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Abstract: Mobile Shelf-based Order Pick (MSOP) systems are gaining significant interest for e-commerce fulfillment due to their rapid deployment capability and dynamic organization of storage pods based on item demand profiles. In this research, we model the MSOP system with class-based storage strategies and alternate pod storage policies using multi-class closed queuing networks. We observe that though closest-open location pod storage policy do not allow to efficiently use the storage spaces in comparison to random location pod storage policy in an aisle, it increases the system throughput for all item classes.

1 Introduction

Mobile Shelf-based Order Pick (MSOP) system is gaining significant interest from e-commerce fulfillment companies. Kiva Systems LLC, which is now a wholly owned subsidiary of Amazon.com, developed the mobile fulfillment system in 2003 (Wurman et al. [2008], D'Andrea and Wurman [2008], Mountz [2012]). The MSOP system is a goods-to-man order-pick system, where the items are stored on movable storage shelves. Robots (also known as drive units) fetch the inventory pods from the storage area and transport them to the pick stations for order picking (see Figure 1). Empty robots can travel underneath the rack locations. This system has been implemented by several large retailers such as The Gap, Walgreens, Staples, and Office Depot. Due to reduced picker travel, the system improves worker pick conditions, and also reduces order pick inaccuracies.

Upto 55% of the operating costs at a distribution center are due to its order pick costs, which include costs associated with its layout (De Koster et al. [2007]). In a goods-to-man system, there are several factors which affect the efficiency of order-picking operations.

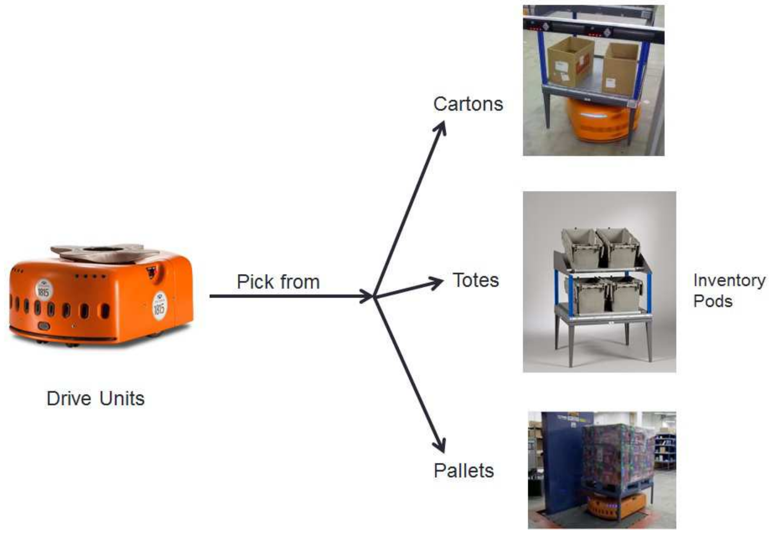

Figure 1: Robot and inventory pods (source: kivasystems.com)

For instance, the location of the SKUs in the storage system and the order pick area layout affect the system responsiveness. The storage location assignment problem consists of allocating the product to the different slots in a warehouse (Heragu et al. [2005]). The basic principle is that the high-demand products have to be allocated to the slots closer to the order-picking stations for reducing the total time in handling. The class-based storage approach is the most effective method, dividing the items into classes and assigning to each class a set of areas in which the products are stored with an objective to reduce the travel time in a warehouse and improve the order picking efficiency (Tompkins et al. [2010]).

The biggest advantage of the MSOP system is 'dynamic layout reorganization.' The pods that are frequently requested by the pick station are gradually positioned closer to the pick stations and the ones that are rarely requested are gradually moved towards the back of the aisles (see Figure 2). Shelves with fast-selling items are indicated in dark whereas slow-selling items are indicated with light squares. This dynamic reorganization of storage shelves based on the SKU demand profile of the pods is one of the most attractive features of the MSOP system.

Several operational decisions affect the MSOP system throughput performance. For instance, the choice of pod storage location, the choice of order assignment to the order pick stations, and the choice of the robot to fetch the pod, affect the throughput performance. Likewise, the design choices such as the number of robots, depth-to-width ratio of the storage area, and maintaining a dedicated or a pooled fleet for order pick and replenishment processes may affect the system performance.

