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Merge Problems with High Speed Sorters

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Merge problems with high speed sorters

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Abstract

To meet customer demands on distribution the necessity of sorting systems for unit loads in regard to quantity and capacity is growing up constantly.

Due to that, manufactures offer powerful sorting machines with a capacity up to 15.000 pieces per hour. Those machines are for instance Crossbelt or Tilt-Tray sorters. The output of these sorters is calculated by the equation of throughput.

$$\lambda = v/s \left[units/h \right]$$

The point of interest is the merge, referring to the overall efficiency of the single sorter. The motion sequence during merging has to be absolutely quick and precise to achieve exact positioning using minimized distances of the single items and high velocities on the main conveyer. Actual research activities at the Institute of Logistics Engineering at the Technical University of Graz will provide improved solutions for this problem.

The mathematical and mechanical models are representing a way how to describe and optimize the merging of single piece items. A basic example will show the transactions while merging in detail and the results will be discussed. Furthermore a layout proposal to simplify and increase the merging of goods will be presented.

1   Introduction

Sorting systems are important parts in the nodes of logistic networks. Parcel services like UPS, baggage handling at the air hubs and distribution centers would not be able to offer the high logistical service without automatically sorting systems. From there the main requirements on sorting systems are throughput, reliability, flexibility, and so on.

Fundamental research offers the basics for a more detailed view on these topics. The focus of this paper is a short view on some research to increase the throughput of a single sorter. Essential for this ambitious intention it is essential to optimize the merges respectively the converging process in material flow systems.

2 The challenge

As mentioned in fig. 1 the throughput of a single sorter depends of conveyor-velocity *v* and packet-distance *s*.

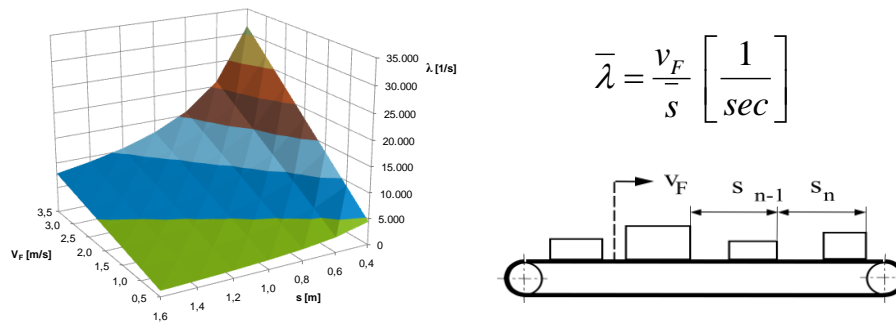$$\overline{\lambda} = \frac{v_F}{\overline{s}} \left[\frac{1}{sec} \right]$$

Figure 1: Relationship of velocity, packet-distance and throughput.

From figure 1 it´s obvious that sorter throughput can only be enlarged by increasing conveyor-velocity *v* and decreasing packet-distance *s*.

One bottle neck for this is the merging area, where the converging process takes place. The merge has to accelerate the tote or the packet from the velocity of the conveyer system (≈1 m/s) to the higher velocity of the high speed sorter (2-4 m/s). Furthermore the tote has to fit in a small gap between the packets and totes on the sorter like illustrated in fig. 2. To increase the throughput the gap has to be as small as possible and the velocity has to be as high as possible.
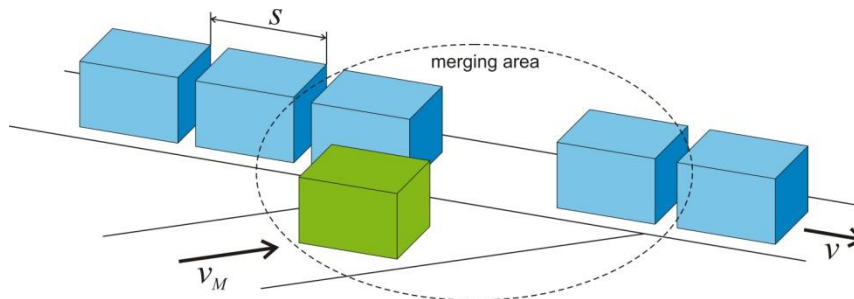

Figure 2: The merge problem.

