Missouri University of Science and Technology
Scholars' Mine

Symposia on Turbulence in Liquids | Chemical and Biochemical Engineering

01 Jan 1971

Parameter for Comparing Anemometer Response

L. M. Fingerson

Follow this and additional works at: https://scholarsmine.mst.edu/sotil

Part of the Chemical Engineering Commons

Recommended Citation

Fingerson, L. M., "Parameter for Comparing Anemometer Response" (1971). *Symposia on Turbulence in Liquids*. 73.
https://scholarsmine.mst.edu/sotil/73

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in Symposia on Turbulence in Liquids by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact scholarsmine@mst.edu.

PARAMETER FOR COMPARING ANEMOMETER RESPONSE

Leroy M. Fingerson
Thermo-Systems Inc.
2500 N. Cleveland Avenue
St. Paul, Minnesota 55113

ABSTRACT

In comparing the theoretical performance of sensors, the term e_b/q (voltage change due to change in heat transfer) was found to be a fundamental sensor parameter. This was true for both sensitivity and frequency response in both constant current and constant temperature systems. Equations and data are presented for calculating this parameter for most wire and film sensors. Wedges, cones, and other non-cylindrical sensors have not been included because of their special problems at low frequencies due to conduction losses[9].

Internal cooling of the sensor is used for high temperature environments. Calculations show that the sensitivity at low frequencies as represented by signal-to-noise ratio decreases with cooled sensors. At high frequency, a significant improvement in frequency response can be attained with cooling, particularly when the heat transfer rate between the sensor and its external environment is very low.

The influence of insulating films on the sensor response is also discussed. Both generalized curves and specific data for quartz and polyester are given. In general, modern techniques make it possible to put on protective films that have negligible influence on frequency response.

The extremely high stability of an anemometer system for steady state measurements is shown. The difficulties of contamination, resistance shifts due to strain, and oxidation on tungsten wires has tended to obscure this basic stability. However, with film type sensors and with hot wires in some environments this stability can often be realized.

INTRODUCTION

The primary goal of this paper is to give the reader a better understanding of the theoretical and practical characteristics (and limitations) of the constant temperature hot-wire (or hot-film) anemometer. Noise, frequency response, and stability are particularly important parameters when determining the ultimate performance of this instrumentation.

When considering the time response of a sensing element exposed to a fluid, the rate the sensor alone responds to environment changes is a basic parameter and is considered first. For a sensor that is a temperature sensitive resistor, adding a bridge circuit and some current is the next step. Just enough current to "sense" the resistance but not enough to heat the sensor gives a resistance thermometer. Increasing the current to heat the sensor above the environment temperature gives an uncompensated constant current anemometer.

In the uncompensated constant current anemometer, it is shown that for small signals the relation between changes in bridge balance, e_b, and changes in heat transfer between the sensor and its environment, q, is a fundamental parameter. It gives the expected electrical signal size for given frequency and magnitude changes in the environment. This "sensitivity" factor is fundamental to both signal to noise ratio and frequency calculations.

A primary point of this paper is that the above parameter is also fundamental to a constant temperature system. For signal to noise calculations, one need only compare the equivalent input noise of the amplifier with the expected signal size. This means that in a constant temperature system it is not necessary to consider the effect of the feedback for noise calculations. Also, for the same bridge circuit and equivalent input noise of the amplifier, the signal to noise ratio of a given signal within the frequency range of the instrument is the same for both constant temperature and constant current anemometers.

The "sensitivity" of various sized hot-wire and hot-film sensors (including cooled-film sensors) is established in terms of heat transfer changes between the sensor and the environment. Using calibration data or heat transfer relations this can then be expressed in terms of velocity sensitivity or, if desired, temperature sensitivity.

Other considerations in anemometry that can be treated theoretically are conduction losses to supports and the influence of electrically insulating coatings on sensors. These are discussed briefly.

