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Stephen P. Boughn
Haverford College, sboughn@haverford.edu

Andrew Dombard '94
Class of 1994, Haverford College

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EXCITATION OF LOW-ORDER JOVIAN p-MODES BY COMETARY IMPACTS

ANDREW J. DOMBARD[1] AND STEPHEN P. BOUGHN
Department of Astronomy, Haverford College, Haverford, PA 19041-1392
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ABSTRACT

Three mechanisms for the impact excitation of low-order p-mode oscillations of Jupiter are examined: a pressure pulse, direct momentum transfer, and a blast wave. The oscillation amplitudes of these low-order modes excited by the impacts of the fragments of P/Shoemaker Levy 9 are estimated to be several orders of magnitude less than current detection limits. Other global oscillation modes, e.g., interfacial, surface, and high-frequency p-modes, are more highly excited by surface impacts but are not the subject of this *Letter*.
Subject headings: comets: individual (Shoemaker-Levy/9) — planets and satellites: individual (Jupiter)

1. INTRODUCTION

The successes of helioseismology have led to the hope that the study of the free oscillation spectrum of the gas giant planets will provide an important tool for the determination of the planets' interiors. Low-order p-modes propagate deep into the interior and, therefore, contain information about interior structure, including the rocky core. Marley (1994) recently discussed the seismological consequences of the collision of comet Shoemaker-Levy/9 with Jupiter, and Lee & Van Horn (1994) estimated the amplitudes of oscillation of a variety of modes and discussed the information provided by the frequency spectrum of such global oscillations. In both of these treatments, only very rough estimates of the levels of excitation were made. Lognonné, Mosser, & Dahlen (1994) used a more realistic model of the impact and concluded that both high-frequency p-waves (10 mHz) as well as surface waves might be detectable as temperature fluctuations. Kanamori (1993) used a treatment similar to that discussed in this *Letter* to obtain an estimate of the excitation due to the superposition of all spheroidal modes up to $l \approx 10{,}000$.

It is the purpose of the present *Letter* to provide estimates of the levels of excitation of a restricted subset of the global modes, namely, the low-order p-modes. In this case "low-order" means those modes whose wavelengths are much greater than the size of the shock wave–produced cavity caused by the impact. Three mechanisms are considered: (1) the pressure pulse resulting from energy deposited in the blast cavity, (2) the direct momentum impulse of the impactor, and (3) the momentum impulse of the downward propagating shock wave. As an example, the amplitudes of excitation of Jupiter by a 10^{30} ergs comet are computed. We used a simple model of Jupiter, a spherically symmetric, $n = 1$ polytrope which provides an approximate description of the interior structure (Hubbard 1984; Marley 1994). The eigenfunctions of the low-order p-modes for this model were computed using a code kindly supplied to us by M. Marley. While this model contains neither a core/mantle discontinuity nor a metallic hydrogen phase transition, the eigenfrequencies of low-order p-modes computed with this model are within 20% of those derived from more detailed models that do take these features into account (Lee & Van Horn 1994). Furthermore, the model does not include rotation and the concomitant splitting of the modes. These differences are not important for order-of-magnitude estimates of the amplitudes of excitations of these modes. The limitations of the model (e.g., no phase boundaries, no rotation, and no magnetic fields) preclude the treatment of entire classes of interfacial and toroidal modes. However, the excitation mechanisms discussed below can be easily extended to all modes with characteristic wavelengths which are significantly larger than the diameter of the blast cavity of the impactor.

2. EXCITATION MECHANISMS

The free oscillations of a spherically symmetric, self-gravitating, compressible fluid can be expressed as a linear combination of normal modes, i.e.,

$$\boldsymbol{x}(\boldsymbol{r}) = \sum_{nlm} a_{nlm}(t)\boldsymbol{\xi}_{nlm}(\boldsymbol{r}) , \qquad (1)$$

where n, l, and m are the radial, spherical and azimuthal eigenvalues. The eigenfunctions $\xi(r, \theta, \phi)$ are of the form

$$\boldsymbol{\xi}_{nlm}(r, \theta, \varphi) = \left[\xi_n^r(r)\hat{r} + \xi_n^t(r)\hat{\theta}\frac{\partial}{\partial\theta} + \frac{\xi_n^t(r)}{\sin\theta}\hat{\varphi}\frac{\partial}{\partial\varphi}\right] Y_l^m(\theta, \varphi) , \qquad (2)$$

