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Order Batching With Time Constraints in a Parallel-aisle Warehouse: a Multiple-policy Approach

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DETERMINATION OF CYCLE TIMES FOR DOUBLE DEEP STORAGE SYSTEMS USING A DUAL CAPACITY HANDLING DEVICE

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Abstract

Double deep storage is an efficient method to improve space utilization in warehouses. Contrary to intuition, it also can be efficient when considering retrieval times, since the aisles of a warehouse with double deep storage may be shorter than comparable warehouses with single deep storage. AS/RS-machines equipped with two load-handling units might further improve the situation, since two storage units (for instance pallets or cases) can be stored and retrieved, effectively allowing quadruple command cycles, bringing in total two storage units into the aisle and retrieving two storage units at the same time. In this paper, we present a method for the computation of cycle times and average cycle times with the assumption of equally distributed access probabilities for AS/RS-machines equipped with two load handling devices.

1 Introduction

Automated storage systems with double deep storage locations using AS/RS-machines with dual load handling units represent a frequently used warehouse solution in recent times. They offer two main advantages compared to traditional storage systems: On the one hand, they allow higher space utilization through double deep storage lanes and on the other hand, they can achieve a higher throughput by allowing quadruple command cycles. Figure 1 shows a rack with double deep storage. In the aisle an AS/RS-machine with two masts and with two load handling units between the masts is serving the storage locations. The two load handling units are moved simultaneously, vertically by a winch and horizontally the AS/RS machine moves on wheels on rails.

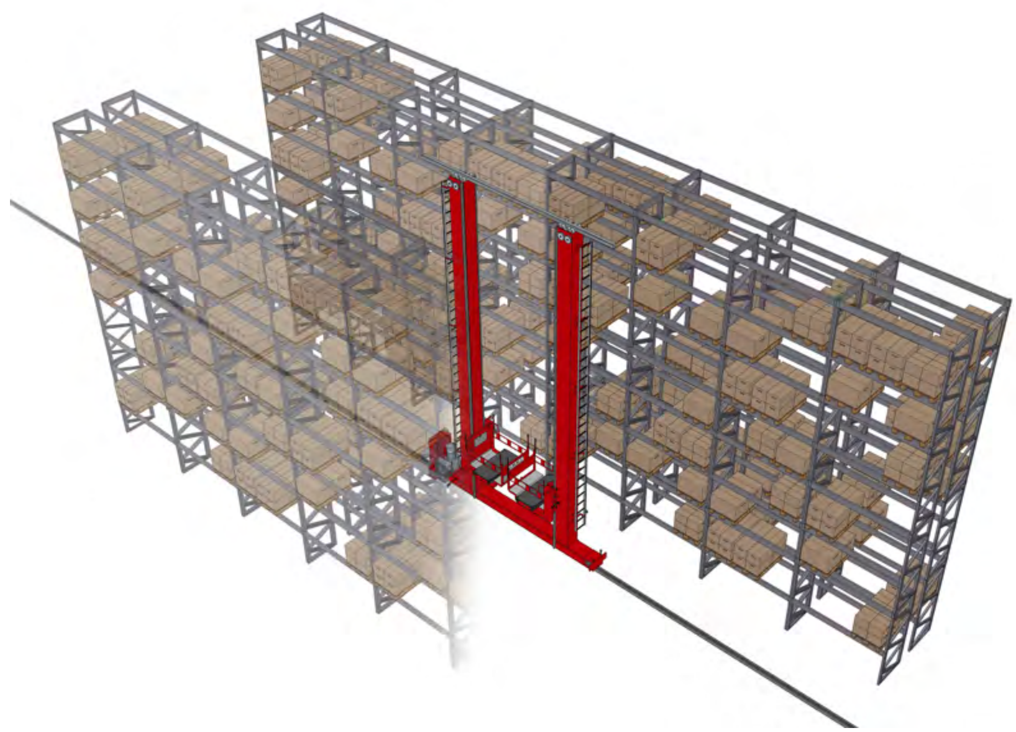


Figure 1: Schematic description of a double deep storage rack with a dual capacity AS/RS machine in the aisle

A quadruple command cycle consists of two storage and two retrieval tasks. These tasks can be processed in two different sequences: either by first storing two storage units and then retrieving two storage units (notation SSRR) or by first storing one storage unit then retrieving one storage unit, storing the other storage unit and finally retrieving another storage unit (notation SRSR).

Formulas for the calculation of cycle times or throughput for such AS/RS systems do not exist yet. Closed form expressions are known so far for configurations with either double deep storage or for machines with dual load handling units. For the frequently used case of double deep storage combined with a dual load handling device, only publications with limited applicability are known: In [1] a double deep storage system with a triple load handling device is considered: They assume for all rearrangement processes to temporarily store the blocking unit on the load handling device and store it back to the same storage lane. Furthermore, it is supposed to first occupy all rear storage positions (up to a fill grade of 50%) before the front positions are used. Also [2] addresses double deep storage configurations with dual and triple load handling devices. What is missing, is a detailed description of the possible processing sequence when performing a quadruple or sextuple command cycle. For the cycle time calculation there are the following limitations: First, for the rearrangement probability the formula of [3] is used, which is primary valid for the singe-capacity load handling device but is not adjusted here. Second, the rearrangement distance is approximated with a marginal value for high values of the fill grade. The latest publication of [4] is also dealing with a double deep storage rack and a dual capacity load handling device. In their model, the authors assume a predefined processing sequence where the two storage operations are always performed prior to the two retrieval operations. Consequently in every cycle a rearrangement by means of the load handling devices is possible if the first retrieval unit is blocked. Again, they suppose to first occupy the rear storage positions (up to a fill grade of 50%) before the front positions are used. This implies that for fill grades of less than 50% a different cycle time model is needed as no rearrangements occur. To close this gap, the focus of this modeling paper is to provide an exact throughput calculation of these systems. We thus derive a formula for calculating the cycle time of a quadruple command cycle based on the principles of Bozer and White[5] and Gudehus[6] having none of the constraints mentioned above.

