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Solitons and the commensurate-incommensurate transition in a convecting nematic fluid

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Quasiperiodic (incommensurate) patterns consisting of an array of solitons are found in a convecting nematic fluid subjected to spatially periodic forcing. The solitons are regions of local compression of the convective rolls and are well described by solutions to the sine-Gordon equation. The commensurate-incommensurate transition is continuous for weak forcing, but becomes discontinuous as the forcing amplitude increases. This behavior indicates the existence of a tricritical point on the phase transition line.

I. INTRODUCTION

Hydrodynamic instabilities generally give rise to spatially periodic patterns in systems with translational invariance, such as Taylor-Couette flow or Rayleigh-Bénard convection. The selection of a particular wave number from the band of possible wave numbers is a process that is not well understood.[1] The dynamics of pattern selection can be elucidated by examining the response of the system to forcing at a wave number that may be different from the naturally selected one. We have performed such an investigation for a convecting fluid. We find that the competition between the natural and imposed periodicities leads to an incommensurate structure in which the roll period is spatially modulated. A suitably defined phase variable shows regions of rapid phase change, periodically arranged to form a soliton lattice, in a way that can be quantitatively described by solutions to the time-independent sine-Gordon equation.

Solitons are known to mediate the transition between commensurate and incommensurate phases in a variety of condensed matter systems (for example, atoms adsorbed on crystalline substrates) and corresponding theoretical models.[2–5] However, solitons have not previously been observed in connection with the selection of patterns resulting from hydrodynamic instabilities, nor have they been predicted.

We discovered a transition from incommensurate states (soliton lattices) in a convecting fluid to nearby commensurate states by varying the ratio of the competing periodicities. We found that the transition is continuous for weak external forcing, but becomes discontinuous as the forcing amplitude is increased. This behavior implies the existence of a tricritical point (in the parameter space formed by the forcing amplitude and the ratio of the competing periodicities) at which the order of the transition changes. Several other novel states were also discovered, including high-order commensurate states and disordered patterns containing dislocation pairs; these will be described briefly.

In order to obtain a system with up to 360 convective rolls that can be conveniently subjected to external forcing, we have utilized an electrohydrodynamic instability in a nematic liquid crystal. (The Rayleigh-Bénard system is less suitable because the period cannot be made as small.) The system consists of a nematic liquid-crystal layer [4-methoxybenzylidene-4′-n-butylaniline (MBBA)] confined between two transparent conductive electrodes with an adjustable separation of 20—120 μm. A potential difference of about 6 V across the layer induces a roll pattern similar to that resulting from the Rayleigh-Bénard instability.[6] To impose a spatially periodic forcing, one of the electrodes is photolithographically separated into two interdigitated regions that are maintained at different potentials. This allows us to create a voltage across the layer that has a component with spatial period $l_1 = 200\ \mu$m. The director orientation of the nematic is aligned parallel to the plates and perpendicular to the electrode fingers by a polymer coating. This causes the rolls to be aligned predominantly parallel to the electrode fingers so that the patterns are essentially one-dimensional. Further information on the experimental arrangement has been given previously.[7]

Two important control parameters are defined as follows. The strength of the external forcing is $\alpha \equiv 2\Delta V/V_c$, where ΔV is the amplitude of the spatially periodic component of the voltage across the plates, and V_c is the threshold for the convective instability. (The average voltage across the cell is maintained at $1.057V_c$; ΔV is small enough that V_c is exceeded everywhere.) The second important parameter is l_0, the period of the instability (width of two rolls) when unforced. It is approximately equal to twice the layer depth and can be adjusted continuously over a wide range.

II. PHASE DIAGRAM AND SOLITON LATTICES

The overall behavior of the system as a function of α and l_0 is shown in Fig. 1. Various commensurate states are seen. For example, in the region labeled 2/3, two periods of the external forcing correspond exactly to three hydrodynamic periods (six rolls). (For values of l_0 smaller than those in the diagram, commensurate states at ratios 1/3, 5/14, 2/5, 3/7, and 1/2 are also seen.)