SENSOR CHARACTERISTICS

1. The Resistance Thermometer

In deriving the response equations, the concept of "thermal impedance" of the sensor will be used[1,2]. This is defined as:

$$Z = \frac{t_s}{q} \tag{1}$$

This term relates how the "sensed" parameter (sensor temperature, t_s) is affected by changes in heat transfer, q, between the sensor and its environment.

For a simple wire (Fig. 1), ignoring end losses, thermal impedance can be expressed as[2]:

$$Z = \frac{1}{i\omega C_T} = \frac{4}{i\omega d_o D_p S_o \rho} \tag{2}$$

Eq. 2 already indicates the desirability of using very fine wires to maximize thermal impedance. The hot wire is considered as a pure thermal capacity in Eq. 2 and the similarity to the equation for the electrical impedance of an electrical capacitor is evident.

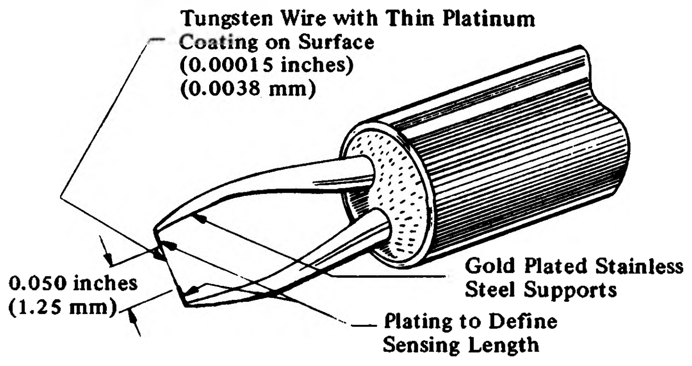


Figure 1 Typical Hot-Wire Sensor

For a film, the thermal impedance was derived by Ling[1] and extended to cooled films by Fingerson[2]. The general expression is:

$$Z = \frac{1}{k_g S_o \sqrt{\omega/\alpha}\ (C+Di)} \tag{3}$$

Figure 2 gives values of C and D as a function of a $\sqrt{\omega/\alpha}$. The asymptotes at low and high frequencies are particularly interesting. At low frequencies, the film sensor behavior approaches that of the simple hot wire since the entire sensor responds to changes

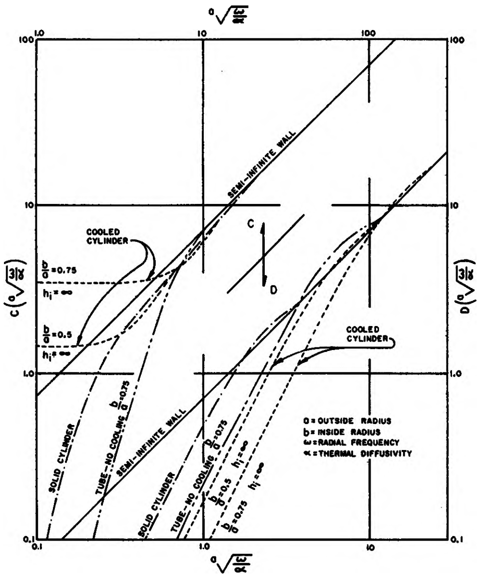


Figure 2 Thermal Impedance Functions C and D

in heat flux (q). At high frequencies ($\omega \to \infty$), both C and D approach $1/\sqrt{2}$. This is the result if the calculation is carried out for a semi-infinite wall. Physically, at high frequencies the "thermal wave" does not penetrate far into the substrate. Under these conditions the radius of curvature and finite diameter of the cylindrical sensor has a negligible effect and the system responds the same as the semi-infinite wall. Fig. 3 gives some typical curves for Z vs frequency for hot wires and cylindrical film sensors.