A simple quadruple command cycle in a single deep storage rack can be described by existing models using a single command cycle plus three times the mean travel distance between arbitrary selected storage locations. However, for the case of double deep storage, this calculation must be adjusted due to rearrangement processes, because there is a positive likelihood, that the unit to be retrieved is in the rear position and blocked by another unit in the front-most position. Therefore, the computation of the rearrangement probability for the particular operating strategy is a central issue in this paper. In our analytical model, we apply a stochastic process to represent the states of the storage locations with a Markov chain approach according to [3].

The remainder of the paper is organized as follows: In section 2, we first explain our assumptions and the used notations for our model. Next, we illustrate the possibilities for randomly performing a quadruple command cycle with the two different rearrangement options. In the following part the analytical model is described by concentrating on the storage lanes' state probabilities. Using a Markov chain for representing the storage and retrieval processes in the warehouse, we can deduct them. Subsequently, this leads to the missing terms for composing the cycle time. Furthermore, we then identify some weaknesses in the modeling that could lead to inaccuracies. A short summary concludes the paper.

2 Analytical model of the quadruple command cycle

2.1 Assumptions and notation

We consider a double deep AS/RS with a dual capacity handling device. There are several S/R machines each operating in one aisle with double deep storage racks on both sides. The assumptions made for the analytical model are as follows:

1. A random storage assignment policy is used. This means every occupied position has an equal probability of access; in a completely occupied lane both units therefore have the same probability of access.
2. We refer to the position closer to the aisle as 'front' position, the position distant from the aisle as 'rear' position. See Figure 2 for illustration.
3. For each retrieval of the unit on the rear position, there is a positive probability that rearrangement is required, if the front position is occupied
4. For rearrangement, the nearest neighbor policy applies, that means the nearest available storage position is chosen for rearrangement.
5. The I/O point is located on the bottom left corner of the rack.
6. A unit is always stored in the rearmost position, e.g. storage lanes that are only occupied in the front are not possible. This also applies when restoring during rearrangements. Therefore state 3 in Figure 2 is excluded in our model.
7. We consider the AS/RS machine to be operated at full capacity, that means there are always storage and retrieval orders waiting.
8. The two load handling devices can be moved independently and are both able to access the front and the rear position of a storage lane
9. We assume the distance between the two load handling units to be equivalent to the distance between two storage positions. So, positioning one load handling device in front of a storage lane means the other load handling device is necessarily positioned to the lane besides.

Furthermore we introduce the following notations:

- $E(SC)$ Expected travel time for a single command cycle according to [5]
- $E(TB)$ Expected travel time for mean travel between distance according to [5]
- $E(DC)$ Expected travel time for a dual command cycle according to [5]
- $E(QC)$ Expected travel time for quadruple command cycle in a single deep storage rack according to [5] and [7]

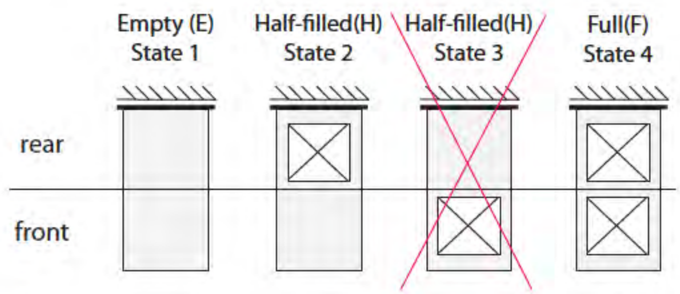


Figure 2: The storage lanes' states and convention of front and rear

$E(QC_{dd})$ Expected travel time for a quadruple command cycle in a double deep storage rack

z Fill grade factor, ranging between 0 and 1

$t_{Rearr.}$ Expected travel time to the rearrangement position and back

t_{Tango} Time to perform a tango movement (rearrangement by temporary storage of the front unit on the load handling device and returning it to the rack)

$t_{LHD,f}$ Time for load handling at the front position

$t_{LHD,r}$ Time for load handling at the rear position

F^* Number of storage lanes in one rack

v_x, v_y Speed of the AS/RS machine in horizontal and vertical direction

a_x, a_y Acceleration/Deceleration of the AS/RS machine in horizontal and vertical direction

L, H Length and height of the storage rack

t_x, t_y The longest horizontal ($t_x = \frac{L}{v_x}$) and vertical travel distance ($t_y = \frac{H}{v_y}$).

$S1, S2$ Different possible storage operations

$R1 - R5$ Different possible retrieval operations

Concerning $E(SC)$, $E(DC)$, $E(TB)$ and $E(QC)$ we refer to normalized travel time models first formulated by [5]. We assume the shape factor of the rack, which takes into account the proportion of t_x and t_y, to be 1. b is defined as $b = min\left(\frac{t_x}{T}; \frac{t_y}{T}\right)$ with $T = max(t_x; t_y)$. To obtain a cycle time in real time units, those formulas need to be denormalized with the rack dimension and the speed of the AS/RS machine. For $b = 1$, both $\frac{L}{v_x}$ and $\frac{H}{v_y}$ can be used equally here.

