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Gravitational correction to vacuum polarization

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We consider the gravitational correction to (electronic) vacuum polarization in the presence of a gravitational background field. The Dirac propagators for the virtual fermions are modified to include the leading gravitational correction (potential term) which corresponds to a coordinate-dependent fermion mass. The mass term is assumed to be uniform over a length scale commensurate with the virtual electron-positron pair. The on-mass shell renormalization condition ensures that the gravitational correction vanishes on the mass shell of the photon, i.e., the speed of light is unaffected by the quantum field theoretical loop correction, in full agreement with the equivalence principle. Nontrivial corrections are obtained for off-shell, virtual photons. We compare our findings to other works on generalized Lorentz transformations and combined quantum-electrodynamic gravitational corrections to the speed of light which have recently appeared in the literature.

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I. INTRODUCTION

The speed of light, in curved space-time, is not as "constant" as one would otherwise imagine. The curvature of space-time, according to classical general relativity (see Appendix A), acts as a refractive medium (without dispersion), giving rise to an effective change in the speed of light (as seen from a global, not local, coordinate system), which reads as

$$\frac{\Delta c}{c_0} = 2\,\frac{\Phi_G(\vec{r})}{c_0^2} = (1+\gamma)\,\frac{\Phi_G(\vec{r})}{c_0^2} < 0. \tag{1}$$

Here, $\Phi_G(\vec{r})$ is the gravitational potential, normalized to zero for two very distant objects, and the γ parameter is introduced (for Einsteinian gravity, we have $\gamma = 1$, see Refs. [1,2]). Throughout this article, we set $c_0 = 299\,792\,458$ m/s equal to the speed of light as consistent with the Einstein equivalence principle, which states that space-time is locally flat. The speed-of-light parameter c_0 is canonically set equal to unity in an appropriate unit system. The time delay formula (1) is valid to first order in the gravitational coupling constant (Newton's constant) G. The concomitant slow-down of light is known as the Shapiro time delay [3–5]. One of the most precise tests has been accomplished with the Cassini spacecraft in superior conjunction on its way to Saturn [6]; it involves Doppler tracking using both X-band (7175 MHz) as well as Ka-band (34 316 MHz) radar.

At high energy, the dispersion relation for a massive particle is not different from that for photons, $E = \sqrt{\vec{p}^{\,2}\,c_0^2 + m^2\,c_0^4} \approx |\vec{p}|\,c_0 = \hbar\,|\vec{k}|\,c_0$, where E is the energy, $\vec{p}$ is the momentum, and $\vec{k}$ is the wave vector of the (light or matter) wave. The modification (1) affects the speed of propagation for photons as well as highly energetic neutrinos. For the central field of the Sun, we have $\Phi_G(\vec{r}) = -G\,M_\odot/r$ where $M_\odot$ is the Sun's mass. In general, Φ_G is negative, implying that light is slowed down due to the bending of its trajectory caused by space-time curvature.

Recently, in Ref. [7], it has been claimed that an additional quantum electrodynamic (QED) correction to the result (1) exists, which is of the functional form

$$\frac{\delta c_\gamma}{c_0} = \chi\,\alpha\,\frac{\Phi_G(\vec{r})}{c_0^2} < 0, \tag{2}$$

where α is the fine-structure constant, and χ is a constant coefficient. For details of the arguments which led Franson to his result given in Eq. (2), we refer the reader to Sec. 3 of Ref. [7]. Essentially, Franson [7] evaluates the vacuum-polarization correction for photons (on shell) in the gravitational field, using a partially noncovariant formalism [the photon energy E is used as a noncovariant variable in the propagators; see Eq. (13) ff. of Ref. [7] for details of Franson's considerations]. It is known from quantum electrodynamic bound-state calculations that even a slight noncovariance in the regularization scheme can induce spurious terms [8]; some scrutiny should thus be applied. Using his calculational scheme, Franson comes to the conclusion that the speed of *photons* [hence the subscript γ in Eq. (2)] is altered due to the gravitational correction to the electron-positron propagators that enter the vacuum-polarization loop calculation. For the coefficient χ, the following result has been indicated in Ref. [7]:

$$\chi = \frac{9}{64} \qquad \text{(according to Ref. [3])}. \tag{3}$$

The decisive point of the analysis presented in Ref. [7] is that the effect described by Eq. (2) is claimed to affect only photons, not neutrinos, thus slowing the photons in comparison to the neutrinos (and other massive fermions). According to Ref. [7], the propagation of otherwise massless photons is influenced by the electron-positron (light fermion) vacuum-polarization effect at one-loop order, that of fermions is not.

A different ansatz for a modification of local Lorentz transformations stems from the work of Vachaspati [9], who claims that in addition to "electromagnetic time," one can define an "absolute time" (on the level of special relativity), which transforms according to a modified Lorentz transformation (referred to here as the Vachaspati transformation), and which, according to Ref. [9], is claimed to be compatible with muon lifetime and Michelson-Morley experiments (see

Appendix B). The Vachaspati transformation also leads to a "speed-of-light" parameter which is dependent on the inertial frame.