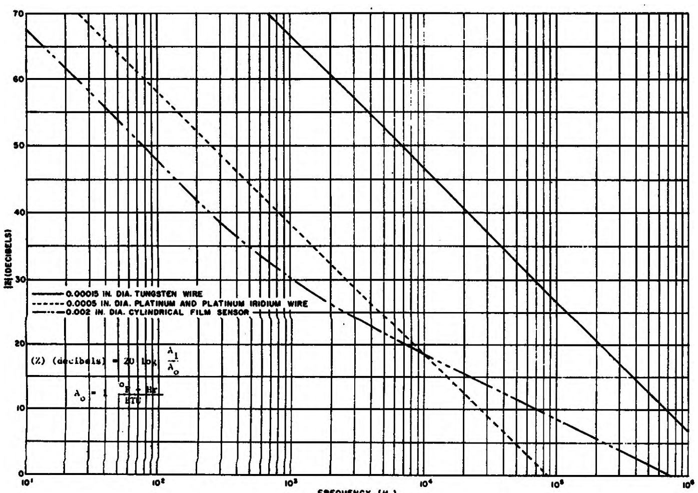
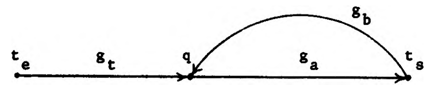


Figure 3 Thermal Impedance as a Function of Frequency for Typical Sensors

Utilizing signal-flow graph techniques[3,4], the diagram of an ideal resistance thermometer can be shown as follows:



Using the rules given in Appendix I:

$$\frac{t_s}{t_e} = \frac{g_t\, g_a}{1 - g_a g_b} \tag{4}$$

If the heat transfer equation between the sensor and its environment is:

$$Q = h_o S_o\ (T_e - T_s) \tag{5}$$

Then, by differentiating Eq. 5:

$$\frac{dQ}{dT_e} = \frac{q}{t_e} = g_t = h_o S_o \tag{6}$$

$$\frac{q}{t_s} = g_b = -h_o S_o \tag{7}$$

and, substituting back into Eq. 4:

$$\frac{t_s}{t_e} = \frac{1}{1 + \frac{1}{Z\, h_o\, S_o}} \tag{8}$$

Substituting Z from Eq. 2 or 3, one can determine the ability of the sensor temperature, t_s, to follow the environment temperature, t_e, for a given frequency, ω.

Figure 4 shows the normal circuit configuration for both a temperature sensor and a constant current anemometer. For "constant" current, $R_3 \gg R_p$ or an electronic constant current source is used rather than the constant voltage source, E, shown.

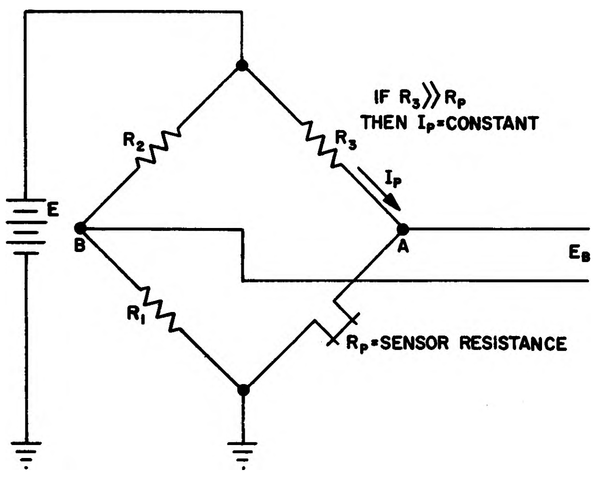
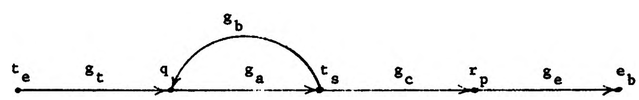


Figure 4 Constant Current System

A complete bridge is not a requirement but is generally convenient for biasing the amplifier and for steady state measurements. At "balance", $E_b = 0$.

For the circuit of Figure 4, the signal flow diagram is:



The associated equation is:

$$\frac{e_b}{t_e} = \frac{g_t\, g_a\, g_c\, g_e}{1 - g_a\, g_b} = \left(\frac{t_s}{t_e}\right) g_c\, g_e \tag{9}$$

This gives the voltage change, e_b, due to a temperature change, t_e, of the environment.