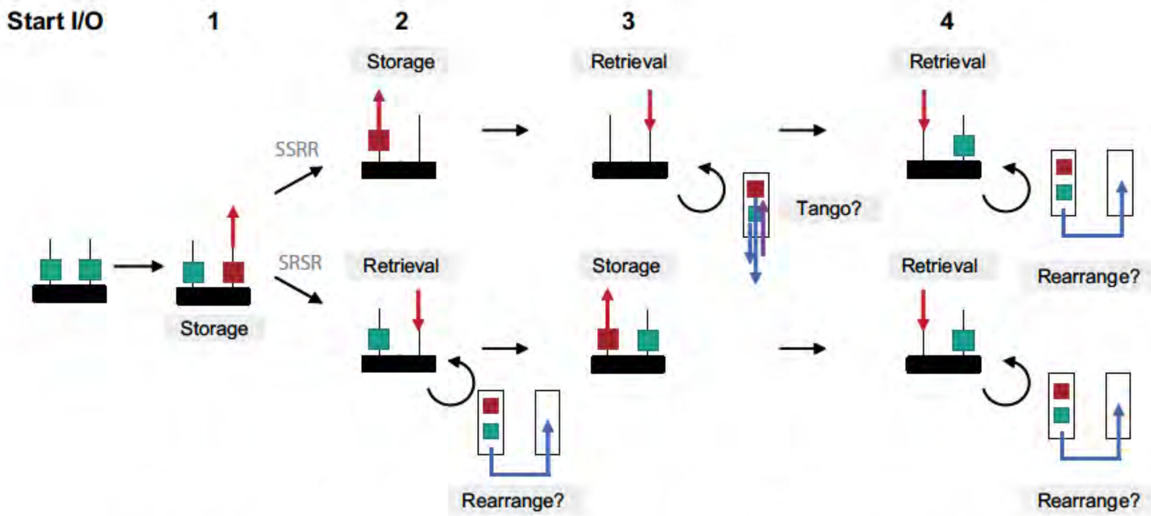


Figure 3: Graphic description of the two possible execution sequences

2.2 Execution of a Quadruple Command Cycle

The execution of the assumed quadruple command cycle is now described. Each cycle begins with the receiving of the two units to be stored and ends with the release of both retrieved units. An obvious condition is that a retrieval is only possible with at least one handling device available which means each retrieval needs at least one preceding storage operation. Consequently there are two actual sequences a cycle can be performed; storage, storage, retrieval, retrieval (SSRR) or storage, retrieval, storage, retrieval (SRSR). See Figure 3 for graphic illustration.

When performing the first sequence both load handling devices are free prior to the next retrieval. If now, the first retrieval unit is blocked, the rearrangement can be performed by the load handling devices: For that purpose the blocking unit in the front is picked (a), afterwards the (formerly) blocked unit in the rear is picked with the second load handling device (b). At last, the blocking unit is restored in the rear position of the same storage lane(c). This process, which we will refer to as 'tango-movement' in the following, is shown in Figure 4. If a rearrangement is required with only one load handling device being free, the blocking unit is picked and stored in the nearest empty storage location. Then, the (formerly) blocked unit is picked for retrieval.

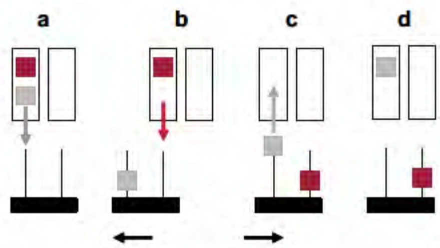


Figure 4: Rearrangement performed by the load handling devices ('tango-movement')

2.3 Analytical model

2.3.1 Composing the cycle time

First, consider a quadruple command cycle for a single deep storage rack. The travel time can be described as a combination of the single command cycle ($E(SC)$) and three times the average travel between distance ($E(TB)$). As we assume a random selection of locations as well as a random selection of the two execution paths in figure 3 with a probability of 0.5 each, this concept is adopted here. In addition we have to take the double deep storing into account.

Depending on the load handling devices' condition different types of rearrangement can occur with every retrieval. Whether a rearrangement is needed depends on the position of the unit within the storage lane and the condition of the storage lane being accessed. Retrieving from the rear while the storage lane is fully occupied leads to rearrangement. For both retrieval operations we first need to consider a probability for conducting a rearrangement or tango-movement and secondly the time the operation takes. For a rearrangement the time is determined by the distance to the next free storage location. The tango-movement provides a deterministic handling time that is only depending on the AS/RS machines' characteristics as the procedure itself doesn't change.

Summarizing, this means the crucial part for determining the cycle time is knowledge about the probabilities of the storage lanes' states as a function of the fill grade. For rearrangement times we can use the approximation Lippolt [3] provided for a nearest neighbor rearrangement; times for the single command cycle and the travel between distance are known. The only path-depending, normalized cycle time can be composed as follows:

$$E(QC_{dd}) = E(QC) + 2 \cdot (P(Rearr.) \cdot t_{Rearr.} + P(Tango) \cdot t_{Tango}) \quad (1)$$

Another distinction of the double deep storage compared to the single deep version is that access times of the load handling device are not known in advance. They depend on the access position within the storage lane which is again dependent on the state condition of the storage lane: When storing in a half-full storage lane, the handling time for accessing the front position is relevant whereas storing in an empty storage lane causes the handling time for the access in the rear. To be more exact, we need to add to equation 1 the mean load handling times for both storage and retrieval, each depending on the fill grade. The same applies for potential rearrangements as load handling times for the restorage also need to be considered. This leads to the path-depending cycle time allowing for load handling times.

$$\begin{aligned} E(QC_{dd}) =& E(QC) + 2 \cdot (P(Rearr.) \cdot (t_{Rearr.} + 2 \cdot E(t^S_{LHD})) + P(Tango) \cdot t_{Tango}) \\ & + 4 \cdot E(t^S_{LHD}) + 4 \cdot E(t^R_{LHD}) \end{aligned} \tag{2}$$

Since for restorage during rearrangement the same assumptions hold true as for storing, the mean load handling time is equivalent. Note that the mean load handling time is referring to a semi fork movement, either extending or retracting, so for one operation the time is multiplied by two.

