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Selina Begum Clemson University

Bill Ferrell Clemson University

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#### Humanitarian Logistics – The First Week

## Selina Begum and Bill Ferrell Clemson University

#### Abstract

Decisions made on material flow during the first week of a natural disaster are critical for victims. Currently, decision makers appears to be making important choices based on experience and intuition with little or no support from quantitative approaches because they do not exist. This research proposes a paradigm and offers two supporting models that will assist decision makers regarding the routing of materials during the first week of a disaster. It explicitly includes information regarding the victims' needs and the degree to which routes are available in a quantitative way that allows updating as information improves. The paradigm involves the use of information gap theory adapted to the this situation for deciding on the types of supplies to send and the Canadian traveler problem for making decisions on the routes to take.

#### **1** Introduction

Humanitarian logistics is important – really important. In 2010, there were 385 natural disasters that killed 300,000 people, impacted another 2 million, and accounted for economic damage estimated to be \$123.9B [1, 2]. To reduce suffering and save lives, relief operations have been launched by a large number of organizations around the world creating an extensive and somewhat disorganized humanitarian relief chain. Whether trying to control this diverse mix of entities or organize efforts in large and mature organizations, logistics and materials handling is the heart of disaster relief. Some researchers have suggested this is the most expensive part of relief operations [3] while others have noted that it is unfortunate that logistics has been considered a cost that organizations had to absorb. In fact, they recommend that logistics be viewed as a strategic component of the relief effort and suggest that this would increase the efficacy of the work humanitarian logisticians performs off and on the field [4].

Material handling – actually handling, storing and routing of relief supplies - has been identified as critical elements of relief operations and has received an increased emphasis in recent years from researchers as illustrated by [5,6]. The literature that we have found focuses on a variety of topics ranging from prepositioning supplies to establishing efficient distribution centers and many aspects of relief operations in between; however, there is a conspicuous gap regarding logistics decisions in the first week after the onset of a disaster. During this initial response phase, operating the disaster supply chain has one goal, responsiveness, while efficiency and cost effectiveness take a back seat. There is pressure to deliver supplies as quickly as possible because their receipt can literally be a matter of life and death. On the other hand, the time immediately after the onset of a disaster is when maximum uncertainly exists. Information about resource availability, characteristics of supplies, status of infrastructure like roads, and delivery schedule of suppliers is limited. For example, there might be several surface routes from the staging areas for relief operations to the disaster site but whether the roads are passable or not is unknown. Despite these obstacles, a disaster supply chain network must be established immediately and relief efforts commenced as quickly as possible. In this research, we begin exploring a paradigm for making decisions during this first week of humanitarian relief efforts when needs are high and information reliability quite low.

The need for models to support decision making regarding the dispatch and routing of relief supplies during the first week is not only common sense need but one that has been well established in the literature [7-9]. Recent major disasters in Haiti 2010, Pakistan 2010, and Japan 2011 are cases where by the efficiency of logistics systems were tested to their fullest and illustrated the complexity faced by the human decision maker in these situations. Better tools to help the decision maker during this chaotic time can save lives immediately and establish a supply chain that can both save lives and improve the situation on the ground more quickly. This research focuses on the response phase of a disaster when prepositioned items need to be deployed after the onset of a disaster and humanitarian operations start distributing relief goods to the beneficiaries. Since this aspect of disaster relief has not been addressed in the literature and anecdotal evidence suggests that this is done in reality on an ad hoc basis, we submit this work fits the definition of a "paradigm paper" since we will be exploring a new framework for decision making in the first week of a disaster.

This research focuses on two aspects of this challenge. This first acknowledges that information about the situation including needs and infrastructure will be very poor; however, better information will be collected every day so it improves – potentially at a rather rapid rate – during the first week. To facilitate decision making during this time, we adopted information gap theory [10] as a framework. The second aspect is selecting routes to deliver the supplies to the affected site. These two aspects are highly connected during the first week of a disaster. For example, it is clear that water, food, shelter and medical supplies are four categories of supplies that are critical to disaster relief. The information that is available about a situation can dictate the strategy for delivering these suppliers; that is, trying to deliver everything immediately might not necessarily the best approach. This can be especially true if little information is known about the more direct routes to affected area. Suppose the most direct routes have a reasonably high probability of not being passable while a longer route has high likelihood of being open. What do you do? These are the types of situations that the new paradigm is being built to address.

### 2 Decisions using Info Gap

Info gap has three primary elements: system model, uncertainty model and performance requirement. It focuses on quantifying the information gap and predicting possible system behavior based on what is already known and the impact of varying parameters in the solution space. In this research, we concentrate on determining which supplies to move to the disaster site in each time interval.

