



26 May 2010, 5:40 pm - 5:50 pm

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Fifth International Conference on

Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium in Honor of Professor I.M. Idriss

May 24-29, 2010 • San Diego, California

SEISMIC RESPONSE OF COLUMNAR REINFORCED GROUND

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ABSTRACT

Ground improvement using stiff columnar reinforcement, such as stone, jet-grout and soil-mix columns, is commonly used for mitigation of seismic damage in weak ground. Seismic shear stress reduction in the reinforced soil mass is often counted on for reducing liquefaction potential. Current design methods assume composite behavior of the reinforced soil, where the shear stress reduction is based on the ratio of the columnar stiffness relative to the soil as well as the area replacement ratio. This implicitly assumes that the stiff columns will deform in pure shear along with the surrounding soft soil. Three dimensional dynamic finite element analyses were performed to better understand the column deformation and shear stress reduction behavior. The analyses focused on the deformation modes of the stiff column during shaking and the stress transfer mechanisms between the column and the surrounding soft ground. These analyses showed that the seismic behavior of columnar reinforced ground is more complicated than widely thought, and importantly, that current design methods may greatly over-estimate the shear stress reduction the columns provide. The study found that stiff columns do not behave as pure shear beams as implicitly assumed by current methods, but that their behavior is a combination of shear and flexural behavior. Further, the results indicate that the mode of deformation of the columns significantly influences their effectiveness in reducing shear stresses in the reinforced soil. For most common applications, the columns deform in combination of flexure and shear. The net effect is that stiff columns typically achieve only a small percentage of the shear stress reduction implied by area-replacement ratio methods that assume composite behavior for reinforced ground. In summary, columnar reinforcement provides little or no seismic shear stress reduction and current methods may be unconservative. The results of this analytical study are presented in this paper and the implications in terms of the current design practice are discussed.

INTRODUCTION

Ground improvement using stiff columnar reinforcement, such as stone columns, jet grout and soil-mix columns is commonly used for mitigation of seismic ground damage in soils susceptible to significant seismic-induced deformation. A number of benefits are gained, such as in-situ densification of loose granular soils where stone columns are installed, and increased bearing support where jet-grout or soil-mix columns are constructed in fine-grained soils that cannot be effectively densified.

Current engineering practices often consider shear stress reduction in the reinforced soil mass a key factor in reducing the liquefaction susceptibility of soils improved with stiff columns. The shear stress reduction mechanism of stiff columns is based on the presumption that the stiff columns attract more of the seismically-induced shear stress than the surrounding softer soil mass. The idea that the column carries a larger shear stress, in proportion to the stiffness ratio, is

implicitly based on the assumption that both the soil and the stiff columns deform compatibly in pure shear, namely undergoing the same shear deformation during shaking. This assumption is further utilized in estimating the reduction of seismically induced shear stresses on the soil (Baez and Martin 1994).

Recent studies suggest that current design methods for shear stress reduction of columnar reinforced ground may be greatly underconservative and should be more closely examined (Martin and Olgun 2007, Olgun and Martin 2008; Goughnour and Pestana, 1998). Three dimensional dynamic finite element analyses were performed to better understand the column deformation and shear stress reduction behavior. The analyses focused on the deformation modes of the stiff column during shaking and the stress transfer mechanisms between the column and the surrounding soft ground.

These analyses showed that the seismic behavior of columnar reinforced ground is more complicated than widely thought, and importantly, that current design methods may greatly over-estimate the shear stress reduction the columns provide. The study found that stiff columns do not behave as pure shear beams as implicitly assumed by current methods, but that their behavior is a combination of shear and flexural behavior. Further, the results indicate that the mode of deformation of the columns significantly influences their effectiveness in reducing shear stresses in the reinforced soil, with shear deformation being the most effective, and flexural being the least. For most common field conditions, the shear stress reduction of the stiff columns was found to be significantly less than predicted by the current design methods such as Baez and Martin (1994) and Priebe (1995). This paper presents the findings from these analyses and describes the mechanisms associated with the shear stress reduction.

BACKGROUND

Studies on the seismic behavior of columnar reinforced ground differ in their explanation of the stress transfer mechanisms between the soil and the stiff columns. Current design methods such as Baez and Martin (1994) implicitly assume that the stiff columns in soft ground behave as shear beams during ground shaking, as the predicted reduction of shear stress in the soil is assumed to be proportional to the area and stiffness of the columns relative to the soil. More recent studies, such as Goughnour and Pestana (1998), suggest that columns behave as flexural beams. If so, this implies that little to no additional shear stress is carried by the columns. Figure 1, illustrates shear and flexural deformation modes of a column; the left side of the figure shows pure shear deformation, and the right side shows pure flexural deformation.

