.97	-2.93 ± 1.30	2

REFERENCES.—(1) Jones *et al.* 1979. (2) Boynton *et al.* 1982.
[a] See text for discussion of errors.

TABLE 2

1983 KITT PEAK OBSERVATIONS

Source (1)	R.A. (1950.0) (2)	Decl. (1950.0) (3)	Time (hr) (4)	θ_c (5)	$F(0)$ (6)	$\delta T_{A,T}$ (mK) (7)	$\Delta I/I\|_0 \times 10^3$ (8)	Reference for θ_c (9)
A1413	$11^h52^m48^s$	+23°39′17″	3.67	1′.8	35%	$+1.99 \pm 0.72$	$+6.22 \pm 2.25$	1
Blank	10 51 48	+23 39 17	3.67					
A2218.............	16 35 44	+66 18 00	35.33	0.97	44%	$+0.08 \pm 0.21$	$+0.20 \pm 0.52$	2
Blank	17 35 44	+66 18 00	37.00					
0016+16	0 15 59	+16 09 24	5.00	0.50	69%	-0.39 ± 0.46	-0.62 ± 0.65	3
Blank	23 15 59	+16 09 24	4.11					

REFERENCES.—(1) Jones *et al.* 1979. (2) Boynton *et al.* 1982. (3) White, Silk, and Henry 1982.

and the pattern repeated. The total integration times after 6 days of observing are listed in column (4) of Table 2 for each source. By comparing the results of these sequences, we placed an upper limit of 0.3 mK antenna temperature on the amplitude of any position dependent offsets, or less than 3% of the size of the offsets that had complicated previous observations using the 11 m primary mirror (Lake and Partridge 1980, Perrenod and Lada 1979). The blank field signal, averaged over the entire observing run, was then subtracted from the corresponding average cluster signal. This difference in antenna temperature between cluster and blank field, given in column (7) of Table 2, was finally corrected for the beam geometry by equation (6) below to provide a limit on the magnitude of the SZ effect in each cluster, listed in column (8) of that table.

While the 12 m mirror is a considerable improvement over the 11 m, the attainable receiver performance in the absence of the fast focal plane chopper allowed only a marginal overall improvement in sensitivity compared with the 1982 results.

IV. CRAWFORD HILL OBSERVATIONS (1981)

As part of an earlier program, independent of the NRAO observations just described, we examined the four clusters of galaxies listed in Table 3 for evidence of the SZ effect with the AT&T Bell Laboratories 7 m telescope at Crawford Hill in 1981 January and March. In good weather, the system used to make these observations gave reliable results at the level of 0.3 mK in antenna temperature. In the observing band, 99.7 to 109.6 GHz, the main beam efficiency of the antenna is 0.90 ± 0.02, and the FWHM beam size is 100″.

The receiver in use was based on a cooled Schottky barrier diode mixer mounted on a 17 K cryogenic refrigerator. The receiver was preceded by a calibration box (Goldsmith 1977) which had a semicircular chopper. This chopper could be used to chop between two beams, separated by 19′ on the sky, or it could be used to chop between a room temperature load and a liquid nitrogen load to calibrate the receiver, or to chop between the liquid nitrogen load and the sky to measure the sky temperature. The chopper was operated at 15 Hz, this frequency being locked to the power line frequency of 60 Hz. Every half chopper cycle had, therefore, exactly two full cycles of 60 Hz hum. The cryogenic refrigerator was powered by a synchronous motor and operated at 1.2 Hz. The beam switching was arranged so that every observation had an even number of refrigerator cycles in order to cancel the noise generated by refrigerator vibrations. The double sideband (DSB) receiver temperature measured at the Cassegrain focus was about 190 K during these observations.

In order to guard against systematic effects caused by weather, we used a fairly complicated chopping scheme, involving nine points on the sky. The nine points were spaced 19′ apart in a regular 3 × 3 grid. The central point was the source "on" position, and the two points 19′ away in azimuth were its "off" positions. The points above and below the center were blank sky "on" positions, observations of which were interleaved with the observations of the central point. Each of the blank sky "on" positions, in turn, was flanked by two blank sky "off" positions. An "on" position was observed using chopping in azimuth to one of its two "off" positions, combined with antenna nodding to the other "off" position to symmetrize the beams. Every observation, therefore, consisted of the measurement of the central, source position and interleaved measurements of the two blank sky positions.

