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2018

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Recommended Citation

Sgarbossa, Fabio; Calzavara, Martina; and Persona, Alessandro, "Economic and Performance Analysis of Dual-bay Vertical Lift Modules" (2018). *15th IMHRC Proceedings (Savannah, Georgia, USA – 2018)*. 16.

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Economic and performance analysis of dual-bay Vertical Lift Modules

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Abstract—Warehouse picking is one of the most time and cost consuming activities in a warehouse, often requiring the presence of human operators, who travel within the aisles to retrieve the items needed by the customers. Several studies demonstrate that the travelling activity can represent even the 50% of the total picking time, with a subsequent creation of a separate storage and picking area for small objects. In the last years, new solutions for order picking systems have been developed, especially for small items. One of these solutions requires Vertical Lift Modules (VLMs), storage columns with extractable trays. In this paper, the employ of dual-bay VLMs, compared to a carton racks warehouse, has been analysed from an economic point of view. Some mathematical formulations have been developed, to estimate the total annual cost and the respective convenience limits of both systems, according to their productivity. Moreover, some useful guidelines for practitioners are derived.

Keywords— *Vertical Lift Module, Warehouse, Order Picking, Cost, Performance*

I. INTRODUCTION

Warehouse picking is the activity of retrieving items from their storage locations to fulfill different customers' orders [1]. Due to the high flexibility and to the lower operative costs, this activity is usually performed by human operators, walking or travelling with a picking cart through the aisles of the warehouse in which the various products are stored [1, 2]. Such picking strategy is also called picker-to-parts picking, and, as widely demonstrated in literature, it is characterized by a high incidence of the travel time, which usually amounts to the 50% of the total order processing time [3]. Moreover, this aspect is even more crucial when the pick and the storage of small items are considered. In fact, small dimension products are often stored in pallets, too, with a consequent waste of space and, hence, time [4]. Therefore, one of the most effective ways for reducing the total picking time and, hence, for reaching a higher system throughput, should consider the decrease of the travel time [5, 6]. This objective can be obtained, for example, by dedicating a different forward area to small objects picking [2], or by introducing new storage systems that ease the picking activity [7]. In this latter case, the new systems could be automated solutions, leading to a parts-to-picker strategy. Some examples are miniloads, other AS/RS systems like the Autostore® [8], or the ones employing particular automated guided vehicles able to move the shelving towards the picker according to the picking

orders, like the KIVA robots developed by Amazon robotics [9]. Of course, the implementation of automated solutions should consider an important trade-off, between the benefits that these systems can carry, and the related emerging costs.

In this paper, a specific parts-to-picker picking system for small objects is introduced and analysed. The system consists of a dual-bay Vertical Lift Module (VLM), used for the storage of the items and for the dynamic picking of the picker. Due to the recently introduced technical modernizations, VLMs are attracting always more attention in several contexts, leading also to new interesting applications [10]. Therefore, it becomes necessary to study this kind of systems, both from a technical and an economical point of view. In fact, even if the implementation of a VLM is not so expensive, especially compared to other automated storage solutions (as for example KIVA robots), it is important to properly consider all the aspects and the characteristics that affect the final cost of such a configuration.

In the present paper, an economical evaluation of a VLM picking system is carried out. Moreover, the proposed mathematical model is used to compare this picking system to a traditional picker-to-parts picking area with traditional shelving and aisles. The modelling of both systems and their critical comparison can lead to the proposal of some useful guidelines, that can help practitioners to understand the real convenience of a VLM system, together with the borders of its adoption.

The introduced mathematical models allow, for the first time, to model a VLM storage system from an economic and from a technical perspective. The performance evaluation takes into account all the most common activities related to warehouse picking (i.e. pick, travel, search and others). Moreover, the comparison between the VLM and the carton racks warehouse turns out in a formula, function of the requested throughput and of the storing volume, which allows to understand the economic sustainability of a VLM.

Such a comparison shows that, first of all, a carton racks warehouse is preferable to a VLM system when the VLM has lower time performances, even if it occupies less space, since a VLM has higher fixed costs. On the other side, in case the VLM is faster than the carton racks system, further analysis is needed to fully understand its applicability. Then, a VLM convenience region is defined, as a function of the throughput ratio of the two systems.

The remainder of the paper is structured as follows. In the next section, a brief literature review concerning small objects picking, vertical lift modules and economic modelling is presented. Then, in Section 3, the economic models, both of the carton racks system and of the VLM picking system, are presented and explained. In Section 4, a parametrical analysis of the economic models and the comparison of the two systems are reported, together with the discussion of the obtained results. Subsequently, Section 5 reports the application of the proposed model to an industrial example. Finally, Sections 6 is for the future researches and for the conclusions.

II. LITERATURE REVIEW

In picker-to-parts warehouses, the pickers travel in the aisles, searching for the items and collecting them to complete their order list. In case of a traditional order picking warehouse, where items are stored on pallets that are positioned on the lower stocking locations of the shelves, pickers use electric pallet trucks to move inside the aisles and to transport one or more mixed pallets, made of the items collected during their order picking activity. Typically, the expected average time per order line of this system is at least about 80-100 s/line, where the main part is related to the travelling and searching activities [3]. Moreover, the pick of the items could have a relevant impact also on the ergonomic level, especially when the operators are picking the last items from the pallet [11].

