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Generation of High-Energy Photons with Large Orbital Angular Momentum by Compton Backscattering

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Usually, photons are described by plane waves with a definite 4-momentum. In addition to plane-wave photons, "twisted photons" have recently entered the field of modern laser optics; these are coherent superpositions of plane waves with a defined projection $\hbar m$ of the orbital angular momentum onto the propagation axis, where m is an integer. In this Letter, we show that it is possible to produce high-energy twisted photons by Compton backscattering of twisted laser photons off ultrarelativistic electrons. Such photons may be of interest for experiments related to the excitation and disintegration of atoms and nuclei, and for studying the photoeffect and pair production off nuclei in previously unexplored experimental regimes.

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Introduction.—An interesting research direction in modern optics is related to experiments with so-called "twisted photons." These are states of a laser beam whose photons have a defined value $\hbar m$ of the angular momentum projection on the beam propagation axis where m is a (large) integer [1]. An experimental realization [2] exists for states with projections as large as $m = 200$. Such photons can be created from usual laser beams by means of numerically computed holograms. The wave front of such states rotates around the propagation axis, and their Poynting vector looks like a corkscrew (see Fig. 1 in Ref. [1]). It was demonstrated that micron-sized Teflon and calcite "particles" start to rotate after absorbing twisted photons [3].

In this Letter, we show that it is possible to convert twisted photons from an energy range of about 1 eV to a higher energies of up to a hundred GeV using Compton backscattering off ultrarelativistic electrons. In principle, Compton backscattering is an established method for the creation of high-energy photons and is used successfully in various application areas from the study of photo-nuclear reactions [4,5] to colliding photon beams of high energy [6]. However, the central question is how to treat Compton backscattering of twisted photons, whose field configuration is manifestly different from plane waves. Below, we use relativistic Gaussian units with $c = 1$, $\hbar = 1$, $\alpha \approx 1/137$. We denote the electron mass by m_e and write the scalar product of 4-vectors $k = (\omega, \boldsymbol{k})$ and $p = (E, \boldsymbol{p})$ as $k \cdot p = \omega E - \boldsymbol{k}\boldsymbol{p}$.

Twisted photon.—We wish to construct a twisted photon state with definite longitudinal momentum k_z, absolute value of transverse momentum $\varkappa$ and projection m of the orbital angular momentum onto the z axis (propagation axis). We start from a plane-wave photon state with 4-momentum $k = (\omega, \boldsymbol{k})$ and helicity $\Lambda = \pm 1$,

$$A^{\mu}_{k\Lambda}(t, \boldsymbol{r}) = \sqrt{4\pi} e^{\mu}_{k\Lambda} \frac{e^{-i(\omega t - \boldsymbol{k}\boldsymbol{r})}}{\sqrt{2\omega}}, \tag{1}$$

where $e^{\mu}_{k\Lambda}$ is the polarization 4-vector of the photon ($e_{k\Lambda} \cdot k = 0$ and $e^*_{k\Lambda} \cdot e_{k\Lambda'} = -\delta_{\Lambda\Lambda'}$, with $\Lambda = -1, 1$). The twisted photon vector potential $\mathcal{A}^{\mu}_{\varkappa m k_z \Lambda}(r, \varphi_r, z, t)$ is obtained after integration over the conical transverse momentum components $\boldsymbol{k}_\perp = (k_x, k_y, 0)$ of the wave vector $\boldsymbol{k} = (k_x, k_y, k_z)$, with amplitude

$$a_{\varkappa m}(\boldsymbol{k}_\perp) = (-i)^m e^{im\varphi_k} \sqrt{\frac{2\pi}{\varkappa}} \delta(k_\perp - \varkappa). \tag{2}$$

Here, φ_k is the azimuth angle of $\boldsymbol{k}_\perp$, and

$$\begin{aligned} \mathcal{A}^{\mu}_{\varkappa m k_z \Lambda}(r, \varphi_r, z, t) &= \int a_{\varkappa m}(\boldsymbol{k}_\perp) A^{\mu}_{k\Lambda}(t, \boldsymbol{r}) \frac{d^2 k_\perp}{(2\pi)^2} \\ &= \sqrt{\frac{4\pi}{2\omega}} e^{-i(\omega t - k_z z)} \int e_{k\Lambda} a_{\varkappa m}(\boldsymbol{k}_\perp) \\ &\quad \times e^{i\boldsymbol{k}_\perp \boldsymbol{r}} \frac{d^2 k_\perp}{(2\pi)^2}. \end{aligned} \tag{3}$$