The largest commensurate (C) region in the phase dia-

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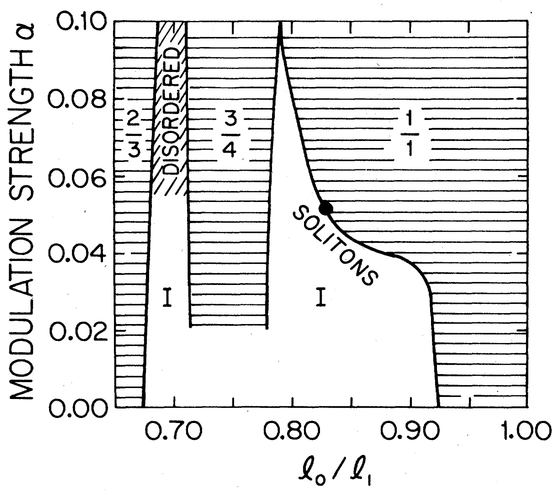

FIG. 1. Phase diagram as a function of the dimensionless modulation strength α and the ratio of competing lengths l_0/l_1. The horizontally shaded regions represent commensurate (C) states. Solitons are found in a portion of the incommensurate (I) region. The dot on the C-I boundary is a tricritical point above which the C-I transition is discontinuous.

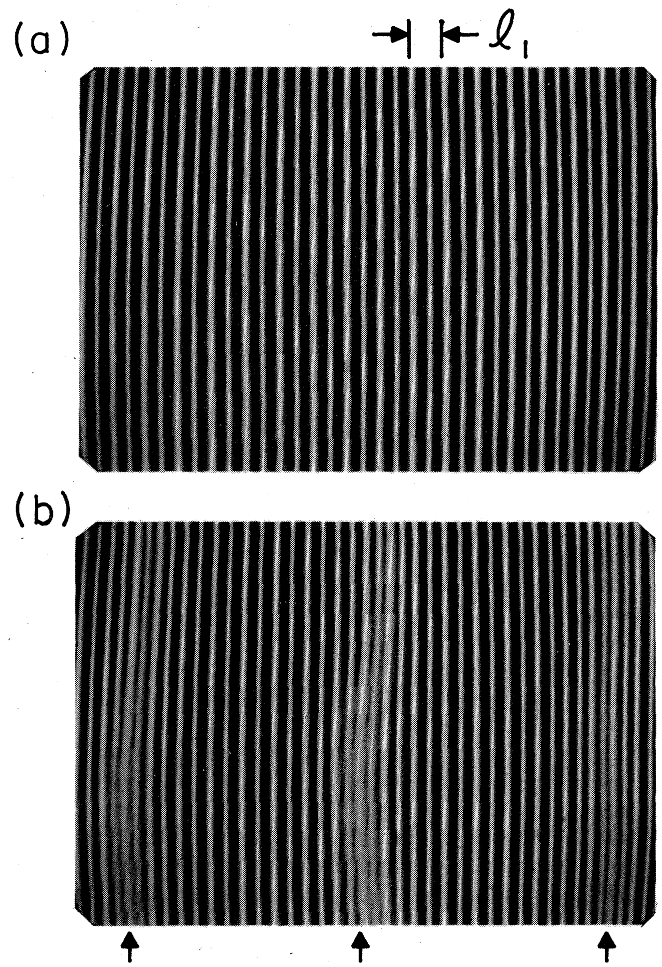

FIG. 2. (a) Photograph of a 1/1 commensurate state. The convective rolls are aligned parallel to the electrode fingers, with two rolls for every period of the modulation. The unforced length ratio is $l_0/l_1=0.941$, and the forcing strength is $\alpha=0.03$. (b) Photograph of a quasiperiodic structure at $l_0/l_1=0.866$ and $\alpha=0.03$. The solitons (indicated by arrows) are regions of compression of the rolls.

gram corresponds to the 1/1 C state. A photograph (taken in transmitted light polarized perpendicular to the electrode fingers) is shown in Fig. 2(a). The electrode fingers are oriented vertically and are not directly visible. Each pair of bright stripes corresponds to a pair of convective rolls. (The long axis of the nematic molecules is tilted out of the plane of the layer by the convection, giving rise to a spatial modulation of the refractive index.) In the 1/1 C state, the interaction between the natural period and the imposed one leads to perfect phase locking over the entire electrode. The rolls are aligned parallel to the fingers, and the roll period (two rolls) is equal to the electrode periodicity. The 1/1 C region in Fig. 1 has a finite width even for $\alpha=0$, because the etched line on the electrode causes a residual perturbation in the electric field with a period of 100 μm.