To achieve an exact position of the goods with a minimized distance the motion sequence must be absolutely precise and rapid. Therefore it`s necessary to have a detailed knowledge of the converging process with all involved forces effecting the packets during the relocation phases.

Previous research activities in the field of motion sequence [1], [2], [3], provide basic theory and experimental investigations. They use some simplifications and assumptions concerning friction where contact area between goods and conveyor and mass distribution has been idealized.

So research objective is to predetermine - with realistic models using actual powerful calculation and modern simulation programs - the exact motion of the goods. With this information the estimated deviation from the normal condition of the packet´s end position can be calculated and with a prior adjustment operation the positioning failure may be compensated.

3   Mathematical model for the converging process

3.1 MBS[1]-Model of a Merge

Fig. 3.illustrates the relocating of a block shaped packet (mass m, angular momentum J_y) from a merge belt (velocity v_M) to a sorter belt (velocity v_S).

The contact surface is divided into a large number of square elements. For a rectangular contact area the indexing of each single element shows j the column and i the line number. The side length ge of an element depends on the requested accuracy of the simulation results. The normal forces N_{ij} and the friction forces R_{ij} are applied at the midpoint of each element.
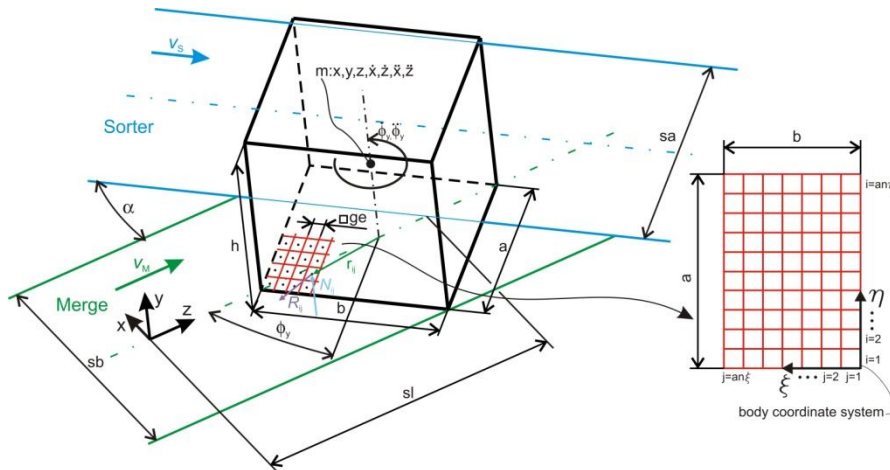

Figure 3: MBS-Model of converging process.

To calculate center of gravity accelerations the equations of motions are read as follows:

$$\ddot{x} = \frac{R_x}{m}; \qquad \ddot{z} = \frac{R_z}{m}; \qquad \ddot{\phi}_y = \frac{M_y}{J_y}$$

To get the overall friction force $\boldsymbol{R}$ all single element friction forces $\boldsymbol{R}_{ij}$ have to be added. The component-by-component summation of the element friction forces follows:

$$R_x = \sum_{i=1}^{an\eta} \sum_{j=1}^{an\xi} R_{xij}; \qquad\qquad R_z = \sum_{i=1}^{an\eta} \sum_{j=1}^{an\xi} R_{zij}$$

And analog for the moment $\boldsymbol{M}_y$ which is caused by the friction forces $\boldsymbol{R}_{ij}$:

$$\begin{bmatrix} r_{xij} \\ 0 \\ r_{zij} \end{bmatrix} \times \begin{bmatrix} R_{xij} \\ 0 \\ R_{zij} \end{bmatrix} = \begin{bmatrix} 0 \\ M_{yij} \\ 0 \end{bmatrix}; \qquad M_y = \sum_{i=1}^{an\eta} \sum_{j=1}^{an\xi} M_{yij}$$

For each element midpoint, the absolute and relative velocities are determined.