Here, we aim to investigate three sets of questions: (i) Is the result given in Eq. (2) compatible with all other astrophysical observations recorded so far in the literature? What bounds can be set for the χ parameter given in Eq. (3)? *Irrespective of the value of* χ, what changes would result from a hypothetical quantum modification of the speed of light, induced according to the functional form Eq. (2), for the description of other physical phenomena? In particular, how would we describe neutrinos in strong gravitational fields, where according to Ref. [7], they propagate faster than the speed of light, even at high energy? How would the result given in Eq. (2) affect the Schiff conjecture [1,2,10]? (ii) The next question then is whether the modification given in Eq. (2) *exists at all.* In Sec. III, we investigate whether or not the calculations reported in Ref. [7], which lead to the quantum effect (2), stand the test of a fully covariant formulation of the gravitational corrections to vacuum polarization, where the virtual fermions in the loop are subject to gravitational interactions. Our calculation is restricted to an analysis of the electron-positron loop insertion into the photon propagator, which is the subject of Ref. [7], and does not take all possible quantum corrections to the photon propagator into account. The analysis in Sec. II thus covers a much more general scope and answers general questions regarding a modification of the speed of light in gravitational fields, induced according to Eq. (2), while the analysis in Sec. III only covers the vacuum-polarization loop with fermion propagators subject to gravitational interactions. (iii) In the context of atomic physics, what phenomenological consequences will result from the gravitational correction to the off-shell (virtual) photon propagator? This is briefly discussed in Sec. IV. The first set of questions also has relevance for the work of Vachaspati [9]. Conclusions are reserved for Sec. V.

II. QUANTUM EFFECTS AND SPEED OF LIGHT

A. Quantum correction and Shapiro time delay

Because the quantum correction (2) is conjectured to be induced by a virtual loop consisting of electrons and positrons lifted from the quantum vacuum, its existence is not excluded by classical theory, i.e., beyond the validity of the original (purely classical) general theory of relativity formulated by Einstein and Hilbert [11–13]. The delay induced by the conjectured modification Eq. (2) for light rays propagating from the Large Magellanic Cloud is claimed to be in agreement [7] with the observed early arrival time of the (still somewhat mysterious) early neutrino burst under the Mont Blanc recorded in temporal coincidence with the SN1987A supernova [14]. Essentially, the paper [7] claims that the apparent superluminality of the "early" neutrino burst could be due to a quantum electrodynamic effect which slows down light in comparison to the neutrinos, in strong gravitational potentials, with a delay induced according to Eq. (2).

However, this result should be compared to other precision measurements of time delays induced by space-time curvature, such as the Shapiro time delay [3–5]. The time delay due to the refractive index of curved space leads to the following formula for a light ray or radar wave as it bounces back from an object close to superior conjunction [see Eq. (49) of Ref. [1] and Fig. 1],

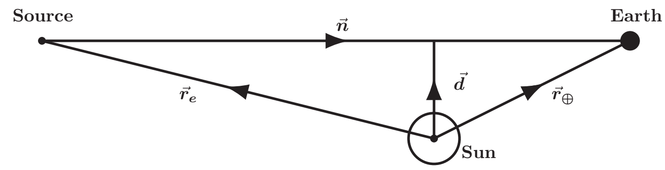

FIG. 1. Geometry for Eq. (4).

$$\delta t = 2(1+\gamma)\frac{G\,M_\odot}{c_0^3}\ln\left(\frac{(r_\oplus + \vec{r}_\oplus\cdot\vec{n})\,(r_e - \vec{r}_e\cdot\vec{n})}{d^2}\right). \tag{4}$$

Here, $\vec{r}_e$ is the vector from the Sun to the source (e.g., the Cassini spacecraft), $\vec{r}_\oplus$ is the vector from the Sun to the Earth, while $\vec{n}$ is the unit vector from the source to the Earth, and d is the distant of closest approach of the light ray as it travels from the Earth to the source and back. The parameter γ is used in order to describe potential deviations from the classical prediction.

The formula (4) is obtained on the basis of the classical result (1). For details of the derivation, we refer to Chap. 4.4 on page 196 ff. of Ref. [15] and exercise 4.8 of page 161 of Ref. [16]. The quantum "correction" given in (2) has the same functional form as the classical result (1) but adds a correction to the prefactor. If we assume the explicit numerical result given in Eq. (3) to be valid, then this leads to a γ coefficient different from unity,

$$\gamma - 1 = \chi\,\alpha = \frac{9\,\alpha}{64} = 1.03\times 10^{-3}. \tag{5}$$

However, the result of the Cassini observations [6] reads as follows:

$$\gamma - 1 = (2.1 \pm 2.3)\times 10^{-5}. \tag{6}$$

The claim (3) thus is in a 44.8σ disagreement with the experimental result (6), which is otherwise consistent with zero. Unless the authors of Ref. [6] have overlooked a significant source of systematic error, the effect described by Eq. (3) thus is in severe disagreement with experiment. Finally, we should remark that the γ parameter also enters the expression for the light deflection formula around a central gravitational center. A 2004 analysis of almost 2 million very-long baseline (VLBI) observations of 541 radio sources, made by 87 VLBI sites, yields the bound [17]

$$\delta\gamma = (-1.7 \pm 4.5)\times 10^{-4}, \tag{7}$$

which also is in disagreement with the claim (2). According to Refs. [18,19], all current VLBI data together yield a value of $\delta\gamma = (0.8 \pm 1.2)\times 10^{4}$, compatible with zero.

Alternatively, we can convert the result (6) into a bound for the χ coefficient,

$$\chi = (2.9 \pm 3.2)\times 10^{-3}\,, \tag{8}$$

consistent with zero. However, quantum effects of the functional form (2), but with a numerically small coefficient compatible with the bound (8), cannot be excluded at present.