In the absence of atmospheric fluctuations, the blank sky measurements should be normally distributed about zero flux, with a dispersion proportional to the system temperature. We used this model to discriminate against atmospheric effects by discarding an observation if the inteleaved blank sky measurements were not within 2 σ of the mean. The observed mean was consistent with zero flux. Typically, only a few percent of the observations were so discarded in clear weather, but this fraction rose markedly when sky conditions deteriorated. This is an independent and unbiased method of discarding data

TABLE 3

1981 CRAWFORD HILL OBSERVATIONS

Source (1)	R.A. (1950.0) (2)	Decl. (1950.0) (3)	θ_c (4)	$F(0)$ (5)	$\delta T_{A,T}$ (mK) (6)	$\Delta I/I\|_0 \times 10^4$ (7)	Reference for θ_c (8)
A576	$7^h17^m20^s$	+55°50′00″	2′.0	63%	$+0.72 \pm 0.42$	$+7.9 \pm 4.6$	1
A1413	11 52 49	+23 29 20	1.8	64%	$+0.21 \pm 9.55$	$+2.3 \pm 6.0$	2
A2009	14 57 59	+21 33 55	...	...	-0.96 ± 0.53	...	
A2218	16 35 43	+66 19 30	0.97	64%	$+0.37 \pm 0.29$	$+4.0 \pm 3.1$	3

REFERENCES.—(1) White and Silk 1980. (2) Jones *et al.* 1979. (3) Boynton *et al.* 1982.

affected by atmospheric effects, because the data to be rejected are selected soley on the basis of the blank sky measurements within an observation.

The limits on the SZ effect resulting from the measurements are summarized in Table 3. Column (6) of that table lists the antenna temperature difference between the source and the mean of the two reference positions, and column (7) gives the reduced fractional intensity change for each cluster derived by the method of § V.

V. A COMPARISON METHOD

The difference in antenna temperature between two beams positions δT_A cannot be used to compare observations of the same cluster made with different equipment because it depends on many experimental parameters: the efficiency of the telescope, the geometry of the beam, the beam-switching pattern, and the measurement wavelength. Conversely, the fractional change in intensity along the line of sight through the cluster center relative to the intensity of the background radiation is an astrophysically interesting quantity that is independent of the geometry of the telescope beam. Furthermore, when evaluated in the Rayleigh-Jeans limit, this intensity change is independent of all instrumental parameters. We will refer to this quantity as the reduced fractional intensity change, $\Delta I/I|_0$. Presently there are no significant observational determinations of the radial profile of the SZ effect in any cluster, and consequently there is no method of correcting the SZ measurements for the effects of the beam geometry that is independent of a model for the intracluster gas. Since it can be shown that this model dependence is rather weak, we have chosen to compare various observations of a cluster within the context of a particular model of the intracluster medium by converting the observed antenna temperature differences δT_A into *reduced fractional intensity changes* $\Delta I/I|_0$. We have applied this procedure to the case of A2218 to produce Table 4.

The magnitude of the SZ effect is proportional to the line-of-sight integral of the electron pressure, which may be computed from a model of the intracluster medium that specifies the spatial distribution of electron density and temperature. Based on X-ray and optical observations of A2218, we have chosen isothermal models of the gas with a galaxy-to-gas scale height ratio τ of 0.5 and the core radii listed in Tables 1, 2, and 3 to calculate these distributions (Cavaliere and Fusco-Femiano 1978; Boynton *et al.* 1982). Since the X-ray data provide information only about the central regions of a cluster, these models will be valid descriptions of only those regions. Indeed, Henriksen and Mushotsky (1985*a*, *b*) have shown that isothermal models cannot provide a satisfactory global description of the gas in many clusters. While the SZ effect in a cluster is more extended than the X-ray emission, we are interested in the difference in the magnitude of the effect in the main and reference beams when reducing beam-switched radio observations. Fortunately, this difference is dominated by the gas distribution in the center of a cluster for the experiments performed to date.