We begin with a brief discussion of the info gap model, specifically, the major components of the model.

#### 2.1 System Model

The system model specifies the functional relationship that connects the input-output structure of the system to the choice of alternatives, the utility, and the uncertainty. The exact nature of the relationship can be as complicated or simple as needed. In disaster relief decision making, a common scenario is that a decision maker has alternative action plans, each with a different utility. The goal is to mobilize resources using one of these alternatives. For example, a simple system model based on expected utility could be valuable for disaster relief like the one presented in equation (1). Historical data can be used to parameterize the model based on previous disasters and the model is sufficiently simple and robust that repeatedly solving it within the info gap framework could easily be accomplished in a short period of time.

$$E[a_j] = \sum_{i=1}^{n} p_i v_{ij} \text{ for } j = 1, 2, \dots m$$
(1)

where

*i* states of the system defined by the nature and intensity of the disaster, i=1,2,...,n

 $a_j$  the alternative *j* chosen from set of alternatives  $\{a_1, a_2, ..., a_m\}$ 

 $p_i$  the probability of a disaster intensity (low, moderate, high or catastrophic)  $v_{ij}$  the utility of a chosen alternative  $a_j$  given a system state *i* 

#### 2.2 Uncertainty Model

Uncertainty is associated with both the available information regarding the state of the system and the utility of each alternative. The uncertainty model captures this data by representing the fact that each alternative may deviate from the estimates. While there are a number of possible implementations, an interval bound model is one choice that seems suitable for this application because of the nature of the problem and the simplicity associated with it that translates into ease of updating and quickly regenerating solutions to the entire model. The interval of uncertainty ( $\alpha$ ) measures the deviation from the observed system.  $U(\alpha, \tilde{p})$  and  $U(\alpha, \tilde{v})$  are defined as the info gap model of uncertainty for probability and utility, respectively. Uncertainties U(.)embody the prior information about the uncertain vector u and each alternative as it captures the deviation from the predicted and observed system. For the predicted system and utility models, uncertainty in probability and utility is an infinite set of values of the vectors p and v, and the model is an unbounded family of nested sets, U(.),  $\alpha \ge 0$ . Equations 2(a) and 2(b) define the absolute functional errors of the estimated probability and utility, and limits these values to an amount  $\alpha$ .

$$U(\alpha, \tilde{p}) = \left\{ p : \left| \frac{p_i - \tilde{p}_i}{\tilde{p}_i} \right| \le \alpha, \quad i = 1, 2, ...n \right\}$$
(2a)

$$U(\alpha, \tilde{v}) = \left\{ v: \left| \frac{v_{ij} - \tilde{v}_{ij}}{\tilde{v}_{ij}} \right| \le \alpha \quad i = 1, 2, \dots, n; j = 1, 2, \dots, m \right\}$$
(2b)

where

- $\alpha$  the horizon of uncertainty
- $p_i$  the states of the system, defined by the intensity of disaster
- $v_{ij}$  the utility of option  $a_j$  given a system state *i*
- $\tilde{p}_i$  the estimated value of state probability
- $\tilde{v}_{ii}$  the estimated value of an alternative option given a system state *i*

Based on the estimates of uncertainty in Equation 2(a) and (b), equations 3(a) and 3(b) frame the uncertainty model where the probability of the system state  $p_i \in [0,1]$  and the utility of alternative given a system state  $v_{ij}$  lies between some maxima and minima dictated by the uncertainty parameter  $\alpha$ .

$$U(\alpha, \tilde{v}) = \left\{ v : \max\left[0, (1-\alpha)\tilde{v}_{ij}\right] \le v_{ij} \le \min\left[1, (1+\alpha)p_i\tilde{v}_{ij}\right] \right\} \qquad \alpha \ge 0$$
(3a)

$$U(\alpha, \tilde{p}) = \left\{ p : 1 = \sum_{i=1}^{n} p_i; \max\left[0, (1-\alpha)\tilde{p}_i\right] \le p_i \le \min\left[1, (1+\alpha)\tilde{p}_i\right] \right\} \qquad \alpha \ge 0 \quad (3b)$$

No uncertainty (i.e., perfect information is available regarding the state of the system or utility of an alternative) is indicated when  $\alpha = 0$  while any value  $\alpha \ge 0$  indicates presence of uncertainty. Equations 3(a) and 3(b) indicate that at the horizon of uncertainty, the values of uncertain probability and utility lie within the range defined by the uncertainty  $\alpha$ .