where $Y_l^m(\theta, \phi)$ are spherical harmonics, and $\xi_n^r(r)$ and $\xi_n^t(r)$ are the radial and transverse waves functions (Unno et al. 1979). The amplitudes, $a_{nlm}(t)$, satisfy the forced harmonic oscillator equation,

$$\ddot{a}_{nlm} + \sigma_{nlm}^2 a_{nlm} = F_{nlm}(t) , \qquad (3)$$

where σ_{nlm} are the eigenfrequencies. In the case of no damping, the σ_{nlm} are real. The effective force, $F_{nlm}(t)$, is related to the physical force density $\boldsymbol{f}(\boldsymbol{r}, t)$ by

$$F_{nlm}(t) = \frac{\iiint \boldsymbol{\xi}_{nlm} \cdot \boldsymbol{f} d^3\boldsymbol{r}}{\iiint \rho(\boldsymbol{r}) | \boldsymbol{\xi}_{nlm}|^2 d^3\boldsymbol{r}} , \qquad (4)$$

where $\rho(\boldsymbol{r})$ is the mass density.

2.1. *Pressure Pulse*

When an impactor collides with a planet, much of its kinetic energy is deposited locally as heat. This heat rapidly increases the pressure in the region surrounding the impact, and the resulting pressure pulse can excite global modes of oscillation. If the size of the region is much smaller than the wavelength of the mode, then the force density of this bubble of overpressure

[1] Postal address: Department of Earth and Planetary Sciences, Washington University, St Louis, MO 63130.

can be effectively modeled by a spherical delta function,

$$f = \delta P(t)\delta(\boldsymbol{r} - \boldsymbol{r}_0)\hat{r}_n \,, \tag{5}$$

where $\delta P(t)$ is the overpressure at the bubble's surface, $\boldsymbol{r}_0$ is the position of the bubble's center relative to the center of the planet, and $\hat{r}_n$ is the outward pointing normal of the bubble's surface. For low-frequency modes, the timescale for energy injection into the bubble is much smaller than the period of the mode, while the time required for the heat to dissipate is much longer than the period. In this case, the time dependence of the pressure pulse can be approximated by a step function, i.e., $\delta P(t) = \delta P \Theta(t)$.

The effective force on a given p-mode is given by equation (4),

$$F_{nlm}(t) = \delta P \Theta(t) \frac{\iiint \xi_{rlm} \cdot \hat{r}_n \delta(\boldsymbol{r} - \boldsymbol{r}_0) d^3\boldsymbol{r}}{\iiint \rho(\boldsymbol{r}) |\xi_{nlm}|^2 d^3\boldsymbol{r}}$$

$$= \delta P \Theta(t) \frac{\oiint \xi_{nlm} \cdot \hat{r}_n dS}{\iiint \rho(\boldsymbol{r}) |\xi_{nlm}|^2 d^3\boldsymbol{r}} \,, \tag{6}$$

where dS is the surface area element of the bubble. By Gauss's Law, $\oiint \xi_{nlm} \cdot \hat{r}_n dS = \iiint \nabla \cdot \xi_{nlm} d^3\boldsymbol{r}$, where the latter integral is over the volume of the bubble. If the radius of the bubble is much less than the characteristic wavelength of the mode in question, then $\iiint \nabla \cdot \xi_{nlm} d^3\boldsymbol{r} \approx V \nabla \cdot \xi_{nlm}|_{\boldsymbol{r}=\boldsymbol{r}_0}$, where V is the volume of the bubble. The effective force becomes

$$F_{nlm}(t) = \frac{(\delta P V)\Theta(t) \nabla \cdot \xi_{nlm}|_{\boldsymbol{r}=\boldsymbol{r}_0}}{\iiint \rho(\boldsymbol{r}) |\xi_{nlm}|^2 d^3\boldsymbol{r}} \,. \tag{7}$$

The term $\delta P V$ is directly proportional to the kinetic energy of the impactor, i.e., $\delta P V = \eta E_I$. The constant η is on the order of unity under a wide variety of physical circumstances. For the case of an ideal monatomic gas, $\delta P V = Nk\delta T$, while $E_I = 3/2 Nk\delta T$ if all the impactor energy goes into heating the gas. Therefore, $\delta P V / E_I = \eta = \frac{2}{3}$. On the other hand, for the disparate case of a pulse of heat deposited locally in a crystalline solid, it is straightforward to show that η is equal to the Grüneisen constant. This constant ranges from about 1 to 3, depending on the crystal (Kittle 1968). If some of the energy goes into breaking chemical bonds, shock waves, etc., instead of raising the temperature, then $\delta P V$ (and η) will be reduced somewhat. In general, one can assume $\eta \lesssim 1$.