Implicit to the Baez-Martin method is the underlying assumption that columnar reinforced ground behaves as a composite mass. Composite mass behavior means that the columns and the surrounding soil undergo the same magnitude shear deformations at any given time during shaking. This deformation compatibility is a result of the of the assumed deformation kinematics of the soil-column mass where the soft soil and the stiff column both undergo pure shear deformations. Inherent to this assumption of pure shear behavior and same magnitude shear deformations, the stiff columns attract significantly larger shear stresses than the surrounding soft ground.

A beam in lateral vibration will undergo predominantly flexural deformations as well outlined in the structural mechanics theory (Chopra 2000). In the classical beam theory (i.e. Euler-Bernoulli beam), a column (or a beam) deforms in pure flexure where the plane sections remain planes and rotate, but still remain perpendicular to the neutral axis (Chopra 2000) as shown in the flexural deformation mode in Figure 1. A refinement to the classical beam theory, which is

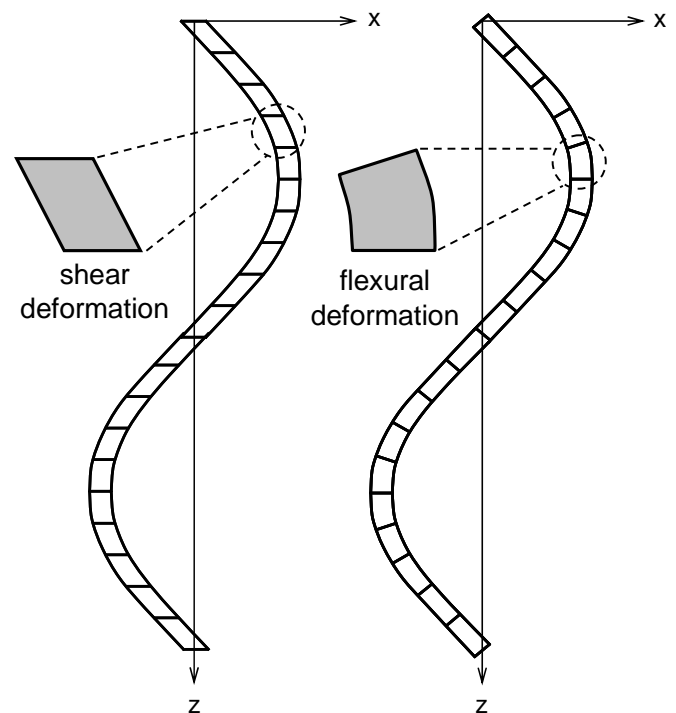


Fig. 1. Shear and flexural deformation modes of a column (Adapted from Goughnour and Pestana 1998).

introduced by Timoshenko, shear deformations need to be considered in addition to the flexural deformations (Timoshenko 1921, 1922). Neither beam theory considers shear deformation as the predominant mode of deformation of a beam in vibration. Therefore it has long been recognized that flexural deformations govern the vibration of a column. Though not critical to this behavior, shear deformations may also need to be considered as a refinement to the classical beam theory in structural mechanics.

In geotechnical earthquake engineering, the shear beam analogy has been the basis for analyzing the response horizontally layered earth systems (i.e. 1-Dimensional soil profile). Such a geometry with horizontal layering results in pure shear deformations of the soil mass where flexural rotations are inhibited due to the geometric/kinematic constraints. It should be recognized that the shear beam theory which has been widely used in geotechnical earthquake engineering applies to these unique geometrical and boundary conditions. Under these conditions the soil mass behaves as a shear beam and undergoes pure shear deformations in response to vertically propagating horizontally polarized shear waves. Against the available beam theories and the lack of a sound theoretical basis, shear beam behavior has been implicitly assumed to also apply for a columnar element embedded in soil. Utilization of the shear beam analogy to analyze columnar reinforced ground is an artifact that stems from one-dimensional site response analysis procedures where unique conditions exist that validate the use of such methods.

However, there is neither a theoretical basis nor a valid reasoning for the use of shear beam theory in modeling the dynamic behavior of columnar reinforced ground during shaking.

In summary, vibrational behavior of a column will be purely flexural as in an Euler-Bernoulli beam. In certain cases that require special attention (i.e. deep beam cross sections) shear deformations in addition to flexural deformations have also been considered in the Timoshenko beam theory. Structural mechanics literature is well established in this field (Timoshenko 1953). A stiff column in soft ground will mainly undergo flexural deformations which predominantly involve rotational deformations. Deformation kinematics suggest that pure shear deformation corresponds to a deformation mode where angular distortions are free from flexure related angular rotations. Pure shear deformation will occur when there is not a rotational deformation mode. While the shear beam behavior may be a valid deformation mode for the soil some distance away from the column, there is not a basis or a theoretical framework to assume that the stiff column and the soil in the vicinity will necessarily deform in pure shear. Such a misconception in the current design methods needs to be clarified and the underlying mechanisms of such behavior need to be further investigated. Numerical analyses have been conducted to make a quantitative assessment of the deformations and stress transfer mechanisms within the soil-column mass.