Incommmensurate or quasiperiodic states containing solitons [see Fig. 2(b)] are found over a substantial range in α and l_0, for example, between the 3/4 and 1/1 C phase boundaries. Over most of the pattern, the rolls are nearly commensurate with the external forcing. Between these regions, the rolls are locally compressed. Figure 2(b) contains three of these compressed regions (marked by arrows), which may be usefully described as solitons. They are equally spaced and form a lattice that is nearly parallel to the rolls.

These patterns are studied quantitatively by digital analysis of the images. We determine the width of each roll by locating the maxima of the light intensity as the pattern is traversed in a direction perpendicular to the electrode. (To obtain good accuracy, parabolas are fitted to the light intensity function near the peaks.) The existence of solitons is demonstrated quantitatively by this technique in Fig. 3. The roll size, shown in Fig. 3(a), varies periodically across the sample, nearly attaining the commensurate value of 100 μm in some regions. The areas of localized compression are the solitons (sometimes called domain walls or discommensurations).

A phase variable ϕ_n may be defined to denote the location x_n of the nth roll pair with respect to the external forcing: $\phi_n/2\pi=(x_n/l_1)-n$. The phase is plotted as a function of position in Fig. 3(b). Each soliton corresponds to a phase change of 2π, or the insertion of an extra roll pair. Most of the phase change occurs over a distance that is small compared to the distance between the solitons, but large compared to l_0. A second example of a quasiperiodic structure with more closely spaced solitons is given in Fig. 4. There are eleven solitons across the sample, of which three are shown in the graph. The solid lines in Figs. 3(b) and 4 are fitted solutions to the sine-Gordon equation, and are described in Sec. IV.

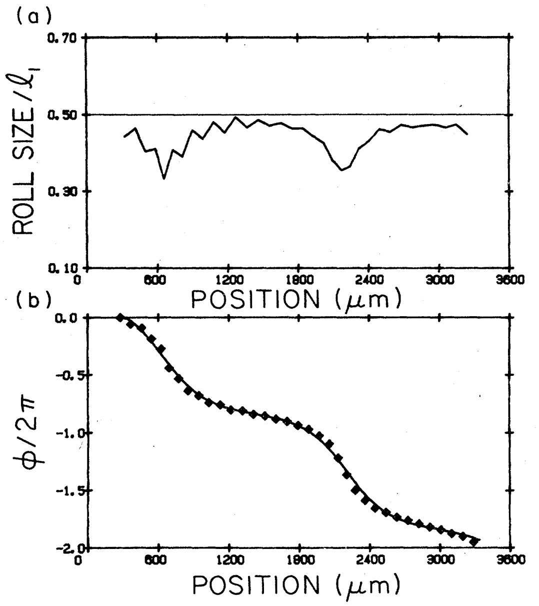

FIG. 3. (a) Variation of the roll size (in units of l_1) across the sample, for the pattern of Fig. 2(b). (b) Variation of the phase of the rolls across the sample. The solid line is a fitted solution of the sine-Gordon equation (see text). Each "step" corresponds to an insertion of two extra rolls and a phase change of 2π.

III. COMMENSURATE-INCOMMENSURATE TRANSITION

This system is ideal for studies of the transition from the commensurate to the incommensurate state as the ratio of the competing periodicities is varied, because the structural changes in the soliton lattice can be seen directly. The soliton spacing s is shown as a function of l_0/l_1 for $\alpha=0.03$ in Fig. 5. It varies smoothly over the range $0.79<l_0/l_1<0.92$. Outside this region, the transition to a C state occurs as follows: (a) s becomes infinite as the system enters the 1/1 C state for $l_0/l_1>0.92$; and (b) s becomes commensurate with l_1 (in fact $s=3l_1$) as the system enters the 3/4 C phase for $l_0/l_1<0.79$. This is the flat region of Fig. 5. Thus the C-I transition can be described in terms of the soliton spacing.