To calculate g_c, the resistance temperature characteristic of the sensor must be known. For present purposes, the following linear relation is adequate:

$$R_p = R_r\,[1 + \lambda(T_s - T_r)] \tag{10}$$

From Eq. 10 one obtains:

$$g_c = \frac{dR_p}{dT_s} = \frac{r_p}{t_s} = \lambda R_r \tag{11}$$

for the transfer function, g_c.

For the remaining transfer function, the voltage across the bridge, E_b, is:

$$E_b = I_p\,R_p - E_F \tag{12}$$

From Eq. 12:

$$\frac{i_p}{r_p} = g_e = I_p\,(1 - \frac{R_p}{R_p + R_3}) \tag{13}$$

Finally, substituting into Eq. 9:

$$\frac{e_b}{t_e} = \frac{\lambda R_r I_p\,(1 - \frac{R_p}{R_p + R_3})}{1 + \frac{1}{Z\,h_o\,S_o}} \tag{14}$$

Eq. 14 gives the change in output voltage, e_b, caused by a change in environment temperature, t_e, for a resistance thermometer.

From Eq. 14 some simplifying assumptions can be made. For the "constant current" case where $R_3 >> R_p$:

$$\frac{e_b}{t_e} = \frac{\lambda R_r\,I_p}{1 + \frac{1}{Z\,h_o\,S_o}} \tag{15}$$

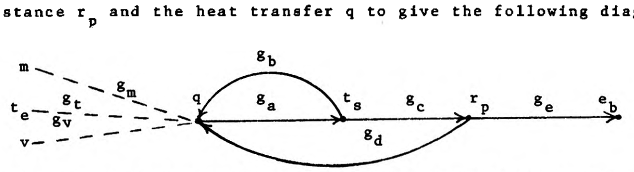
For the "steady state" case, where $Z \to \infty$ as $\omega \to 0$, we have:

$$\frac{e_b}{t_e} = \lambda R_r\,I_p \tag{16}$$

Therefore, in the simplest case the sensitivity of the resistance thermometer depends on the product of the sensor reference resistance and temperature coefficient of resistance, and the current in the sensor. The maximum current is, of course, limited to the maximum allowable self-heating. Frequency response is still represented by Eq. 8, since all that the additional terms did was give a magnitude for the voltage signal. In other words, no frequency dependent terms were added.

2. Constant Current Operation

For a constant current anemometer, the "sensing" current, I_p, is large enough to heat the sensor above the environment temperature. This adds a feedback loop, g_d, between the sensor resistance r_p and the heat transfer q to give the following diagram:



Once the sensor is heated, composition changes and velocity changes affect heat transfer, q, as well as temperature changes. For analysis, it is convenient to deal only with changes in heat transfer, q. The relation between changes in the environment and the heat transfer, q, can then be handled independently.

The equation for the above diagram is:

$$\frac{e_b}{q} = \frac{g_a\,g_c\,g_e}{1 - g_a\,g_b - g_a\,g_c\,g_d} \tag{17}$$

The equation for electrical heat input to the sensor is:

$$Q = I_p^2\,R_p \tag{18}$$

then:

$$g_d = \frac{q}{r_p} = I_p^2\,(1 - \frac{2R_p}{R_p + R_3}) \tag{19}$$

Putting this back into Eq. 17 gives:

$$\frac{e_b}{q} = \frac{\lambda R_r\,I_p(1 - \frac{R_p}{R_p + R_3})}{\frac{1}{Z} + h_o S_o - \lambda R_r I_p^2(1 - \frac{2\,R_p}{R_p + R_3})} \tag{20}$$

Eq. 20, as is true of the entire derivation above, is not new or unique. However, generally Eq. 2 has been substituted for thermal impedance, Z, and velocity is substituted for q using heat transfer relations. The form of Eq. 20 keeps it very general.