ANALYTICAL STUDY AND NUMERICAL MODELING

In the current analytical study, it was important to first clearly understand the basic mechanics of column behavior and the fundamental differences between deformation modes. A major focus of this analytical investigation was to distinguish the deformation modes of the stiff column within soft ground. Clarification of the deformation modes during shaking was a fundamental step in determining how the soil mass responds seismically, and ultimately, how much shear stress reduction is achieved in the soil.

The modeling involved three-dimensional dynamic finite element analyses that simulated the seismic response of soft ground reinforced with stiff columnar elements. The analyses were performed using the computer code DYNAFLOW (Prevost 1981). Shown in Figure 2 is plan view of the finite element mesh used to model the representative profile developed for this study. The finite element mesh was 1.8 m x 1.8 m in plan view, 12 m deep, and contained approximately 14,000 elements. The soil profile consisted of 6 meters of soft soil underlain by 6 m of relatively stiffer material. Shear wave velocities of the soil in the upper and the lower 6 meters of the profile were 150 m/s and 250 m/s with unit weights of 16.7 kN/m³ and 17.6 kN/m³, respectively. The upper 6 meters of the soil profile was reinforced with 6-m long columns, 90-cm in diameter with a 180-cm center-to-center spacing, corresponding to a spacing-to-diameter (S/D) ratio of 2 and an

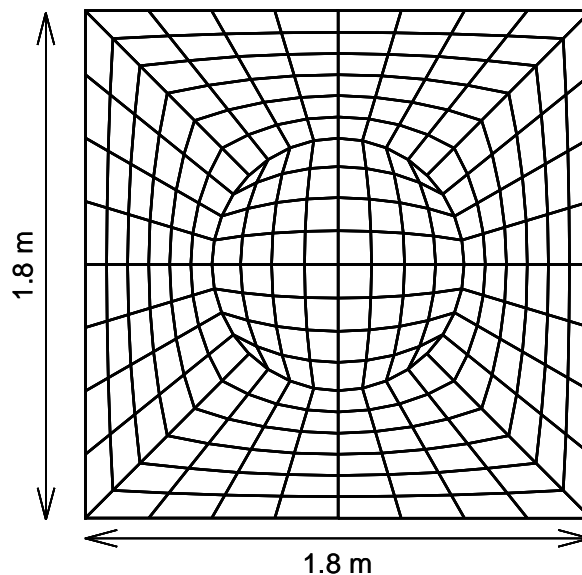


Fig. 2. Plan view of the finite element mesh used in the analyses

area replacement ratio of about 20%. Column-to-soil stiffness ratio ($G_{\text{column}}/G_{\text{soil}}$) was taken as 10 in the baseline analysis of the analytical studies.

This geometry and stiffness ratio is typical of stone-column reinforced ground. This representative profile was used for the benchmark analyses to understand the basic mechanisms of column behavior related to deformation modes and shear stress reduction mechanisms. Subsequently, after the behavior was understood for this base case, key parameters such as column-to-soil stiffness ratio and column diameter were varied in an additional set of parametric analyses to show their effect on shear stress reduction.

The analyses considered a linear elastic stress-strain relationship for the soil and the stiff column. Linear elastic modeling was preferred mainly due to its simplicity, and because the main issues of concern for this particular study are sufficiently captured by linear behavior assumptions. Any further sophistication in modeling the material behavior probably would not have added to the findings. The analyses were performed with total stress analyses where pore pressure generation in the soil was not considered.

In terms of boundary conditions along the sides, the three-dimensional model was assumed to be surrounded by an infinitely repeating sequence of identical 1.8 m x 1.8 m reinforced soil sections. This was achieved by assigning the opposite nodes on each face of the model to be equivalent. By assigning nodal equivalency to node couples at the same elevation they share the same set of equations of motion, and therefore undergo the same motion. This equivalency imposes dynamic symmetry along each vertical face of the model and therefore a repeating sequence of columnar reinforcement is

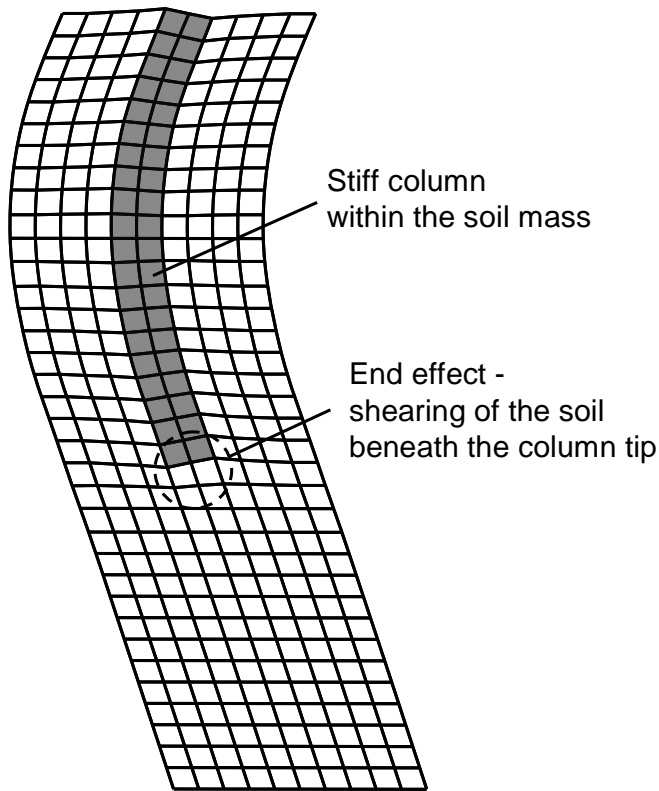


Fig. 3. Deformed shape of the soil-column system – Schematic view of the cross-section

