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Smoothness of the 2.2 micron background: constraints on models of primeval galaxies

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THE SMOOTHNESS OF THE 2.2 MICRON BACKGROUND: CONSTRAINTS ON MODELS OF PRIMEVAL GALAXIES[1]

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ABSTRACT

We have searched for fluctuations in the near-infrared background in the K band ($\lambda = 2.2$ μm). The data show no statistically significant signal and imply 90% confidence level upper limits on brightness fluctuations of 7×10^{-20} ergs s^{-1} cm^{-2} Hz^{-1} sr^{-1} on scales from 10″ to 30″ and 3×10^{-20} ergs s^{-1} cm^{-2} Hz^{-1} sr^{-1} on scales from 60″ to 300″. We discuss the implications of these measurements on models of primeval galaxies.

Subject headings: cosmic background radiation — galaxies: evolution — galaxies: formation — galaxies: photometry — infrared: sources

I. INTRODUCTION

The formation and early evolution of galaxies are two important unsolved problems in cosmology. Eggen, Lynden-Bell, and Sandage (1962) pointed out that if few heavy elements were produced in the Big Bang (which is supported by the fact that halo stars in our Galaxy are metal-poor), an early generation of stars must have been consumed in order to form the metals that are seen in the older disk stars. This process must happen during the initial collapse of a protogalaxy, and the short time scale involved led Partridge and Peebles (1967*a*) to conclude that, during this phase, galaxies should be highly luminous as they radiate away the binding energy of the nuclei that form. They estimated that these young galaxies should be visible today as relatively large (> 5″) infrared objects whose luminosities would peak in the 1–3 μm wavelength range. Sunyaev, Tinsley, and Meier (1978) argued that young galaxies would instead appear as centrally condensed, almost stellar, objects that might be mistaken for quasars. Whatever their size might be, one could hope to detect the contribution of young galaxies to the extragalactic background light; Partridge and Peebles (1967*b*) suggested that this could possibly be detected in the wavelength range from 5 to 15 μm.

A number of observers have (unsuccessfully) attempted to detect primeval galaxies (Partridge 1974; Davis and Wilkinson 1974; Koo and Kron 1980), as well as their combined effect on the extragalactic background light (Roach and Smith 1968; Lillie 1971; Mattila 1976; Dube, Wickes, and Wilkinson 1977, 1979; Spinrad and Stone 1978; Toller 1983). All these observations were performed at wavelengths below 1 μm. In this paper we report on observations made in the K band (2.2 μm). We derive upper limits on the anisotropy of the sky brightness in this band and discuss the limitations that they impose on models of primeval galaxies.

II. SCENARIO

From an observational viewpoint, the most important characteristics of primeval galaxies are their number density, their luminosity, and the epoch at which they formed.

Partridge and Peebles (1967*a*) argued that the Galaxy formed soon after the protogalactic gas stopped expanding with the background matter, at the time

$$t_p \approx \left(\frac{\pi^2 r_p^3}{8GM}\right)^{1/2} , \tag{1}$$

where G is Newton's constant, M is the mass of the Galaxy, and r_p is its radius at the time of maximum extent. Recent observations of globular clusters (Hartwick and Sargent 1978) suggest $r_p \approx 60$ kpc and $M \approx 6 \times 10^{11}$ $M_\odot$ and perhaps as large as 1.2×10^{12} $M_\odot$. This yields $t_p \approx 2.2$–3.2×10^8 yr. If the cosmological constant vanishes, the time t corresponds to a redshift

$$1 + z = 16.44(h^2 \Omega t_8^2)^{-1/3} , \tag{2}$$

where we have followed the usual convention of writing Hubble's constant as $100h$ km s^{-1} Mpc^{-1}; Ω is the density parameter, with $\Omega = 1$ corresponding to a flat universe; and t_8 is the time expressed in units of 10^8 yr. Adopting $\Omega = 0.2$ and $h = 0.75$ yields a redshift corresponding to t_p in the range $14 < z < 19$; allowing any combination of cosmological parameters in the range $0.2 < \Omega < 1.0$ and $0.5 < h < 1.0$ produces a range $6 < z < 25$.