Therefore, the effective force can be expressed as

$$F_{nlm}(t) = \frac{\eta E_I \nabla \cdot \xi_{nlm}|_{\boldsymbol{r}=\boldsymbol{r}_0} \Theta(t)}{\iiint \rho(\boldsymbol{r}) |\xi_{nlm}|^2 d^3\boldsymbol{r}} \,. \tag{8}$$

The solution to equation (3) for the normal mode amplitude is then

$$a_{nlm} = \frac{-\eta E_I \nabla \cdot \xi_{nlm}|_{\boldsymbol{r}=\boldsymbol{r}_0}}{\sigma_{nlm}^2 \iiint \rho(\boldsymbol{r}) |\xi_{nlm}|^2 d^3\boldsymbol{r}} \cos \sigma_{nlm} t \,. \tag{9}$$

In the spherically symmetric approximation, it can be assumed without loss of generality that the impactor collides with the planet at the "north pole" ($\theta = 0$, $\phi = 0$). In this case, the pressure pulse only couples to the $m = 0$ modes, and the rms radial displacement of the surface is

$$(\delta r)_{\rm rms} = \frac{\eta E_I \nabla \cdot \xi_{nl0}|_{\boldsymbol{r}=\boldsymbol{r}_0}}{\sqrt{2}\, \sigma_{nl0}^2 \iiint \rho(\boldsymbol{r}) |\xi_{nl0}|^2 d^3\boldsymbol{r}} \xi_r(R) Y_l^0(\theta, \phi) \,. \tag{10}$$

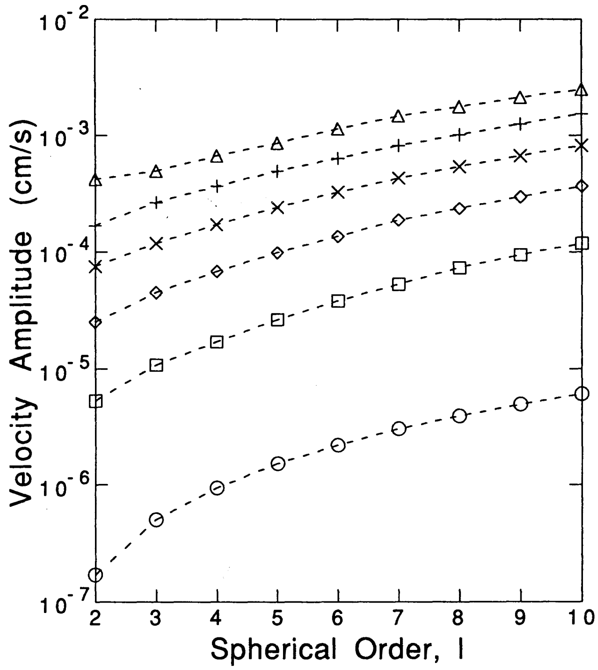

FIG. 1.—Velocity amplitudes (cm s^{-1}) for a 10^{30} ergs impactor from the pressure pulse assuming $\eta = 1$. The different curves are for radial eigenvalues $n = 0$ (*circles*), 1 (*squares*), 2 (*diamonds*), 3 (*crosses*), 4 (*plus signs*), and 5 (*triangles*). The current observational limit is about 100 cm s^{-1}.

Figure 1 shows estimates of the rms radial velocities of low n and l p-modes of Jupiter averaged over the surface of the planet. An impactor energy of 10^{30} ergs and a point of impact at an atmospheric pressure of 50 bars (Zahnle & MacLow 1994) were assumed. These results are not very sensitive to the location of the impact point. For example, the rms velocities for an impact at 12 bars differ by less than 25% from those in Figure 1. The frequencies of modes in Figure 1 are in the range 0.12 to 1.3 mHz.