$$\begin{bmatrix} \dot{x} \\ 0 \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\phi}_y \\ 0 \end{bmatrix} \times \begin{bmatrix} r_{xij} \\ 0 \\ r_{zij} \end{bmatrix} = \begin{bmatrix} v_{xij} \\ 0 \\ v_{zij} \end{bmatrix}; \qquad \begin{bmatrix} v_{xij} \\ 0 \\ v_{zij} \end{bmatrix} - \begin{bmatrix} v_{Mx} \\ 0 \\ v_{Mz} \end{bmatrix} = \begin{bmatrix} v_{rxij} \\ 0 \\ v_{rzij} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} v_{xij} \\ 0 \\ v_{zij} \end{bmatrix} - \begin{bmatrix} v_{Sx} \\ 0 \\ v_{Sz} \end{bmatrix} = \begin{bmatrix} v_{rxij} \\ 0 \\ v_{rzij} \end{bmatrix}$$

If the contact pressure per unit area $p_{(\xi,\eta)}$ is known, for the normal forces N_{ij} can be calculated by a double integral:

$$N_{ij} = \int_{bo_\xi}^{bo_\xi + ge} \int_{bo_\eta}^{bo_\eta + ge} p_{(\xi,\eta)} d\eta \, d\xi; \qquad bo_\xi = ge \cdot (j-1); \qquad bo_\eta = ge \cdot (i-1)$$

The Coulomb friction model (friction coefficient μ_{ij}) is also used in this model to get the element friction forces $\boldsymbol{R}_{ij}$.

$$\begin{bmatrix} R_{xij} \\ 0 \\ R_{zij} \end{bmatrix} = N_{ij} \cdot \mu_{ij}\left(v_{Slip}\right) \cdot \begin{bmatrix} -v_{rxij} \\ 0 \\ -v_{rzij} \end{bmatrix} \cdot \frac{1}{\sqrt{v_{rxij}^2 + v_{rzij}^2}} \; ; \qquad\qquad v_{Slip} = \sqrt{v_{rxij}^2 + v_{rzij}^2}$$

Derivations:

$$\frac{dx}{dt} = \dot{x} \; ; \; \frac{d\dot{x}}{dt} = \ddot{x} \; ; \; \frac{dy}{dt} = \dot{y} \; ; \; \frac{d\dot{y}}{dt} = \ddot{y} \; ; \; \frac{d\phi_y}{dt} = \dot{\phi}_y \; ; \; \frac{d\dot{\phi}_y}{dt} = \ddot{\phi}_y$$

This model can be transferred into a computer program with less effort. The most important advantages of the model are:

- proper numerical handling (important for computational implementations)
- high accuracy of results (realistic implementation of friction effects)
- it is possible to consider roller conveyors or other belt configurations
- extendable for non- block shaped packets
- non-constant surface pressure distributions between packet and belt implemented

3.3   Additional comments on the implementation of friction

Friction problems with "static" friction are difficult to handle numerically.
     "The Problem is that the differential equations (ODEs) describing the motion of the masses in such a system must be discontinuous in the sense that there is a jump discontinuity in the right-hand side of the ODEs as the relative velocity of the surfaces in contact goes through zero. This, in turn, is because the force changes the relative velocity passes through zero." [4]
     Considering this fact there are existing approaches to approximate the static friction to get much better numerical characteristics. For further use in the MBS-Model a velocity proportional definition of the friction coefficient μ is selected. Fig. 4 illustrates a higher order polynomial which presents the friction coefficient μ depending from the slip velocity v_{Slip}. This kind of polynomial is called STEP-function. Values for dynamic velocity v_s and slip velocity v_d have to be adapted to the overall velocities in the MBS-System. The dynamic velocity v_s slip and the velocity v_d are characteristic variables of the STEP-polynomial (corresponding polynomial approach according to arctan formulation in [5]).
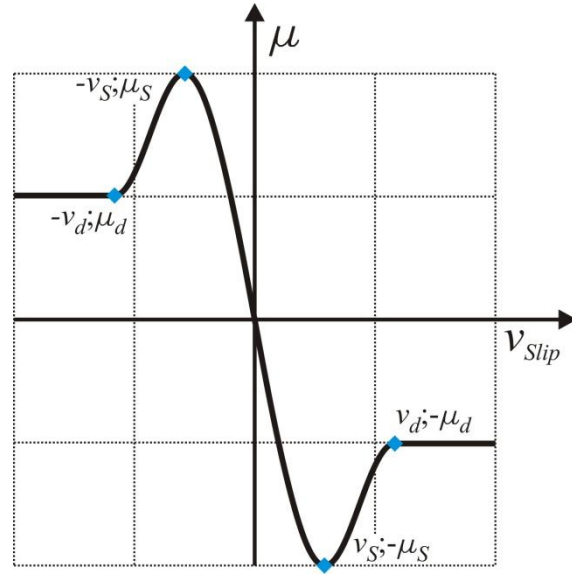
Figure 4: Velocity dependent definition of the friction coefficient μ (STEP-function).