If a galaxy of mass M processed a fraction ΔX of its initial hydrogen to heavy elements during a period of time Δt, its mean bolometric luminosity during this phase is

$$L_{\rm PG} \approx 7 \times 10^{-3} Mc^2 \Delta X / \Delta t . \tag{3}$$

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Partridge and Peebles (1967*a*) argued that $\Delta t \approx 3 \times 10^7$ yr and $\Delta X \approx 0.02$. Assuming the mass of a bright galaxy to be $\sim 1.2 \times 10^{12} M_\odot$, its luminosity during this phase would have been

$$L_{PG} \sim 3 \times 10^{47} \text{ ergs s}^{-1} . \tag{4}$$

This is a rather uncertain estimate, as a substantial fraction of the mass of bright galaxies remains unseen (Boughn and Saulson 1983, and references therein); if this "dark matter" is made of nonbaryonic matter or did not participate in the initial flash, the relevant mass could be one order of magnitude lower. Besides, the flash could have lasted much longer if the initial collapse was dominated by gas dynamics (Larson 1974) and the initial star formation rate was smaller than assumed by Partridge and Peebles; this could make $\Delta t \approx 10^8$ yr and correspondingly lower the luminosity of the primeval galaxy during this phase.

If the emission is dominated by O and B stars, its spectrum will correspond to that of a blackbody with an effective temperature of about 3×10^4 K. Ezer and Cameron (1971) have argued that, because the first generation of stars does not contain metals, the effective temperature should be higher, possibly in the range 5–10 $\times 10^4$ K. The intrinsic spectra could be modified by self-absorption, with a cutoff at the Lyman continuum wavelength of 912 Å and about one-third of this light possibly being reemitted as Lyα photons (1216 Å) (Partridge and Peebles 1967*a*). In our discussion, we compare our results with models with effective temperatures 3×10^4, 5×10^4, and 10^5 K with and without the absorption cutoff and corresponding emission of Lyα photons.

Assuming the present density of bright galaxies to be $\eta_0 = 8 \times 10^{-3} h^3$ Mpc^{-3} (Davis and Huchra 1982), and that all of them underwent a similar bright phase, the expected areal density of primeval galaxies is

$$N = 3 \times 10^{-4} \eta_0 (1 + z_{PG})^3 d_A^2 c \Delta t_{PG} \text{ deg}^{-2} , \tag{5}$$

where z_{PG} is the redshift of the bright phase which lasts Δt_{PG}, and d_A is the "angular diameter" distance (Weinberg 1972). For a flat ($\Omega = 1$) universe, this corresponds to

$$N = 6 \times 10^2 \left(\frac{\eta_0}{0.008}\right)\left(\frac{\Delta t_8}{0.3}\right)\left(\frac{h}{0.75}\right)(\sqrt{1+z} - 1)^2 , \tag{6a}$$

or 5400 deg^{-2} if $z = 15$, with the indicated choice for the parameters; whereas, if $\Omega = 0.2$,

$$N = 1.5 \times 10^6 \left(\frac{\eta_0}{0.008}\right)\left(\frac{\Delta t_8}{0.3}\right)\left(\frac{h}{0.75}\right) \times \frac{[0.1z - 0.9(\sqrt{1+0.2z} - 1)]^2}{1+z} , \tag{6b}$$

or 34,000 deg^{-2} if $z = 15$, again assuming the indicated values for the remaining parameters.

III. OBSERVATIONS

Data were taken with the 4 m telescope at CTIO on the nights of 1981 September 14–15 and 15–16 and with the 1.3 m telescope at KPNO on the nights of 1983 March 30–31 through April 2–3. We used standard K-band ($\lambda_0 = 2.22$ μm, $\Delta\lambda = 0.42$ μm) filters and InSb photovoltaic photometers mounted at the Cassegrain foci.

We searched for brightness fluctuations in selected "blank" areas using the telescopes in the standard "chop-wobble" IR photometry mode. A nutating secondary reflector switches the beam rapidly (8 Hz) between two positions, A and B, on the sky ("chop"), and the difference between the signals at these two positions is taken, $\delta S_{AB} = S_A - S_B$. After a given integration time (we used between 20 and 60 s), the telescope is moved ("wobble") slightly, and two new positions are compared, producing $\delta S_{CD} = S_C - S_D$. Finally, the difference between the two differences is evaluated,

$$\Delta S = \delta S_{AB} - \delta S_{CD} = S_A + S_D - (S_B + S_C) . \tag{7}$$

The procedure eliminates nearly all the effects due to variable offsets produced by atmospheric and telescope emission.