defined. The model was shaken at the base in both horizontal directions simultaneously. The EW and NS horizontal components of the strong ground motions recorded in Izmit (IZT station) during the 1999 Kocaeli Earthquake were used for this purpose.

NUMERICAL ANALYSIS RESULTS

The mechanics of deformation and the interaction of the stiff column with the surrounding soil during shaking were of

primary interest of the analytical study. A schematic of the deformed soil-column system during the dynamic analysis is shown in Figure 3. Shown is a simplified sketch of the two-dimensional planar section of the deformed finite element mesh. As can be seen, the stiff column bends within the soft soil mass while the soil elements within the reinforced zone mainly deform in shear with the exception of the soil in the vicinity of the column. Apparently the soil near the column undergoes some rotational deformations along with the column which is deforming mainly in flexure. Also, the rotation of the column base causes some additional deformation of the soil beneath as seen in the figure. A closer look at the modes of shear and flexural deformation is necessary to identify the mechanics of columnar behavior.

In an effort to understand the modes of deformation along the column sets of four quadrilateral points along the center of the model were taken as illustrated in Figure 4 and the shear and flexural deformations were investigated. The magnitudes of shear deformation (γ) and flexural deformation (θ) were calculated using Equations (1) and (2) along the height of the 6-m column, as well as the lower half of the model that was unreinforced.

$$\text{Shear deformation } \gamma = \frac{\gamma_L + \gamma_R}{2} \approx \frac{\Delta d_1 - \Delta d_2}{2} \cdot \frac{d}{h \cdot w} \quad (1)$$

$$\text{Flexural deformation } \theta = \frac{z_2 - z_1}{w} \quad (2)$$

The cumulative shear and flexural deformations were computed by integrating the absolute values of shear deformation (γ) and flexural deformation (θ) over the course of shaking. Progression of shear and flexural deformations along both directions at three elevations along the column is shown in Figure 5. The relative magnitude of shear and flexural deformations throughout shaking remains unchanged as a constant ratio between the two parameters is maintained. As can be seen, near the top of the column at a depth of 1.1m,