B. Fermion wave equation

Let us analyze the problem of a fermion wave equation for a local Lorentz frame in which photons propagate slower than high-energy fermions. We remember that the Lorentz violation induced by Eq. (2) actually is quite subtle; the effect is not excluded by classical physics and vanishes globally in the absence of gravitational interactions, i.e., it does not perturb the speed of light in globally flat (Minkowski) space-time. In order to write a wave equation describing fermions, we have to carefully distinguish between the flat-space speed of light c (in the absence of gravitational interactions), the classical "correction" Δc (which is compatible with the Einstein equivalence principle and does not preclude the existence of the local Minkowskian frame of reference), and the quantum correction δc_γ given in Eq. (2), which changes the speed of light in a "local" reference frame to

$$c_{\rm loc} = c_0 + \delta c_\gamma = c_0 - |\delta c_\gamma|. \tag{9}$$

We recall that, physically, the speed of light is the speed which describes the propagation of the transverse components of the electromagnetic field, which enter the Maxwell equations. (The necessity of a careful separation of transverse and longitudinal components has recently been highlighted in a consideration of the photon wave functions, given in Ref. [20].)

As already stated in Sec. I, according to Ref. [7], the modification (2) is supposed to slow down photons in strong gravitational fields, not neutrinos or electrons. Let us therefore investigate the question of a correct equation to describe fundamental fermions in strong gravitational fields (deep potentials), on the basis of a (possibly generalized) Dirac equation. One possibility is to postulate that the local Lorentz transformation has to be modified to include the local quantum modification of the speed of light, while the formalism of classical general relativity is unaltered by the quantum modification. Let us also assume that the "local Lorentz transformation," under the presence of the quantum correction (2), is formulated to be the transformation which preserves the light element

$$dx^\mu\, dx_\mu = c_{\rm loc}^2 dt^2 - d\vec{r}^{\,2} = 0, \tag{10}$$

where $dx^\mu = (c_{\rm loc}\, dt, d\vec{r})$ is a space-time interval, and $c_{\rm loc}$ is the speed of light in the local coordinate system. The correction Δc given in Eq. (1) is compatible with the Einstein equivalence principle of a locally flat space-time and therefore does not change the Dirac equation (with parameter c) in the usual Dirac equation for fermion wave packets, but the quantum correction δc_γ, given in Eq. (2), leads to the replacement $c_0 \to c_{\rm loc}$.

According to Eq. (2), high-energy fermions are faster than light rays at high energy, by an offset $|\delta c_\gamma|$, making them effectively superluminal, thus leading to an explanation for the early neutrino burst from the supernova 1978A (see Refs. [7,14]). The preferred way to describe highly energetic fermions (neutrinos) which travel faster than light is via the tachyonic Dirac equation [21], which in the local reference frame reads as

$$\left(i\hbar\, \gamma^\mu \frac{\partial}{\partial x^\mu} - \gamma^5 m c_{\rm loc} \right) \psi(t,\vec{r}) = 0, \tag{11}$$

where $\psi(t,\vec{r})$ is the fermion wave function. The projector sums for the tachyonic spinor solutions have recently been investigated in Refs. [22,23]. The main problem here does not lie in the tachyonic equation, but in the description of highly energetic neutrinos because of their uniform velocity offset $|\delta c_\gamma|$ at high energy from photons. This offset prevents them from reaching the photon mass shell in the local coordinate system. To see this, let us note that the particles described by Eq. (11) fulfill the dispersion relation

$$E = \sqrt{\vec{p}^{\,2}\, c_{\rm loc}^2 - (m c_{\rm loc}^2)^2}, \tag{12a}$$

$$E = \frac{m c_{\rm loc}^2}{\sqrt{v^2/c_{\rm loc}^2 - 1}}, \tag{12b}$$

$$|\vec{p}| = \frac{m v}{\sqrt{v^2/c_{\rm loc}^2 - 1}}, \tag{12c}$$

where $v \approx c_0 > c_{\rm loc}$ is the propagation speed of highly energetic neutrinos, required for the explanation of the early arrival time of the neutrinos according to Ref. [7]. The energy can thus be expressed as

$$E = \frac{m c_{\rm loc}^2}{\sqrt{v^2/c_{\rm loc}^2 - 1}} \approx \frac{m\, c_{\rm loc}^{5/2}}{\sqrt{2|\delta c_\gamma|}}, \tag{13a}$$

$$v = c_{\rm loc} + |\delta c_\gamma| \approx c_0. \tag{13b}$$

We are now in a dilemma: On the one hand, the energy of a highly energetic neutrino is not bounded from above, but even for a neutrino traveling *exactly* at the speed of light $v = c_0$, the right-hand side of Eq. (13a) only contains the fixed parameters $c_{\rm loc}$ and $|\delta c_\gamma|$. Hence, the only way to make Eq. (13a) compatible with Eq. (11) is to assume a universal mass "running" of the tachyonic mass parameter in Eq. (11), linear with the energy scale, of the functional form

$$m \to m(E) \propto E = \frac{\sqrt{2|\delta c_\gamma|}}{c_{\rm loc}^{5/2}}\, E. \tag{14}$$

It thus becomes clear that the mere existence of a "local" gravitational quantum correction of the functional form (2) would induce severe problems in the description of high-energy fermions in local reference frames in strong gravitational fields ("deep potentials"). In other scenarios of Lorentz breaking mechanisms in local reference frames [24–26], the Lorentz-breaking terms are not required to run with the energy scale. The same is true for small Lorentz-violating admixture terms to Dirac equations in free space [27–29].