The CTIO observations were done with an aperture of 10".6 in diameter with a chopping angle of 30" in the north-south direction, and the telescope was wobbled 30" in the east-west direction. The resulting beam pattern is shown in Figure 1*a*.

The observations at KPNO were made with apertures of 54" and 70" in diameter with both the chop and wobble angles being 300" along the east-west direction. In this case, positions A and D coincide, and the resultant signal is $\Delta S = 2S_A - S_B - S_C$. The corresponding beam pattern is shown in Figure 1*b*.

We observed 45 blank areas at high Galactic latitude which were selected such that no sources were located in or near the beams to the limits of several Schmidt and prime focus 4 m plates. Table 1 contains the approximate coordinates of the

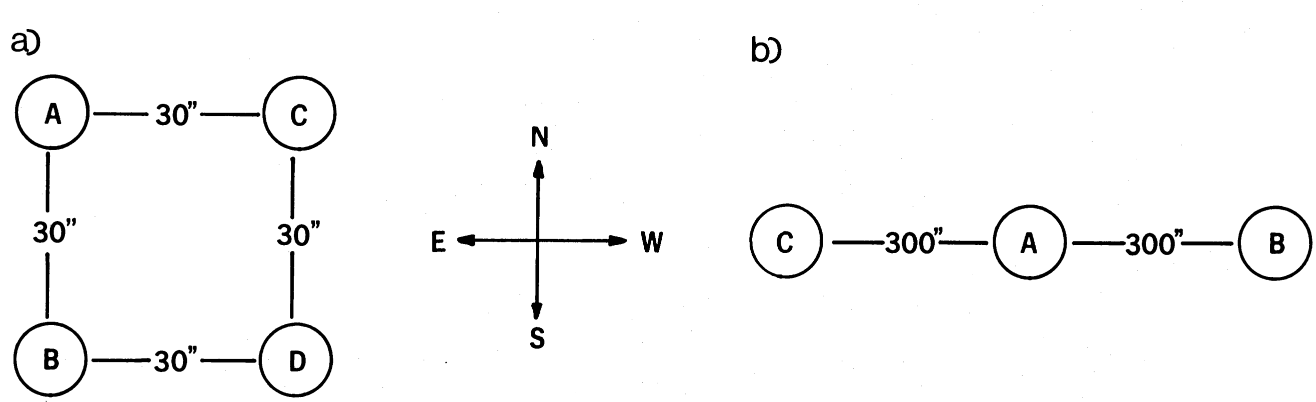

FIG. 1.—Beam pattern for the (*a*) CTIO and (*b*) KPNO observations

TABLE 1

AREAS OBSERVED

α, δ (1950)	b	Number of Blank Areas	Aperture	Plate (limit mag)	Beam Pattern
21^h40^m, $-40°$	$-50°$	6	10.″6	IIIa-J, ESO survey (23)	Fig. 1*a*
23 34 , −40	−70	10	10.6	IIIa-J, 4 m prime focus (24)	Fig. 1*a*
03 53 , −42	−50	6	10.6	IIIa-F, 4 m prime focus (23)	Fig. 1*a*
10 15 , +47	+55	10	54″, 70″	Palomar Survey-red (19.5)	Fig. 1*b*
13 07 , +31	+86	13	54″, 70″	Palomar Survey-red (19.5)	Fig. 1*b*

areas searched. Each blank area was observed for 30–60 minutes.

IV. DATA ANALYSIS AND RESULTS

The data consists of 56 measurements, $M_i = \Delta S_i + N_i$, of the flux differences ΔS_i obtained as described in § III. The quantity N_i is the contribution due to the noise associated with each measurement. Eleven of the 45 blank areas were observed with two apertures, 54″ and 70″. Somewhat less than 2% of the KPNO data were deleted due to interference that had been noticed at the telescope and traced to a loose BNC connector in the photometer's preamplifier circuit.