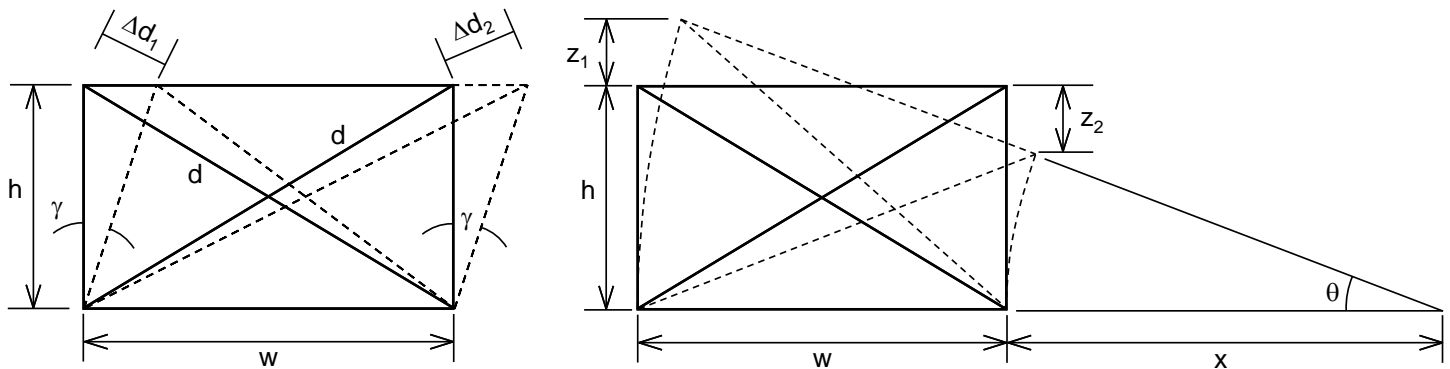


Fig. 4. Schematic of shear and flexural deformations

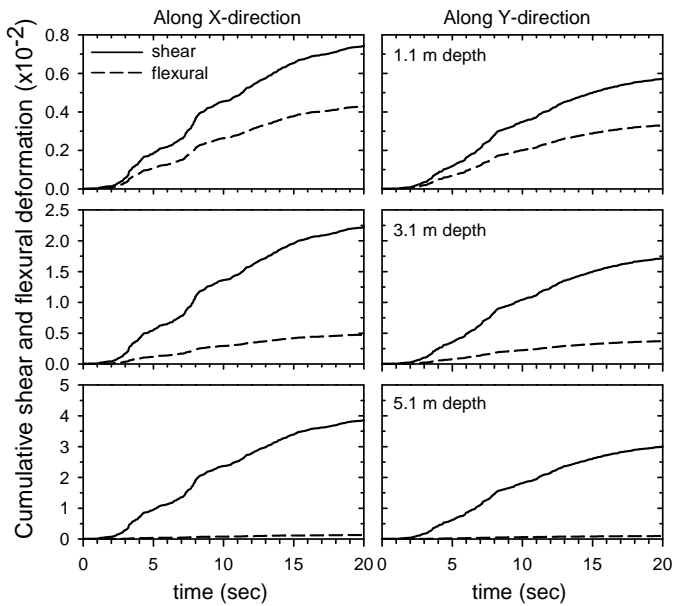


Fig. 5. Progression of shear and flexural deformations along the column

flexural deformation is considerable, about slightly more than half the shear deformation along both horizontal directions. The relative magnitude of flexural deformation in proportion to shear deformation is smaller at mid-depth and near the bottom end of the stiff column.

The contribution of shear deformation with respect to the total of shear and flexural deformation is calculated along the length of the 6-m columns, and continuing along a vertical section through the underlying unreinforced soil down to a depth of 12 m. Shear deformation contribution is defined using Equation 3 below using the respective magnitudes shear (γ) and flexural (θ) deformations. Had the assumption of pure shear beam behavior that forms the basis of current design procedures held, we would expect the shear deformation contribution to be 100%.

$$\text{Shear deformation contribution (\%)} = \frac{\gamma}{\gamma + \theta} \cdot 100(\%) \quad (3)$$

Calculated values of shear deformation contribution are shown in Figure 6 at the center along the height of the finite element model. Contribution of shear deformation along the stiff column increases with depth, indicating it behaves more as a shear beam at deeper levels. Additionally, the contribution of shear deformation beneath the column below a ~1 m transition zone base quickly reaches 100% as expected. With the exception of this transition zone, which is attributed to the end effects imposed by the 90 cm diameter column to the soil underneath, the unreinforced soil acts beneath the stiff column behaves like a pure shear beam as expected.

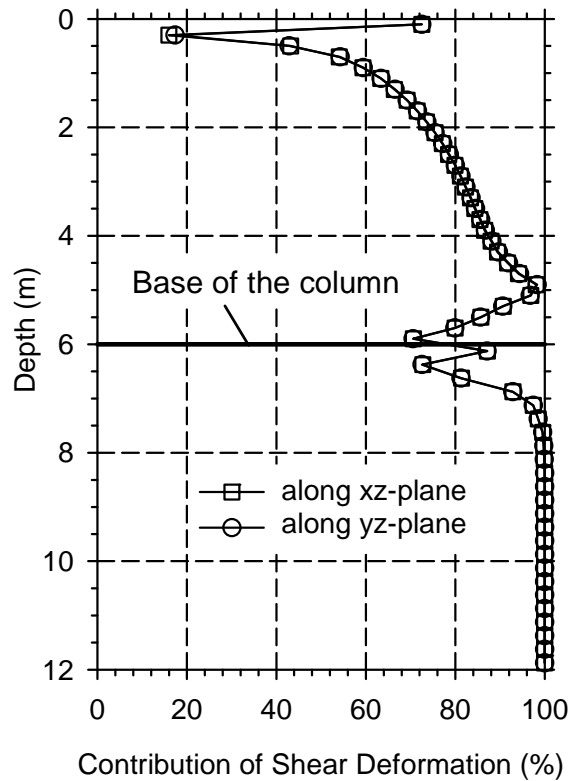


Fig. 6. Relative contribution of shear deformations along the center of the model

These results indicate that for the considered geometry (90 cm diameter column with 180 cm center-to-center spacing) and column-to-soil stiffness ratio of 10, the stiff reinforcement element behaves differently than a pure shear beam which is the underlying assumption in the current design guidelines (Baez and Martin, 1994). It is of primary interest how such deviations affect the stress transfer mechanisms of the soil-column system. Of practical concern is the effect of these unanticipated flexural deformations on the relative magnitudes of shear stresses carried by the soil and the stiff column. If pure shear behavior assumption held, we would expect the stiff column and the soil carry shear stresses in proportion to their stiffnesses. Even though the magnitudes of flexural deformations are small compared to the shear deformations for the stiffness ratio investigated, as presented below, even such small values of flexural deformation have significant implications in terms of the shear stresses carried by the stiff columns.