Roy et al. [2014] investigate the effect of pod storage policies ("random open location storage" and "closest open location storage") on system throughput performance with a

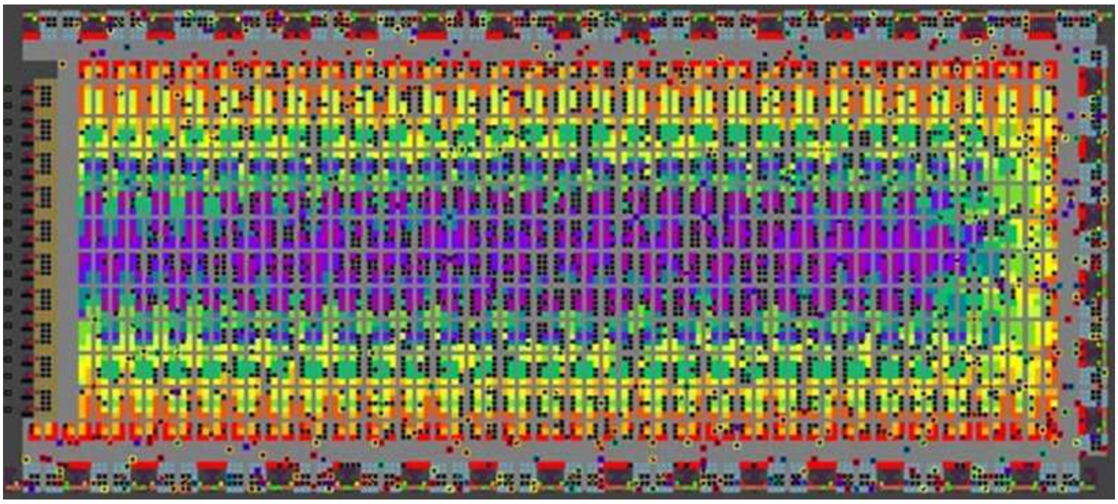

Figure 2: Distribution of pods in the MSOP system (source: wired.com)

single class of items. In this research, we extend our previous work by investigating the performance of the MSOP system with class-based storage and in combination with two pod storage policies within the zone: "random open location storage" and "closest open location storage." The random open location storage may seem to be the most efficient and commonly used storage policy, where any pod (after an order pick) is equally likely to be stored in any of the open locations. The primary objective of this policy is to maximize space utilization. In the closest open location storage policy, the pod is stored in the closest open location within the aisle that has been chosen for retrieval. The closest open location storage policy may be useful in reducing the travel time of the robot. We derive the aisle travel time expressions and develop a multi-class closed queuing network model to analyze two pod storage policies and three item (pod) classes A, B and C.

The rest of this paper is organized as follows. In Section 2, we describe the order pick process in the MSOP system. The analytical model for the order pick system with class-based storage is presented in Section 3. The results from the numerical experiments and conclusions from this study are drawn in Sections 4 and 5 respectively.

2 System Description

The storage area consists of an even number of aisles $\mathcal{A}$. A single pod is assumed to be l meter long and w meter deep. The order pick area is D meters long and W meters deep. The warehouse shape factor is characterized by depth/width, which is the ratio between D and W. D represents number of pod storage locations per aisle and W represents the the number of aisles . In this research, we consider the MSOP system with one order-pick station located in the middle of the cross-aisle (in front of the aisles). This area is called the order pick area.

The storage and retrieval requests are assumed to be independent, and processed on