As the data will be used to look for evidence (or lack thereof) of a statistical sky signal that would have inflated the dispersion of the data points from the respective mean values by more than expected from the statistical noise alone, it is crucial to estimate accurately the errors associated with each one of the data points (Uson and Wilkinson 1984). Extensive spectral analysis showed the noise in each measurement to be white, Gaussian, and consistent with the assumption of being drawn from a single parent population. Therefore, we assign to each measurement a formal error σ_i, which is the standard deviation of the mean for the sequence of data points for that particular blank area. (The integration times for each data point were between 40 s and 2 minutes, and the observations of each blank area lasted 30 minutes; some of them were observed more than once, as indicated above.) The average values are $\bar{\sigma}_i = 1.3 \times 10^{-28}$ ergs s^{-1} cm^{-2} Hz^{-1} for the CTIO data and $\bar{\sigma}_i = 3.3 \times 10^{-27}$ ergs s^{-1} cm^{-2} Hz^{-1} for the KPNO data.

One small correction was applied to the KPNO data. Because the chop and wobble angles were large (300″), the change in the sky brightness with zenith angle produced a small atmospheric contribution to the data. The sky brightness was determined by monitoring the DC output of the photometer as a function of zenith angle about once every 2 hr during each night. These measurements conformed to the expected secant law model for the atmosphere and were used to correct the data. The corrections systematically increased the measured values, with the largest correction amounting to only 0.4 standard deviations. Nevertheless, the corrections bring the overall mean for the KPNO data to $\overline{\Delta S} = -0.5 \pm 8.1 \times 10^{-28}$ ergs s^{-1} cm^{-2} Hz^{-1}. The overall mean of the CTIO data is $\overline{\Delta S} = -1.9 \pm 2.9 \times 10^{-29}$ ergs s^{-1} cm^{-2} Hz^{-1}. These values show that the observing procedure does indeed achieve a good cancellation of systematic effects.

The data are displayed in Figure 2 as histograms of the measurements normalized by their estimated standard deviation. The curves are the Gaussian distributions expected if the measured fluxes are due entirely to noise, i.e., $\Delta S_i \equiv 0$. An underlying sky signal, $\Delta S_i \neq 0$, which should be uncorrelated with the measurement noise, would have produced shallower and broader histograms; this is clearly not the case here: the largest deviation of the 56 measured flux differences from zero was only 2.4 standard deviations.

Since there is no statistically significant evidence that the flux differences ΔS_i differ from zero, we estimate upper limits on the luminosity of primeval galaxies that would still be consistent with the data. We assume a simple two-parameter model with a Poisson distribution of primeval galaxies characterized by an average number density of N galaxies per square degree with each one contributing the same flux S.

The most robust statistic is the likelihood ratio

$$\lambda \equiv \frac{P_1(M_i)}{P_0(M_i)} \quad (8)$$

where $P_1(M_i)$ is the probability density that the data M_i come from an ensemble with primeval galaxies characterized by N and S, and Gaussian noise characterized by σ_i, whereas $P_0(M_i)$ is the probability density that the M_i come from an ensemble with no signal and only noise σ_i (Whalen 1971). The probability distribution of the statistic λ is difficult to compute, so we used the Monte Carlo technique and generated a sequence of realizations of λ from simulated data which were drawn from a Poisson distribution of "primeval galaxies" characterized by N and S with a "measurement" noise contribution derived from a Gaussian distribution characterized by its standard deviation σ_i and zero mean. From the distribution of values of λ we can determine the likelihood that our measurements are consistent with a given model of primeval galaxies. The results of the Monte Carlo investigation are illustrated in Figure 3. The shaded region of the graph indicated those models for which the likelihood ratio has less than a 10% chance of being as small as the value calculated from our data. Those models are therefore excluded at the 90% confidence level. The boundary of the left portion of this region is determined by the data taken with the larger apertures at KPNO, while the boundary of the right portion of the excluded region is due to the data taken at CTIO which were obtained with the smaller aperture. Each set of observations is insensitive to models in which the number of primeval galaxies per square degree is so small that there is less than a 90% probability of having one of them falling in at least one of our beams.