Average values of shear strain and shear stress within the column and the soil at each elevation are calculated and the maximum values of these average strains and stresses throughout shaking are plotted in Figure 7. As mentioned the column is 10 times stiffer than the surrounding soft soil. As can be seen in plot (c), the stiff columns were not strained as hard as the soil around them – they experienced negligible shear strains, while peak strains in the reinforced soil mass

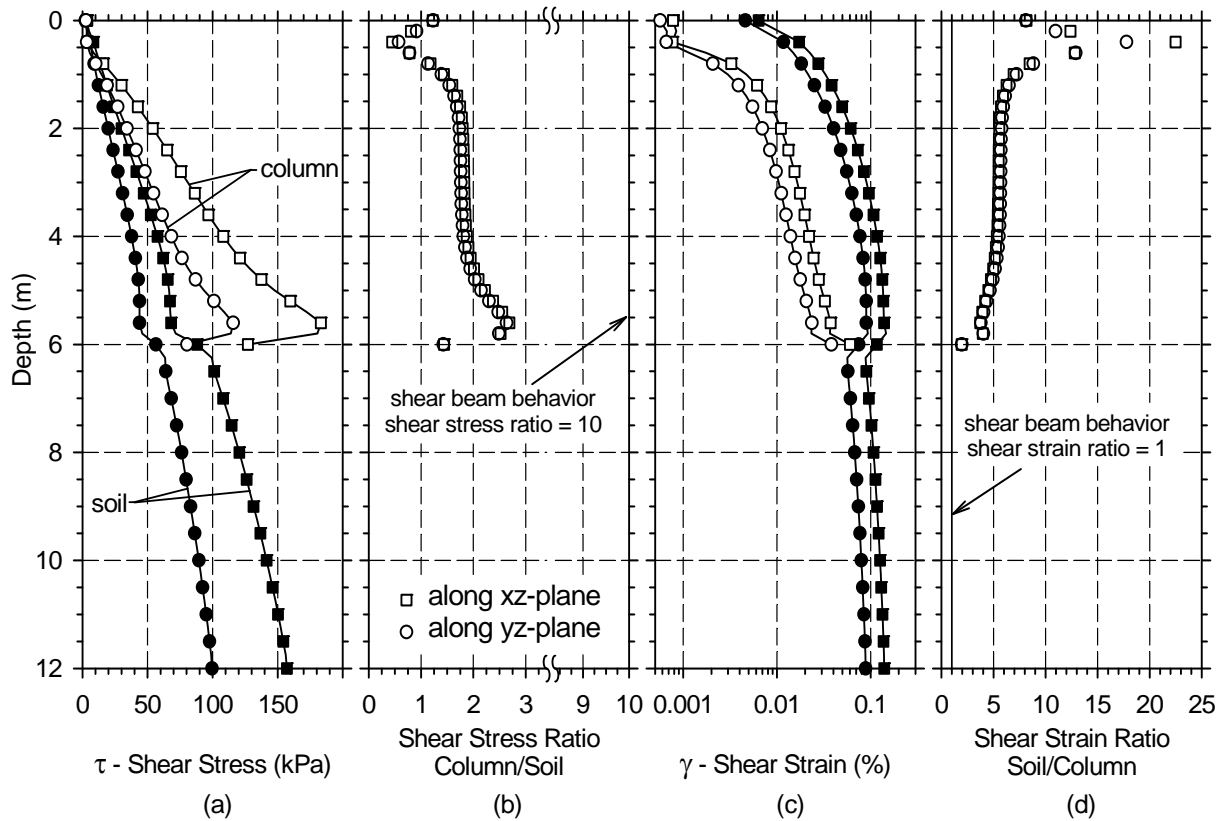


Fig. 7. Peak values of shear stresses and strains within the soil and the column

approached 0.1%. If the stiff column behaved as a pure shear beam, the column and the soil would deform compatibly and would undergo the same magnitude of shear strains. However, as seen in plot (d), the stiff column on average is deforming 5 times less than the soil. The stiff column is not being strained as hard as it would be strained as a pure shear beam. If the column behaved as a shear beam and underwent the same magnitude shear strains as the surrounding soil we would expect it to attract 10 times the shear stresses carried by soil, in proportion to their stiffness ratio. As a result, it is not attracting as much stress as anticipated by current design methods as described below.

Predicted peak seismic shear stresses are shown in plot (a) in Figure 7. The peak stresses in the stiff columns (120-180 Pa) were consistently higher than those in the soil mass (50-70 kPa) in the reinforced zone, as would be expected because the columns are stiffer and attracted more load; however, they did not attract nearly enough shear stress to significantly reduce the shear stresses in the reinforced soil mass. The stiff columns picked up only a small percentage of the shear stresses implied by methods such as Baez and Martin (1994) that assume composite shear behavior. That is, if the basic assumptions behind the shear stress reduction for composite behavior were valid then the stiff column should have carried 10 times more shear stress than the soil. The peak value of seismic shear stress on the column is only about two-to-three

times larger than the shear stress induced on the soil as seen in plot (b). In essence, one might say this behavior indicates that the column is not very “efficient” in reducing seismic shear stresses as anticipated by Baez and Martin (1994). This finding is consistent with results reported by Goughnour and Pestana (1998) based on their analysis of ground reinforced with stone columns. They also suggested that the columns should provide little, if any, shear stress reduction in most cases. The main implication is that commonly-used design approaches based on assumptions of composite behavior for ground reinforced with discrete columnar elements may greatly over-estimate seismic shear stress reduction.