An estimate of the surface temperature fluctuations associated with these modes can be made by assuming that the density fluctuations are adiabatic. For a perfect gas (Lognonné et al. 1994),

$$\frac{\Delta T}{T} = (\gamma - 1)\frac{\Delta \rho}{\rho} = (1 - \gamma)\nabla \cdot [(a_{nlm} \xi_{nlm}(\boldsymbol{r})] \,,$$

where γ is the ratio of the specific heat at constant pressure to that at constant volume. The temperature fluctuations corresponding to the excitation levels in Figure 1 are all less than 1 μK and therefore undetectable.

2.2. *Direct Momentum Transfer*

An impactor clearly delivers a direct momentum impulse to the surface of a planet. Assuming that the impactor trajectory is radial (the optimum case), the average force per unit volume is

$$f = -\frac{1}{\Delta V}\frac{\Delta P}{\Delta t}\hat{r} \,, \tag{11}$$

where ΔV is the volume of the impact region, ΔP is momentum of the impactor, and Δt is the characteristic timescale of the impact. The effective force is (from eq. [4])

$$F_{nlm}(t) = -\frac{(\Delta P/\Delta t) \iiint (1/\Delta V)\xi_n^r Y_l^m d^3r}{\iiint \rho(r) |\xi_{nlm}|^2 d^3r} . \quad (12)$$

If $(\Delta V)^{1/3}$ is small compared to wavelength of the mode, then $1/\Delta V$ approximates a three-dimensional Δ function. Again assuming the impact occurs at the pole, the effective force becomes

$$F_{nlm}(t) = -\frac{(\Delta P/\Delta t)\xi_n^r(r_0)\sqrt{(2l+1)/4\pi}}{\iiint \rho(r) |\xi_{nlm}|^2 d^3r}\delta_{mo} . \quad (13)$$

For impactor velocities characteristic of the solar system, the impact timescale, Δt, is on the order of seconds (Zahnle & MacLow 1994), that is, much smaller than the periods of low-order *p*-modes. Therefore, the impulse approximation is appropriate, and the solution to equation (3) is

$$a_{nl0} = -\frac{\Delta P \xi_n^r(r_0)\sqrt{(2l+1)/4\pi}}{\sigma_{nl0} \iiint \rho(r) |\xi_{nl0}|^2 d^3r} \sin \sigma_{nl0} t . \quad (14)$$

Figure 2 indicates the rms surface velocities for an impactor with momentum of 3×10^{23} g cm s^{-1} (i.e., an energy of 10^{30} ergs and velocity of 60 km s^{-1}).

2.3. *Blast Wave*

Another mechanism by which momentum is transferred to the planet is the shock created by the explosive deposition of the energy of the impactor. The blast wave from a localized explosion sweeps up mass in a way that conserves energy (Zeldovich & Raizer 1968). The result is that the momentum per unit area carried by the shock increases with time until the flow becomes sonic. For strong shocks, i.e., the velocity of the blast wave, v_{BW}, is much greater than the local velocity of sound, v_s, most of the mass encountered is swept up into a thin shell at the shock front. From the model of strong shocks of Zeldovich & Raizer (1968) with parameters appropriate for the Jovian atmosphere, roughly 30% of the energy in the shock is in the form of translational kinetic energy. The rest is in the internal energy of the shocked material. Thus,

$$\tfrac{1}{2} M_{BW} v_{BW}^2 = 0.3 E_{BW} , \quad (15)$$

where M_{BW} is the mass swept up by the shock, and E_{BW} is the shock energy. Then the momentum carried by the shock is $0.6E_{BW}/v_{BW}$, which increases as the shock propagates and slows down. An upper limit to the momentum is obtained by setting $v_{BW} = v_s$. Actually, the strong shock conditions fail for velocities greater than this, so the maximum momentum acquired by the shock will be less than this value. Roughly $\frac{1}{4}$ of the shock momentum will be directed inward, so the momentum impulse is

$$\Delta p < 0.075(E_I/v_s) , \quad (16)$$

where it is assumed that $E_{BW} = (1/2)E_I$, probably an overestimate. For a comet energy of 10^{30} ergs and a sound speed of 9×10^4 cm s^{-1}, the momentum impulse is less than 8×10^{23} g cm s^{-1}, which is about a factor of 3 larger than the original momentum of the impactor.