Therefore, it is often suggested to store small-dimensions items in a separate forward area, in which items are not stored in pallets but in other specific storage systems [2, 7]. In fact, it has already been demonstrated in literature that this approach can significantly speed up the picking process, thanks to a reduction of the storage space dedicated to every single item and, then, to a reduction of the travelled distances [4]. Moreover, the use of alternative storage solutions for small objects can improve the picking activity, both in terms of time and ergonomic effort [12].

The most common systems used for the storage and the picking of small-dimensions items can be divided into two main categories: static picker-to-parts solutions and dynamic parts-to-picker ones [7, 13]. Static systems usually require the storage of goods in racks or in other structures that are fixed in one place and, therefore, usually simple and not expensive. Some examples of static systems are: carton racks, often equipped with specific devices (containers, dividers etc.), modular drawer cabinets, movable aisle systems, flow rack systems. On the other side, in a dynamic system the items are brought to the picker by an equipment, that is usually supported by automated systems, as well as computer software tools. Dynamic solutions can typically assure higher space utilization, also taking advantage of the vertical space, that is normally not used very well in the static solutions. Examples of dynamic systems are: vertical carousels, horizontal carousels, single-bay and dual-bay Vertical Lift Modules, miniloader AS/RS systems, A-frames and picking machines, like the robots developed by Amazon robotics [9, 13].

All these solutions present different advantages and disadvantages, that could lead to different possible applications, according to the aspects that you want to give priority to. Generally, the factors that have to be considered are the dimensions of the stored bins, the allocation of each product

code and its picking frequency [2, 4]. According to the picking frequency of the item and to its storage allocation, one storing solution can turn out to be more suitable than another. It is then important to estimate the throughput of each alternative, together with their costs, to understand their most proper applicability field.

A Vertical Lift Module (VLM) is a dynamic storage solution composed of several trays, in which the items are stored, and of an automated storage and retrieval system, needed to retrieve, transport and deliver a tray at a time in front of the operator. Thanks to the recent development of such storing systems, the employ of VLMs has interestingly expanded, also in warehouse picking contexts [10, 14]. In particular, the introduction of dual-bay VLMs allows the picker to work in parallel to the system: while the picker has in front of him a certain tray, the AS/RS can independently store the previous tray and retrieve the following one. Of course, this can lead to an improved throughput of the picking activity, since the picker does not have to walk to reach the items to pick, and also the search and the pick of the item are eased [15].

Although Vertical Lift Modules possible applications are promising, they have received until now very few consideration in literature. The first relevant contribution, dealing with single-bay VLMs is by [16]. It is focused on the proposal of formulations that can be useful to estimate the storage and retrieval cycle times of the system. Another more recent research is by [10], in which exactly dual-bay VLMs are studied. Finally, [14] propose to employ dual-bay VLMs for a fast processing of small-objects picking orders, by introducing the so-called VLM fast picking system, and by studying some possible solutions that can speed up the overall configuration, like class-based storage assignment of the items, batch retrievals of the trays and order batching. All these contributions are mainly investigating the performance of a VLM, in terms of times estimation and throughput improvement. On the other side, for now there are no studies that are dealing with the economic impact that a VLM can have in a warehouse, especially compared to a traditional warehousing system.

The study of the economic contribution of a warehousing system generally should consider its most relevant costs items. For example, [7] suggest to take into account the building, the equipment within it, the value of the material to be stored and the cost of the operation. On the other hand, [17] propose to focus on the initial investment, on the shortage costs and on the costs associated to the storage policy. In [18] the authors state that the warehouse layout and configuration can effectively affect its construction and maintenance costs, the material handling costs, as well as the storage capacity, the space utilization and the equipment utilization.

As far as warehouse picking is concerned, the comparison of different picking approaches from an economic perspective has not received, for now, a proper attention. [19] develop design guidelines for a case-picking warehouse, through a statistical-based methodology and considering the number of labour hours. Other contributions are more focused on the operational aspects of a picking warehouse, like the ones related to forward area dimensioning, items allocation, replenishment impact and related costs [20, 21, 22].

On the other side, some researches state that the most important costs of a picking warehouse are related to the time needed for processing a picking order [1, 18]. Therefore, researches on this topic mainly propose to reduce costs by reducing the picking time [23]. This can be achieved, for example, by reducing the travel time, through the reduction of the distances travelled by the operators, or by using paperless picking devices, that can decrease the search and pick time [3, 6, 24].

The literature review highlights the current lack in mathematical models for the evaluation of small items storage systems, above all if dual-bay VLMs are considered. Therefore, the present paper aims at proposing new formulations useful to understand the applicability of a VLM storage system, both in terms of costs and performance. In particular, the proposed approach is based on the comparison of a VLM with a carton racks warehouse, also deriving some interesting guidelines for a proper use of a VLM system, as shown in the following sections.