At any angle θ from the cluster center, the fractional intensity of the effect relative to the background is given by the pressure integral over the line of sight (Sunyaev and Zel'dovich 1969):

$$\frac{\Delta I}{I}(\theta, \nu) = \frac{k\sigma_T}{m_e c^2} f\left(\frac{h\nu}{kT}\right) \int dl n_e(\theta, l) T_e(\theta, l) , \tag{1}$$

where $f(h\nu/kT) \equiv f(x) = xe^x[x \coth(x/2) - 4]/(e^x - 1)$, $T = 2.7$ K is the background temperature, and n_e and T_e are the electron density and temperature of the intracluster plasma. The observed antenna temperature difference will be proportional to the convolution of equation (1) with the combined pattern G of the two switched beams, normalized by the solid angle of one beam. We write the expected antenna temperature profile of the cluster as

$$\delta T_A(\theta, \nu) = \eta T_B(\nu) \frac{\Delta I}{I}(\theta = 0, \nu) F(\theta) , \tag{2}$$

where η is the telescope efficiency, $T_B(\nu)$ is the Rayleigh-Jeans equivalent brightness temperature of the background, and $F(\theta)$ is the (frequency-independent) convolution

$$F(\theta) \equiv \int_0^{2\pi} d\phi' \int_0^{\pi/2} \sin\theta' d\theta' G(\theta, \theta', \phi'; \theta_b, \theta_t, \xi) \times \frac{\Delta I}{I}(\theta', \nu)\left[\frac{\Delta I}{I}(\theta = 0, \nu)\right]^{-1} . \tag{3}$$

Here θ is the angle from the cluster center to the main beam axis, θ_t is the angle between the main and reference beams, θ_b is the full width at half maximum (FWHM) of each beam, and ξ is the parallactic angle. The angular extent of the Sunyaev-Zel'dovich effect is often comparable with the beam switching angle θ_t, necessitating careful consideration of the signal received in the reference beam (Boynton *et al.* 1982). Details of this convolution and the assumed beam pattern G are presented in the Appendix. With this definition, $F(0)$ is the efficiency with which a given beam geometry samples the angular dis-

TABLE 4

COMPARISON OF MEASUREMENTS OF A2218[a]

Group (1)	ν (GHz) (2)	θ_b (3)	θ_t (4)	$F(0)$ (5)	$\Delta I/I(0, \nu) \times 10^4$ (6)	$\Delta I/I|_0 \times 10^4$ (7)
Lasenby and Davies 1983	5.0	8′ × 10′	30′	31%	$+2.22 \pm 6.95$	$+2.13 \pm 6.66$
Birkinshaw, Gull and Northover 1981	10.6	4.′5	15	40%	-10.22 ± 2.04	-9.37 ± 1.87
Schallwich 1982	10.7	1.2	8.2	57%	-8.44 ± 1.73	-7.73 ± 1.59
Birkinshaw and Gull 1984	10.7	3.3	14.4	46%	-3.33 ± 1.67	-3.05 ± 1.53
Birkinshaw, Gull, and Hardebeck 1984	20.3	1.78	7.15	48%	-2.26 ± 0.64	-1.93 ± 0.55
Perrenod and Lada 1979	31.4	3.5	8	36%	-9.71 ± 4.48	-7.69 ± 3.55
Lake and Partridge 1980	31.4	3.6	9	37%	$+9.91 \pm 4.84$	$+7.85 \pm 3.83$
1981 Crawford Hill observations	100–110	1.67	19	64%	$+6.4 \pm 5.0$	$+4.0 \pm 3.1$
1983 Kitt Peak observations	89.6	1.17	4.23	44%	$+3.2 \pm 8.2$	$+2.0 \pm 5.2$

[a] Based on the X-ray data of Boynton *et al.* 1982, the expected reduced fractional intensity change is $\Delta I/I|_0 = -5.24^{-1.92}_{+2.83} \times 10^{-4}$.

tribution of a particular cluster, and $F(\theta)$ expresses the shape of the expected drift scan profile.