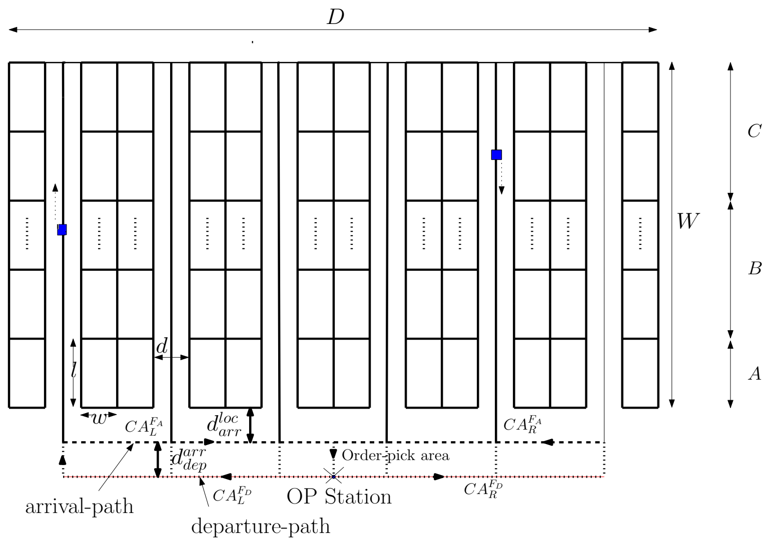

Figure 3: Layout of a storage with an order pick station

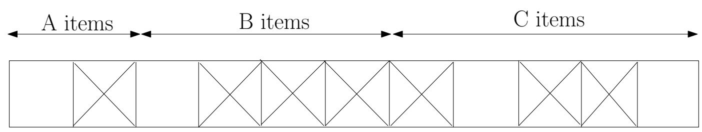

Figure 4: Dedicated area for A, B and C items in the side of an aisle

a FCFS (first come, first served) basis. The robot first fetches the pod (that stores the requested order item) and travels to queue at the order pick station. After the item is picked from the pod, the robot stores the pod back in the order pick area. In our layout, the robot uses the single-directional arrival path to fetch the pod and uses the single-directional departure path to store the pod back (Figure 3).

The robots use a dual command cycle to process transactions, i.e., after completing the storage transactions, the robot immediately proceeds to fetch another pod for the order picking process. The items are stored in pods according to the classes namely A, B, and C i.e., pods store one of the three items A, or B, or C only. Class A items are relatively few in numbers but account for a large amount(70-80%) of the order pick activity whereas class C items are relatively large in number but account for a relatively small account of the order-pick activity. The volume of order-pick activity for class B items lie in between the volumes for class A or class C items.

Each side of an aisle has a dedicated storage area for storing pods with A, B, and C class items. In Figure 4, we show an aisle side with storage locations dedicated to A, B, and C class items storage. Each pod storage segment also has a number of open locations, which are used by the robot to store the pod before fetching a pod of the same or another class.

3 Queuing Network Model

The MSOP system is modeled as a closed queuing network with a simplification of the vehicle movement and the Markov-chain based models are developed to analyze the travel time. In Roy et al. [2014], we derive the mathematical expressions for the travel time where a robot stores a pod in the open location and then retrieves a pod from another storage location in a warehouse. To analyze class-based storage policies, we extend the single item (pod) class approach to estimate the first two moments of the robot travel time in an aisle to three item (pod) classes. We maintain dedicated robots for handling different classes of pods and hence the queuing model is a multi-class network. The models are evaluated using approximate mean value analysis. Using analytical models, we optimize different design parameters of a warehouse to improve system performance. The performance measures obtained from the models include robot utilization, system throughput and the expected throughput time for order-picking.

3.1 Network Nodes

The order pick (OP) station is located at the middle of the cross-aisle in the departure path. We divide the cross-aisle departure path into two equal segments ($CA_L^{F_D}$ and $CA_R^{F_D}$ corresponding to the left and right segment of the cross-aisle in the order pick area). Likewise, the arrival path along the cross-aisle is divided into two equal segments ($CA_L^{F_A}$ and $CA_R^{F_A}$ corresponding to the left and right segment of the cross-aisle in the arrival path). Each segment of the cross-aisle is modeled as an Infinite Server (IS) queue. A robot starts its service and accesses either the left or the right side of the segments on the departure path (with equal probability, 1/2). It then chooses any one of the aisles with equal probability $p = \frac{1}{\mathcal{A}/2}$, and then accesses either pick face of the aisle with equal probability and moves towards an open rack location to store a pod (based on the pod storage strategy). Next it moves towards a pick-up location, retrieves the pod and exits the aisle. As soon a robot exits the aisle it accesses the arrival path segment based on its current position from either the left or the right side of the cross aisle.

Based on the aisle protocol explained in Roy et al. [2014], only one robot can enter the aisle at a time; therefore, each aisle is modeled as a single server queue with an infinite buffer size. The pick station is modeled as a single server queue with service rate μ_{OP}. Since we assume that orders are always waiting to be served by the robot, we model the system as a closed queueing network (see Figure 5). In this model we consider two pod storage strategies, random and closest open location storage strategy, either each storage zone. Using Approximate Mean Value Analysis (AMVA), we obtain the expected cycle time for order picking $E[CT_{op}]$, the throughput of order picking X_{op}, and expected queue lengths at various nodes.

We now explain the approach to estimate the aisle service times and then discuss the queuing model.