III. COST MODELS FOR SMALL ITEMS WAREHOUSING SOLUTIONS

In this Section the two cost models for the evaluation of the considered storing systems are presented. They refer to a carton racks warehouse and to a Vertical Lift Module system, respectively (Figure 1).

These cost models allow to compare the two systems not only from a performance point of view, like partially already done in previous researches [10, 14, 15, 16], but also from an economic perspective, understanding the impact of the various costs that usually emerge in the adoption of these storage systems. In fact, this would help the comprehension of their possible applicability, as well as of the circumstances in which one is preferable to the other, as also shown in the following sections.

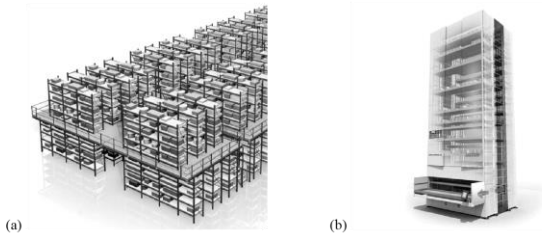


Fig. 1. Analysed systems. (a) Carton racks warehouse, (b) Dual bay Vertical Lift Module.

The introduced models take into account different cost components and they are characterized by a fix term and a variable one:

$$TC^s = C_{fix}^s + C_{var}^s \quad (1)$$

where $s = W$ for the carton racks warehouse and $s = V$ for the dual bay VLM.

The fix cost component refers to the space occupied by the system and to the eventual investment costs for facilities and devices; on the other side, the variable cost component mainly depends on the hourly operator cost and on the throughput of the system, in terms of required picks per year.

Thus, this permits to compare the two solutions both from an economic perspective and from a performance one, measured by the throughput of the two alternatives.

The replenishment activity, needed for refilling the storage locations with items, is assumed to be performed in an additional time with a similar strategy for both systems [2].

The input parameters and the variables of the models are reported in Table 1.

TABLE I. NOTATIONS

Notation	Description
Q [picks/h]	Total hourly required throughput, $Q = \sum_{i=1}^n Q_i$
V [m ³]	Total storage volume, $V = \sum_{i=1}^n V_i$
H [m]	Plant height
SL^s	Storage system saturation level, $s \in \{W, V\}$
Q^s [picks/h]	Storage system hourly throughput, $Q^s = 3600/t_i^s$, $s \in \{W, V\}$
t_i^s [s]	Average cycle time per line $s \in \{W, V\}$
V^V [m ³]	Storage volume of one VLM
A^V [m ²]	Operating area of one VLM, including the VLM area and the space for the operator
C^V [€/year]	VLM annual cost
k^s	Storage system space cost coefficient, $s \in \{W, V\}$
C_{sp} [€/m ² year]	Annual space cost per square meter
C_{op} [€/h]	Hourly operator cost
h_y [€/year]	Number of working hours in a year
TC^s [€/year]	Annual storage system total cost, $s \in \{W, V\}$
C_{fix}^s [€/year]	Annual storage system fix costs, $s \in \{W, V\}$
C_{var}^s [€/year]	Annual storage system variable costs, $s \in \{W, V\}$

A. Carton racks warehouse cost model

The first storing solution here analysed is a traditional warehouse suitable for small items picking, made of bin-shelving and aisles [2]. Here, it is considered to have a set of static racks on the ground floor and a further set of racks on a mezzanine system (Figure 1a). In such a picker-to-parts system, the picking operators walk within the various aisles to retrieve the items reported on their picking list and to put them in their picking carts.

The formulation of the total cost for this traditional warehouse TC^W is composed of two terms. The first term refers to the space cost, while the second one is for the workforce cost:

$$TC^W = C_{fix}^W + C_{var}^W = C_{sp} \cdot k^W \cdot \frac{V}{SL^W \cdot H} + C_{op} \cdot \frac{Q}{Q^W} \cdot h_y \quad (2)$$

In addition to the parameters defined before, $Q^W = 3600/t_i^W$ is the throughput of the picking system, where t_i^W is the average cycle time per line (pick). These values can be estimated mainly thanks to direct measurement of the pickers' activities during a significative period, and usually t_i^W turns out

to be within a range of values from 30 to 60 seconds per line [2, 3, 5].

B. Dual-bay VLM cost model

The second picking system for small items considers the employ of a dual-bay Vertical Lift Module. A Vertical Lift Module (VLM) is a closed storage column with various trays containing the stored items (Figure 1b). These trays are stored and retrieved by an automated system: according to the picking list, the system brings the required tray to the picking bay, so that the picker can process his order. In this case, the picking strategy is parts-to-picker, with the operator standing in front of the VLM picking bay, waiting for the retrieval of the trays containing the required items to pick. Moreover, the dual-bay allows the picker to work in parallel with the storage and retrieval system: while the operator is picking from a certain tray, the crane can store the previous tray and then retrieve the following one, resulting in a higher system throughput [14].