2.3.2 The storage lanes' state probabilities

We start with the investigation of the different state conditions a single storage lane can achieve. In relation to the stated assumptions and the presented execution of the quadruple command cycle, there are three different states a storage lane can have: Either the storage lane is empty, the position in the rear is occupied, or the storage lane is fully occupied on both positions. The state with the position in the front being occupied and the rear one being empty is excluded because it is unreasonable from a practical perspective.

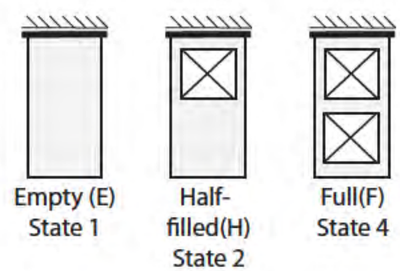


Figure 5: Possible state conditions of a storage lane

Depending on the fill grade the probability of being in one of those states differs. Once the warehouse is in operation, the state of the storage lanes continuously changes from one of the possible states to another one by storing and retrieval operations. In fact, we can identify two storing operations and five retrieval operations that cause state transitions. When storing, a unit can either be stored in an empty storage lane (state E) resulting in transition S1 to state H or in a half-full storage lane (H), thus resulting in a fully occupied storage lane, state F (transition S2), as shown in Figure 6.

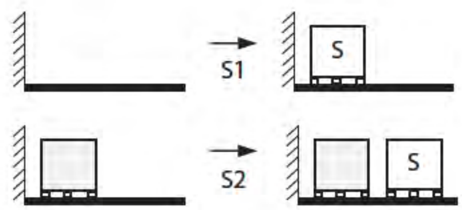


Figure 6: State transitions caused by storing operations

For retrievals, there are two straightforward cases when no rearrangement is needed: A unit can either be retrieved from a half-full storage lane (transition R1) or from a fully occupied storage lane (transition R2) as shown in Figure 7.

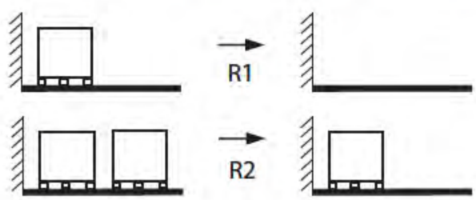
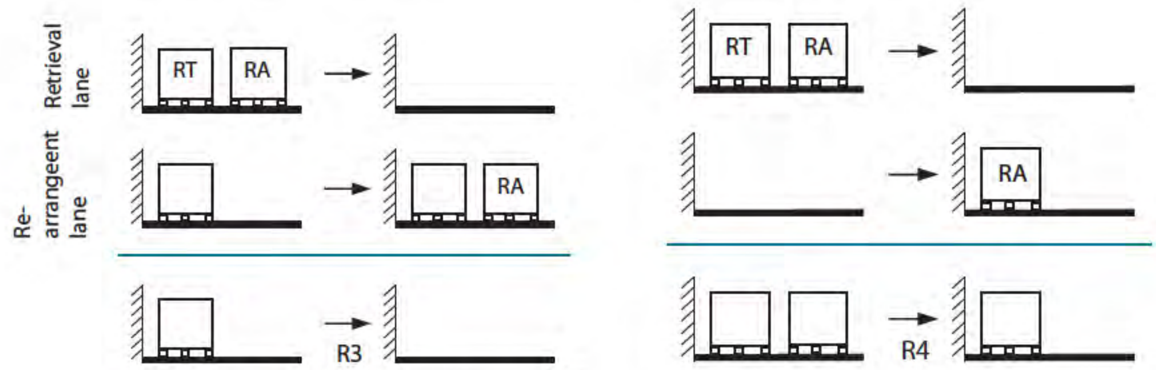


Figure 7: State transitions caused by retrieval operations

In addition, retrievals can provoke rearrangements that affect the state of different storage lanes. Every time the rear position in a fully occupied storage lane should be accessed, the unit in the front position must be restored in another storage lane. The restorage can either be performed into an empty storage lane or a half-full storage lane (see storage transitions S1 or S2). As the actual retrieval unit is also removed from the storage lane, the result is one empty storage lane and either one half-full or a fully occupied lane. Consolidating the state transitions means, in one case a semi-occupied storage lane disappears and an empty storage lane emerges (transition R3). In the other case, a full storage lane disappears and a half-full storage lane emerges (transition R4). See Figure 8 for graphic illustration and better understanding of both cases, where RT shows the retrieval unit and RA the blocking unit which is rearranged.

The last case for retrievals refers to the tango-movement (rearrangement when both load handling devices are free). As the blocking unit in the front is restored in the same storage lane, this lane changes from fully- to semi-occupied as shown in



(a) State transitions during R3 (b) State transitions during R4

Figure 8: The two possible state transitions caused by retrieval operations with rearrangement

Figure 9. We can express the probabilities of those events depending on the storage