The polynomial in fig. 4 is a piecewise function:

$$\mu\left(v_{\text{Slip}}\right) = \begin{vmatrix} -\text{sign}\left(v_{\text{Slip}}\right) \cdot \mu_d & \left|v_{\text{Slip}}\right| > v_d \\ -\text{STEP}\left(\left|v_{\text{Slip}}\right|, v_d, \mu_d, v_s, \mu_s\right) \cdot \text{sign}\left(v_{\text{Slip}}\right) & v_s < \left|v_{\text{Slip}}\right| < v_d \\ \text{STEP}\left(\left|v_{\text{Slip}}\right|, -v_s, \mu_s, v_s, -\mu_s\right) & -v_s < v_{\text{Slip}} < v_s \end{vmatrix}$$

$$= \begin{vmatrix} -\text{sign}\left(v_{\text{Slip}}\right) \cdot \mu_d & \left|v_{\text{Slip}}\right| > v_d \\ -\left[\mu_d + \left(\mu_s - \mu_d\right) \cdot \left(\dfrac{\left|v_{\text{Slip}}\right| - v_d}{v_s - v_d}\right)^2 \cdot \left(3 - 2\dfrac{\left|v_{\text{Slip}}\right| - v_d}{v_s - v_d}\right)\right] \cdot \text{sign}\left(v_{\text{Slip}}\right) & v_s < \left|v_{\text{Slip}}\right| < v_d \\ \mu_s - 2\mu_s \cdot \left(\dfrac{v_{\text{Slip}} + v_s}{2v_s}\right)^2 \cdot \left(3 - 2\dfrac{v_{\text{Slip}} + v_s}{2v_s}\right) & -v_s < v_{\text{Slip}} < v_s \end{vmatrix}$$

4   Numerical realization of the MBS-Model

The software to simulate the merging procedure is written in MATLAB (Program for mathematics). But in fact that no higher MATLAB functions like special solvers were used in this software, the program coding would also be possible in a common programming language like C, C++, or FORTRAN. Fig. 5 exhibits the sequence of operation diagram and shows a picture from the program´s graphical user interface (GUI).
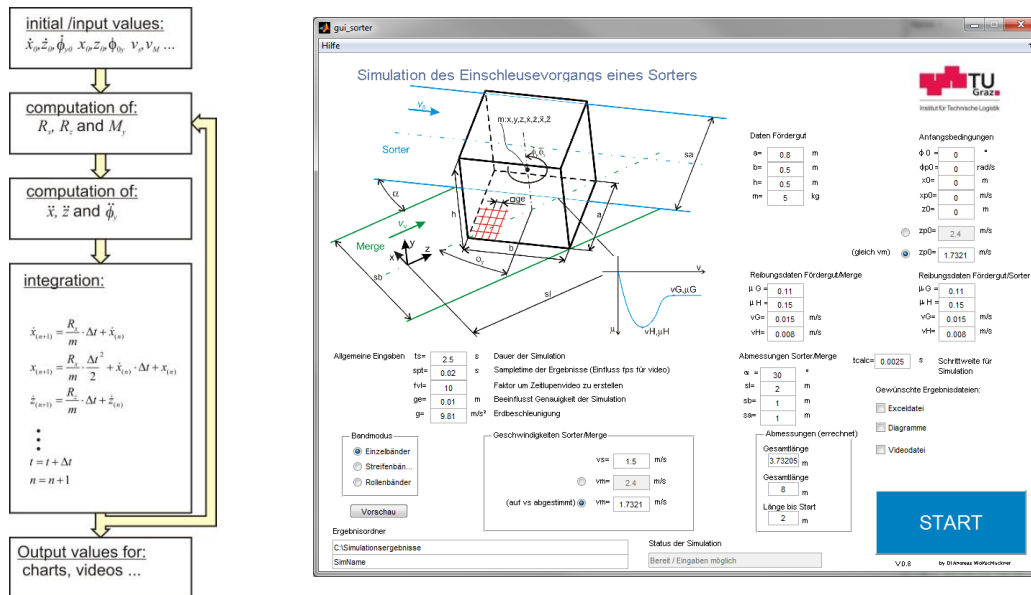