To perform the picking of Q items and to stock the total storage volume V , a certain number of VLMs N^V have to be installed to work in parallel. This number can be calculated as follows:

$$N^V = \max\left(\left\lceil \frac{Q}{Q^V} \right\rceil; \left\lceil \frac{V}{V^V} \right\rceil\right) \quad (3)$$

where V^V is the storage capacity [m³] and Q^V is the throughput [items/h] of a single VLM.

However, in order to minimize the cost of investment, the total number of installed VLM is typically defined following the equality $N^V = \left\lceil \frac{Q}{Q^V} \right\rceil = \left\lceil \frac{V}{V^V} \right\rceil$. In fact, if $\left\lceil \frac{Q}{Q^V} \right\rceil < \left\lceil \frac{V}{V^V} \right\rceil$ it means that it is possible to install a slower VLM system, with a lower investment cost; on the other side, when $\left\lceil \frac{Q}{Q^V} \right\rceil > \left\lceil \frac{V}{V^V} \right\rceil$, a smaller VLM system can be used, with a lower investment cost.

Similarly to the formulation of the total cost for a carton racks warehouse, also the formulation for a dual-bay VLM system is composed of two terms, the *space cost* and the *workforce cost*, multiplied by the number N^V :

$$\begin{aligned} TC^V &= C_{fix}^V + C_{var}^V = \\ &= N^V \cdot \left[(C_{sp} \cdot A^V + C^V) + C_{op} \cdot \frac{Q}{N^V \cdot Q^V} \cdot h_y \right] \end{aligned} \quad (4)$$

Here, the *space cost* includes both the space occupied by the VLM, obtained multiplying C_{sp} by the area A^V (sum of the VLM area and of the working space of the operator) and the annual cost of the VLM C^V .

On the other hand, in the *workforce cost* the only difference with respect to TC^W is related to $Q^V = 3600/t_l^V$, where t_l^V is the average cycle time per line in case of picking from VLM system, usually expressed in seconds.

Then, by introducing

$$k^V = \frac{C_{sp} + C^V/A^V}{C_{sp}} = 1 + \frac{C^V}{C_{sp} \cdot A^V} \quad (5)$$

as the floor space cost coefficient for the VLM, and considering

$$A^V = \frac{V^V}{SL^V \cdot H} \quad (6)$$

with SL^V saturation level of the VLM, Equation (4) can be rewritten as

$$TC^V = N^V \cdot \left[\left(C_{sp} \cdot k^V \cdot \frac{V^V}{SL^V \cdot H} \right) + C_{op} \cdot \frac{Q}{N^V \cdot Q^V} \cdot h_y \right] \quad (7)$$

The average cycle time per line t_l^V depends on some characteristics of the storage system (i.e. the dual command time of the VLM crane) and on the picker's performance [14]. In fact, since a VLM order picking system consists in a VLM working in parallel with a picker, the resulting cycle time derives from the comparison between the time spent by the crane to perform a dual command and the time spent by the picker to perform his/her activities, such as picking the items from the trays and other tasks like counting, weighing or stocking the items to new locations. Usually, since this system is evaluated as an alternative solution of the carton racks warehouse, the average cycle time per line t_l^V is not estimated through direct measurements but with mathematical or simulation models.

From previous scientific contributions [10, 14, 15], typical values of t_l^V are between 30 to 40 seconds per line. As for the t_l^W , it considers the time spent in picking activities and in the other activities, such as order setup, searching and travel.

IV. SYSTEMS ECONOMIC-PERFORMANCE EVALUATION AND COMPARISON

Once that the two cost models are defined, it is possible to use them to evaluate and compare the two storage systems, and, therefore, the two different picking approaches.

A. Economic comparison and analysis

As far as the scope of the present paper is concerned, the following focuses on an economic comparison of the two systems, carton racks warehouse and dual-bay VLM.

This starts through the definition of the following ratio:

$$R_{TC} = \frac{TC^V}{TC^W} \quad (8)$$

whose calculation can easily allow to understand the convenience of the VLM with respect of a carton racks warehouse (i.e. $R_{TC} \leq 1$).

Considering Equations 2 and 4, it is easy to notice that for the *space cost* component the carton racks warehouse C_{fix}^W is always preferable to the VLM, C_{fix}^V , since for this latter system there is a further cost related to the VLM annual cost C^V , even if the required space is lower. This is simply verified for typical values of the annual space cost per square meter C_{sp} (about 80-120 €/m²/year) and k^W coefficient (about 1.5-2), VLM annual cost C^V (about 18,000-24,000 €/year) and saturation levels $SL^W \approx 10\%$ and $SL^V \approx 30\%$.

Therefore, if there are not particular restrictions on space availability, the convenience of the VLM mainly depends on its performance in terms of system throughput Q^V , and the traditional carton racks warehouse is always preferable when its throughput is higher even if it occupies more space.

When VLM outperforms the carton racks warehouse it is interesting to understand when it is also convenient from an economic point of view. For this reason, in the next section a *VLM convenience region* is defined based on the technical data and cost factors of the two systems, such as: throughput, storage volume, working hours per year, annual floor space cost per square meter, hourly operator cost, VLM and carton racks system cost per year.

B. VLM convenience region definition and analysis

In this section a formulation of the *VLM convenience region* is introduced, based on the most influential factors. This results in a set of conditions in which the employ of a VLM picking system is preferable than a carton racks warehouse.