Furthermore, $\omega = |\boldsymbol{k}| = \sqrt{\varkappa^2 + k_z^2}$. For further analysis we introduce the three four-vectors

$$\eta^{(\pm)} = \mp \frac{1}{\sqrt{2}}(0, 1, \pm i, 0), \qquad \eta^{(z)} = (0, 0, 0, 1). \tag{4}$$

The initial twisted photon is composed of wave vectors of the form

$$\boldsymbol{k} = \omega(\sin\alpha_0 \cos\varphi_k, \sin\alpha_0 \sin\varphi_k, -\cos\alpha_0), \tag{5}$$

which for $\alpha_0 = 0$ propagate in the negative z direction. Here, $\theta = \pi - \alpha_0$ and φ_k are the polar and azimuth angles of the initial photon, and we have $\tan\alpha_0 = k_\perp/(-k_z)$. The polarization vectors can be expressed as

$$\begin{aligned} e_{k\Lambda} = \eta^{(-\Lambda)} e^{i\Lambda\varphi_k} \cos^2\left(\frac{\alpha_0}{2}\right) + \eta^{(\Lambda)} e^{-i\Lambda\varphi_k} \sin^2\left(\frac{\alpha_0}{2}\right) \\ + \frac{\Lambda}{\sqrt{2}} \eta^{(z)} \sin\alpha_0. \end{aligned} \tag{6}$$

Integration leads to

$$\int e_{k\Lambda} a_m(\boldsymbol{k}_\perp) e^{i\boldsymbol{k}_\perp \boldsymbol{r}} \frac{d^2 k_\perp}{(2\pi)^2} = \frac{\Lambda}{\sqrt{2}} \eta^{(z)} \psi_{\varkappa m}(r, \varphi_r) \sin\alpha_0 + i^\Lambda \eta^{(-\Lambda)} \psi_{\varkappa, m+\Lambda}(r, \varphi_r) \cos^2\left(\frac{\alpha_0}{2}\right) - i^\Lambda \eta^{(\Lambda)} \psi_{\varkappa, m-\Lambda}(r, \varphi_r) \sin^2\left(\frac{\alpha_0}{2}\right), \tag{7}$$

with the scalar twisted particle wave function

$$\psi_{\varkappa m}(r, \varphi_r) = \frac{e^{im\varphi_r}}{\sqrt{2\pi}} \sqrt{\varkappa} J_m(\varkappa r). \tag{8}$$

The vector field $\mathcal{A}^\mu_{\varkappa m k_z \Lambda}(r, \varphi_r, z, t)$ describes a photon state with projections of the orbital angular momentum on the z axis equal to $m-1$, m, $m+1$. For large m, the restriction to $(m-1, m, m+1)$ means that the twisted state is a state with "almost defined angular momentum projection m" (see Fig. 1), and we denote it as $|\varkappa, m, k_z, \Lambda\rangle$.

The usual S matrix element for plane-wave (PW) Compton scattering involves an electron being scattered from the state $|p, \lambda\rangle$ with 4-momentum p and helicity $\lambda = \pm\frac{1}{2}$ to the state $|p', \lambda'\rangle$ and a photon being scattered from the state $|k, \Lambda\rangle$ to the state $|k', \Lambda'\rangle$,

$$S^{(\mathrm{PW})}_{fi} \equiv \langle k', \Lambda', p', \lambda' | S | k, \Lambda, p, \lambda\rangle. \tag{9}$$

In view of Eq. (3), the S matrix element $S^{(\mathrm{TW})}_{fi}$ for the scattering of a twisted (TW) photon $|\varkappa, m, k_z, \Lambda\rangle$ into the state $|\varkappa', m', k'_z, \Lambda'\rangle$ needs to be integrated as follows,

$$S^{(\mathrm{TW})}_{fi} \equiv \langle \varkappa', m', k'_z, \Lambda'; p', \lambda' | S | \varkappa, m, k_z, \Lambda; p, \lambda\rangle = \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} a^*_{\varkappa' m'}(\boldsymbol{k}'_\perp) S^{(\mathrm{PW})}_{fi} a_{\varkappa m}(\boldsymbol{k}_\perp). \tag{10}$$