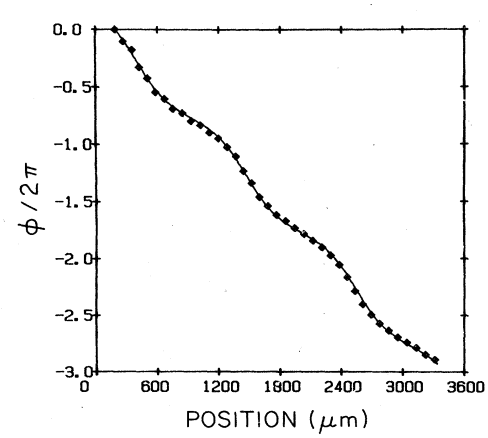

FIG. 4. Variation of the phase of the rolls across the sample farther from the C-I transition ($l_0/l_1=0.816$ and $\alpha=0.03$).

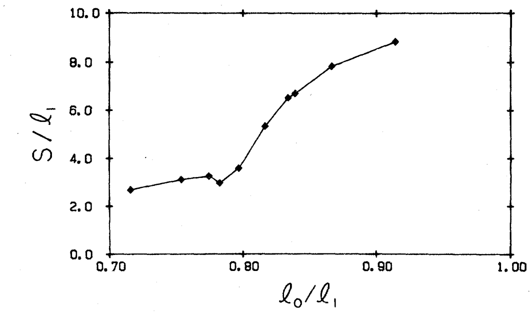

FIG. 5. Variation of the soliton spacing s (in units of l_1) as a function of the unforced ratio of competing lengths l_0/l_1 (at $\alpha=0.03$). The curved portion corresponds to the soliton lattices. The nearly flat portion at $l_0/l_1<0.79$ is the commensurate 3/4 state.

We find that the nature of the C-I transition changes as the strength α of the periodic potential is varied: the transition occurs continuously for small α, and discontinuously for large α. This behavior can be most easily demonstrated by introducing a variable l, which is the inverse of the *mean* wave number of the pattern. We determine this quantity by Fourier analysis of digitized images of the patterns, and its behavior as a function of the unforced period l_0 is shown in Fig. 6, for various values of α. In the figure, both l and l_0 are measured in units of the forcing period l_1. When $\alpha=0$, l is proportional to l_0. As α is increased, a step appears at the 1/1 C state, but the curve apparently remains continuous. Finally for $\alpha\geq 0.05$, the variation of l with l_0 becomes *discontinuous* at the step, and the size of the jump increases with α. We conclude that there is a point on the boundary of the 1/1 C state in the phase diagram of Fig. 1 where the transition changes from second order (low α) to first order (high α). This is the signature of a tricritical point.

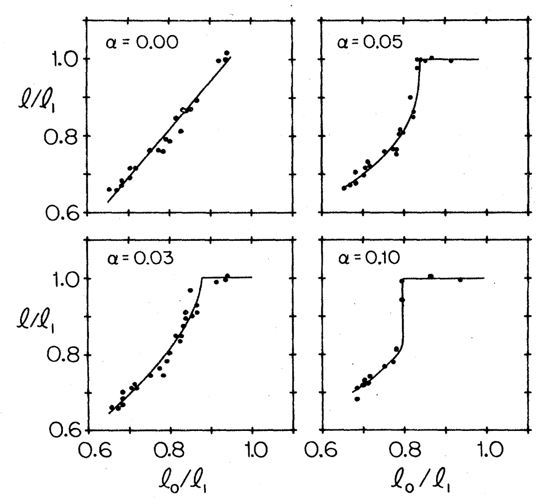

FIG. 6. Variation of the inverse l of the mean wave number with the unperturbed roll size l_0 (in units of l_1), for various forcing strengths α. The C-I transition is continuous for $\alpha<0.05$, and discontinuous for $\alpha>0.05$.

For values of the forcing strength $\alpha < 0.05$, where the C-I transition is continuous, l varies with l_0 approximately as a power law:

$$l/l_1 = 1 - A[(l_c - l_0)/l_1]^{\beta} ,$$

where l_c is the unforced roll period at the edge of the step. We estimate that $\beta = 0.4 \pm 0.1$ near the tricritical point ($\alpha \approx 0.05$). For $\alpha = 0.03$, the exponent β increases to 0.6 ± 0.1. Unfortunately, the precision of the data at present does not allow accurate determination of the exponents to be made.

If the variation of l with l_0 is examined over a wide range (i.e., from about $l_0/l_1 = 0.33$ to 1.0), a graph resembling the "devil's staircase"[3] is obtained with steps at the commensurate values listed earlier.

