Volume 14 | Issue 1

Article 5

1968

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THE POWER INDEX AND THE ELECTORAL COLLEGE: A CHALLENGE TO BANZHAF'S ANALYSIS

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DISCUSSIONS OF THE ELECTORAL COLLEGE have tended to be dominated by arguments that are in two senses unduly partial. There is the incomplete account which illuminates one part of a complex institution with illusions of balanced consideration — the Electoral College favors the smaller States because of the bonus votes, says one, or the larger States because of a peculiar distribution of the nation's Democrats and Republicans, according to another, for example. And then there is the defense of parochial interests in universal terms — a small-State Senator predictably advances the district plan in a Symposium in this *Review*.[1]

As the Symposium demonstrated, it is possible even now to make contributions to the debate on the Electoral College with perspective and some impartiality. Neal Peirce's accounting of the quirks of the present system[2] is a good case in point, certainly. John Banzhaf's intricate mathematical model of power in the presidential electoral system,[3] to which I shall devote my attention, is harder to assess. It has its uses, I think, but its view of the electoral process is distorted. Banzhaf's Article brings modern insights to bear on a venerable problem at a time when my own research suggests that the legal community is drawing less and less on nonlegal information from disciplines such as mathematics, economics, psychology, and sociology. Mathematical models at their best are comprehensive and objective, usefully relating large amounts of information with unflagging precision, but they, like nonquantitative explanations, are subject to bias. In borrowing mathematical techniques, and in interdisciplinary analysis generally, the most is made of new insights by holding them to the critical standards of both the borrower's and the lender's disciplines.

A number of models of electoral power have been applied to the Electoral College, with different results. How are we to choose among them? (1) Conventional wisdom credits each vote in a State with

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1. Mundt, *Reflections on the Electoral College — Comment*, 13 VILL. L. REV. 336 (1968). For a description of the district plan see Banzhaf, *One Man, 3.312 Votes: A Mathematical Analysis of the Electoral College*, 13 VILL. L. REV. 304, 320 (1968).

2. Peirce, *Reflections on the Electoral College — Comment*, 13 VILL. L. REV. 342 (1968).

equal power and each vote in the Electoral College with equal power, so that the power of a voter can be expressed as:

$$\frac{\text{number of electoral votes in the State}}{\text{number of voters in the State}}$$

or the voter's share of the State's electoral power. (A variation considers total population rather than voters.) (2) The Shapley-Shubik a priori index, widely used by students of political behavior and the basis of every study of power cited by Banzhaf, other than his own, arranges voters in every possible permutation (ABC, ACB, BAC, BCA, etc.), counts off from the first of each permutation to the voter completing the required majority, and credits him with pivotal power in that permutation and with overall power equal to the proportion of the list of permutations in which he is pivotal.[4] (3) The Banzhaf index of electoral power consists in arranging the voters in any one permutation, tabulating all possible combinations of votes for and against a candidate or issue,[5] and finding the proportion of combinations in which a voter has the power to change the outcome by switching his vote.[6] Two very different ways of expressing this proportion, which I shall call versions "A" and "B", have appeared in print.[7]

The Shapley-Shubik and Banzhaf "A" indexes prove preferable in bloc- or weighted-vote situations, wherever the electoral rules require voters to act in concert (as States now typically instruct presidential electors) or give a voter a number of ballots to cast together (as in the case of legislative bodies where one member casts three votes, another five, and so forth.) Contrary to conventional wisdom, the indexes show that the power of a weighted or bloc vote may exceed or fall short of its simple numerical share of the total. We may take some obvious examples. If there are three voters in a simple-majority system, with weights of 3, 3, and 1, all have the same effective power because each may combine with another to form a winning majority. On the other hand, with votes weighted 3, 3, 3, and 1, the latter has no effective power because he can never change a losing to a winning coalition. The Shapley-Shubik and Banzhaf "A" indexes comprehend these situations

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4. Shapley & Shubik, *A Method for Evaluating the Distribution of Power in a Committee System*, 48 AM. POL. SCI. REV. 787 (1954).

5. For example where (+) indicates a vote for an issue or candidate and (—) indicates a vote against the issue or candidate, some of the possible voting combinations are: A(+) B(+) C(+), A(+) B(+) C(—), A(+) B(—) C(—), and A(—) B(—) C(—).

6. Banzhaf, *supra* note 1, at 314 n.32.

7. For version "A" see Banzhaf, *Weighted Voting Doesn't Work: A Mathematical Analysis*, 19 RUTGERS L. REV. 317, 334 (1965); for "B" see Banzhaf, *Multi-Member Electoral Districts — Do They Violate the "One Man, One Vote" Principle,*

and all of the less spectacular ones between, giving closely similar values in small electorates or legislative bodies. For example, to a body of five with weights of 5, 5, 3, 3, and 1, Banzhaf's version "A" ascribes power in shares of 2/7, 2/7, 1/7, 1/7, and 1/7, respectively,[8] not unlike the Shapley-Shubik distribution of 9/30, 9/30, 4/30, 4/30, and 4/30;[9] and for a body whose members cast votes weighted 4, 2, 1, 1, and 1, respectively, the Banzhaf "A" proportions of 7/11, 1/11, 1/11, 1/11, and 1/11[10] parallel Shapley and Shubik's 6/10, 1/10, 1/10, 1/10, and 1/10.[11]) Their general approach, ascribing shares of power more realistically than common sense and expressing them as fractions of one, is rightly regarded as a major contribution to the understanding of elections.