## 2.3 Performance Criteria

In relief operation the performance of an organization can be measured in terms of time to mobilization and number of the beneficiaries reached. In this research we selected utility of the service provided which considers the utility of the alternative. A performance measure is usually a value derived from the process model. So, for example, a simple performance criterion could be requiring it to exceed a minimum threshold:

$$EU \ge EU_c$$
 (4)

## 2.4 Robustness Function

Info gap theory seeks to identify the strategy that is good enough but simultaneously prevents an unwanted outcome. The robustness function identifies the degree of resistance to both uncertainty and immunity to failure. A robustness measure like the one presented in equation (5) has potential in this application.

$$\hat{\alpha}(a_{j}, EU_{c}) = \max\left[\alpha : \min_{\substack{v \in U(\alpha, \bar{v}) \\ p \in U(\alpha, \bar{p})}} E[a_{j}] \ge EU_{c}\right]$$
(5)

Selecting this function is important because it determines the quality of the solution in an uncertain environment like that encountered in the first week of a disaster. Large robustness values for any alternative indicate that selecting it will satisfy the critical requirements of the model even if the system model is prone to error. On the other hand, a low robustness implies that the outcome is vulnerable to model uncertainty. As you can see, info gap was selected as the framework for part of this paradigm because it is rather flexible in construct, especially in how it handles the way parameters are varied which is important in the first week of disaster relief because information is so unreliable.

The second part of the paradigm deals with selecting surface routes during the first week. Once the choice has been made to respond to a disaster and the items have been secured, a decision must be made regarding the routes to take to reach the disaster area, mode of transportation, and quantities to be delivered. To address this issue, we focus on the problem of how routes can be selected so that crews and materials safely reach the disaster site to conduct relief and rescue activities in a reasonable amount of time. The motivation for this work is rather obvious; during a disaster, particularly in the early stages, accurate information about the extent of the damage to the infrastructure is very scarce yet effectively and quickly deploying resources in a way that they safely reach the affected areas can be a matter of life and death for many people. For example, should all of the water be sent along the shortest path when it contains segments with a high likelihood of being impassable and backtracking can take a much longer amount of time that choosing a longer but more secure path? Or, should part of the water be sent along the shortest path and the rest sent along the route with the highest likelihood of being passable? Or, should all be send along the most likely passible route? We explore this aspect of decision making in the first week using the Canadian traveler problem (CTP) [11] as a starting point. The CTP is a stochastic variation of the shortest path problem where the goal is to provide decision makers with information regarding travel time and alternative routes that will improve the efficiency and effectiveness of their efforts.

## **3** Routing Decisions using CTP

The basic problem supposes that a traveler has to go from site *S* to site *T* in the undirected graph G(V, E). The graph G(V, E), where the nodes *V* corresponds to the set of sites and edges E correspond to the set of roads between sites, is known to the traveler. Each edge  $e \in E$  has a non-negative length associated with it and length can be, interpreted as the time it takes to traverse the road or the cost to traverse. An edge is accessible or blocked with probability p or (1-p), respectively. From the above description of parameters an instance of the CTP can defined as a 6-tuple [12]  $I = \langle V, E, p, c, v_o, v_* \rangle$  where

 $\langle V, E \rangle$  is a connected undirected graph with set of vertex V and set of edges E

 $p: E \rightarrow [0,1)$  is the probability that a road (edge) is not accessible

 $c: B \rightarrow N_{t}$  defines the travel cost of the road

 $v_0, v_* \in V$  are the locations of source and destination location of the traveler.

Humanitarian logisticians face the problem of incomplete information during the initial phase of disaster. Although a map of the location is available and some general information might be known, it is not known with certainly to the decision maker if a path is traversable or not. We assume that once an area becomes accessible, it remains accessible for the rest of the time while the organization is there conducting relief work and propose that the CTP can be adapted to this situation. As such, the solution can be used by the decision maker to direct the relief efforts. Finding meaningful solutions to this problem are an interesting challenge. Several approaches have been investigated and a modified Dijkstra's is one interesting possibility.

# 3.1 Resolving the CTP

The CTP falls in the category of online algorithms since it seeks to minimize the cost of reaching a target in a weighted graph where some of the edges are unreliable and the traveler only learns that after they reach an adjacent node and can pass no further. Here, online algorithms refer to problems in which decisions must be based solely on information that is available at a point in time whereas offline algorithms generate optimal solution given complete information of the problem; hence, the difference between the shortest path problem (SPP) and the CTP. The fact that online algorithms so closely match real decision making in the first week of a humanitarian crisis and offline algorithms are so ill suited is the key reason the CTP was chosen. Another reason is, it captures critical question of exploiting the information that a decision maker must make in that first week. Since there is a cost associated with gathering more information in terms of time and resources, CTP addresses the challenge of balancing making an immediate decision and investing further in exploring the region to gather better information.