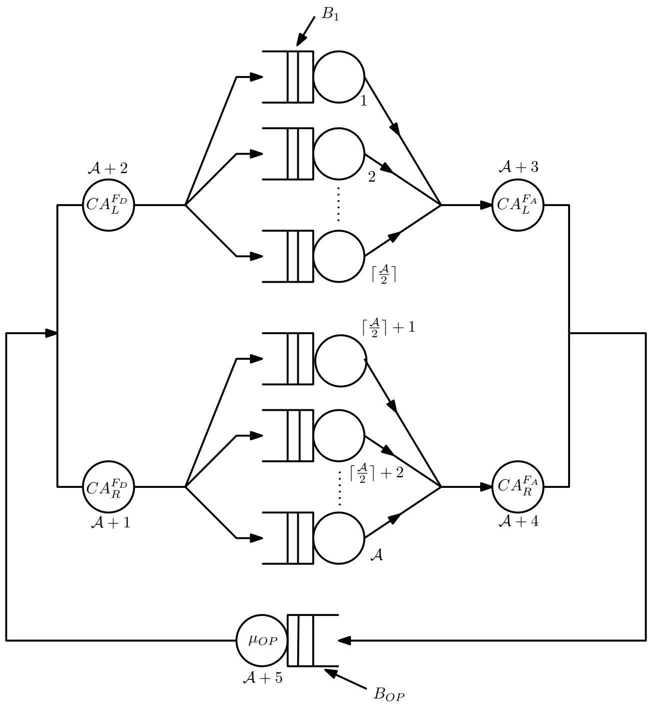

Figure 5: Single-class queueing network model for order picking using an MSOP system

3.2 Aisle Service Time Estimation for A, B and C Pod Class with Random Pod Storage

Let N_A, N_B and N_C denote the total number of storage locations for A, B and C pod classes, respectively, in each side of an aisle , out of which m_A, m_B and m_C locations are open for storing A, B and C class pods, respectively.

Let us denote the service time for class A, B and C robots in an aisle using the terms $t^A_{Aisle,R}$, $t^B_{Aisle,R}$ and $t^C_{Aisle,R}$, respectively, for the random open location pod storage strategy. The travel time of a robot includes the time to store a pod and retrieve another pod, including the return travel ($t^i_{WL,R}$) for i^{th} class robot, the handling time to store a pod (t_{store}), the handling time to retrieve a pod ($t_{retrieval}$) and the time associated with travelling between the arrival-path (onward as well as return) of a cross-aisle to the starting point of the aisle location, d^{loc}_{arr} with a robot velocity v_r.

$$t^A_{Aisle,R} = t^A_{WL,R} + t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r} \tag{1}$$

$$t^B_{Aisle,R} = t^B_{WL,R} + \frac{2N_A l}{v_r} + t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r} \tag{2}$$

$$t^C_{Aisle,R} = t^C_{WL,R} + \frac{2(N_A + N_B)l}{v_r} + t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r} \tag{3}$$

Note that the travel time associated with the travel between the arrival and the departure paths is included in the cross-aisle service time. Roy et al. [2014] show that the first and the second moments of the distance travelled to store and retrieve a pod within the locations in an aisle segment with N storage locations is given by Equations 4 and 5.

$$E[D_R] = \frac{(4N+1)l}{3} \tag{4}$$

$$E[D_R^2] = \frac{(6N^2+2N-1)l^2}{3} \tag{5}$$

The expected service time by a robot in aisle for the dedicated area A, B and C can be obtained by taking the expectations of the terms in (1), (2) and (3), respectively, which is given as:

$$E[t^A_{Aisle,R}] = E[t^A_{WL,R}] + t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r} \tag{6}$$

$$E[t^B_{Aisle,R}] = E[t^B_{WL,R}] + \frac{2N_A l}{v_r} + t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r} \tag{7}$$

$$E[t^C_{Aisle,R}] = E[t^C_{WL,R}] + \frac{2(N_A+N_B)l}{v_r} + t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r} \tag{8}$$

Since $t_{WL,R} = \frac{D_R}{v_r}$, the first and second moments of $.t^A_{WL,R}$, $t^B_{WL,R}$ and $t^C_{WL,R}$ can be obtained.

The second moments of the three classes of robot service time in an aisle are calculated as follows:

$$\begin{aligned} E[t^{A^2}_{Aisle,R}] &= E[t^{A^2}_{WL,R}] + \left(t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r}\right)^2 \\ &+ 2\left(t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r}\right) E[t^A_{WL,R}] \end{aligned} \tag{9}$$

$$\begin{aligned} E[t^{B^2}_{Aisle,R}] &= E[t^{B^2}_{WL,R}] + \left(\frac{(2N_A)l}{v_r} + t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r}\right)^2 \\ &+ 2\left(\frac{(2N_A)l}{v_r} + t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r}\right) E[t^B_{WL,R}] \end{aligned} \tag{10}$$

$$\begin{aligned} E[t^{C^2}_{Aisle,R}] &= E[t^{C^2}_{WL,R}] + \left(\frac{2(N_A+N_B)l}{v_r} + t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r}\right)^2 \\ &+ 2\left(\frac{2(N_A+N_B)l}{v_r} + t_{store} + t_{retrieval} + \frac{2d^{loc}_{arr}}{v_r}\right) E[t^C_{WL,R}] \end{aligned} \tag{11}$$

Using the first and second moment of the robot service times in an aisle, the coefficient of variation (CV) of the all classes of robot service times in an aisle is obtained.