When fitted to this expected profile F, the run of antenna temperature differences for each drift scan $\delta T_{A,DS}$ yields a profile amplitude

$$A_{DS}(\nu) = \delta T_{A,DS}(\theta, \nu)/F(\theta) , \tag{4}$$

and the reduced fractional intensity change then follows directly from equation (2):

$$\left.\frac{\Delta I}{I}\right|_{0,DS} = \frac{A_{DS}(\nu) f(x=0)}{\eta T_B(\nu) f(x)} . \tag{5}$$

Formulas (4) and (5) were used to calculate the values of $\Delta I/I|_0$ in column (6) of Table 1 from our 1982 data.

In the case of tracking observations, the antenna temperature difference $\delta T_{A,T}(\nu)$ is measured with one beam pointed at the cluster center, and the reduced fractional intensity change also may be obtained from equation (2):

$$\left.\frac{\Delta I}{I}\right|_{0,T} = \frac{\delta T_{A,T}(\nu) f(x=0)}{\eta T_B(\nu) F(0) f(x)} . \tag{6}$$

The quantitites in Tables 2 and 3 were derived from our data using equation (6).

Abell 2218 is the cluster perhaps most studied for the SZ effect. Birkinshaw, Gull, and Northover (1981); Perrenod and Lada (1979); Lake and Partridge (1980); Lasenby and Davies (1983); Birkinshaw and Gull (1984); Birkinshaw, Gull, and Hardebeck (1984) all made measurements of the cluster by tracking the source while beam switching in azimuth. Schallwich (1982), on the other hand, made declination scans through the cluster, also while beam-switching in azimuth. In this case, the antenna temperature difference measured at the center of the scans is equivalent to that obtained in a tracking measurement. Using equation (6) to convert the observed antenna temperature differences to reduced fractional intensity changes, we have prepared a summary of existing observations of A2218 in Table 4. Column (5) in that table lists the fractional intensity changes at the observing frequency, and column (6), the reduced fractional intensity changes in the Rayleigh-Jeans limit. The expected magnitude of the SZ effect listed in Table 4 was calculated from equation (A9) on the basis of X-ray and optical measurements.

VI. DISCUSSION

In deriving the reduced fractional intensity changes from the observed antenna temperature differences, we have removed the effects of the differences in beam geometry and observing wavelength from the results. This allows a direct comparison of the results of different measurements, but at the expense of introducing some model dependence to the comparison. The only model-dependent factor in this reduction is the convolution of beam and cluster profiles, $F(0)$, that enters into the denominator of equation (6). This model-dependent factor $F(0)$ is sensitive only to the shape of the radial distributions of the electron temperature and density, so the reduced fractional intensity change $\Delta I/I|_0$, computed directly from the radio observations, is free from the large uncertainty in the central electron temperature that hinders the prediction of $\Delta I/I|_0$ from the combined X-ray and optical data.

Examination of Table 4 reveals significant differences between the values of $\Delta I/I|_0$ derived from the various experiments. The two measurements at $\lambda = 9$ mm are made with the same instrument, yet Lake and Partridge report an increase in intensity of the same magnitude as the decrease reported by Perrenod and Lada. Furthermore, the recent measurements at Owens Valley by Birkinshaw and Gull, at $\lambda = 3$ cm, and by Birkinshaw, Gull, and Hardebeck, at $\lambda = 1.5$ cm, yeild reduced fractional intensity changes that are significantly smaller than the results of earlier work by Birkinshaw, Gull, and Northover and by Schallwich at $\lambda = 3$ cm, and by Perrenod and Lada at $\lambda = 9$ mm. Our own data are consistent with all this work but clearly cannot resolve these apparent discrepancies.

The reduced fractional intensity change predicted from a combined study of X-ray and optical data on A2218 is consistent with the results of all the microwave experiments except that of Lake and Partridge. Consequently, the discrepancy between the Owens Valley results and the earlier work is not illuminated by that combined study, primarily because of the large uncertainty in the electron temperature derived from the velocity dispersion of the galaxies and the properties of the X-ray image (Boynton *et al.* 1982). Combination of the central X-ray surface brightness with the observed magnitude of the SZ effect also allows the calculation of the electron temperature in the cluster, but in doing this with their $\lambda = 3$ cm data, Birkinshaw and Gull erred in the conversion of X-ray flux to central surface brightness.[10] A corrected calculation based on their data yields $T_0 = 14.5 \pm 7.3$ keV, in agreement both with the temperature determined in the X-ray imaging study, $T_0 = 24^{+8}_{-12}$ keV, and with the temperature expected on the basis of X-ray spectroscopy of similar clusters (Mushotsky 1984). By contrast, the large SZ effect reported by Birkinshaw, Gull, and Northover, by Schallwich, and by Perrenod and Lada implies a much higher electron temperature.