CTP is PSPACE complete and there are many algorithms that solve this particular problem optimally, some of which are described in [12]. In this research, Dijkstra's algorithm for resolving the SPP is modified (henceforth called CTP-D) to resolve the CTP.

Dijkstra's algorithm optimally solves a single source SPP problem under the assumption that all edges have non-negative weights using a greedy approach. The algorithm starts at the source node and grows in a greedy manner until the destination node is reached and all nodes reachable from the source are considered. Dijkstra's algorithm requires that the lengths along all edges are known a priori and it systematically moves from the source node to every other node in the network so that at each step the shortest distance from the source to another node is determined. The algorithm terminates if there is no more nodes that can be reached. Although Dijkstra's algorithm can provide shortest distance between any two points in the network, it has limitations in terms of not being able to handle the stochastic nature of the CTP so it is modified in this research. To resolve the CTP, every node is assigned a probability of being blocked and, as the algorithm executes, one or more nodes might get blocked. To include this feature in the CTP, a random number generator is used in conjunction with the probability of a node being blocked to determine which nodes, if any, are blocked during an iteration of the algorithm. If a node is blocked, the algorithm calculates a revised cost of finding an alternative route or waiting at the previous node or both. The pseudo code for the CTP-D algorithm is found in Figure 1

As CTP-D progresses, a random number determines if a node is permanently or partially blocked thereby increasing the travel time or forcing the algorithm to reevaluate the node and finding the shortest path under given probability of block.

#### **3.2** Numerical Example

A sample data set is used to illustrate features of this methodology. The shortest path route is calculated using both Dijkstra's algorithm for the SPP assuming deterministic information known *a priori* and using the CTP-D when paths can be blocked at random during execution of the algorithm. Four different sources and 13 different destinations with 36 vertices and 85 edges are considered. Table 1 reports the results of initial experimentation using these scenarios with the time required to travel from all origins to all destinations using Dijkstra's algorithm in the SPP and the modified Dijkstra's algorithm on the CTP.

1 <b>function</b> CTP_Dijkstra( <i>Graph</i> , <i>source</i> , <i>probBlock</i> ):								
2 <b>for each</b> vertex v in <i>Graph</i> :								
dist[v]:=infinity;								
4 $\operatorname{dist[source]} := 0;$								
Q: = the set of all nodes in <i>Graph</i> ;								
6 <b>while</b> <i>Q</i> <b>is not</b> empty:								
u: = vertex in Q with smallest distance in dist[];								
Remove $u$ from Q;								
<b>for each</b> neighbor $v$ of $u$ :								
10 ran_num = random();								
11 if ran_num >= probBlock								
12 $alt: = dist[u] + dist_between(u, v);$								
13 else								
14 $alt: = dist[u] + dist_between(u, v) + ran_num;$								
15 <b>if</b> $alt < dist[v]$ :								
dist[v]:=alt;								
17 previous[ $v$ ]:= $u$ ;								
decrease-key $v$ in $Q$ ;								
19 return dist[]								
20 end CTP_Dijkstra								

Figure 1: Modified Dijkstra's algorithm (CTP-D) applied to CTP

Due or the fundamental nature of the problem, it is clear that the online algorithm can never perform better than the offline algorithm using known data and, in fact, solving the SPP optimally provides a lower bound on CTP-D. Because of this fact, online algorithms are usually evaluated using the competitive ratio which reflects how closely the solution found by the online algorithm follows the offline counterpart. The competitive ratio is defined as the worst case ratio between the minimum distance as reflected by the solution using the online algorithm and the length of the shortest source target path [13].

The competitive ratios for the scenarios explored in this numerical example are shown in Table1.

From	Origin 1			Origin 2		
То	SPP	СТР	Ratio	SPP	СТР	Ratio
Destination 1	366	502	1.37	369	573	1.55
Destination 2	549	697	1.27	446	594	1.33
Destination 3	643	814	1.27	479	627	1.31
Destination 4	538	622	1.16	541	849	1.57
Destination 5	611	641	1.05	212	386	1.82
Destination 6	771	1002	1.30	455	619	1.36
Destination 7	639	982	1.54	436	619	1.42
Destination 8	377	620	1.64	541	814	1.50
Destination 9	308	387	1.26	472	797	1.69
Destination 10	283	283	1.00	552	812	1.47
Destination 11	1021	1467	1.44	560	741	1.32
Destination 12	724	1073	1.48	322	573	1.78
Destination 13	824	1293	1.57	363	538	1.48

Table 1: Time required traveling between origins and destinations using the Dijkstra (SPP) and CTP-D.