Another useful model with which to compare our data is one in which the fluxes of the observed sky positions are statistically independent and derive from a single Gaussian distribution with standard deviation σ_0. The mean of the distribution is the average background flux, but our observing scheme is insensitive to it. Our measurement yields upper limits to σ_0. Monte Carlo calculations similar to those described above set 90% confidence level upper limits of $\sigma_0 < 1.4 \times 10^{-28}$ ergs cm^{-2} s^{-1} Hz^{-1} on scales from 10″ to 30″, given by the CTIO data, and $\sigma_0 < 2.0 \times 10^{-27}$ ergs cm^{-2} s^{-1} Hz^{-1} on scales from 60″ to 300″ using the KPNO data. Taking

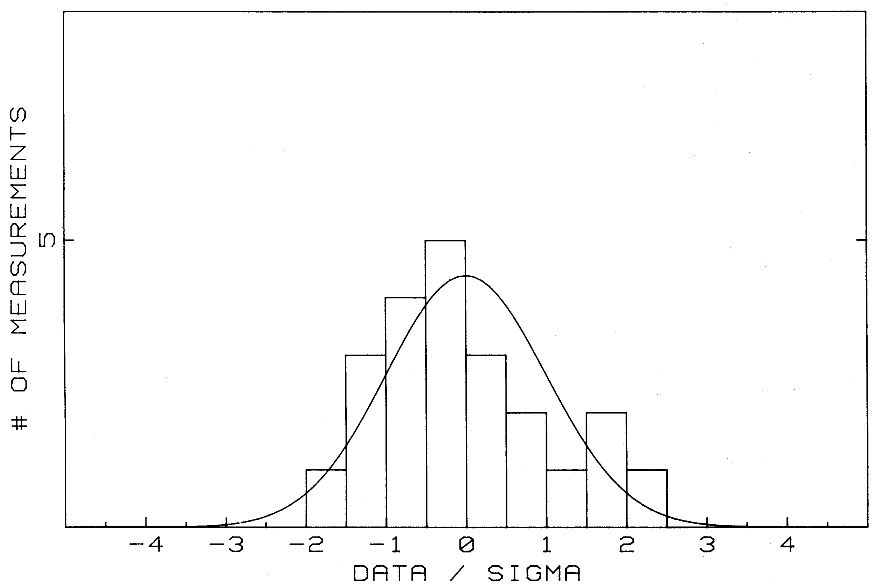

FIG. 2*a*

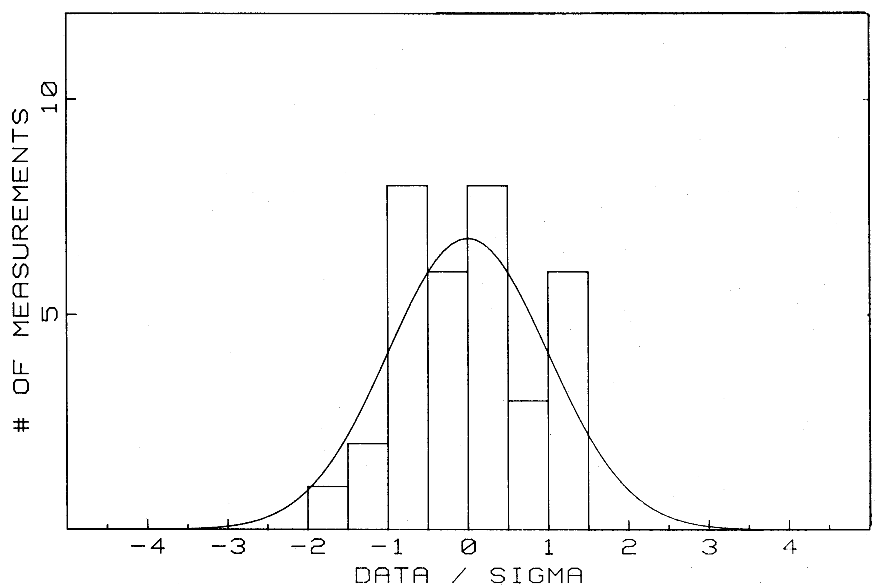

FIG. 2*b*

FIG. 2.—Histograms of normalized measurements M_i/σ_i for (*a*) the CTIO data and (*b*) the KPNO data. The Gaussian curves show the expected shapes if the data are only due to the measurement noise N_i, i.e., if $\Delta S_i \equiv 0$.

into account the solid angle of the beams, these correspond to 90% confidence level upper limits on brightness fluctuations of 7×10^{-20} ergs cm^{-2} s^{-1} Hz^{-1} sr^{-1} and 3×10^{-20} ergs cm^{-2} s^{-1} Hz^{-1} sr^{-1} respectively, on the same angular scales.