As a final remark, let us note that according to Ref. [7], high-energy neutrinos would be traveling faster than light, but not faster than electrons. Hence, the analog of Cerenkov radiation emitted by neutrinos, namely, the reaction $\nu \to \nu + e^+ + e^-$, cannot occur; according to Ref. [30], this process constitutes the main decay channel of tachyonic neutrinos. Genuine Cerenkov radiation $\nu \to \nu + \gamma$ is suppressed for the electrically neutral neutrinos and must proceed via a W loop. The slow-down of light in comparison to high-energy fermions according to Eq. (2), though, would lead to Cerenkov radiation from highly energetic charged leptons [e.g., synchrotron losses at the Large Electron-Positron Collider (LEP)]. Within the models studied in Refs. [31–33], rather stringent bounds have been obtained for certain Lorentz-violating parameters. All of these results, though, are model dependent. E.g., the dispersion relation $E = p\,v_\nu$, assumed in Ref. [30], is different from the dispersion relation that is generally assumed for tachyonic neutrinos [see Eq. (12a) here in the paper and independently Ref. [21]]. The Lorentz violation induced by the slow-down of light due to a radiative correction proposed in Ref. [7] is quite subtle; however, the functional form (2) allows for a direct model-independent comparison with bounds on the γ parameter introduced in Eq. (1), as discussed in Sec. II A.

C. Equivalence principle and Schiff conjecture

The Schiff conjecture (see Sec. 2.2.1 of Ref. [10]) is connected with two different forms of the equivalence principle, namely, the weak equivalence principle and the Einstein equivalence principle. Originally, Newton stated that the property of a body called "mass" ("inertial mass") is proportional to the "weight" (which enters the gravitational force law), a principle otherwise known as the "weak equivalence principle" (WEP). The Einstein equivalence principle (EEP) states that (i) WEP is valid, (ii) the outcome of any local nongravitational experiment is independent of the velocity of the freely falling reference frame in which it is performed (local Lorentz invariance, LLI), (iii) the outcome of any local nongravitational experiment is independent of where and when in the universe it is performed (local position invariance, LPI).

It is obvious to realize that the existence of a local modification of the speed of light in deep gravitational potentials according to (2) would lead to a (very slight, but noticeable) violation of point (iii) of the EEP. Namely, because the shift δc_γ affects only photons, not neutrinos or electrons (according to Ref. [7]), one could measure the local propagation velocity of high-energy fermion versus photon wave packets. The former propagate at velocity c_0 in a local reference frame, whereas the latter are affected by the correction $\delta c_\gamma \propto \Phi_G$. The potential Φ_G depends on the position in the Universe where the experiment is performed (for reference values of Φ_G in different regions, see Table 1 of Ref. [7]).

According to Sec. 2.2.1 of Ref. [10], the Schiff conjecture states that for self-consistent theories of gravity, WEP necessarily embodies EEP; the validity of WEP alone guarantees the validity of local Lorentz and position invariance, and thereby of EEP. The question of whether the correction δc_γ violates the WEP is a matter of interpretation because δc_γ affects only massless objects, namely, photons; it is, as already emphasized, a *quantum* effect which goes beyond the scope of classical mechanics in which the weak equivalence principle was first formulated (in its original form by Newton).

One could perform a thought experiment and enter a region of deep gravitational potential with three freely falling, propagating wave packets, one describing a photon, the others describing a very highly energetic neutrino and a very highly energetic electron, respectively. The latter two propagate at a velocity (infinitesimally close to) c_0. If a correction of the form δc_γ exists, then photons will have been decelerated to a velocity $c_0 - |\delta c_\gamma|$ within the deep gravitational potential, whereas *both* fermions will have retained a velocity (infinitesimally close to) c_0. If we regard the photons as particles (the photon being a concept introduced into physics after the WEP was first introduced by Newton), then we could argue that a "force" must have acted onto the photon, causing deceleration, even though the particles were in free fall. This might indicate a violation of the WEP but only if the photon were regarded as a normal "particle" in the sense of Newton's idea (which is not fully applicable because of the vanishing rest mass of the photon). Alternatively, we could interpret any change in velocities relative to the local speed of light as an "acceleration" and thus interpret the faster propagation of the electrons and neutrinos in comparison to the photon within the region of deep gravitational potential as the result of a force which must have acted on the fermions. *Both* neutrinos and electrons retain a velocity very close to c_0 and have thus been accelerated by the same velocity $|\delta c_\gamma|$; because of their different rest mass, the force acting on them must have been different, thus violating the WEP.

Today, one canonically understands the WEP as not being tied to "massive" objects, stating that free-fall at a given point in space-time is the same for all physical systems, and that photons, electrons, and neutrinos in a gravitational potential all act as if they are in the same accelerated coordinate frame. In that sense, if a theory predicts that gravitational potentials make the local photon velocity different from the local limiting velocity of high-energy massive particles, then that theory violates the WEP.

Thus, depending on the interpretation, one might conclude that Schiff's conjecture holds true, in the sense that the correction (2) violates *both* the WEP as well as the EEP. The caveat must be stated because strictly speaking, photons do not have a rest mass, and thus, the WEP in the original formulation is not fully applicable. One should also bear in mind that slight violations of fundamental laws and symmetries of nature are being discussed and all we can do is establish bounds for violating parameters [24–29]. For the scenario studied by Vachaspati (see Appendix B and Ref. [9]), the violations of the EEP and the WEP would be of order unity; the "light speed measured in absolute time" can be different from the "light speed measured in electromagnetic time," depending on the relative velocity of the moving frames v_A.