Figure 9: State transitions caused by tango-movement

lanes' valid state conditions. First formulated in [3], a part of them can be transferred to our model:

$$P(S1) = \frac{P(E)}{P(E) + P(H)} \tag{3}$$

$$P(S2) = \frac{P(H)}{P(E) + P(H)} \tag{4}$$

$$P(R1) = \frac{P(H)}{P(H) + 2P(F)} \tag{5}$$

$$P(R2) = \frac{P(F)}{P(H) + 2P(F)} \tag{6}$$

Equations 3, 4, 5 and 6 express the probabilities for the different events to occur. This means equation 3 represents the probability to choose an empty storage lane from all storage lanes that are available for storing. Remember, when storing, it can be chosen from all empty (E) and semi-occupied storage lanes (H). For example, equation 5 shows the probability to choose a semi-occupied storage lane from all storage lanes that are possible for retrieval. In the denominator, the probability of P(F) is multiplied by two as the probability to choose a fully occupied storage lane is twice as high as choosing a half-full one. In other words: Since the storage lane is occupied by two units, the access probability of both positions adds up. For the other events we need to define new probabilities in contrast to [3], to take into account the dual capacity of the handling device. The rearrangement events can be determined as follows:

$$P(R3) = \frac{P(F)}{P(H) + 2P(F)} \cdot \frac{P(H)}{P(E) + P(H)} \cdot \frac{3}{4} \tag{7}$$

$$P(R4) = \frac{P(F)}{P(H) + 2P(F)} \cdot \frac{P(E)}{P(E) + P(H)} \cdot \frac{3}{4} \tag{8}$$

The first part of equation 7 and 8 refer to the retrieval from a fully occupied storage lane, the second part represents the restorage of the blocking unit into an empty or semi-occupied storage lane and the last part is required due to the quadruple command cycle: $\frac{3}{4}$ is the probability that, if rearrangement is needed, a 'regular' rearrangement is performed. As illustrated in Figure 3, there are four different process steps where a retrieval is performed; in the third or fourth step of the top path and in the second or fourth step of the bottom path. First of all, for all four steps, access to a blocked unit is possible and therefore, for all four steps a rearrangement can be needed. Within a path, for one single retrieval both retrieval steps are equally likely, so for both steps the probability is 0.5. Also both paths have an equal probability to be chosen for execution. That means, each of those four retrieval steps has the same probability of $\frac{1}{4}(= \frac{1}{2} \cdot \frac{1}{2})$. For three steps a regular rearrangement is performed, resulting in the term $\frac{3}{4}$ in equation 7 and 8.

For one step (upper path, step 3) the tango-movement is possible, which leads to the term $\frac{1}{4}$ in the following expression for R5:

$$P(R5) = \frac{P(F)}{P(H) + 2P(F)} \cdot \frac{1}{4} \tag{9}$$

Equation 9 also denotes the probability for a tango-movement $P(Tango)$, the probability for a regular rearrangement is the sum of equation 7 and 8:

$$P(Rearr.) = P(R3) + P(R4) = \frac{3}{4} \cdot \frac{P(F)}{P(H) + 2P(F)} \tag{10}$$

2.3.3 Stochastic process and Markov chain

We know the probabilities for both rearrangement processes in dependence of the storage lanes' states. To determine the probabilities of those states, we model the procedure of storage and retrieval operations as a stochastic process. For the corresponding Markov chain, every state can be reached from any other state, and therefore has a stationary distribution. This distribution gives the average state conditions, which are the probabilities we are looking for. To formulate a time-discrete Markov chain the transition probabilities of all state transitions are needed. Also state transitions from a state to itself need to be considered here. The transition probabilities represent the probability of an event (state transition) subject to the condition that the system is in a certain state. For example, the transitions probability $P(S \mid H)$ denotes the probability of a storage operation if the system (the corresponding storage lane respectively) is in the half-full state (H). By means of the previously defined probabilities (equations 3, 4, 5, 6, 7, 8, 9) and the definition of the conditional probability (see [8]) we can define those transition probabilities.(Note that the probabilities defined above do not represent the needed transition probabilities, but the probability that a storage lane's state condition is affected given that a certain operation takes place.) The state transition probabilities are calculated as follows:

$$
\begin{aligned}
P(S \mid H) &= \frac{P(H \mid S) \cdot P(S)}{P(H)} = \frac{P(S2) \cdot P(S)}{P(H)} = \frac{P(H) \cdot P(S)}{(P(E) + P(H)) \cdot P(H)} \\
&= \frac{P(S)}{P(E) + P(H)}
\end{aligned}
\tag{11}
$$

The remaining state transition probabilities are calculated analogously. The complete Markov chain with all state transitions is shown in Figure 10. The transition

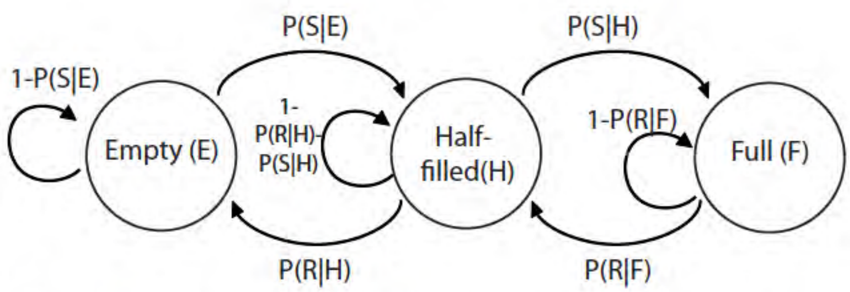


Figure 10: Markov chain with all possible state transitions

probabilities can be represented as a matrix, the so called transition matrix where each row is summing up to 1. It leads to a system of linear equations that we need to solve. In the equilibrium, the probability of a storage operation is equal to the probability of a retrieval operation, thus we can use the relation $P(S) = P(R)$. As described in [3] by adding the filling grade factor as a function of the storage lanes' state conditions and the normalization condition of the state probabilities, we obtain the following system of linear equations:

$$\begin{aligned} 1 &= P(E) + P(H) + P(F) \\ z &= \frac{2 \cdot P(F) + P(H)}{2} \\ P(F) &= \frac{P(H)^2}{2 \cdot P(E) - \frac{3}{4} \cdot P(H)} \end{aligned} \tag{12}$$