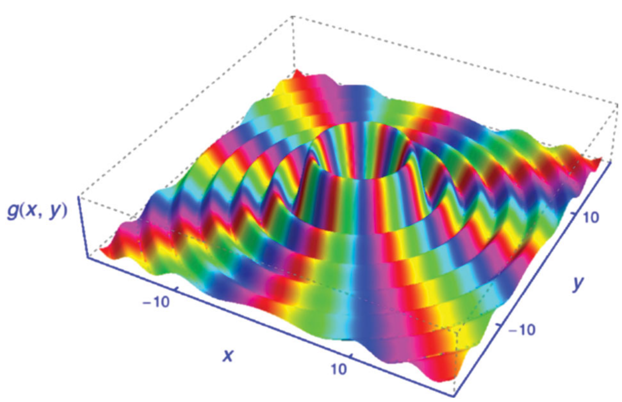

FIG. 1 (color). The twisted photon vector potential component $\mathcal{A}^\mu_{\varkappa m k_z \Lambda}(t, x, y, z)$ is shown for $\mu = 2$ (y component), $\varkappa = 1$, $m = 5$, $k_z = \sqrt{24}$, and $\Lambda = 1$. The plot displays $g(x, y) = |\mathcal{A}^\mu_{\varkappa m k_z \Lambda}(0, x, y, 0)|^2$ as a function of x and y. The complex phase of the vector potential, which is a superposition of terms proportional to $e^{4i\varphi_r}$ and $e^{6i\varphi_r}$, is indicated by the variation of the color of the wave function on the rainbow scale.

Compton scattering of plane-wave photons.—We investigate the collision of an ultrarelativistic electron with 4-momentum $p = (E, 0, 0, vE)$, $v = |\boldsymbol{p}|/E$, and $\gamma = E/m_e$, propagating in the positive z direction, and a photon of energy ω and three-momentum given by Eq. (5). After the scattering, the 4-momentum of the electron is p', and the scattered photon has energy ω' and three-momentum $\boldsymbol{k}' = \omega'(\sin\theta' \cos\varphi'_k, \sin\theta' \sin\varphi'_k, \cos\theta')$, where θ' and φ'_k are the polar and azimuth angles of the final photon. From the equation $(p + k - k')^2 = m_e^2$, we obtain

$$\omega' = \frac{m_e^2 x}{2E(1 - v\cos\theta') + 2\omega(1 + \cos\beta)}, \tag{11}$$

where $x m_e^2 = 2p \cdot k = 2\omega E(1 + v\cos\alpha_0)$ and $\cos\beta = \cos\alpha_0 \cos\theta' - \sin\alpha_0 \sin\theta' \cos(\varphi_k - \varphi'_k)$. The S-matrix element for plane waves is

$$S^{(\mathrm{PW})}_{fi} = i(2\pi)^4 \delta(p + k - p' - k') \frac{M_{fi}}{4\sqrt{EE'\omega\omega'}}, \tag{12}$$

where the amplitude M_{fi} in the Feynman gauge is

$$M_{fi} = 4\pi\alpha\left(\frac{A}{s - m_e^2} + \frac{B}{u - m_e^2}\right), \tag{13a}$$

$$A = \bar{u}_{p'\lambda'} \hat{e}^*_{k'\Lambda'} (\hat{p} + \hat{k} + m_e) \hat{e}_{k\Lambda} u_{p\lambda}, \tag{13b}$$

$$B = \bar{u}_{p'\lambda'} \hat{e}_{k\Lambda} (\hat{p}' - \hat{k} + m_e) \hat{e}^*_{k'\Lambda'} u_{p\lambda}, \tag{13c}$$

and $s - m_e^2 = x m_e^2$, $m_e^2 - u = 2p \cdot k' = 2\omega' E(1 - v\cos\theta')$. The bispinors $u_{p\lambda}$ and $u_{p'\lambda'}$ describe the initial and final electrons, and $e_{k\Lambda}$ and $e_{k'\Lambda'}$ are the polarization vectors of the initial and final photon. We denote the Feynman dagger as $\hat{p} = \gamma^\mu p_\mu$. Using Dirac algebra, we may write $A = A_1 + A_2$ and $B = B_1 + B_2$ with