The analyses suggest significant strain incompatibility between the soil and columns which were 10 times stiffer in shear relative to the soil. Such incompatibility was also evident in the deformed mesh shapes as shown earlier, which showed that the columns tended to flex back and forth within the soil profile and rotate at the ends during shaking rather than shearing along with the surrounding soil. As such, they clearly did not behave as shear beams as tacitly assumed. Therefore, even though the columns were much stiffer, they did not strain sufficiently in shear to attract a significant portion of the shear loading.

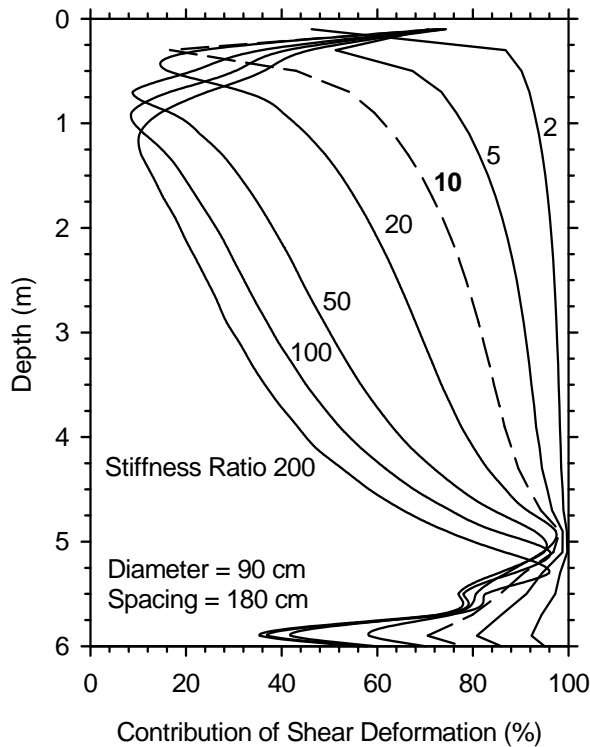


Fig. 8. Shear contribution for a series of stiffness ratios

PARAMETRIC ANALYSES

After establishing and understanding basic column-soil behavior using the representative case above, detailed parametric studies were performed to better understand the effects of reinforcement-to-soil stiffness ratio and column diameter on seismic shear stress reduction. The results are summarized in the following figures.

Contribution of shear deformation along the length of the column for a variety of stiffness ratios is presented in Figure 8. Column stiffness was varied in additional analyses as the shear modulus of the native soil was held constant and a range of column-to-soil stiffness ratios were achieved. The graph is shown for stiffness ratios ranging from 2 to 200. (For reference, typical column-to-soil stiffness ratios for stone columns are about 5-10, and about 50-100 for soil-mix columns, and 100-150 for jet-grout columns). The case for the stiffness ratio of 10, shown with a dashed line, corresponds to the base case presented earlier. It is shown in the figure that as column stiffness increases, the column progressively behaves more as a flexural beam, and the contribution of shear deformation decreases. The column behavior consistently also changes with depth, having more shear beam behavior toward the bottom of columns, and more flexural behavior near the top.

As the contribution of shear deformation varies along the column length the average values of shear contribution along

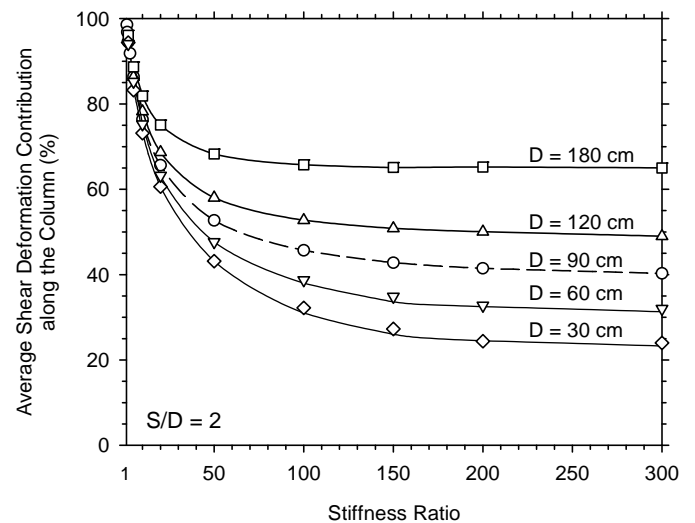


Fig. 9. Average shear contribution for a series of stiffness ratios and a variety of column diameters

the column length were computed for a variety cases. The results are shown in Figure 9 for a range of stiffness ratios and column diameters. As shown, stiffness ratio has a significant effect for ratios less than about 20. In particular, for stiffness ratios of less than 3, the columns have more than 80% shear beam behavior, and exponentially approach 100% as the stiffness further decreases. As stiffness ratios increase, especially after about 20, increased stiffness ratio has little effect on shear deformation in the columns. However it should be mentioned that a predominantly shear deformation behavior occurs for stiffness ratios less than 3, a level at which even if fully efficient shear the stress reduction potential would be minimal.