Figure 5: Sequence of motion diagram and screenshot of the MATLAB program.

The program returns all relevant data to make a full analysis of the motion process during packets relocation. Therefore exact packet position and velocity are evaluated for every single time step. To get an overall understanding of the merging procedure the friction forces and the friction moment for every time step are calculated and plotted.

Fig. 6 shows the path of the packet and the trajectories for each corner in a general example. Packets rotation about the y-axis during the converging is evident. To analyze this phenomenon a closer look to the forces and velocity diagrams is necessary.

Fig. 7 illustrates velocity over time (left-hand) and friction forces over time (right-hand). The fip line in the left diagram represents the packet´s angular rate about the y-axis. It is obvious, when the packet begins to rotate and when the circular motion stops. The xp and zp lines represent the velocities of the center of gravity in x- and z-direction for the packet and Rx and Rz the friction forces in x- and z- direction.

Table 1: Input parameters of an example Simulation Run.

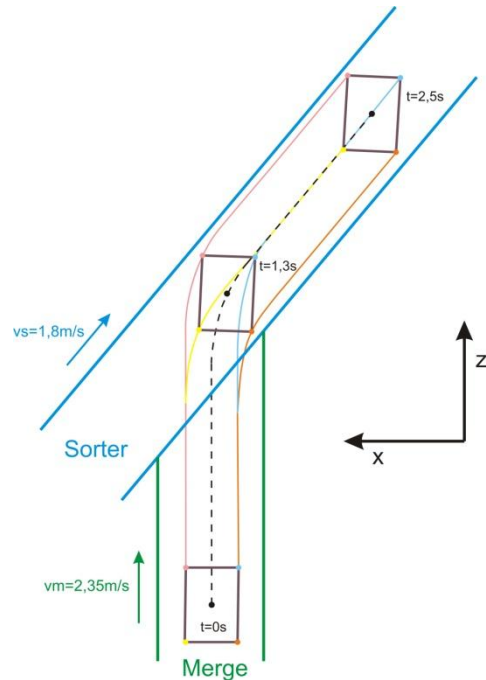| Input data for a simulation (example)	
Mass of the packet: *m*=30kg	*Friction coefficient sorter/packet:* μ_d=0.3; μ_s=0.35
Packet dimensions: *a* x *b* x *h*=0.7 x 0.5 x 0.5m	*Friction coefficient merge/packet:* μ_d =0.3; μ_s =0.35
Angel Sorter/Merge: α=40°	*Simulation time:* *tsim*=2.5s
Speed of sorter: v_S=1.8m/s	*Element size:* *ge*=0.005m
Speed of Merge: v_M=2.35m/s	*Sample time:* *spt*=0.02s

Figure 6: Trace curves of the relocation process.