III. DIRAC EQUATION AND GRAVITATIONAL COUPLING

The far-reaching consequences of any correction of the form (2) to the speed of light in deep gravitational potentials together with the bound formulated in Eq. (8) for the χ coefficient stimulate a recalculation of the leading gravitational correction to vacuum polarization, supplementing the analysis of Ref. [7]. Recently, the gravitationally coupled Dirac equation has been investigated [34–37], with particular emphasis on the Dirac-Schwarzschild problem, which is the equivalent of the Dirac-Coulomb problem for electrostatic interactions and describes a particle bound to a central gravitational field. From now on, for the remainder of this article, we revert to natural units with $\hbar = c_0 = \epsilon_0 = 1$, because we no longer consider a conceivable "correction" of the form (2). In leading order, the Hamiltonian which governs the gravitational interaction is given by [see Eq. (12) of Ref. [34]]

$$H = \vec{\alpha} \cdot \vec{p} + \beta\, m\, w(r), \tag{15}$$

$$w \approx 1 - \frac{r_s}{2r} = 1 - \frac{G\,M}{r} = 1 + \Phi_G, \tag{16}$$

where $r_s = 2GM$ is the Schwarzschild radius. Here, r is the Eddington coordinate in the Eddington form [38] of the Schwarzschild metric, which however is equal to the radial coordinate in the original Schwarzschild metric in the limit $r \to \infty$ (i.e., in the limit of a weak gravitational field). We use the vector of Dirac $\vec{\alpha}$ matrices, and the β matrix, in the standard representation [39].

After a Foldy-Wouthuysen transformation, the Hamiltonian (15) takes the form (in the leading order in the relativistic expansion)

$$H \approx \beta \left(m + \frac{\vec{p}^{\,2}}{2m} - \frac{r_s}{2r} \right) = \beta \left(m + \frac{\vec{p}^{\,2}}{2m} + \Phi_G \right). \quad (17)$$

Here, the β matrix describes the particle-antiparticle symmetry [35], while the latter form shows that the gravitational potential can be inserted into the Schrödinger equation "by hand" in the leading order (the somewhat nontrivial relativistic corrections involve the gravitational *Zitterbewegung* term, and the gravitational spin-orbit coupling [35]).

The leading gravitational term in Eq. (15), in the fully relativistic formalism, corresponds to a position-dependent modification of the Dirac mass of the electron, which is present only if one departs from the local Lorentz frame (locally flat space-time) and aims to describe the Dirac particle globally, in the curved space-time. Defining the effective mass m_G of the electron as

$$m_G = m\, w(r) \approx m\,(1 + \Phi_G)\,, \quad (18)$$

one can carry out the calculation of the vacuum polarization insertion as described in the literature. One possibility is to use the covariant formalism described in Chap. 7 of Ref. [39], which relies on a Feynman parameter integral. A recent, particularly clear formulation given in Sec. 5 of Ref. [40] clarifies that the additional mass terms introduced in Pauli-Villars regularization do not affect the calculation of the vacuum-polarization tensor, which depends only on the physical, local, effective mass of the electron. An alternative possibility is given in Chap. 113 of Ref. [41], where a subtracted dispersion relation is used in order to circumvent parts of the problems associated with regularization and renormalization, and leads to a dispersion integral which starts at the pair production threshold $(2m_G)^2$. The result of all these approaches invariantly reads as follows, in terms of a modification of the photon propagator $D_{\mu\nu} = g_{\mu\nu}/k^2$:

$$\frac{g_{\mu\nu}}{k^2} \to \frac{g_{\mu\nu}}{k^2\,[1 + \overline{\omega}^R(k^2)]}, \qquad k^2 = \omega^2 - \vec{k}^2. \quad (19)$$

A straightforward application of the formalism of covariant quantum electrodynamics then leads to the renormalized (superscript R) vacuum-polarization insertion, written in terms of the effective mass m_G of the electron,

$$\overline{\omega}^R(k^2) = \frac{\alpha k^2}{3\pi} \int_{4m_G^2}^{\infty} \frac{dk'^2}{k'^2}\, \frac{1 + 2m_G^2/k'^2}{k'^2 - k^2} \sqrt{1 - \frac{4m_G^2}{k'^2}}. \quad (20)$$

We note that $\overline{\omega}^R(k^2)$ vanishes for $k^2 = \omega^2 - \vec{k}^2 = 0$, thus leaving the speed of light of on-shell photons invariant. For $k^2 \neq 0$ (off-shell, virtual photons), we note the asymptotic behavior

$$\overline{\omega}^R(k^2) = \frac{\alpha}{15\pi}\frac{k^2}{m_G^2} + O(k^4), \qquad k^2 \to 0\,, \quad (21a)$$

$$\overline{\omega}^R(k^2) = -\frac{\alpha}{3\pi}\ln\left(-\frac{k^2}{m_G^2}\right) + \frac{5\alpha}{3\pi} + O\left(\frac{\ln(-k^2)}{k^2}\right), \quad k^2 \to \infty. \quad (21b)$$

These are in principle familiar formulas (see Chap. 7 of Ref. [39]), and we identify the leading gravitational effect on vacuum polarization to be given by the gravitationally corrected mass. The conclusions of Ref. [7], and the result (2), can thus be traced to an inconsistent evaluation of the vacuum polarization integral, which relies on a relativistically noncovariant formulation [see the discussion surrounding Eq. (6) of Ref. [7]], and bears an analogy with similar problems encountered in bound-state quantum electrodynamics [8].