This *VLM convenience region* starts from the following condition of the previous equation (8):

$$R_{TC} \leq 1 \quad (9)$$

Then, the term $R_Q = \frac{t_l^W}{t_l^V} = \frac{Q^V}{Q^W}$ is introduced, defined as a throughput ratio.

Moreover, as previously described, $N^V = \left\lfloor \frac{Q}{Q^V} \right\rfloor = \left\lfloor \frac{V}{V^V} \right\rfloor$; thus, by substituting equations (2) and (4) in equation (8) and assuming the value of $V^V = \frac{V}{N^V}$, after some mathematical elaborations the final formulation for the systems comparison turns out to be:

$$\frac{C_{sp} \frac{V^V}{H} \left(\frac{k^V}{SL^V} \frac{k^W}{SL^W} \right)}{C_{op} \cdot h_y \cdot (R_Q - 1)} \leq \frac{Q}{N^V \cdot Q^V} \leq 1 \quad (10)$$

where $\frac{C_{sp} \frac{V^V}{H} \left(\frac{k^V}{SL^V} \frac{k^W}{SL^W} \right)}{C_{op} \cdot h_y \cdot (R_Q - 1)} = \frac{Q^*}{Q^V}$ is the threshold value and it is verified if the throughput ratio is as follow:

$$R_Q > 1 + \frac{C_{sp} \frac{V^V}{H} \left(\frac{k^V}{SL^V} \frac{k^W}{SL^W} \right)}{C_{op} \cdot h_y} \quad (11)$$

Therefore, the adoption of a VLM system made of N^V machines is convenient from an economic point of view when

$$Q^* \leq Q/N^V \leq Q^V \quad (12)$$

A similar finding can be stated considering the total cost of the VLM system, which is lower than the one of the carton racks warehouse if the throughput ratio R_Q is higher than the *convenient value* R_Q^* , expressed by the following equation:

$$R_Q > R_Q^* = 1 + \frac{C_{sp} \frac{V^V}{H} \left(\frac{k^V}{SL^V} \frac{k^W}{SL^W} \right)}{C_{op} \cdot h_y} \quad (13)$$

The formulations previously introduced can be used to perform an analysis that can help the derivation of some preliminary results, reported through different graphs, reported in the following. All graphs illustrate the trend of Q^*/Q^V between 0% and 100%, representing the maximum value for this ratio, as imposed in Formula (10).

This trend is represented according to the varying of other parameters: V^V , variable between 40 m³ and 60 m³ with a step

of 2 m³, C^V , equal to 18,000 €/year or 24,000 €/year, and C_{sp} , equal to 80 €/m² or 120 €/m². Moreover, it has been considered that operators work in two work shifts, resulting in $h_y=3,600$ h/year, or in only one, with $h_y=1,800$ h/year. Different hourly cost of the operator C_{op} has been analysed as well (15, 20, 25 and 30 €/h). The throughput ratio R_Q is equal to 1.25 and 1.5, corresponding, for example, to $t_l^W=37.5$ s and $t_l^V=30$ s or $t_l^W=45$ s and $t_l^V=30$ s, respectively.

Each line in the graph represents the limit of the VLM convenience region according to a certain set of input parameters, which lays in the upper delimited part of the graph area.

Figure 2a shows the trend of Formula (10), representing the threshold choice between the traditional carton racks warehouse and the VLM, with $C^V = 24,000$ €/year, $C_{sp} = 80$ €/m², $h_y=3,600$ h/year and $R_Q=1.5$.

Figure 2b, instead, reports the same kind of analysis but with a higher annual floor space cost per square meter, $C_{sp} = 120$ €/m².

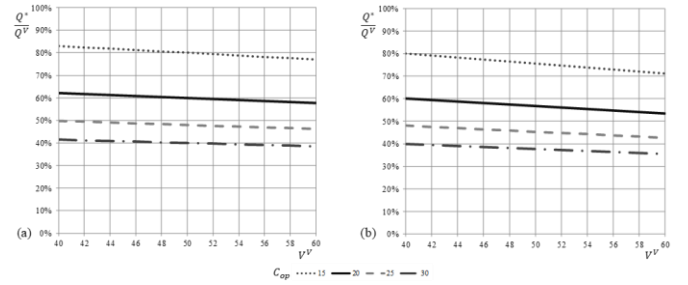


Fig. 2. Trend of Q^*/Q^V and VLM convenience region for $C^V=24,000$ €/year, $h_y=3,600$ h/year, $R_Q=1.5$ and (a) $C_{sp}=80$ €/m² or (b) $C_{sp}=120$ €/m².

First of all, it can be seen that the threshold changes according to the hourly cost of the operator; in particular, it is higher for a lower hourly cost. Furthermore, the increase of V^V leads to a decreasing trend of Q^*/Q^V : the decrease is then steeper for the lower values of C_{op} .