IV. DISCUSSION AND COMPARISON WITH THEORETICAL MODELS

The patterns described in Secs. II and III are essentially one dimensional. However, it is important to recognize that defects do occur in the soliton lattices. Sometimes the solitons are not parallel to the rolls, and they occasionally end at dislocations in the roll pattern. An example of this phenomenon is shown in Fig. 7. In some regions of the phase diagram (for example, the diagonally shaded region of Fig. 1), the dislocations are so numerous that we classify the patterns as disordered. These states will be described elsewhere.[8]

Even though these experiments are conducted on a convecting nematic liquid crystal, we believe that, neglecting experimental difficulties, similar phenomena would also be obtained in spatially modulated Rayleigh-Bénard convection. There has been some consideration of the problem of periodically forced thermal convection from the standpoint of linear stability theory. However, only commensurate states were considered.[9] (It is possible that the phase modulation seen in our experiments is related to the phase modulation expected for the Eckhaus instability of convection rolls.[10]) This problem is worthy of further theoretical work based either on the full hydrodynamic equations, or on simplified model equations of the type that have been devised to study convective patterns.

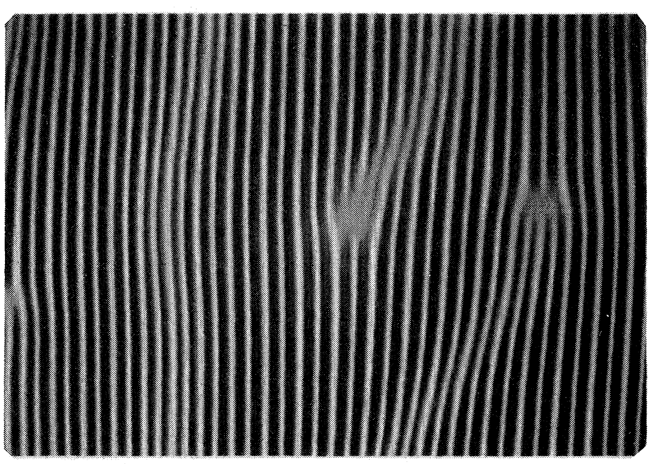

FIG. 7. Photograph of a dislocation pair that is a defect in the soliton lattice ($l_0/l_1 = 0.834$ and $\alpha = 0.03$).

Although a theoretical treatment applicable to the present experiments is lacking, we have found a strong correspondence between our observations and the behavior of the Frenkel-Kontorova (FK) model, in which a one-dimensional array of particles is connected by harmonic forces in the presence of a spatially periodic external potential.[2] It is known that for suitable potentials, solitons mediate the C-I transition in this model. Since the convective rolls are characterized by a preferred size, it is not implausible to suppose that compression of the rolls might be resisted by elastic forces. (Of course the dissipation present in our convective system will certainly limit the applicability of the model to steady-state phenomena. Relaxation in the roll positions would not be properly described by the FK model.)

We have quantitatively compared our observations with the behavior of the FK model for a weak sinusoidal potential. In this limit, the phase variable ϕ_n may be regarded as a continuous function of position, $\phi(x)$. Frank and van der Merwe[11] showed that in this continuum limit, $\phi(x)$ is a solution of a sine-Gordon equation. We found that excellent nonlinear fits to our data can be constructed using general solutions to the one-dimensional time-independent sine-Gordon equation. These solutions have the form

$$\frac{\phi(x)}{2\pi} = a_1 - \frac{1}{\pi} \mathrm{am} \left[\frac{2K}{s} x + \delta \right] ,$$

where am(u) is the amplitude of an elliptic integral of the first kind. In this expression, s is the distance between solitons, and K, a_1, and δ are constants. The minus sign is chosen to describe compression of the rolls. The solid lines in Figs. 3(b) and 4 represent nonlinear least-squares fits to our measured phase variation. We find that the data are quantitatively described by this function for $\alpha < 0.05$, thus supporting the use of the term "soliton" and the applicability of the FK model for weak external forcing.