2.3.4 Calculation of the components of the cycle time

When solving the system, we obtain the following results for the storage lanes' state probabilities as a function of the fill grade factor z. In fact, there are two solutions for each formula because of the squared term. As the solutions we are looking for represent probabilities, only the positive solutions are relevant and the solutions yielding negative values can be rejected.

$$P(E) = -\frac{1}{2}\sqrt{-7 \cdot z^2 + 40 \cdot z + 16} + \frac{1}{2} \cdot z + 3 \tag{13}$$

$$P(H) = \sqrt{-7 \cdot z^2 + 40 \cdot z + 16} - 3 \cdot z - 4 \tag{14}$$

$$P(F) = -\frac{1}{2}\sqrt{-7 \cdot z^2 + 40 \cdot z + 16} + \frac{5}{2} \cdot z + 2 \tag{15}$$

Substituting those values in the equations 9 and 10 results in a formula for the probability of rearrangement and tango-movement depending on the fill grade factor. In the same way we can calculate equations for the different storage and retrieval processes (formulas 3 to 9) as a function of the fill grade factor.

$$P(Rearr.) = P(R3) + P(R4) = \frac{3 \cdot [-\sqrt{-7 \cdot z^2 + 40 \cdot z + 16} + 5z + 4]}{16z} \tag{16}$$

$$P(Tango) = P(R5) = \frac{-\sqrt{-7 \cdot z^2 + 40 \cdot z + 16} + 5z + 4}{16z} \tag{17}$$

After determining the probability for rearrangement and tango, what is left with regard to section 2.3.1 (equations 1 and 2) are the handling times for the load handling devices and the rearrangement. Lippolt provided both values in dependence of the state probabilities which can be also applied in our model.

The fact of having two load handling devices performing a quadruple command cycle in our case, does not affect how handling times and rearrangement positions depend on state conditions. Thus, we apply the values of 13, 14 and 15 to the formulas provided. The mean load handling times, which depend on the state probabilities, can then be specified in the following way:

$$E(t_{LHD}^{S}) = \frac{1}{2} \cdot (t_{LHD,f} \cdot (1 + P(S2)) + t_{LHD,r} \cdot (1 - P(S2))) \tag{18}$$

$$E(t_{LHD}^{R}) = \frac{1}{2} \cdot (t_{LHD,f} \cdot (1 + P(R2)) + t_{LHD,r} \cdot (1 - P(R2))) \tag{19}$$

In both formulas all events of storage and retrieval are incorporated according to the position of access within the storage lane. Inserting the probabilities respectively gives the load handling times as a function of the fill grade factor:

$$\begin{aligned} E(t_{LHD}^{S}) = & \\ & \frac{1}{2} \cdot (t_{LHD,f} \cdot (1 + \frac{-3z + \sqrt{-7 \cdot z^2 + 40 \cdot z + 16} - 4}{-\frac{5}{2}z + \frac{1}{2}\sqrt{-7 \cdot z^2 + 40 \cdot z + 16} - 1}) \\ & + t_{LHD,r} \cdot (1 - \frac{-3z + \sqrt{\cdots} - 4}{-\frac{5}{2}z + \frac{1}{2}\sqrt{\cdots} - 1})) \end{aligned} \tag{20}$$

$$\begin{aligned} E(t_{LHD}^{R}) = & \frac{1}{2} \cdot \left(t_{LHD,f} \cdot (\frac{9z - \sqrt{-7 \cdot z^2 + 40 \cdot z + 16} + 4}{4z}) \right) \\ & + \frac{1}{2} \cdot \left(t_{LHD,r} \cdot (\frac{-z + \sqrt{-7 \cdot z^2 + 40 \cdot z + 16} - 4}{4z}) \right) \end{aligned} \tag{21}$$

Concerning the time needed for rearrangement the distance to the next available storage position for rearrangement (nearest neighbor) apparently depends on the fill grade: At a low fill grade a storage lane right beside has a high probability of being in an empty or semi-occupied state. The higher the fill grade, the less this probability becomes, so the nearest available position moves further away. An approximation of the nearest neighbor rearrangement distance is provided by [3] with the following expression:

$$E(RD) = 2 \cdot \left(\frac{7}{15}\right)^{1-\frac{pU}{F^*}} \cdot \frac{1}{\sqrt{pU}} \tag{22}$$

with pU being the number of potential storage lanes available for rearrangement: Resulting from multiplying the probability that a storage lane is qualified as rearrangement location (e.g. being in state 1 or 2) with the number of storage lanes in the rack. With the state probabilities provided above this yields to the mean rearrangement distance:

$$E(RD) = 2 \cdot \left(\frac{7}{15}\right)^{1-\frac{pU}{F^*}} \cdot \frac{1}{\sqrt{(\frac{1}{2}\sqrt{-7 \cdot z^2 + 40 \cdot z + 16} - \frac{5}{2} \cdot z - 1) \cdot F*}} \tag{23}$$

and

$$pU = \left(\frac{1}{2}\sqrt{-7 \cdot z^2 + 40 \cdot z + 16} - \frac{5}{2} \cdot z - 1\right) \cdot F^* \tag{24}$$

For the normalized calculation of the cycle time and $b = 1$ equation 23 denotes $t_{Rearr.}$.

Now, all components of equations 1 and 2 are known and we are able to compute the cycle time of the quadruple command cycle under the given assumptions.