Figure 2b shows some differences compared to Figure 2a: if the space has a higher cost, the threshold choice between the carton racks warehouse and the VLM is lower, especially for higher V^V . Hence, the VLM turns out to be the best choice also for lower values of Q^*/Q^V , because it allows to store the items more efficiently, with a higher saturation of the available space.

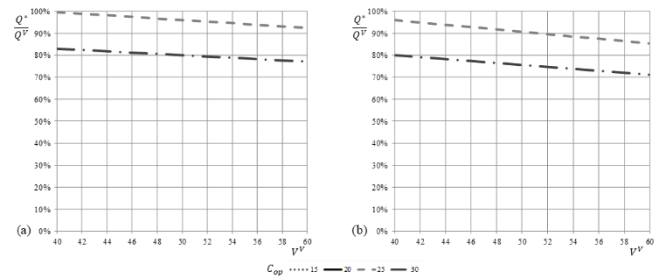


Fig. 3. Trend of Q^*/Q^V and VLM convenience region for $C^V=24,000$ €/year, $h_y=3,600$ h/year, $R_Q=1.25$ and (a) $C_{sp}=80$ €/m² or (b) $C_{sp}=120$ €/m².

In Figure 3 all input parameters are the same of the ones of Figure 2, except of R_Q , which is here equal to 1.25. This is related to a lower difference in the respective cycle times, $t_l^W = 37.5$ s and $t_l^V = 30$ s.

In this particular case, it can be seen that the VLM system is convenient only when the hourly cost of the operator is higher. Here, it is relevant to note the important impact of the ratio R_Q on the definition of the convenience region.

Figures 4a and 4b show how the thresholds change when the cost of the VLM is lower ($C^V = 18,000$ €/year), fixing $C_{sp} = 80$ €/m² and $h_y = 3,600$ h/year for the two values of R_Q .

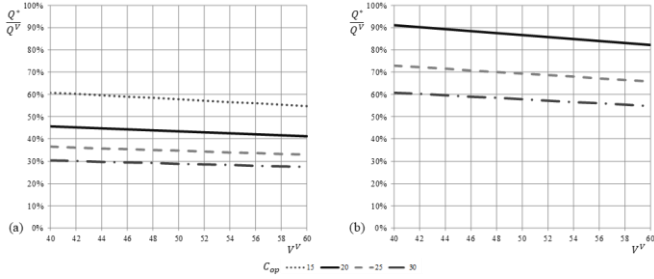


Fig. 4. Trend of Q^*/Q^V and VLM convenience region for $C^V=18,000$ €/year, $h_y=3,600$ h/year, $C_{sp}=80$ €/m² and (a) $R_Q=1.5$ or (b) $R_Q=1.25$.

In this case, of course, the threshold moves down, with a wider convenience region with respect to the previous results shown in Figures 2a and 3a. Reducing the cost of VLM of 25%, the threshold value decreases about of the same value.

Finally, Figures 5a and 5b show the same scenario reported in Figures 2a and 3a but considering that the systems are used for only one daily work shift ($h_y = 1,800$ h/year).

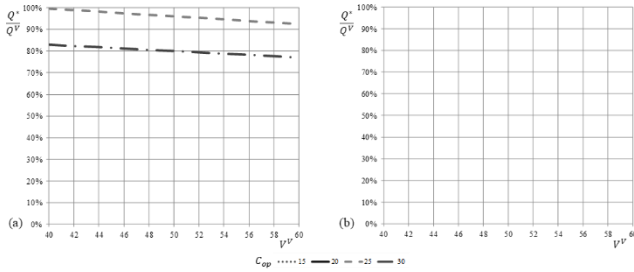


Fig. 5. Trend of Q^*/Q^V and VLM convenience region for $C^V=24,000$ €/year, $h_y=1,800$ h/year, $C_{sp}=80$ €/m² and (a) $R_Q=1.5$ or (b) $R_Q=1.25$.

The convenience regions of the two systems change in favour of the carton racks warehouse: if the systems are less used, then it is more probable that the carton racks warehouse is the best option. When the ratio between the throughputs of the systems is lower, the threshold is always higher than 100%; therefore, the VLM is never convenient.

This analysis and the graphs reported in the present section show that, generally, the ratio Q^*/Q^V is not very sensible to the storage capacity V^V and to the annual floor space cost per square meter, C_{sp} . On the other side, the change of the annual cost of the VLM C^V leads to an important shift of the convenience

threshold: if the VLM is more expensive, then the border for the adoption of this solution increases.

Another interesting aspect is represented by the ratio of the cycle times R_Q . When this ratio increases, hence, when the VLM is faster than the carton racks warehouse, the line of the Q^*/Q^V ratio moves down, enlarging the VLM convenience region. On the other side, if the R_Q is lower the VLM system is always worse than the traditional solution.

Finally, the number of work shifts and the number of working hours per year h_y can have an influence on the results. If the systems are used only for one work shift, it could turn out that the VLM is too expensive for low picking rates.