In the absence of weighted or bloc voting, where equally-weighted votes may be cast independently, and of multicameral configurations, it may be added, the conventional, Shapley-Shubik, and Banzhaf "A" indexes are equally valid, giving the trivial result that each voter has an equal share of the available power. But the Banzhaf "B" version differs radically. In it, he assigns each voter a power "percentage" equaling the share of pivotal votes he casts in the entire schedule of combinations — 50 percent in systems with three voters, 37.5 percent in systems with five, and so on. The "percentage" times the number of people to be elected equals the voter's total power or influence within his electoral district. It is version "B" that Banzhaf has applied first to multimember districting[12] and now as a component in a compound index to the Electoral College, neither making it clear, I think, that he has turned from "A" to a new formula nor making the essential distion between electoral power in bloc- or weighted-vote situations and those (like multimember legislative districts or the popular vote stage of presidential elections) that lack these characteristics and have autonomous voters with equally-weighted ballots instead.

In his study of multimember districting, Banzhaf has applied his "B" formula as if the legislators elected from each multimember district voted en bloc in the legislature — or, to put it differently, as if the question of whether they were forced by the rules to do so were irrelevant — an assumption that goes against reality. In his study of the Electoral College he assumes more realistically that a State's elec-

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8. Banzhaf, *Weighted Voting Doesn't Work: A Mathematical Analysis,* 19 RUTGERS L. REV. 317, 334 & n.38 (1965).
9. Computed from the a priori power index set forth in Shapley & Shubik, *supra* note 4.
10. Banzhaf, *supra* note 8, at 334 & n.39.
11. Computed from the a priori power index set forth in Shapley & Shubik, *supra* note 4.
12. Banzhaf, *Multi-Member Electoral Districts — Do They Violate the "One

toral votes will typically be cast en bloc. But in this case he counts the bloc effect twice — squaring the advantage large blocs have over the small — and he therefore comes to an inflated assessment of the bias in the Electoral College.[13] The Shapley-Shubik index avoids this error; it can be applied to the Electoral College in a number of ways (the main choice is whether to use permutations of individual voters or of States), but in no case is the large-State advantage squared. In the Electoral College study, Banzhaf actually combines the Shapley-Shubik index as applied to States with his formula "B" as applied to individual voters within the States. Even if we assume that version "B" and the Shapley-Shubik index are interchangeable, to analyze presidential elections as two stages, find a large-State bias in each stage, and multiply the advantage by itself is a procedure few are likely on reflection to endorse. In Electoral College analysis, the difference between the Shapley-Shubik index applied to individuals or States and Banzhaf's use of version "B" *times* Shapley-Shubik's State power analysis is the difference between bias of tens and of hundreds of percentage points.

Still another difficulty occurs in version "B": it ignores combinations with more than a bare majority on either side. According to this test, power inheres only in precisely marginal votes; a vote in which one side outnumbers the other by more than one is power-free by definition. If the effect of focusing on a fraction of all combinations were only to sample the universe of hypothetical votes representatively, the measure's selectivity would be unobjectionable, but instead the sample size, the proportion of elections which are found marginal, is itself a factor in the final datum — the "percent influence" (expressing a relation of the voter to the number of combinations in the electoral system, with and without power, rather than to other voters as in the typical weighted-vote analysis).

The other ingredient of the "percent influence" in version "B" is pivotal power, the power to change a vote. Unlike Shapley and Shubik's pivotal power, which begins with the assumption that each participant has an equal chance to occupy a pivotal position and distributes credits

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13. The squaring is described in general terms by Banzhaf, *supra* note 1, at 312–13:
    The analysis of the existing Electoral College is a two step process. First, all of the different possible arrangements of *electoral* votes are examined . . . and a determination is made of those arrangements in which any given state, by a change in the way it casts its bloc of electoral votes, could change the outcome of the election. As the second step one looks to the people of the state, and determines in how many of the different voting combinations involving people of that state any given resident could affect, by changing his vote, how that state's bloc of electoral votes would be cast. Finally, the results of the two steps are combined . . . . (footnote omitted).
In this way a formula attributing an advantage to large States is multiplied by a formula attributing an advantage to voters in such States, counting a single characteristic of the electoral system twice.

first to one, then to the next, and so forth, permutation by permutation, Banzhaf's pivotal power is the shared threat of defection from a bare majority. Power so conceived is common to a majority, although the threat can be carried out by only one person at a time, and it follows that the sole instance in which pivotal power belongs to one voter uniquely is a one-voter district. In other words, the Banzhaf "B" measure of power or participation first focuses on a fraction of the combinations (in elections of real-world proportions the percentage of precisely marginal votes in small) and then ascribes decisive power in each of these elections to members of the entire majority. By contrast, the Shapley-Shubik index finds power in all possible voting arrangements but ascribes pivotal power to only one participant in each. Since the size of the majority increases more sharply than the frequency of bare majorities decreases, as one moves from smaller to larger districts or States, the Banzhaf index shows a net increase which Banzhaf calls an increase in influence or power. One may well ponder the relevance of this index to electoral power.

It should be understood that there is no available empirical proof or disproof of any of these measures as applied to the Electoral College. One must analyze them for plausibility and consistency in the abstract. That the large States apparently have been favored (in the parties' choice of candidates and issues, for example) proves nothing about the extent of a large-State bias in the rules of the game, since it is not hard to suggest other reasons for the phenomenon.

The reader with patience and inclination is encouraged to work through the literature cited by Banzhaf. It has been my limited purpose to offer a caveat: even a mathematical model may be less than fully explanatory or fully objective, with disabilities paralleling those of the less arcane qualitative approaches. Biases can be numerical as well as political. There is still need for careful mathematical and nonmathematical exposition of the workings of the Electoral College.