We have estimated the background fluctuations which would be expected due to nearby ($z < 1$) galaxies which were too dim to appear on our finding charts. These estimates were generated by translating the (red) number counts of Koo; Couch and Newell; and Shanks, Stevenson, Fong, and MacGillivray (Shanks, *et al.* 1984) to the *K* band, using the appropriate *K*-corrections and galaxy colors. We assume that our selection of blank fields eliminated galaxies brighter than $m_R = 19.5$ for the KPNO observations or $m_J = 24.0$ for the CTIO measurements. The expected fluctuations are $\sigma_G \approx 1.6 \times 10^{-28}$ ergs cm^{-2} s^{-1} Hz^{-1} for the CTIO measurements and $\sigma_G \approx 1.6 \times 10^{-27}$ ergs cm^{-2} s^{-1} Hz^{-1} for the KPNO data. These expected contributions are somewhat uncertain in view of the extrapolations we have made to compute them. Nevertheless, they suggest that an increase of the sensitivity of this experiment might pick up a contribution due to this background of nearby galaxies.

We estimated as well the contamination due to stars in our Galaxy; it was found to be an order of magnitude smaller than the contribution due to faint galaxies.

V. DISCUSSION

The predicted flux densities f_ν (expressed as *K*-band magnitudes, $m_K = -2.5 \log f_\nu - 50.5$) are displayed in Figure 3

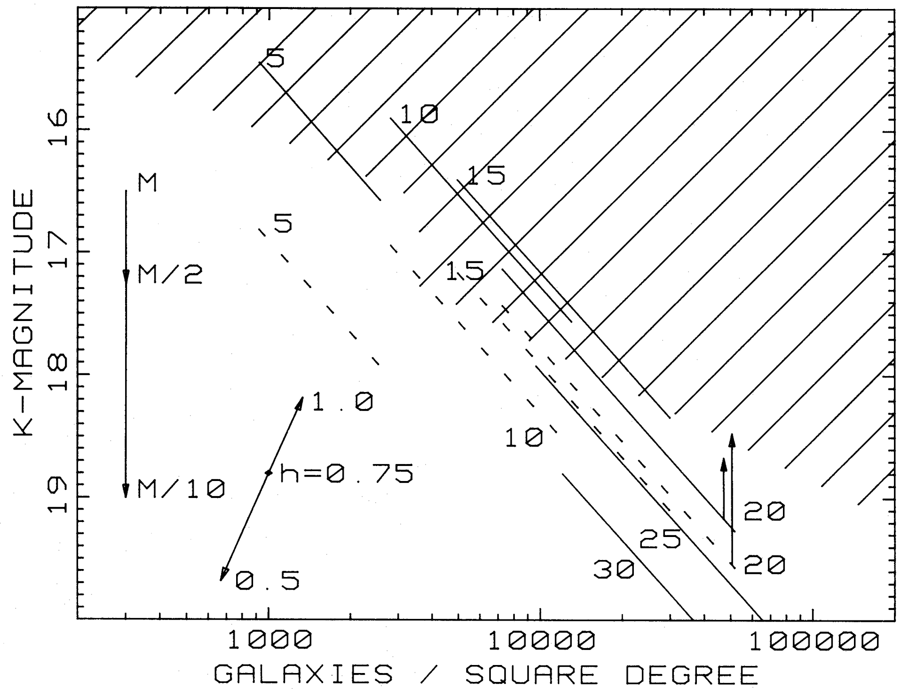

FIG. 3.—Models of primeval galaxies excluded by these measurements (*shaded region*). The lines correspond to theoretical predictions for $T = 3 \times 10^4$ K (*solid lines*) and $T = 5 \times 10^4$ K (*dashed lines*) (see § V).

versus areal density of primeval galaxies for several values of the redshift, $1 + z$. We have adopted the fiducial values discussed in § II, namely $M = 1.2 \times 10^{12}\ M_{\odot}$, $\eta_0 = 8 \times 10^{-3} h^3$ Mpc^{-3}, and $\Delta t = 0.3 \times 10^8$ yr with a Hubble constant $h = 0.75$. The upper left side of each line corresponds to a flat universe, whereas the lower right side corresponds to $\Omega = 0.2$. As indicated in the graph, halving the mass that participates in the bright flash lowers the curves by 0.75 mag, with a factor of 10 in mass corresponding to 2.5 mag. Also indicated is the effect of modifying the adopted value of Hubble's constant: a faster expanding universe ($h = 1.0$) shifts all the lines toward the upper right by the indicated amount, whereas, if $h = 0.5$, all curves are shifted toward the lower left by the increment shown. Finally, the effect of increasing Δt is to shift the predictions *along* the curves toward the lower right corner, by an amount that conforms to the corresponding change in the surface densities as derived from equations (6a) and (6b).