Figure 6 reports the trend of R_Q^* varying V^V for the different set of parameters already shown in Figures 2a and 2b, 4 and 5. Generally, these plots show how R_Q is not very sensible to V^V , while it can change according to the hourly cost of the operator C_{op} . Moreover, comparing 6a and 6b, it can be derived that the space cost has a low influence on the definition of the threshold. Different is the effect of a change on C^V , as demonstrated by the comparison of 6a and 6c: if the VLM has a lower cost, the convenience threshold moves down, and the VLM turns out to be a possible solution also for lower values of R_Q^* . Finally, the comparison of 6a and 6d shows how the number of working hours h_y influences R_Q^* : if the VLM is used only for one working shift the threshold is higher, and it is then more probable that the carton racks warehouse is the best storing configuration.

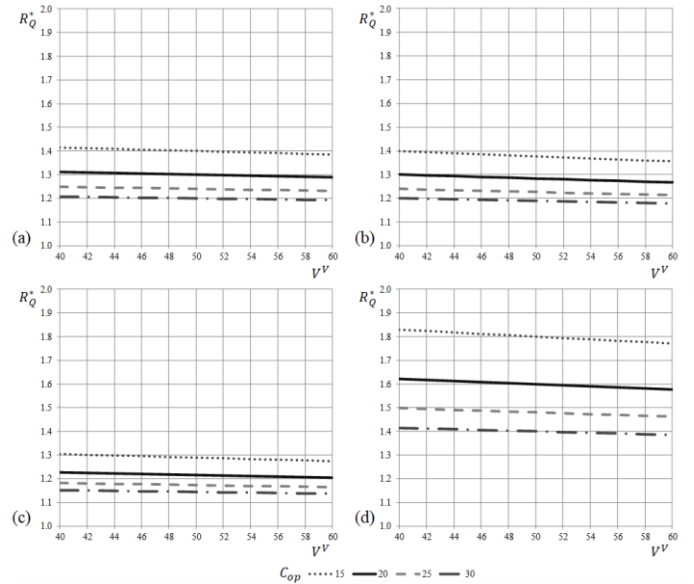


Fig. 6. Trend of R_Q^* varying V^V for $C^V=24,000$ €/year, $h_y=3,600$ h/year and (a) $C_{sp}=80$ €/m² or (b) $C_{sp}=120$ €/m² and for (c) $C^V=18,000$ €/year, $h_y=3,600$ h/year, $C_{sp}=80$ €/m² and (d) $C^V=24,000$ €/year, $h_y=1,800$ h/year, $C_{sp}=80$ €/m².

V. MODELS APPLICATION

The present economic model has been applied to a real case study to understand the applicability of VLM systems as alternative of a pick from carton warehouse.

The analysed company stocks two different categories of products in a carton racks warehouse, with a storage area of about $26 \times 10 \text{ m}^2$ with a mezzanine. The two groups of items are stocked in the two separated levels. General information about the warehouse and the pickers is reported in Table 2; it is considered $k^W=1$ since the two groups are stocked in the two separated areas, and $SL^W \approx 0.128$. The picking time per line has been estimated with direct measurements on a period of 2 weeks, dividing the total amount of time spent by the pickers in the warehousing operations and the total amount of line performed in the period. The annual space cost per square meter C_{sp} is $120 \text{ €}/(\text{hm}^2)$ and the hourly operator cost C_{op} is 20 €/h .

TABLE II. INFORMATION ABOUT THE CASE STUDY

	Group A	Group B
Type of products	Merchandising products (t-shirts, cap, gloves etc.)	Small metal parts (sealings, small bearings, etc.) and kits for motorcycle engines
# of stored items	About 2,500	About 3,500
$V \text{ [m}^3\text{]}$	About 165 m^3 in the mezzanine floor	About 165 m^3 in the ground floor
$Q \text{ [picks/h]}$	250	340
$t_l^W \text{ [s]}$	40.9 s	46.6 s
$h_y \text{ [h]}$	1,800 h	3,600 h

Considering that the useful height of the plant is about 10 meters, the possible VLM system can be 10 meters high, with a storage volume V^V of about 55 m^3 . The t_l^V is 34.5 s for the items belonging to group A and 31.3 s for the ones of group B, based on different order size, setup time. The actual performance of the VLM machine has been estimated using the formulation developed by [14, 15].

Then, considering $t_l^W=40.9 \text{ s}$ for group A and $t_l^W=46.6 \text{ s}$ for group B, it can be estimated the throughput ratio R_Q between the traditional solution and the VLM system for both cases: 1.19 for the group A and 1.49 for group B.

Based on the typical values of $k^V \approx 11$ and $SL^V \approx 0.3$ as reported in the previous section, it is simple to understand if the installation of a VLM system could be convenient or not, by calculating the value of R_Q and controlling if condition (13) is verified or not.

In particular, for the products of group A, $R_Q=1.19$, while $R_Q^* = 1.53$. Therefore, since $R_Q < R_Q^*$, a VLM system turns out not to be convenient for this kind of products.

For the group B, since the t_l^W is higher than the previous case and the t_l^V is lower, the throughput ratio is higher, $R_Q=1.49$. In this case, the warehouse working shifts are equal to two per days, resulting in $h_y=3,600 \text{ h/year}$, and $R_Q^* = 1.27$. As a consequence, $R_Q > R_Q^*$ and, therefore, the installation of VLM

systems turns out to be preferable compared to the current carton racks warehouse.