IV. BOUND-STATE ENERGIES

A final word on bound-state energies is in order. With the mass of the electron assuming the value $m \to m_G$, the vacuum polarization potential (Uehling, one loop), derived from the virtual exchange of space-like Coulomb photons ($k^2 = -\vec{k}^2$), is easily derived as (in units with $\hbar = c_0 = \epsilon_0 = 1$)

$$V_{\rm vp}(\vec{r}) = -\frac{4\alpha}{15}\frac{Z\alpha}{m_G^2}\,\delta^{(3)}(\vec{r}), \quad (22)$$

where Z is the nuclear charge number. However, the gravitationally corrected mass also enters the Dirac-Coulomb Hamiltonian $H = \vec{\alpha}\cdot\vec{p} + \beta\, m_G - Z\alpha/r$, where the Dirac matrices are used in the standard representation, and r denotes the electron-proton distance [39]. By consequence, after a Foldy-Wouthuysen transformation, the gravitationally corrected mass parameter m_G also enters the Schrödinger wave function, and the probability density of S states with orbital angular momentum $\ell = 0$ at the origin becomes proportional to $(Z\alpha m_G)^3$. The gravitationally corrected energy shift reads as

$$\langle V_{\rm vp}(\vec{r})\rangle = -\frac{4\alpha}{15\pi}\frac{(Z\,\alpha)^4\, m_G}{n^3}\,\delta_{\ell 0}. \quad (23)$$

The energy shift is proportional to the effective mass of the electron, which also enters the Schrödinger spectrum $E_n = -(Z\alpha)^2 m_G/(2n^2)$, where n is the principal quantum number. [We recall that the Dirac-δ potential $\delta^{(3)}(\vec{r})$ is formulated with respect to the central electrostatic potential generated by the nucleus of charge number Z, not the gravitational center, while n and ℓ denote the principal and orbital angular momentum quantum numbers of the state.] The scaling with the effective mass of the electron thus affects the vacuum polarization energy shift as much as the leading Schrödinger term and thus does not shift atomic transitions with respect to each other.

The gravitational correction to bound-state energy levels due to fluctuations of the electron position in the gravitational field of the Earth can easily be estimated as follows. Namely, the atomic electron coordinate fluctuates over a distance of a Bohr radius about the position in the gravitational field. If we denote by $\vec{R} = \vec{r}_N + \vec{r}$ the electron coordinate from the Earth's center (with the Earth mass being denoted as $M_\oplus$), where r_N

is the proton coordinate, then the fluctuations of the electron about the gravitational center of the atom cause an energy shift of the order of

$$-\frac{G\,m_e\,M_\oplus}{|\vec{r}_N+\vec{r}|}+\frac{G\,m_e\,M_\oplus}{|\vec{r}_N|}\sim\frac{G\,m_e\,M_\oplus\,a_0}{\vec{R}^2}=2.9\times10^{-21}\ \text{eV}. \tag{24}$$

This effect influences typical atomic transitions (with transition frequencies on the order of one eV) at the level of one part in 10^{21}.

V. CONCLUSIONS

The main results of the current investigation can be summarized as follows: Both the Vachaspati transformation (see Appendix A and Ref. [9]) as well as the Franson time delay [see Eq. (2) and Ref. [7]] are in disagreement with the Einstein equivalence principle (EEP, see the discussion in Sec. II C). The Franson time delay, which affects only photons, not fermions, is a subtle effect, and the violation of the EEP due to the Franson time delay would be at the quantum level (hence a small correction) as opposed to the Vachaspati transformation. Hence, it is warranted to establish an astrophysical bound on the magnitude of the χ parameter introduced in Eq. (2). This is done is Sec. II A. Furthermore, the description of fermions in deep gravitational potentials, under the assumption of a time delay δc_γ for photons according to Eq. (2), is studied in Sec. II B. It is shown that the description of fermions in such a deep gravitational potential will require a mass term that "runs" with the energy and thus is more problematic than a superficial look at the "small" correction term (2) would otherwise suggest.

In Sec. III, we analyze the leading gravitational correction to vacuum polarization using a fully covariant formalism and find that, with on-mass-shell renormalization, the effect can be described by a mass term modification which depends on the value of the gravitational potential in the vicinity of the virtual electron-positron pair. It vanishes on shell and thus does *not* lead to a nonvanishing χ coefficient in the sense of Eq. (2). Finally, in Sec. IV, we analyze conceivable shifts for atomic bound-state levels, caused by off-shell virtual photons in the vacuum-polarization loops. We find that the effect, at least within the approximations employed in Sec. IV, does not shift spectral lines with respect to each other because it can be absorbed in a prefactor of the vacuum-polarization term which is also present in the leading Schrödinger binding energy. Finally, we estimate the leading gravitational correction to atomic energy levels, which depends on the quantum numbers, in terms of fluctuations of the electron and nucleus coordinates in the gravitational field of the Earth, and come to the conclusion that the term induced by the coordinate fluctuations within the binding Coulomb potential is of relative order 10^{-21}.

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APPENDIX A: GLOBAL REFERENCE FRAME AND SPEED OF LIGHT

Let us motivate the Shapiro time delay on the basis of the Schwarzschild metric [42], in isotropic form (Sec. 43 of Chap. 3 of Ref. [38]),

$$\begin{aligned} ds^2 = {} & \left(\frac{1-r_s/(4r)}{1+r_s/(4r)}\right)^2 dt^2 \\ & -\left(1+\frac{r_s}{4r}\right)^4 (dr^2+r^2d\theta^2+r^2\,\sin^2\theta\,d\varphi^2). \end{aligned} \tag{A1}$$

We use units with $\hbar = c_0 = \epsilon_0$ for the entire Appendix A. Light travels on a null geodesic, with $ds^2 = 0$, and so

$$\left(\frac{1-r_s/(4r)}{1+r_s/(4r)}\right)^2 dt^2-\left(1+\frac{r_s}{4r}\right)^4 d\vec{r}^{\,2}=0. \tag{A2}$$

One obtains

$$\left(\frac{d\vec{r}}{dt}\right)^2=\frac{[1-r_s/(4r)]^2}{[1+r_s/(4r)]^6}=\left[1-2\frac{r_s}{r}+O\left(\frac{1}{r^2}\right)\right]. \tag{A3}$$

We now consider the limit of large distance r. Using the relation $r_s = 2GM$, the local speed of light, expressed in terms of the global coordinates, is

$$\left|\frac{d\vec{r}}{dt}\right|=1-\frac{2GM}{r}=1+2\,\Phi_G(\vec{r}), \tag{A4}$$

where we identify $\Phi_G(\vec{r}) = -GM/r$ with the gravitational potential. One can easily generalize the derivation [see Chap. 4.4 on page 196 ff. of Ref. [15], Eq. (4.43) of Ref. [16], and Sec. 4.5.2 as well as the discussion on page 160, and exercise 4.8 on page 161 of Ref. [16] as well as Ref. [43]]. The effect is known as the Shapiro time delay [3–5]. The application to the travel time of particles stemming from the SN1987A supernova is discussed in Refs. [44,45].