Column diameter was found to have some effect on column behavior. For smaller columns in the range of 30 cm diameter, there was a maximum of 20% shear beam contribution for a range of stiffnesses. For larger columns of 180 cm diameter, there was at least 70%-80% shear beam behavior for most stiffnesses. This is consistent with what would be expected. An infinitely wide column would correspond to a pure shear beam, and thus larger diameter columns behave more like shear beams than smaller columns.

Because shear stress was the main interest, the ratios of the average shear stress in the columns relative to the shear stress in the soil mass were computed. These ratios were computed for various stiffnesses and column diameters. As shown in Figure 10, the column shear stresses are only up to about 2.5 times higher than those in the soil for a wide range of stiffnesses and column diameters. Only a modest amount of shear stress was attracted by the stiffer reinforcement. This is a key finding, because if pure shear beam behavior were occurring, the shear stresses in the stiff column relative to the soil mass would be in proportion to the column-to-soil stiffness ratio. In other words, the column which is 100 times

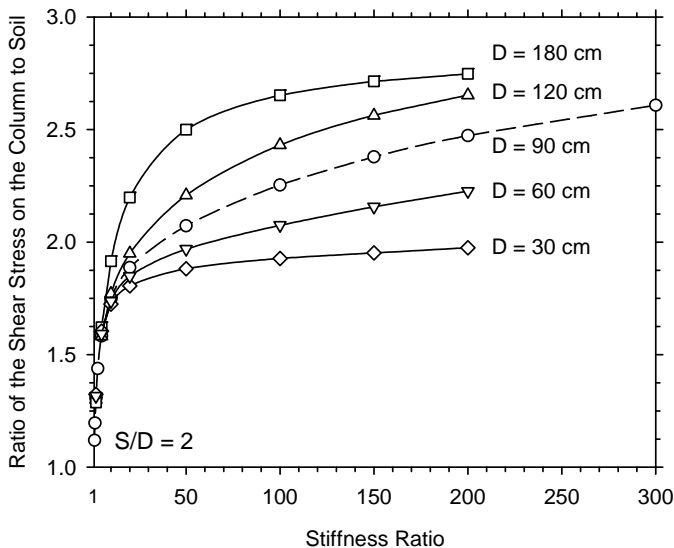


Fig. 10. Average column-to-soil stress ratio for a series of stiffness ratios and a variety of column diameters

stiffer than the soil (about typical of a soil-mix column in loose sand) would attract 100 times the stress carried by the soil if the column behaved as a pure shear beam and assumptions made by Baez and Martin (1994) were valid. However the results presented in this figure indicate that such a column would only carry 1.8 to 2.6 times the shear stress carried by the soil for the range of column diameters investigated.

CONCLUSIONS

Current methods used to predict seismic shear stress reduction in soft soil profiles reinforced with columnar elements assume composite behavior. This implicitly assumes a shear mode of deformation of the columns as well as the soil. Three-dimensional dynamic finite element modeling using DYNFLOW was performed for a 12-m deep soft soil profile reinforced with stiff columnar elements. An $S/D = 2$ was assumed, along with a column diameter of 90 cm, a soil-to-stiffness ratio of 10, and column lengths of 6 m. These improvement geometries and stiffnesses are typical of stone-column reinforced ground. Analyses were first performed for this representative case to investigate the essential behavior and stress transfer mechanisms. This was followed by parametric analyses to show the effects of column diameter and column-to-soil stiffness ratio.

The analyses indicate that the deformation behavior of the columnar elements is more complicated than thought. The columns deform in a combination of both shear and flexure during seismic loading. The net effect is that stiff columns typically achieve only a small percentage of the shear stress reduction implied by area-replacement ratio methods, such as Baez and Martin (1994) that assume composite behavior for

reinforced ground. Parametric analyses show that as the column-to-soil stiffness ratio increases, the tendency for flexural deformation of the columns increases, and thus the shear contribution of the columns becomes less. The “efficiency” of the columns to behave as shear beams and produce shear stress reduction decreases with increasing column stiffness.

For the spacings (i.e., $S/D=2$), diameters, and column-to-soil stiffnesses ratios seen in most common field situations, such as for stone columns and jet-grout columns, there is relatively little shear stress reduction achieved in the soil mass. Commonly-used design approaches based on assumptions of composite behavior for ground reinforced may greatly overestimate the actual level of seismic improvement in terms of shear stress reduction.

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