The actual momentum carried by the blast wave is probably considerably less than the above limit. Kompaneets (1960) showed that a blast wave in an exponential atmosphere propagates downward for only 1.4 scale heights before its energy rapidly dissipates. A rough estimate of the downward momentum of the blast wave after it has propagated for 1.4 scale heights can be obtained by computing the swept-up mass in a cube 1.4 scale heights (~22 km) on a side. If the fragment deposits its energy at an atmospheric pressure of 50 bar (Zahnle & MacLow 1994), then the swept-up mass is on the order of 3×10^{17} g, and the momentum is $\Delta p = (0.075E_I M_{BW})^{1/2} = 1 \times 10^{23}$ g cm s^{-1}, which is somewhat less than the initial momentum of the impactor. The upward component of the shock wave is unconfined (Marley 1994) and does not couple to the normal modes.

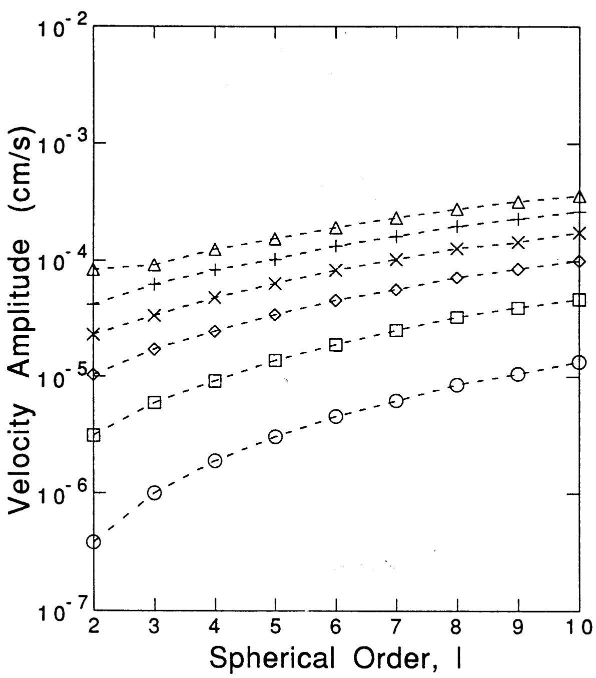

FIG. 2.—Velocity amplitudes (cm s^{-1}) for a 3×10^{23} g cm s^{-1} impactor from direct momentum transfer. The different curves are for radial eigenvalues $n = 0$ (*circles*), 1 (*squares*), 2 (*diamonds*), 3 (*crosses*), 4 (*plus signs*), and 5 (*triangles*). The current observational limit is about 100 cm s^{-1}.

3. DISCUSSION

Figures 1 and 2 imply that the pressure pulse mechanism is more effective at exciting low-order *p*-modes by a factor of ~3 over the direct momentum impulse mechanism. However, owing to the fact that the pressure pulse excitations are probably overestimates and that the results of this *Letter* are clearly approximations, the conclusion is that the two mechanisms give the same order of magnitude excitations. This is a coincidence since the two depend differently on the velocity of the impactor. For very high velocities the pressure pulse will dominate, and for very low velocities momentum transfer dominates. The blast wave excitation level depends both on the impactor energy and the characteristics of the atmosphere, and it is again a coincidence that this mechanism also results in the same order-of-magnitude level of excitation.

The excitation amplitudes in Figure 1 are from 10^{-5} to 10^{-6} times smaller than the upper limits given by Lee & Van Horn (1994). They arrived at these limits by assuming that all the energy of the impactor is deposited in a single mode. Our estimates indicate that on the order of from 10^{-10} to 10^{-12} of the impactor energy is deposited in each low-order *p*-mode.

These small fractions are not surprising and are due to what is essentially an impedance mismatch between the impactor and the normal mode. Since current detection limits of p-modes are on the order of 100 cm s^{-1} (Deming et al. 1989), which is 10^5 to 10^7 times larger than the estimates in Figure 1, low-order p-mode excitation by cometary impacts of the order of magnitude of the Shoemaker-Levy impacts will not likely prove to be useful in probing the interior structures of the gas giants. Higher frequency p-modes, surface modes, and interfacial modes are more highly excited by surface impacts but are not the subject of this *Letter*.

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