APPENDIX B: VACHASPATI TRANSFORMATION

Vachaspati [9] distinguishes between "absolute time" t_A and "electromagnetic time" t_E. The Lorentz-Vachaspati transformation resembles the Lorentz transformation, but with a variable "speed-of-light parameter" u_0, which, in the primed system, transforms into u_0':

$$u_0'\,t_A' = \gamma_A(u_0\,t_A+\beta_A\,x), \tag{B1a}$$

$$x' = \gamma_A(x+\beta_A u_0\,t_A). \tag{B1b}$$

The backtransformation formally carries a resemblance to the Lorentz transformation,

$$u_0\,t_A = \gamma_A(u_0'\,t_A'-\beta_A\,x'), \tag{B2a}$$

$$x = \gamma_A(x'-\beta_A u_0'\,t_A'). \tag{B2b}$$

The relativistic factors carry a different functional form,

$$\gamma_A=\sqrt{1+\left(\frac{v_A}{c_0}\right)^2},\quad \beta_A=\frac{v_A}{c_0}\sqrt{1+\left(\frac{v_A}{c_0}\right)^2}. \tag{B3}$$

One verifies that

$$x'^2 - {u'_0}^2\, t_A^2 = x^2 - {u_0}^2\, t_A^2. \tag{B4}$$

For the absence of time dilation, one considers events 1 and 2, with coordinates

$$x'_1 = 0, \quad t'_{A,1} = 0, \quad x_1 = 0, \quad t_{A,1} = 0, \tag{B5a}$$

$$x'_1 = v_A t_A, \quad t'_{A,2} = t_A, \quad x_2 = 0, \quad t_{A,2} = \tau. \tag{B5b}$$

One obtains

$$t_{A,2} = \tau = \left(\gamma_A \frac{u'_0}{u_0} - \frac{v_A^2}{c_0\, u_0}\right) t_A. \tag{B6}$$

There is no time dilation if one chooses the parameter u'_0 to read as

$$u'_0 = \frac{c_0\, u_0 + v_A^2}{c_0\, \gamma_A}. \tag{B7}$$

If $u_0 = c_0$, then $u'_0 = (1 + v_A^2/c_0^2)^{1/2}\, u_0 = \gamma_A\, u_0$.

For comparison (we briefly recall textbook material), let us consider the Lorentz transformation,

$$c_0\, t'_E = \gamma_E (c_0\, t_E + \beta_E\, x), \tag{B8a}$$

$$x' = \gamma_E (x + \beta_E\, c_0\, t_E), \tag{B8b}$$

where the subscript E stands for the "electromagnetic" events according to Vachaspati [9]. The backtransformation reads as $c_0\, t_E = \gamma_E (c_0\, t'_E - \beta_E\, x')$ and $x = \gamma_E (x' - \beta_E\, c_0\, t'_E)$. The relativistic factors have the familiar functional form,

$$\gamma_E = \left[1 - \left(\frac{v_E}{c_0}\right)^2\right]^{-1/2}, \quad \beta_E = \frac{v}{c_0}. \tag{B9}$$

One verifies that $dx'^2 - c_0^2\, d{t'_E}^2 = x^2 - c_0^2\, dt_E^2$. For the derivation of time dilation, we consider events 1 and 2,

$$x'_1 = 0, \qquad t'_{E,1} = 0, \qquad x_1 = 0, \qquad t_{E,1} = 0, \tag{B10a}$$

$$x'_1 = vt, \qquad t'_{E,2} = t, \qquad x_2 = 0, \qquad t_{E,2} = \tau. \tag{B10b}$$

One immediately obtains the familiar time dilation formula, $t_{E,2} = \tau = t/\gamma_E$.

Vachaspati's formalism identifies the absolute time t_A as a formally different parameter from the electromagnetic time $t = t_E$. Furthermore, the speed-of-light parameter u_0 has to be adjusted for the relative speed of the primed and unprimed coordinate systems. The relationship of the u_0 and u'_0 to the observed, physical speed of light in both coordinate systems and the (claimed) reconciliation of the Vachaspati transformation with the Michelson-Morley experiment are discussed in Ref. [9]. The Vachaspati transformation reproduces the Galilei transformation in the limit $v_A \to 0$ and constitutes an alternative to the Lorentz transformation, with a variable speed-of-light parameter. In the primed system, the parameter u'_0 can be larger than c_0.

The muon lifetime measurement [46], which demonstrates that fast-moving muons live longer, is "reconciled" in Ref. [9] with the concept of absolute time by pointing out that the lifetime of muons is determined by the electromagnetic time, or "electroweak time" t_E, which need to be equal to the absolute time t_A. It is doubtful if the concept of an absolute time has any physical interpretation beyond its occurrence in the Lorentz-like transformation law (B1). However, the Vachaspati transformation is indicated here in order to demonstrate that a Lorentz-like transformations with a variable speed-of-light parameter u_0, whose value depends on the inertial frame, have been discussed in the literature. The Vachaspati transformation is different from the modification of the speed of light proposed by Franson [7] in that the variation of the speed is introduced at the classical as opposed to the quantum level.