Dependent on the technical and structural conditions it is possible that both storage units are received and released at the same time. In that case, the time for load handling at the I/O position must be subtracted twice from the composed cycle time because both when receiving and releasing one load handling time is saved. The previously defined formula can be adjusted accordingly:

$$\begin{aligned} E(QC_{dd}) =& E(QC) + 2 \cdot (P(Rearr.) \cdot (t_{Rearr.} + 2 \cdot E(t^S_{LHD})) + P(Tango) \cdot t_{Tango}) \\ & + 4 \cdot E(t^S_{LHD}) + 4 \cdot E(t^R_{LHD}) - 2 \cdot t_{LHD,f} \end{aligned} \tag{25}$$

The result from equation 25 gives the mean travel time to randomly chosen positions and possible rearrangement positions as well as related load handling times. To get the real time required for the cycle we need to incorporate dead times like reaction times and those for acceleration as well as deceleration. Typically there is one general addition for dead times often referred to as t_0. Per movement of the AS/RS machine for acceleration and deceleration the term $\frac{1}{2} \cdot \left(\frac{v_x}{a_x} + \frac{v_y}{a_y}\right)$ is added. Considering these aspects the denormalized time for the quadruple command cycle is as follows:

$$\begin{aligned} E(QC_{dd}) =& t_0 + \frac{5}{2} \cdot \left(\frac{v_x}{a_x} + \frac{v_y}{a_y}\right) + E(QC) \cdot \frac{L}{v_x} \\ & + 2 \cdot (P(Rearr.) \cdot (t_{Rearr.} + 2 \cdot E(t^S_{LHD})) + P(Tango) \cdot t_{Tango}) \\ & + 4 \cdot E(t^S_{LHD}) + 4 \cdot E(t^R_{LHD}) - 2 \cdot t_{LHD,f} \end{aligned} \tag{26}$$

with

$$t_{Rearr.} = t_{0,Rearr.} + (\frac{v_x}{a_x} + \frac{v_y}{a_y}) + \left(2 \cdot \left(\frac{7}{15}\right)^{1-\frac{pU}{F^*}} \cdot \frac{1}{\sqrt{pU}}\right) \cdot \frac{L}{v_x} \tag{27}$$

2.4 Sources of inaccuracies

2.4.1 Inaccuracies regarding rearrangement time

The determination of the rearrangement distance yields three possible sources of inaccuracies: First in the nearest neighbor distance approximation provided by [3]

(See Equation 22). However, the author claims the approximation to be sufficiently accurate as the deviation in travel time is less than one second.

The second source is caused by the actual rearrangement distances. Up to a fill grade of 90% the position for rearrangement is in a distance of 1.5-times storage positions. For most configurations of rack-dimensions and AS/RS machine properties the AS/RS machine can not reach its maximum speed in the distance of 1.5 positions, but still a full acceleration phase is assumed in Equation 27. This results in an overestimation of the actual travel time to the rearrangement position. The extent of the deviation can not be determined generally as it is highly depending on the individual configurations. With high acceleration and deceleration specifications the overestimation is lower as compared to AS/RS machines with lower values that take much more time to reach its maximum speed. On the other side, the quality of the approximation highly depends on dimension and specifications of the rack. Thus, in our example the deviations for exact travel times from simulation and analytical results vary between the different sample rack configurations. Furthermore there is another issue regarding the dimension of storage positions, especially if they do not have the same proportion as the storage rack itself. In this case, the analytical determined horizontal rearrangement distance differs from the vertical distance and can not be specified clearly. To better demonstrated this, we chose two sample rack configurations with two versions for the second configuration (2a and 2b) for our examples. With 2a having storage positions in the proportion of the rack and 2b having storage positions with different proportions (See Table 3 in the Appendix).

	Simulative			Analytical		
Fill grade	1	2a	2b	1	2a	2b
0.90	1.4920	1.7834	1.4982	1.340	1.340	x: 1.6406, y: 1.0938
0.91	1.5510	1.8492	1.5521	1.394	1.394	x: 1.7076, y: 1.1384
0.92	1.6183	1.9215	1.6192	1.460	1.460	x: 1.7882, y: 1.1921
0.93	1.7047	2.0203	1.7133	1.541	1.541	x: 1.8874, y: 1.2583
0.94	1.8142	2.1463	1.8210	1.643	1.643	x: 2.0126, y: 1.3417
0.95	1.9586	2.3270	1.9706	1.777	1.777	x: 2.1764, y: 1.4509
0.96	2.1770	2.5581	2.1731	1.961	1.961	x: 2.4020, y: 1.6013
0.97	2.4900	2.9553	2.5040	2.235	2.235	x: 2.7378, y: 1.8252
0.98	3.0377	3.5763	3.0589	2.702	2.702	x: 3.3097, y: 2.2064
0.99	4.3349	5.2049	4.3735	4.234	4.234	x: 4.6197, y: 3.0798

Table 1: Simulative and analytical rearrangement distances

Table 1 shows the results for the rearrangement distances from the analytical model and the simulation for three different rack configurations in the fill grade range of 0.9 to 0.99. For configurations 1 and 2a the analytical distance is identical, as both configurations comprise a quadratic number of storage lanes in the same proportions of the storage rack. Not having this characteristic, for configuration 2b the mean rearrangement distance according to the formula provides different results in x- and y-dimension, shown in the last column of table 1.

	Simulative		**Analytical**		
Configuration	Rearr. distance	Exact travel time considering acc. + dec.[s]	Rearr. distance	Exact travel time considering acc. + dec.[s]	Travel time acc. to formula [s]
1	x: 1.4920	3.0901	1.3396	2.9280	3.5358
	y: 1.4920	2.1850	1.3396	2.0704	3.5358
2a	x: 1.7846	10.3478	1.3396	8.9653	10.5063
	y: 1.7846	4.8780	1.3396	4.2263	10.5063
2b	x: 1.4988	7.7428	1.6406	8.1009	10.2604
	y: 1.4988	4.4703	1.0938	4.6770	10.2604

Table 2: Rearrangement distances and corresponding travel times for fill grad 90%