Figure 7 shows the VLM convenience region for latter analysed case, since as just demonstrated for the group A, there is no region where VLM system is preferable. For group B, it can be seen a clear VLM convenience region and for Q^*/Q^V values higher than about 64% to 67%. In fact, in this particular case, for the storage volume V^V of about 55 m^3 , the condition $\frac{Q}{N^V \cdot Q^V} > \frac{Q^*}{Q^V}$ is verified because $\frac{Q}{N^V \cdot Q^V} = 0.984$.

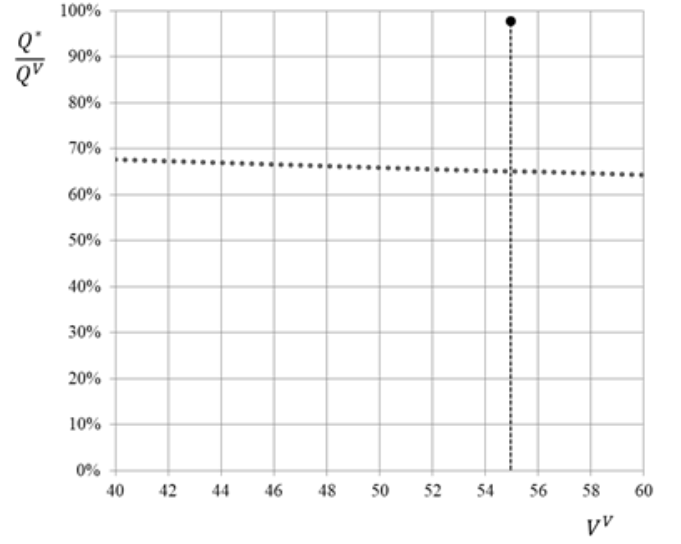


Fig. 7. VLM convenience region for the group B. The dot represents the position of the adopted solution for the storage volume $V^V = 55 \text{ m}^3$

This can be verified also comparing the total cost of the two solutions. In case of the traditional one, the total cost is

$$TC^W = C_{sp} \cdot k^W \cdot \frac{V}{SL^W \cdot H} + C_{op} \cdot \frac{Q}{Q^W} \cdot h_y =$$

$$= 15,600.00 + 317,194.54 = 332,794.54 \text{ €/year}$$

On the other side, the required number of VLMs to be installed N^V is

$$N^V = \max\left(\left\lceil \frac{Q}{Q^V} \right\rceil; \left\lceil \frac{V}{V^V} \right\rceil\right) = \max\left(\left\lceil \frac{340}{115} \right\rceil; \left\lceil \frac{165}{55} \right\rceil\right) = 3$$

and the total cost can be estimated as:

$$TC^V = N^V \cdot \left[\left(C_{sp} \cdot k^V \cdot \frac{V^V}{SL^V \cdot H} \right) + C_{op} \cdot \frac{Q}{N^V \cdot Q^V} \cdot h_y \right] =$$

$$= 79,200.00 + 212,500.00 = 291,700.00 \text{ €/year}$$

It can be then also derived that the total saving obtained by installing the VLM systems is about 12.3%, corresponding to about $41,000 \text{ €/year}$.

VI. CONCLUSIONS AND FUTURE RESEARCH

The present paper has proposed a mathematical formulation useful to compare two storage systems for order picking of small items: a carton racks warehouse and a dual-bay Vertical Lift Module. The model starts with the proposal of two cost models, which consider, for both systems, the most common emerging costs, like the occupied space cost, also depending on its saturation, together with the workforce cost. The derived formulations represent an easy as well as effective tool to properly compare the possible emerging costs, according to the throughput of the systems and to their storage capacity. Thanks to the proposed model, it is possible to identify the most suitable application ranges of both systems. In fact, the two cost models have been put into relation, to derive a single synthetic formulation. This depends on the annual floor space cost per square meter, on the annual operator cost, on the volume saturation levels of the two systems and on the throughput ratio of the two systems. The application of the formulation in the parametrical analysis showed that the ratio Q^*/Q^V is not very sensible to the total volume of the stocked items V , while it is to the annual floor space cost per square meter and to the annual cost of the VLM. On the other side, the application of the model to an industrial case showed that differences in the required throughput Q , in the number of working hours per year h_y and/or in the number of lines per order N can affect the applicability of the VLM.

Besides obtaining these results, the same formulations can be used to do the design and the sizing of the two storage solutions. Moreover, the model introduced for the dual-bay VLM could be applied also for the evaluation of the single bay VLM, by changing the system picking times and the related throughput Q .

As already stated, this paper represents a first study for contributing to the mathematical modelling of small items storage systems. It would be interesting to add in future researches the terms useful to better consider the refilling activity, which is different for the two systems and which could have an impact on the storage allocation, on the travelled distances and, then, on the overall time [2, 4]. Moreover, it would also be interesting to extend the analysis to other small-items storage systems, to derive a complete tool for their evaluation and comparison. This could help the choice of their most proper application in real warehouse picking contexts.

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