The charts in fig. 7 show duration of converging process and kinetic results. The total converging time consumes 0.9 s (from 0.6 s to 1.5 s).The friction moment M causes the rotary motion of the packet and is plotted in the friction forces over time diagram. Because of the linear combination of angular and linear velocities and velocity dependency of friction there is a peak of about 10 Nm at the end of converging.
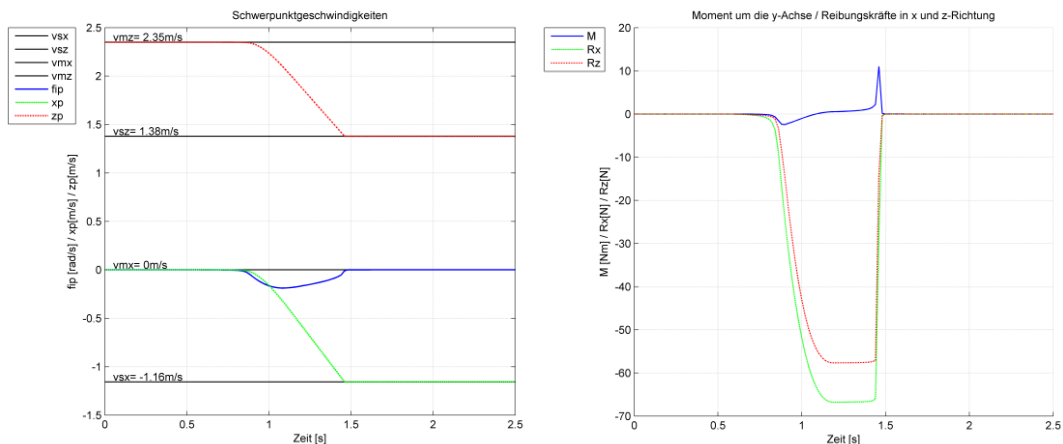

Figure 7: Velocity/time and Force/time diagrams.

One important result is that the packet changes it´s angle about the y-axis during the relocation process. This is relevant for practical implementations of merges. One has to keep in mind that sorter velocity, merge velocity, angle (sorter/merge) and the friction coefficients (packet/belts) have to be well balanced. In fig. 8 three case studies are shown. In the first two pictures, the angle (sorter/merge) is increased by 10°. If the velocities are not adapted to this new situation, the packet would fall from the sorter belt (central picture). In the right picture the velocities are adjusted.

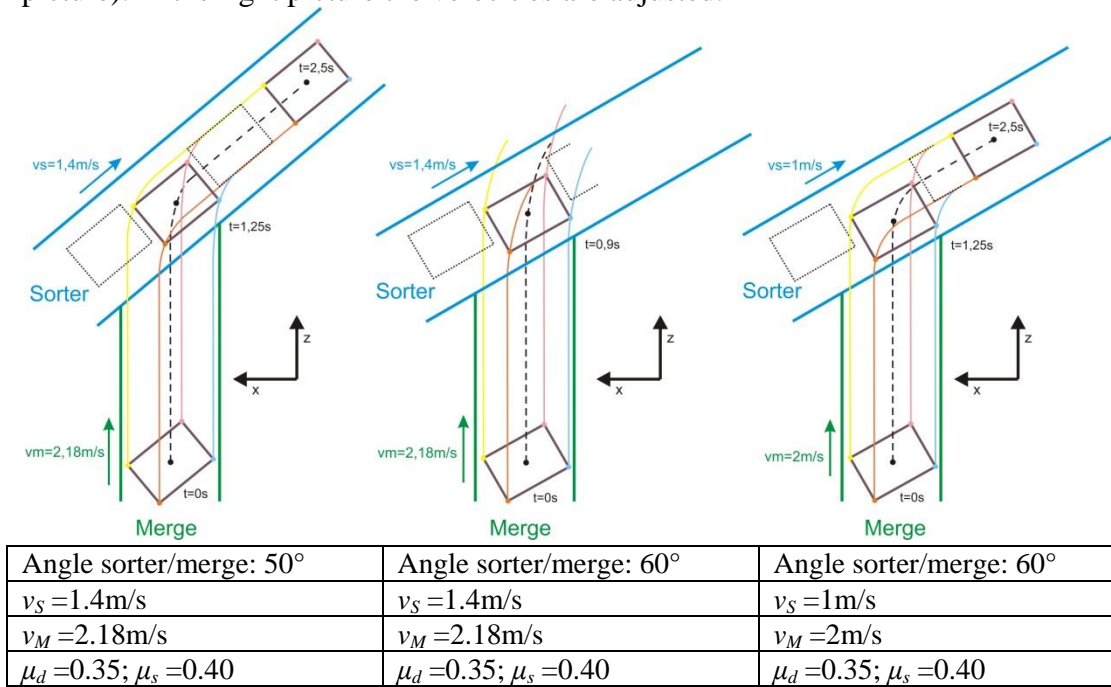| Angle sorter/merge: 50°	Angle sorter/merge: 60°	Angle sorter/merge: 60°
v_S =1.4m/s	v_S =1.4m/s	v_S =1m/s
v_M =2.18m/s	v_M =2.18m/s	v_M =2m/s
μ_d =0.35; μ_s =0.40	μ_d =0.35; μ_s =0.40	μ_d =0.35; μ_s =0.40

Figure 8: Examples for different motion sequences.