As expressed above, the restrictions on Eq. 20 are few. Dynamic effects of conduction losses to the supports can be considered in the thermal impedance term. Bridge resistors are assumed non-reactive. Cable resistance can be included in a more complex, frequency dependent, expression for R_p. Figure 5 shows comparisons of sensitivities, e_b/q, of typical sensors as a function of frequency.

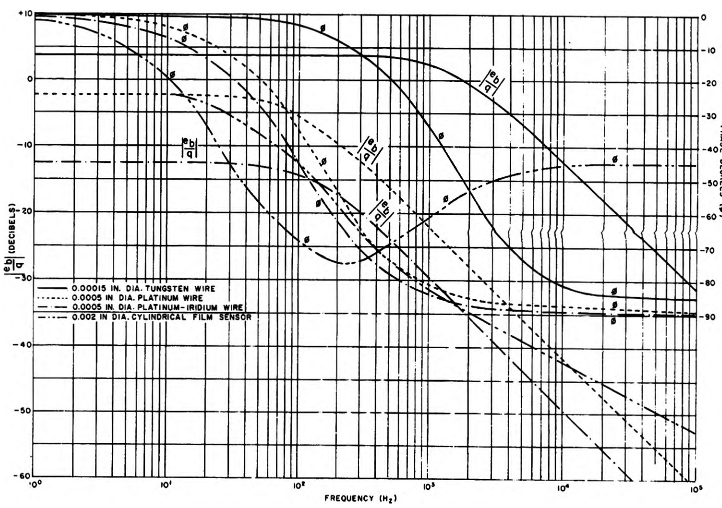


Figure 5 Sensitivity Comparison for Typical Anemometer Sensors

It should be emphasized that Eq. 20 only expresses the voltage output due to a change in heat transfer to the "sensitive" portion of the sensor. Measurement errors can occur from frequency dependent terms in the fluid boundary layer or in the conduction losses to the supports. These errors are associated with frequency dependent changes in the relation between velocity, for example, and heat transfer to the sensitive portion of the sensor. Figure 6 shows effects of velocity on sensitivities of a typical wire probe and a typical cylindrical film probe.

Equation 20 gives the "signal" for signal to noise calculations in constant current anemometry. Once the results of Eq. 20 are known, one needs only to know the noise characteristics of the

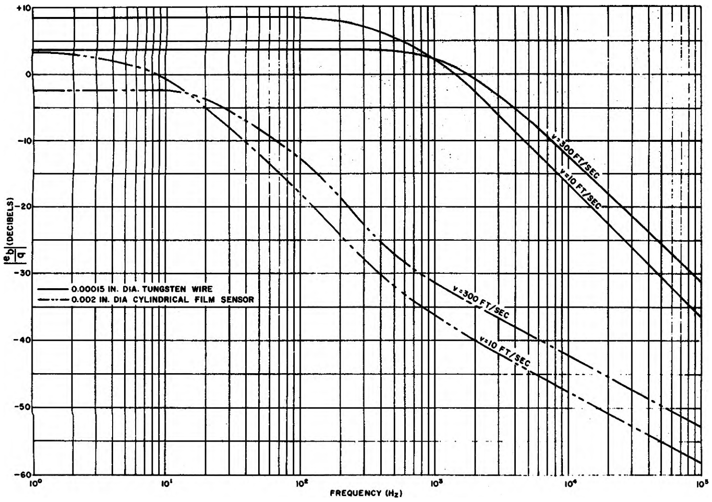


Figure 6 Effect of Velocity on Sensitivity of Typical Sensors

amplifier being used to determine the signal to noise ratio of the anemometer system.