From	Origin 3			Origin 4		
То	SPP	СТР	Ratio	SPP	СТР	Ratio
Destination 1	284	458	1.61	284	874	3.08
Destination 2	467	615	1.32	716	1152	1.61
Destination 3	561	858	1.53	749	1074	1.43
Destination 4	456	604	1.32	811	1006	1.24
Destination 5	529	875	1.65	482	630	1.31
Destination 6	689	758	1.10	725	831	1.15
Destination 7	557	626	1.12	706	758	1.07
Destination 8	362	441	1.22	811	1179	1.45
Destination 9	293	454	1.55	742	911	1.23
Destination 10	373	525	1.41	822	1439	1.75
Destination 11	939	1469	1.56	549	697	1.27
Destination 12	642	1053	1.64	339	582	1.72
Destination 13	742	1041	1.40	380	380	1.00

In all the cases except one the ratio lies between 1.0 and 2.0, indicating consistence performance. The model was applied to other example reflecting differing scenarios and the worst competitive ratio was 3.0. It is important to

understand that the intention here is merely to illustrate the performance of the CTP-D relative to the deterministic SPP using Dijkstra's algorithm for a few scenarios. There is no implication that more general conclusions can be drawn because they cannot. There is much additional research that must be performed in this area. On the other hand, we strongly believe that these results suggest that CTP-D can be a important part of the new paradigm for helping decision makers during the first week of a disaster and prosing this new paradigm is the main contribution of this research.

Looking at the solutions provided by the algorithms reinforces intuition about the underlying problem and the difficulty that decision makers face. Deciding on how to route relief supplies from point A to point B in the first week of a disaster are difficult. The solutions associated with routing supplies from Origin 2 to Destination 1 in this numerical example is now considered and illustrated in Figure 2. At the outset, the decision maker has knowledge of the possible paths between origins and destinations as well as the degree to which they can be traversed before the disaster occurs. Figure 2a illustrates the minimum distance route found by solving the SPP with Dijkstra's algorithm when it is assumed that all edges are traversable and remain so during the entire time. The minimum distance is 300.

Now, the decision maker knows that in reality three things can happen: all of the edges remain traversable, some of the edges become partially traversable, and/or some of the edges can become completely blocked. In this new paradigm, CTP-D is used to investigate rerouting for partially blocked or blocked edges. Figure 2b illustrated how CTP-D finds a new shortest path when the traveler finds that the edge from the fourth node is completely blocked. The algorithm finds an alternative path involving rerouting (black node) and it calculates the revised cost for the rerouted path. In Figure 2c, some edges are partially blocked and one node is completely blocked resulting in the worst performance of the three options. In all cases the edges blocked/partially available are shown in dashed lines. Also in the figure 2c, the solid node indicates that the solution includes different nodes to reach the destination because rerouting was cheaper than waiting for the edge to become available.



Figure 2: A visual illustration of paths resulting from Dijkstra's algorithm and CTP-D

## 4 Conclusions

In conclusion, this research explores a paradigm for addressing material flow in humanitarian logistics during the first week after the onset of a disaster. Info gap theory and the Canadian traveler problem are adapted to assist in the making decisions on which items to send, the origins to send them from, when to send them, and the route to use in the face of uncertain information. This paradigm is quantitative so results are repeatable and understandable but flexible because the information available to the decision maker at the onset of the disaster can be very sketchy and unreliable but can improve dramatically as the week progresses. As such, it is important for the paradigm and supporting models to accurately translate the types of information that will most likely be updated in useful output. For example, it is likely that more precise information on the types of supplies needed at various destinations and the degree to which road segments can be passed will be improved dramatically as the first relief workers move towards and arrive at different disaster areas. The associated material flow decisions that must be made are exactly which suppliers to send from which locations and along which routes. As this paradigm and the supporting models become more fully developed and tested, we believe they will provide decision makers and logisticians in the field with valuable insights into available alternatives along with measures of effectiveness and chances of a success. This approach will quickly translate updated information on the situation into information the decision maker can use to guide his or her actions resulting in better decisions and improved operations to assist the victims of natural disasters in the first week.

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