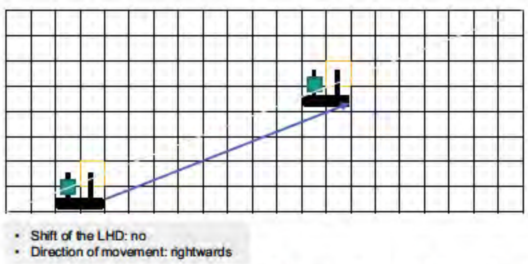
Table 2 shows return travel times (in seconds) for the related rearrangement distances. Also, it provides an indication regarding differences between simulation and analytical results as well as exact calculated analytical value and the formula's value with fixed amount for acceleration and deceleration. Besides the exact travel time is calculated separately for x- and y-dimension showing the actual values when moving in different directions. However, the higher value (here in x-dimension) is more relevant for calculating mean rearrangement times as the rearrangement distance gives a mean value and assumes a movement in both direction. The travel time in x-dimension exceeds the travel time in y-dimension because in all sample configurations the storage positions have a greater extend in width than in height while acceleration in y-dimension is higher. Nevertheless, in reality it occurs that the shorter time is relevant in cases where the rearrangement position is directly above or below the initial position.

The last source of inaccuracy is the fact, that the probabilities of finding an empty or half-empty position in a neighborhood is different for those storage positions, which are at the perimeter of the rack. The influence of this factor is depending on the relation between the number of storage locations at the perimeter vs. the total number of storage locations of the rack, though

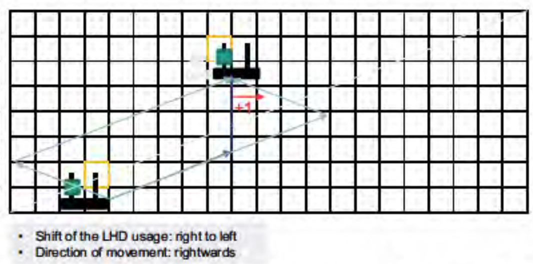
The rearrangement approximation may seem poorly exact because of the reported inaccuracies between the analytical model and the simulation results. The approximation is justified by the inevitable differences trough discretization of the pick face and it's long established usage introduced by [3], though.

2.4.2 Inaccuracies regarding the position of the dual load handling device

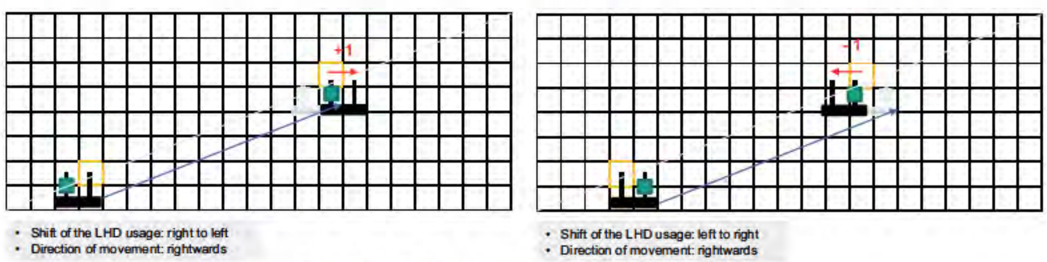
The established travel time models are developed for traditional storage systems having one load handling device. There is no question of positioning as the load handling device is always located in front of the rack position that is accessed. Positioning becomes more complex with the AS/RS system having two load handling devices. As there can be a shift between the two load handling devices from one event to another, real travel distances vary accordingly. Of course with no shift, nothing changes compared to the traditional AS/RS machine, as shown in Figure 11a.



(a) Travel distance with no shift of LHD



(b) Travel distance with vertical distance determining



(c) Travel distances with shift of LHD

Figure 11: The state transitions caused by the two rearrangement possibilities

Depending on the type of the shift (from left to right or right to left) and the direction of movement of the whole AS/RS machine, the horizontal travel distance of the AS/RS machine can be longer or shorter than the actual distance between the concerned positions (See Figure 11c). There can be an extended or minimized travel distance in the dimension of the distance between the two load handling devices (which we assume to be the width of one storage lane). However this only applies when the horizontal travel distance is greater than the vertical travel distance, which means the horizontal distance between the positions is determining. While, if the vertical distance is determining, the positioning neither consumes more time nor yields time savings. This is illustrated in Figure 11b, which shows that the shift of one horizontal distance is easily possible within the no-cost zone.

Results from our simulation model show that the error is less than 0.2 or 0.25% of the cycle time, where the analytical model is underestimating the actual cycle time.

3 Conclusion

For double deep storage systems using a dual capacity load handling devices there were no travel time models for determining cycle times. In this paper, we developed an analytical model based on Lippolts' [3] approach. Thus, we succeeded in deriving a travel time model for the given storage configuration operated in a randomly executed quadruple command cycle under random storage policy. Finally, we do not omit to mention possible inaccuracies of our model which lead the way to further fields of research.

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Appendix

Parameter	**Config. 1**	**Config. 2a**	**Config. 2b**
Rack dimension (HxL)	12.4 x 24.8 m	12 x 36 m	12 x 36 m
Speed in x-direction	$4\frac{m}{s}$	$3\frac{m}{s}$	$3\frac{m}{s}$
Speed in y-direction	$2\frac{m}{s}$	$1\frac{m}{s}$	$1\frac{m}{s}$
Acc. and Dec. in x-direction	$2\frac{m}{s^2}$	$0.4\frac{m}{s^2}$	$0.4\frac{m}{s^2}$
Acc. and Dec. in y-direction	$2\frac{m}{s^2}$	$0.6\frac{m}{s^2}$	$0.6\frac{m}{s^2}$
Dimension of shelves	0.4 x 0.8 m	0.5 x 1.5 m	0.5 x 1.0 m
Number of storage positions	1922	1152	1728

Table 3: Specification of the three considered rack configurations in the examples

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