$$A_1 = -\bar{u}_{p'\lambda'} \hat{e}^*_{k'\Lambda'} \hat{e}_{k\Lambda} \hat{k} u_{p\lambda}, \tag{14a}$$

$$A_2 = 2(e_{k\Lambda} \cdot p) \bar{u}_{p'\lambda'} \hat{e}^*_{k'\Lambda'} u_{p\lambda}, \tag{14b}$$

$$B_1 = \bar{u}_{p'\lambda'} \hat{k} \hat{e}_{k\Lambda} \hat{e}^*_{k'\Lambda'} u_{p\lambda}, \tag{14c}$$

$$B_2 = 2(e_{k\Lambda} \cdot p') \bar{u}_{p'\lambda'} \hat{e}^*_{k'\Lambda'} u_{p\lambda}. \tag{14d}$$

M_{fi} as defined in Eq. (13a) can thus be written as

$$M_{fi} = M^{(1)}_{fi} + M^{(2)}_{fi}, \tag{15a}$$

$$M^{(1,2)}_{fi} = 4\pi\alpha\left(\frac{A_{1,2}}{s - m_e^2} + \frac{B_{1,2}}{u - m_e^2}\right). \tag{15b}$$

For a head-on collision of a plane-wave photon and electron, the relativistic kinematics then imply the following differential cross section, for the unpolarized cross section (summation over the outgoing and averaging over the incoming electron and photon polarizations),

$$\frac{d\sigma}{d\Omega'} = \frac{2\alpha^2\gamma^2}{m_e^2}F(x,n), \qquad n \equiv \gamma\theta', \qquad x = \frac{4\omega E}{m_e^2},$$
$$F(x,n) = \left(\frac{1}{1+x+n^2}\right)^2\left[\frac{1+x+n^2}{1+n^2} + \frac{1+n^2}{1+x+n^2} - 4\frac{n^2}{(1+n^2)^2} + \mathcal{O}(\gamma^{-1})\right]. \tag{16}$$

According to Eq. (3), a twisted photon is a superposition of plane-wave photons with the same energy and conical momentum spread. We thus expect that the twisted and plane-wave photon scattering cross section will be related. Indeed, for the mixed (m) case where the initial photon is twisted but the outgoing one is a plane-wave photon, one finds

$$S_{fi}^{(m)} \equiv \langle k', \Lambda', p', \lambda' | S | \varkappa, m, k_z, \Lambda; p, \lambda\rangle = \int \frac{d^2k_\perp}{(2\pi)^2} S_{fi}^{(\mathrm{PW})} a_{\varkappa m}(\boldsymbol{k}_\perp), \tag{17}$$

and the corresponding cross section is given by Eq. (16) with the only replacement

$$x = \frac{4\omega E}{m_e^2} \to \frac{4\omega E \cos^2\alpha_0}{m_e^2}. \tag{18}$$

For strict backscattering geometry, the differential Compton cross section (16) and the energy of the scattered photon attain maxima, and additional simplifications are possible because the azimuth angle of the photon $\varphi_k' = \varphi_k$ is conserved, as discussed in the following.

Compton backscattering of twisted photons.—For twisted photons, the final m' photon is a superposition of plane waves with small transverse momentum $\boldsymbol{k}'_\perp = \boldsymbol{k}_\perp$ and very small scattering angle $\theta' = k'_\perp/\omega' \lesssim (1+x)/(4\gamma^2)$ (see Fig. 2). In this limit, $\omega' = xE/(1+x)$. For strict backward scattering, several quantum numbers in Eq. (10) are thus conserved under the scattering for twisted (TW) photons,

$$S_{fi}^{(\mathrm{TW})} = 2\pi i^{m'-m+1}\delta(\varkappa - \varkappa')\delta(E + \omega - E' - \omega') \times \delta(p_z + k_z - p_z' - k_z') \times \int_0^{2\pi} e^{i(m-m')\varphi_k} \frac{M_{fi}^{(1)} + M_{fi}^{(2)}}{4\sqrt{EE'\omega\omega'}} d\varphi_k, \tag{19}$$

where we have used the decomposition (15).