For strong external forcing, it is unclear whether the FK model is relevant. It is perhaps worth noting that the C-I transition in the FK model does become discontinuous as the forcing strength is increased.[3] Multicritical points have also been noted in a theoretical model of systems exhibiting transitions between disordered and incommensurate states.[12]

It would be desirable to compare the widths of the commensurate regions observed experimentally with the FK (and other) models. These widths have been calculated for the FK model with a piecewise parabolic potential.[3] Presumably the results for a sinusoidal potential would be similar for strong forcing, where the masses are confined near the minima of the potential. Both the model and our experiments show a wide 1/1 commensuration and a narrower 3/4 commensuration at high α. Commensurate states between these should be too narrow to observe if the forcing is strong, as we find experimentally. In order to make a quantitative comparison of commensurate widths with model predictions, it would be necessary to make much more detailed (and time-consuming) measurements of the shapes of the commensurate boundaries. However, the quantitative description of the structure of the soliton lattice (for weak forcing) by solutions to the sine-Gordon equation suggests that models of this type may be appropriate, despite the fact that they were intended to describe equilibrium systems.

The primary result of this investigation, the discovery of soliton lattices in a periodically forced convecting fluid, is of course independent of any particular theoretical model.

ACKNOWLEDGMENTS

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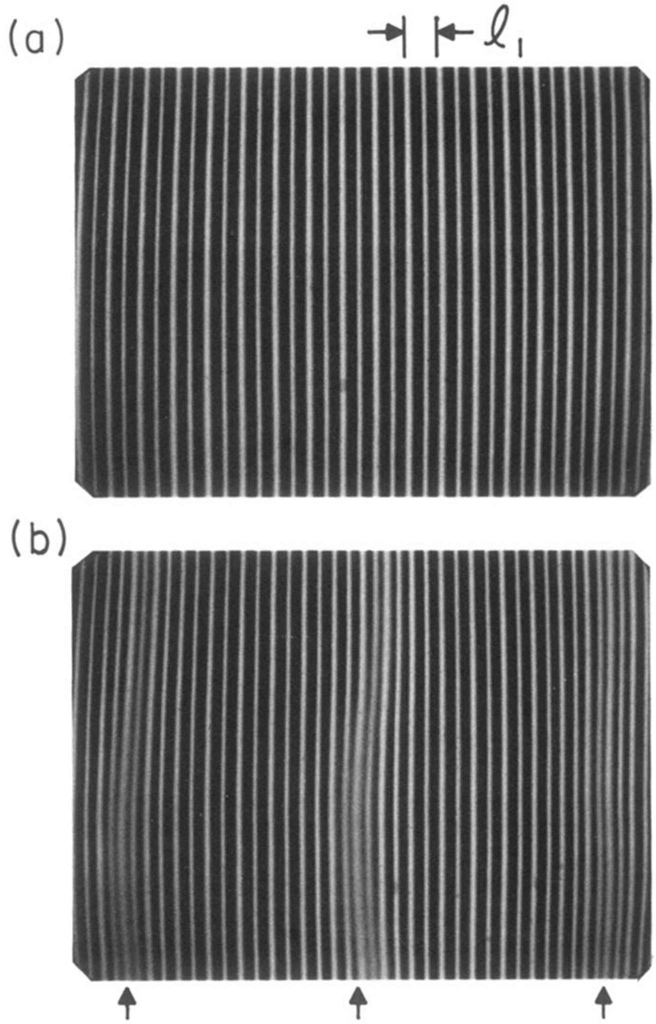

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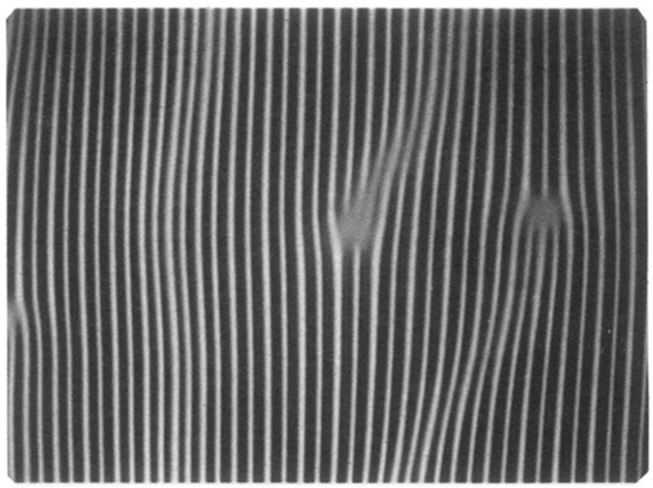

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