The service time parameters of the cross-aisle travel are identical to the ones shown in Roy et al. [2014]. Roy et al. [2014] also discuss the procedure to obtain the aisle service time expressions for the closest open location pod storage policy.

Table 1: System Dimensions and Operation Parameters

Symbol	Description	Values
d_{arr}^{dep}	Distance between arrival & departure path	1.2 meter
d_{arr}^{loc}	Distance between arrival path & starting point of rack locations	1 meter
d	Aisle width	1 meter
l	Gross length of pod location	0.99 meter
w	Gross depth of pod location	1 meter
v_r	Speed of a robot	3 meter/sec
t_{store}	Time needed to store a pod	5 sec
$t_{retrieval}$	Time needed to retrieve a pod	5 sec
t_{pick}	Time needed for order picking at OP station	15 sec

Table 2: Performance Measures of MSOP system using three classes of robots

Input						Output: Performance Measure									
Policy	N_s	N_A	N_B	N_C	V	Q_{OP}	Q_{Aisle}	U_{OP}	U_{Aisle}	$E[CT_A]$ (sec.)	$E[CT_B]$ (sec.)	$E[CT_C]$ (sec.)	X_A (picks/hr.)	X_B (picks/hr.)	X_C (picks/hr.)
Random	1000	25	15	10	20	0.7	1.8	50%	86%	560.6	669.7	716.9	89.6	21.2	9.7
					30	0.8	2.8	56%	97%	767.6	874.5	896.9	98.2	24.4	11.8
					40	0.9	3.8	59%	99%	981.3	1086.9	1083.4	102.6	26.2	12.9
		10	15	25	20	1.2	1.8	70%	86%	398.5	470.9	601.6	126.3	30.2	11.8
					30	1.5	2.7	78%	98%	542.7	610.7	730.2	138.9	35.2	14.7
					40	1.8	3.7	83%	99%	691.8	756.9	865.2	145.4	37.8	16.5
Closest	1000	25	15	10	20	0.8	1.8	55%	86%	514.5	605.9	647.4	97.9	23.7	10.8
					30	0.9	2.8	61%	97%	703.0	792.1	811.2	107.2	27.0	12.9
					40	1.0	3.8	65%	99%	897.8	985.3	981.3	111.9	29.1	14.4
		10	15	25	20	1.3	1.8	73%	86%	385.5	446.6	554.7	130.6	32.0	12.9
					30	1.7	2.7	82%	97%	524.0	580.2	677.0	144.0	37.0	15.8
					40	1.9	3.7	87%	99%	667.2	720.0	805.3	150.8	39.9	17.6

4 Numerical Experiments

To perform the numerical experiments, we use the system dimension data given in Table 1. We deploy 70%, 20%, and 10% of the robots available in the system to fetch an A, B and C pod class, respectively, from the aisle storage location. Class switching is not allowed which means that a robot which stores an A class pod cannot retrieve another class pod (i.e., B or C). We first compare the performance of the order pick system by storage policy (random and closest open location storage strategy). It is expected that the closest open location storage strategy would take less residence time in the aisles than the random storage. The analysis results (provided in Table 2) show that order pick operations with closest open location storage policy takes about 3%-9% less residence time ($E[CT_i]$, $i \in \{A, B, C\}$) than the random open location storage policy for A, B, and C class robots. Therefore, a system can gain a slightly higher throughput capacity with the closest open location storage policy than random open location storage policy.

To study the effect of the number of storage locations in the dedicated area, we fix the number of locations in the dedicated area B and vary the locations in the area A and C. We observe that as the number of locations in area A decreases and the number of locations in the area C increases, the queue length Q_{OP} at order pick station increases but the queue length Q_{Aisle} at aisle nodes decreases, which results in reduction in cycle time and increase in throughput for all three classes.

5 Conclusions

In this research, we analyze alternate class-based storage strategies in an MSOP system using multi-class closed queuing networks. We analyze the system with two pod storage strategies. We see that though the open locations are not efficiently used in the closest open location storage policy, the throughput time estimates are always more than the throughput time estimates obtained using the random open location storage policy for all the three classes. Relaxing some of the assumptions of the model, such as retrieving the pod other than the same side of the aisle as storage, is subject of future research.

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