In order to carry out the integration over φ_k, we have to analyze the dependence of the polarization vectors $e_{k\Lambda}$ and $e_{k'\Lambda'}$ on the azimuth angle. To this end, we write the polarization vector of the final photon in the scattering amplitude M_{fi} in the form $e_{k'\Lambda'} = -\Lambda'(e^{(x')} + i\Lambda' e^{(y')})/\sqrt{2}$, where the unit vector $e^{(x')} = (0, \boldsymbol{e}^{(x')})$ is in the scattering plane, defined by the vectors $\boldsymbol{p} \parallel \boldsymbol{p}'$ and $\boldsymbol{k}'$,

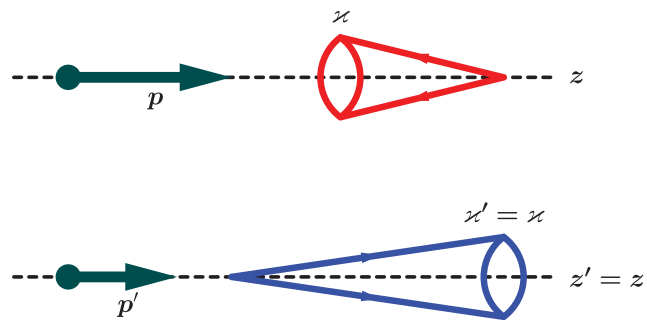

FIG. 2 (color). Initial (above) and final (below) states for the head-on Compton backscattering geometry of a twisted photon. According to Eq. (24), the conical momentum spread $\varkappa$ of the initial twisted photon is preserved ($\varkappa' = \varkappa$) during the scattering, but the propagation energy increases: $\omega' = \sqrt{k_z'^2 + \varkappa'^2} \gg \omega = \sqrt{k_z^2 + \varkappa^2}$.

while the unit vector $e^{(y')}$ is orthogonal to it: $\boldsymbol{e}^{(x')} \parallel (\boldsymbol{p} \times \boldsymbol{k}') \times \boldsymbol{k}'$ and $\boldsymbol{e}^{(y')} \parallel (\boldsymbol{p} \times \boldsymbol{k}')$. As a result, we have in 4-vector component notation

$$e_{k'\Lambda'} = -\frac{\Lambda'}{\sqrt{2}}\begin{pmatrix} 0 \\ \cos\theta'\cos\varphi_k - i\Lambda'\sin\varphi_k \\ \cos\theta'\sin\varphi_k + i\Lambda'\cos\varphi_k \\ -\sin\theta' \end{pmatrix}. \tag{20}$$

Omitting terms of the order of θ', we obtain

$$e_{k'\Lambda'} = -\frac{\Lambda'}{\sqrt{2}}(0, 1, i\Lambda', 0)e^{-i\Lambda'\varphi_k} = \eta^{(\Lambda')}e^{-i\Lambda'\varphi_k}. \tag{21}$$

The polarization vector $e_{k\Lambda}$ of a "conical" component of the initial twisted photon (as a function of φ_k) is obtained by setting $\theta' = \pi - \alpha_0$ in $e_{k'\Lambda'}$ and coincides with Eq. (6). Substituting the expressions for $e_{k'\Lambda'}$ and $e_{k\Lambda}$ given in Eqs. (21) and (6) into the definitions of A_1 and B_1 given in Eqs. (14a) and (14c), we find for twisted photons,

$$A_1 = 2\omega\sqrt{EE'}[(1 - \Lambda\Lambda'\cos\alpha_0)(1 + \cos\alpha_0) + 2\lambda\Lambda\sin^2\alpha_0]\delta_{\lambda\lambda'}\delta_{2\lambda,-\Lambda'}, \tag{22a}$$
$$B_1 = -2\omega\sqrt{EE'}[(1 - \Lambda\Lambda'\cos\alpha_0)(1 + \cos\alpha_0) - 2\lambda\Lambda\sin^2\alpha_0]\delta_{\lambda\lambda'}\delta_{2\lambda,\Lambda'}. \tag{22b}$$