5 Impact of theoretical considerations on merge and sorting capacity

Fig. 9 shows the effect inadequate parameters (packet speed, packet initial orientation, belt speed...) have. If the packets angular deviation (discussed in the chapter above) is not compensated, more space on the sorter tray or the conveyor belt is needed. In fig. 9 the length required for a packet is c (optimized process) and respectively c+Δaf (not optimized process). Another consequence of inappropriate converging is an undefined packet end position on the sorter tray or conveyor belt. That means larger sorter trays and bigger gaps (s+Δpf) between the packets as an optimized process (gap size s) would need.
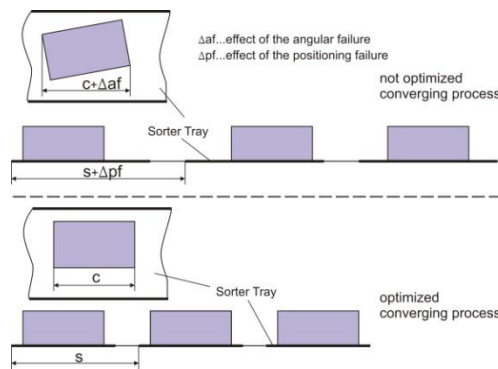
Figure 9: Effect of optimizing the converging parameters.

The attainable effects can be derived from Figure 2. Assuming, for example, totes with the dimensions 400 x 600mm and a sorting speed of 3 m/s it´s common to have a packet distance s of about 600mm or a gap of 200mm.

Reducing the gap by only one centimeter 300 additional packets per hour can be sorted. This also brings a reduction of 40 meters total length for a 4,000 tray sorter.

The estimated potential of increasing the overall efficiency, by adopting parameters in the merging area, is up to 1÷2%. This is of special interest, because of the easy and cheap way to implement the research results.

Fig. 10 illustrates an arrangement of materials handling equipment that considers the angle modification of the packet during the converging process. Using an additional adjustment device and normally existing acceleration device the angular displacement and inadequate velocities can be compensated. These devices are located prior to the merge area of a sorter.
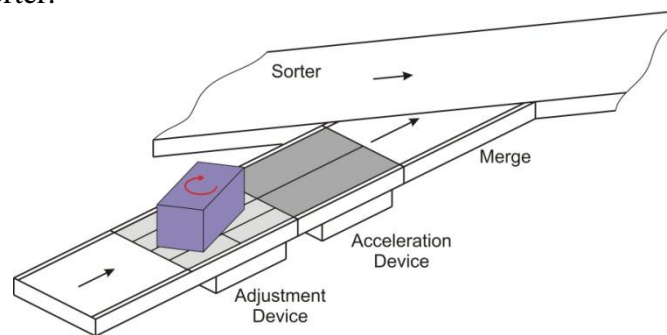

Figure 10: Sorter with adjustment and acceleration device

6   Prospects

The further research steps are the validation of these models, using general multi-body-simulation software (e.g. ADAMS) and the development of a test stand. This will offer several possibilities to validate theoretical considerations. Using modern image processing techniques will give a deeper view on motion sequences in various operating conditions.

Actual, the MATLAB program is in a very early stage. Many things like a full implementation of roller conveyors or other belt configurations are not available yet. Continuous improvements of the software are in progress to extend the functional range by:

- full support for roller conveyors or other belt configurations (e.g. strip merges)
- realistic geometry modeling (e.g. gap between merge and sorter)
- support for non-block shaped packets
- support for non-constant surface pressure distribution between packet and belt

But not only sorters benefit from optimizing the converging parameters. Further possibilities to increase throughput are offered by the use of *continuous* merges instead of *stop and go* merges. These systems are basing on similar theory as sorter merges. In general, all merges in material flow systems benefit from this approach and the operational throughput of the system can be increased significantly.

References

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