The possible redistribution of Lyman photons discussed in § II does not affect the predictions for redshifts $1 + z < 16$. For redshifts in the range $16 < 1 + z < 22$, the absorbed photons would have missed our bandpass anyway, but the expected Lyα emission does contribute to our signal, increasing the predicted fluxes by about 0.5 mag for a 3×10^4 K effective blackbody temperature (*solid lines*), and by about 1 mag if the effective temperature was 5×10^4 K (*dashed lines*). Finally, for redshifts above $1 + z = 22$, the Lyα line falls outside our band and therefore causes no effect; while Lyman absorption would reduce the predicted fluxes by 0.4 mag ($1 + z = 25$) to 0.7 mag ($1 + z = 30$) for both assumed temperatures.

For the 3×10^4 K model, the predictions exceed the upper limits deduced from our data for redshifts $1 + z < 15$, if $h \geq 0.75$. For $h = 0.5$, all models are still (barely) allowed. Models with $T = 5 \times 10^4$ K are only excluded if $h = 1.0$; although if Lyα emission is considered, the data exclude models with $z \approx 20$ even if $h = 0.75$. If $T = 10^5$ K, the predicted curves fall slightly below those for $T = 5 \times 10^4$ K.

For the excluded models, the simplest conclusion is that not all the mass in the galaxies participated in the bright flash. Besides, assuming larger values for Δt brings most models out of the excluded region, unless $h = 1.0$.

Absorption by dust along the line of sight will reduce the flux of primeval galaxies. Although the effect is smaller at K-band than at optical wavelengths, it might have prevented us from detecting a signal above the excluded level. We have made a rough estimate of the expected extinction, using the method of Wright (1981) with a dust extinction curve from Savage and Mathis (1979). We assume the dust is contributed mainly by irregular and spiral galaxies (half the number of galaxies used above) and produces 0.2 mag of visual extinction per galaxy. Not all galaxies would be seen face-on; however, the higher contribution due to other orientations is statistically compensated by a lower chance of intersecting them along the line of sight. The model overestimates the effect if the (comoving) density of dust was smaller in the past (which seems likely), as the dominant contribution corresponds to the higher redshift range. The calculation yields $\Delta m_K \approx 0.17 \Omega^{-0.4} h^2$ if $1 + z = 10$ (0.10 and 0.18 for $h = 0.75$ and $\Omega = 1$ and 0.2 respectively), with the prediction scaling by factors of 2, 4, 8, and 12 for $1 + z = 15$, 20, 25, and 30 respectively. This could have drastically reduced our possibilities of detecting even the brightest models considered if $1 + z \geq 25$. Current upper limits on the density of intergalactic dust (Stein and Soifer 1983) allow, in principle, much higher values for the extinction due to this smooth component, although there is no evidence that it exists in any significant amount at all.

Clustering of primeval galaxies would have increased the fluctuations and enhanced our possibilities of detecting them. The effect is uncertain, as there are no reliable estimates of the evolution of the clustering of galaxies with time. Galaxies could have formed at random locations, with clustering developing after the bright phase; therefore, the effects of clustering have been ignored in our analysis.

Primeval galaxies could be larger on the sky than the 10."6 aperture that we used at CTIO, although most of the light might be emitted by a central core of a few arcseconds in diameter (Sunyaev, Tinsley, and Meier 1978). In any case, a possible diminution of the signal expected from each primeval galaxy due to this effect is compensated by the fact that the beam would intersect more of them. Overall, this would not affect the excluded region in Figure 3, except that the cutoff in the sensitivity of the CTIO data would move toward the left, as there would be a greater chance of intersecting primeval galaxies along the line of sight.

As indicated above, an increase in sensitivity of this technique should produce significant signals due to the infrared light of faint nearby objects. Such a contribution could be separated by multicolor observations of any "blank area" that showed a signal. Any candidate "primeval galaxy" would then be observed spectroscopically.

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