Among the measurements summarized in Table 4 there is a distinct set of higher precision experiments (Birkinshaw, Gull, and Northover 1981; Schallwich 1982; Birkinshaw and Gull 1984; Birkinshaw, Gull, and Hardebeck 1984) that appear particularly discrepant. That is, they exhibit a substantial contrast between the size of their reported internal errors and the range of values of $\Delta I/I|_0$. Specifically, the dispersion in these four measurements is 2.4 times larger than the pooled rms uncertainty based on the experimental errors quoted by each group.

Since the values of $\Delta I/I|_0$ presented in Table 4 depend only on the gas model and not on the experimental details, it is clear that this discrepancy is *not* explained by differences in the way the various experiments measure the gas distribution assumed for this cluster. But what if the model were chosen differently? By calculating $\Delta I/I|_0$ using a grid of polytropic gas models with $0.3 \leq \tau \leq 0.7$, $1.0 \leq \gamma \leq 1.6$, and $0'.43 \leq \theta_c \leq 1'.5$, we find that the aforementioned ratio of dispersions describing these four experiments is never less than 2.3 for any of these models.[11] Thus the model dependence of the reduction of the radio observations to $\Delta I/I|_0$ is very weak, particularly as this dependence influences the comparison of observations of the same cluster made with different equipment. We conclude that the discrepancy between these results must arise from undiagnosed measurement errors, since it cannot be ascribed to the nature or distribution of the intracluster gas.

[10] They neglected to consider the angular variation of the X-ray surface brightness over the interval in which the flux is integrated. Their eq. (16) should read $I_0 = S_x(1+z)^4\theta_c^{-2}[\ln(1+\theta_x^2/\theta_c^2)]^{-1}$, where θ_x is the radius on the sky within which the flux is measured.

[11] The ranges in τ and θ_c correspond to 2 σ limits on these parameters determined from the combined X-ray and optical data (Boynton *et al.* 1982).

VII. CONCLUSIONS

We have explored the current limits of the NRAO millimeter observatory for measurements of faint continuum sources. In an attempt to overcome the large position-dependent offsets that had been observed earlier with the 11 m telescope, we used a drift scan technique in 1982, despite the low observing efficiency of this method. Our data were badly contaminated, however, by as yet unexplained excess noise, reducing the sensitivity of our measurements substantially. The observations we undertook in 1983 show the problem of systematic offsets is significantly alleviated with the new 12 m optics, allowing observations in a tracking mode. This technique provides a factor of 4 reduction in the integration time necessary to reach a given sensitivity level, when compared with the drift scan mode we used with the 11 m telescope. A temporary side effect of the new optical configuration is an order-of-magnitude reduction of the maximum beam-switching frequency, leading to almost a fourfold increase in the noise level of the receiver. Even so, these 1983 results are encouraging because they demonstrate that the performance of the new reflector is greatly improved when compared with its predecessor. When the noise level of the receiver is restored to its previous level by increasing the maximum beam-switching frequency, the Sunyaev-Zel'dovich effect should be observable with the NRAO facility.

The observing scheme used on the AT&T Bell Labs antenna worked well, but that experiment was limited by poor weather conditions. The new SIS receiver (Stark 1983), which replaces the Schottky barrier diode receiver used in 1981, has a DSB receiver temperature under 50 K over a 500 MHz bandpass at 104–107 GHz. This should result in an order-of-magnitude decrease in the integration time necessary for these observations and allow better use of brief periods of excellent weather.