[1] C. M. Will, Living Rev. Relativ. **9**, 3 (2006).
[2] C. M. Will, Living Rev. Relativ. **17**, 4 (2014).
[3] I. I. Shapiro, Phys. Rev. Lett. **13**, 789 (1964).
[4] I. I. Shapiro, G. H. Pettengill, M. E. Ash, M. L. Stone, W. B. Smith, R. P. Ingalls, and R. A. Brockelman, Phys. Rev. Lett. **20**, 1265 (1968).
[5] I. I. Shapiro, Rev. Mod. Phys. **71**, S41 (1999).
[6] B. Bertotti, L. Iess, and P. Tortura, Nature (London) **425**, 374 (2003).
[7] J. D. Franson, New J. Phys. **16**, 065008 (2014).
[8] H. Persson, S. Salomonson, and P. Sunnergren, Adv. Quantum Chem. **30**, 379 (1998).
[9] Vachaspati, Ind. J. Pure Appl. Phys. **42**, 5 (2004).
[10] C. M. Will, Living Rev. Relativ. **4**, 4 (2001).
[11] A. Einstein, Königlich Preussische Akademie der Wissenschaften (Berlin), Sitzungsberichte, 799 (1915).
[12] D. Hilbert, Nachrichten der Königlichen Gesellschaft der Wissenschaften (Göttingen), Math.-phys. Klasse, 395 (1915).
[13] A. Einstein, Ann. Phys. (Leipzig) **354**, 769 (1916).
[14] V. L. Dadykin, G. T. Zatsepin, V. B. Karchagin, P. V. Korchagin, S. A. Mal'gin, O. G. Ryazhskaya, V. G. Ryasnyi, V. P. Talochkin, F. F. Khalchukov, V. F. Yakushev, M. Aglietta, G. Badino, G. Bologna, C. Castagnoli, A. Castellina, W. Fulgione, P. Galeotti, O. Saavedra, J. Trincero, and S. Vernetto, JETP Lett. **45**, 593 (1987) [Pis'ma Zh. Eksp. Teor. Fiz. **45**, 464 (1987)].
[15] H. C. Ohanian and R. Ruffini, *Gravitation and Spacetime* (W. W. Norton, New York and London, 1994).
[16] T. Padmanabhan, *Gravitation: Foundations and Frontiers* (Cambridge University Press, Cambridge, England, 2010).
[17] S. S. Shapiro, J. L. Davis, D. E. Lebach, and J. S. Gregory, Phys. Rev. Lett. **92**, 121101 (2004).
[18] S. B. Lambert and C. Le Poncin-Lafitte, Astron. Astrophys. **499**, 331 (2009).
[19] S. B. Lambert and C. Le Poncin-Lafitte, Astron. Astrophys. **529**, A70 (2011).
[20] P. J. Mohr, Ann. Phys. (NY) **325**, 607 (2010).
[21] A. Chodos, A. I. Hauser, and V. A. Kostelecky, Phys. Lett. B **150**, 431 (1985).
[22] U. D. Jentschura and B. J. Wundt, Eur. Phys. J. C **72**, 1894 (2012).
[23] U. D. Jentschura and B. J. Wundt, J. Phys. G **41**, 075201 (2014).

[24] Q. G. Bailey and V. A. Kostelecky, Phys. Rev. D **74**, 045001 (2006).
[25] Q. G. Bailey, Phys. Rev. D **80**, 044004 (2009).
[26] V. A. Kostelecky and J. D. Tasson, Phys. Rev. D **83**, 016013 (2011).
[27] D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997).
[28] J. S. Diaz, V. A. Kostelecky, and M. Mewes, Phys. Rev. D **80**, 076007 (2009).
[29] V. A. Kostelecky and M. Mewes, Phys. Rev. D **85**, 096005 (2012).
[30] A. G. Cohen and S. L. Glashow, Phys. Rev. Lett. **107**, 181803 (2011).
[31] F. R. Klinkhamer and M. Schreck, Phys. Rev. D **78**, 085026 (2008).
[32] B. Altschul, Phys. Rev. D **80**, 091901(R) (2009).
[33] M. A. Hohensee, R. Lehnert, D. F. Phillips, and R. L. Walsworth, Phys. Rev. Lett. **102**, 170402 (2009).
[34] U. D. Jentschura, Phys. Rev. A **87**, 032101 (2013); **87**, 069903(E) (2013).
[35] U. D. Jentschura and J. H. Noble, Phys. Rev. A **88**, 022121 (2013).
[36] U. D. Jentschura, Ann. Phys. (Berlin) **526**, A47 (2014).
[37] U. D. Jentschura, Phys. Rev. A **90**, 022112 (2014).
[38] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, Cambridge, England, 1924).
[39] C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
[40] P. Indelicato, P. J. Mohr, and J. Sapirstein, Phys. Rev. A **89**, 042121 (2014).
[41] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics*, Volume 4 of the Landau Course on Theoretical Physics, 2nd ed. (Pergamon Press, Oxford, UK, 1982).
[42] K. Schwarzschild, Preussische Akademie der Wissenschaften (Berlin), Sitzungsberichte, 189 (1916).
[43] N. Ashby, Phys. Today **55**, 41 (2002).
[44] M. J. Longo, Phys. Rev. Lett. **60**, 173 (1988).
[45] L. M. Krauss and S. Tremaine, Phys. Rev. Lett. **60**, 176 (1988).
[46] B. Rossi and D. B. Hall, Phys. Rev. **59**, 223 (1941).