One may write the neglected contribution $M_{fi}^{(2)}$ as $M_{fi}^{(2)} = -4\pi\alpha\bar{u}_{p'\lambda'}\hat{e}^*_{k'\Lambda'}u_{p\lambda}(e_{k\Lambda})_z\epsilon/\omega$. It is negligible for our relativistic kinematics ($\gamma \gg 1$), because

$$\epsilon = \omega\left(\frac{p_z}{k\cdot p} - \frac{p_z'}{k\cdot p'}\right) = \frac{x(x+2)}{2\gamma^2(1+\cos\alpha_0)^2} \ll 1. \tag{23}$$

Therefore, we have $|M_{fi}^{(2)}| \ll |M_{fi}^{(1)}|$ for strict backward scattering, and the S matrix element reads

$$S_{fi}^{(\mathrm{TW})} \approx i(2\pi)^2 \delta_{mm'} \delta(\varkappa - \varkappa') \delta(E + \omega - E' - \omega') \times \delta(p_z + k_z - p_z' - k_z') \frac{M_{fi}^{(1)}}{4\sqrt{EE'\omega\omega'}}, \tag{24}$$

with $M_{fi}^{(1)}$ given in Eqs. (15) and (22). This result states that for strict backscattering, the angular momentum projection $m' = m$ and the conical momentum spread $\varkappa' = \varkappa$ of the twisted photons are conserved and confirms the principal possiblity for the frequency upconversion of twisted photons under strict Compton backscattering. A technique for the registration of electrons scattered at small (even zero) angles after the loss of energy in the Compton process is implemented, for example, in the device for backscattered Compton photons installed on the VEPP-4M collider (Novosibirsk) [4].

Certainly, it is interesting to estimate the admixture of different $m' \neq m$ twisted photon states if the electron carries away a small transverse momentum $p'_\perp \ll \varkappa$ under the scattering. A solution of the relativistic kinematic equations then implies that the azimuthal angle of the scattered twisted photon component is not conserved but acquires a phase slip,

$$\varphi_{k'} = \varphi_k + \Delta, \qquad \Delta \approx \frac{p'_\perp}{\varkappa} \sin\varphi_k. \tag{25}$$

Taking into account this phase slip we obtain a distribution approximately given by Eq. (24) under the replacement

$$\int_0^{2\pi} \frac{d\varphi_k}{2\pi} e^{i(m\varphi_k - m'\varphi_{k'})}|_{\varphi_{k'}=\varphi_k} = \delta_{mm'} \rightarrow \int_0^{2\pi} \frac{d\varphi_k}{2\pi} e^{i[(m-m')\varphi_k - m'\Delta]} = J_{m-m'}(m'p'_\perp/\varkappa). \tag{26}$$

This yields a distribution where the scattered twisted photon angular momenta m' are displaced from the initial twisted photon angular momentum m by $\delta m \equiv |m' - m| \sim mp'_\perp/\varkappa$. Finally, as the initial twisted photon is obtained by an integration over a conical angular distribution of plane-wave components, the energy of the final twisted photon for the case of nonstrict backscattering can be obtained from Eq. (11).

Conclusions.—The general convoluted invariant matrix element (10) for Compton scattering of twisted photons, which can be evaluated numerically for arbitrary scattering geometry, is found to take a particularly simple form for strict backscattering (see Fig. 2), according to Eqs. (15) and (22). For that geometry, the energy of the final twisted photon is increased most effectively ($\omega'/\omega \sim \gamma^2 \gg 1$). According to Eq. (24), the magnetic quantum number $m' = m$ and the conical momentum spread $\varkappa' = \varkappa$ are preserved under strict backscattering. This implies that the conical angle θ' of the scattered twisted photon is very small, $\theta' \approx \varkappa'/\omega' \sim 1/\gamma^2$.

High-energy photons with large orbital angular momenta projections can be used for experimental studies regarding the excitation of atoms into circular Rydberg states, and for studying the photo-effect and the ionization of atoms, as well as the pair production off nuclei. As ion traps for highly-charge ions are currently under construction (e.g., Ref. [7]), one of the most interesting experiments would concern the question of whether nuclear fission can be achieved by the absorption of fast rotating nuclei, via the absorption of one or more twisted high-m photons at energies below the giant dipole resonances [8] which are typically in the range of ~10–30 MeV. Such a study might reveal fundamental insight into the dynamics of a fast rotating quantum many-body system.

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