This study has placed upper limits on the magnitude of the Sunyaev-Zel'dovich effect at $\lambda = 3$ mm in seven rich clusters of galaxies. We have used a scheme which is only mildly model-dependent to reduce our measurements of antenna temperature differences to estimates of the fractional intensity change relative to the background intensity that would be measured at the cluster center with a beam of infinitesimal solid angle and unit efficiency in the Rayleigh-Jeans limit. Using this formalism, we have compared all the existing measurements of the SZ effect and find that all except Lake and Partridge (1980) are consistent with the magnitude of the effect expected from a cluster model based on a study of the combined X-ray and optical properties of the cluster (Boynton *et al.* 1982). Additionally, we have shown that the inconsistency between the results of Birkinshaw, Gull, and Northover 1981); Schallwich (1982); Birkinshaw and Gull (1984); and Birkinshaw, Gull, and Hardebeck (1984) cannot be explained by the physics of the intracluster gas within the context of the polytropic models allowed by the X-ray data.

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APPENDIX

This Appendix outlines the calculation of the convolution $F(\theta)$ and the reduced fractional intensity change $\Delta I/I\,|_0$, defined in § V for the case of an isothermal model of the intracluster gas. In such a model the electron temperature is, of course, constant, and in cylindrical coordinates centered on the cluster the variation of the electron density may be written

$$n_e = n_0(1 + \theta^2/\theta_c^2 + l^2/r_c^2)^{-3\tau/2} , \tag{A1}$$

in terms of the galaxy-to-gas scale height ratio τ (Cavaliere and Fusco-Femianio 1978) and the cluster core radius, $r_c = \theta_c c[z + \frac{1}{2}(1 - q_0)z^2]/H_0(1 + z)^2$ (Weinberg 1972). The angular profile of the SZ effect may then be expressed as

$$\frac{\Delta I/I(\theta, \nu)}{\Delta I/I(\theta = 0, \nu)} = \frac{\int_0^\infty dl n_e(\theta, l)}{\int_0^\infty dl n_e(\theta = 0, l)} = (1 + \theta^2/\theta_c^2)^{(1-3\tau)/2} . \tag{A2}$$

The telescope beam may be treated as approximately Gaussian with FWHM θ_b and is switched through an angle θ_t in azimuth. A drift scan traverses the cluster in right ascension, so the switching direction is inclined to the drift direction by the parallactic angle ξ. When the main beam is separated from the cluster center by an angle θ, the overall pattern of the switched beams may be written

$$G(\theta, \theta', \phi'; \theta_b, \theta_t, \xi) = \frac{4 \ln 2}{\pi\theta_b^2}\left[\exp\left(-\frac{4 \ln 2\psi_m^2}{\theta_b^2}\right) - \exp\left(-\frac{4 \ln 2\psi_r^2}{\theta_b^2}\right)\right], \tag{A3}$$

with

$$\psi_m^2 = \theta'^2 + \theta^2 - 2\theta'\theta \cos \phi' ,$$
$$\psi_r^2 = \theta'^2 + \theta_r^2 - 2\theta'\theta_r \cos \phi_r' ,$$
$$\theta_r^2 = \theta^2 + \theta_t^2 + 2\theta\theta_t \cos \xi ,$$

and

$$\phi_r' = \phi' - \sin^{-1}(\theta_t \sin \xi/\theta_r) .$$

Using the small angle approximation, $\sin\theta \approx \theta$, in equation (3) and the Bessel function identity,

$$I_0(z) = \frac{1}{\pi}\int_0^{\pi} d\phi e^{\pm z\cos\phi}\,, \tag{A4}$$

to integrate over ϕ gives

$$F(\theta) = 2\beta\int_0^{\infty} w' dw'(1 + w'^2)^{(1-3\tau)/2}[I_0(2\beta w w')e^{-\beta(w^2+w'^2)} - I_0(2\beta w_r w')e^{-\beta(w_r^2+w'^2)}]\,, \tag{A5}$$

with the substitutions $w \equiv \theta/\theta_c$ and $\beta \equiv 4\ln 2/w_b^2$. This function has been integrated numerically when needed in the data reduction described in the text. The dependence of $F(\theta = 0)$ on θ_b and θ_t with $\tau = \frac{1}{2}$ is indicated in Figure 3*b* of Boynton *et al.* (1982), albeit with a different normalization,

$$F(\theta = 0) \equiv 4\sqrt{\pi}\,\Gamma(\tfrac{3}{4})\langle \mathrm{ff}\rangle_{\mu}/\Gamma(\tfrac{1}{4})\,, \tag{A6}$$

where $\langle \mathrm{ff}\rangle_{\mu}$ is defined in that paper and illustrated in that figure.

The reduced fractional intensity change expected from this model is found from equation (1) in the Rayleigh-Jeans limit:

$$\left.\frac{\Delta I}{I}\right|_0 = \frac{k\sigma_T}{m_e c^2} f(x=0) r_c n_0 T_0^2 \int_0^{\infty} d\zeta (1 + \zeta^2)^{-3\tau/2}\,. \tag{A7}$$

The X-ray flux S_x from the cluster due to thermal bremsstrahlung measured within a radius θ_x of the cluster center is (Boynton *et al.* 1982)

$$S_x = C_x n_0^2 T_0^{1/2} \bar{g}(T_0) r_c^3 D^{-2}\int_0^{\theta_x/\theta_c} u\,du \int_0^{\infty} d\zeta (1 + u^2 + \zeta^2)^{-3\tau}\,, \tag{A8}$$

where $C_x = (16\pi/3)(2m_e c^2/3\pi k)^{1/2}\alpha^3\Lambda_c c(\Sigma N_Z Z^2/N_e)$, $\bar{g}(T_0)$ is the Gaunt factor averaged over the passband of the detector, the cluster distance is given by $D = c[z + \frac{1}{2}(1 - q_0)z^2]/H_0$, $\alpha = e^2/\hbar c$, $\Lambda_c = \hbar/m_e c$, and $\Sigma N_Z Z^2/N_e = 1.23$ for a fully ionized plasma of cosmic abundances. By combining these equations, we can write the expected reduced fractional intensity change in terms of observables:

$$\left.\frac{\Delta I}{I}\right|_0 = \frac{k\sigma_T}{m_e c^2} C_x^{-1/2} f(x=0) S_x^{1/2} T_0^{5/4}[\bar{g}(T_0)]^{-1/2}(1+z)(D/\theta_c)^{1/2} R(\tau, \theta_x/\theta_c)\,, \tag{A9}$$

where the ratio of integrals,

$$R(\tau, \theta_x/\theta_c) \equiv 2\int_0^{\infty} d\zeta (1+\zeta^2)^{-3\tau/2}\left\{\int_0^{\theta_x/\theta_c} u\,du \int_0^{\infty} d\zeta (1 + u^2 + \zeta^2)^{-3\tau}\right\}^{-1/2}\,, \tag{A10}$$

may then be evaluated:

$$R\left(\tau = \frac{1}{2}\right) = \sqrt{2}\,\pi\,\frac{\Gamma(1/4)}{\Gamma(3/4)}\left[\ln\left(1 + \frac{\theta_x^2}{\theta_c^2}\right)\right]^{-1/2}\,, \tag{A11}$$

$$R\left(\tau \neq \frac{1}{2}\right) = \sqrt{2}\,\pi\,\frac{\Gamma[(3\tau-1)/2]}{\Gamma(3\tau/2)}\left\{\frac{3\Gamma(3\tau)(1/2-\tau)}{\sqrt{\pi}\,\Gamma(3\tau - 1/2)[(1 + \theta_x^2/\theta_c^2)^{3(1/2-\tau)} - 1]}\right\}^{1/2}\,. \tag{A12}$$

We have taken $H_0 = 100$ km s^{-1} Mpc^{-1}, $q_0 = \frac{1}{2}$, and the X-ray observables from Boynton *et al.* (1982) in evaluating the expected intensity change for A2218 given in Table 4.

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PAUL E. BOYNTON: Department of Physics, FM-15, University of Washington, Seattle, WA 98195

STEPHEN S. MURRAY: Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

R. BRUCE PARTRIDGE: Haverford College, Haverford, PA 19041

SIMON J. E. RADFORD: Department of Astronomy, FM-20, University of Washington, Seattle, WA 98195

ROBERT A. SCHOMMER: Department of Physics and Astronomy, Rutgers University, P.O. Box 849, Piscataway, NJ 08854

ANTONY A. STARK and ROBERT W. WILSON: AT&T Bell Laboratories, Crawford Hill, HOH L-231, Holmdel, NJ 07733

BOBBY L. ULICH: Multiple Mirror Telescope Observatory, University of Arizona, Tucson, AZ 85711