For making comparisons some simplifying assumptions are helpful. For example for constant current operation ($R_3 >> R_p$), Eq. 20 becomes:

$$\frac{e_b}{q} = \frac{\lambda R_r\ I_p}{\frac{1}{Z} + h_o S_o - \lambda R_r I_p^2} \tag{21}$$

If one substitutes for $h_o S_o$ from Eqs. 5, 10, and 18, then for steady state conditions ($\omega \to 0$, $Z \to \infty$):

$$\frac{e_b}{q} = \frac{1}{I_p}\left(\frac{R_p}{R_e} - 1\right) \tag{22}$$

For high frequencies ($\omega \to \infty$), the $\frac{1}{Z}$ term dominates and Eq. 21 becomes:

$$\frac{e_b}{q} = \lambda Z\ R_r\ I_p \tag{23}$$

Both Eqs. 22 and 23 are useful for comparison purposes.

Equation 23 states that the temperature coefficient, reference resistance, sensor current, and thermal impedance are the only factors that determine the high frequency response of a sensor. It must be remembered that we are talking about response to a given change in heat transfer to the sensor, q, and not a velocity change. Also, both I_p and Z are dependent on sensor size.

For the steady state signal, only the sensor current is size dependent. Therefore, for a given "overheat", R_p/R_e, the sensitivity goes down as current increases. Again, it is important to remember that "q" is a given change in heat transfer - which presumably has a smaller influence on a large sensor. For a given sensor and environment, an increase in I_p actually increases e_b/q because of the influence on sensor temperature (and therefore R_p).

3. Constant Temperature Operation

The constant temperature anemometer system utilizes a bridge circuit similar to that of Fig. 4 but adds negative feedback as shown in Fig. 7. In this system, rather than the current being constant (or nearly so), the current is varied to maintain the bridge at (or near) balance. A balanced bridge implies that the sensor temperature is constant. It is for this reason the system is called a constant temperature system.

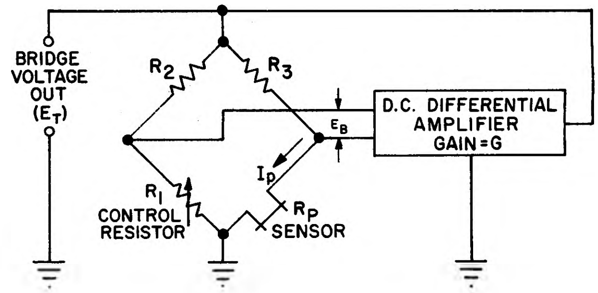
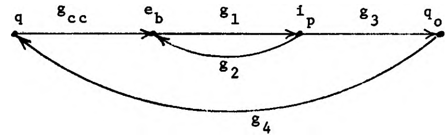


Figure 7 Schematic of Constant Temperature Anemometer System

Three additional transfer functions are added for the constant temperature system as shown in the following signal flow diagram:



In the above, g_{cc} is the transfer function (e_b/q) derived for the constant current system (Eq. 20), without the $R_3 >> R$ assumption. The transfer function g_1 is the transconductance of the amplifier. With an initial bridge "off-balance", a change in current through the sensor will affect the off-balance (e_b) and is represented by the feedback loop g_2. The transfer function g_3 is the change in heat transfer to the sensor due to a change in sensor current, i_p, caused by the feedback from the amplifier. This change is compared with the input change in q to determine closed loop gain.

The closed loop gain of the system (G_{CL}) is

$$G_{CL} = \frac{g_{cc}\ g_1\ g_3}{1 - g_1 g_2 - g_{cc}\ g_1 g_3 g_4} = \frac{q_o}{q} \tag{24}$$

The transfer functions g_1, g_2, g_3 and g_4 are derived in Appendix II. Substituting for these, the closed loop gain is:

$$G_{CL} = \frac{-1}{-\frac{(R_p + R_3)K}{2 I_p\ R_p\ G E_t (e_b/q)} + 1} \tag{25}$$

For perfect response, $G_{CL} \to -1$. Equation 25 is useful both for determining frequency response and for stability considerations. For the latter the frequency dependence of the amplifier and possible reactance terms in the cable, bridge resistors, etc., must be included. For present purposes, only frequency response is being considered. Figure 8 gives some calculated curves of G_{CL} assuming the sensor as the only frequency dependent component.

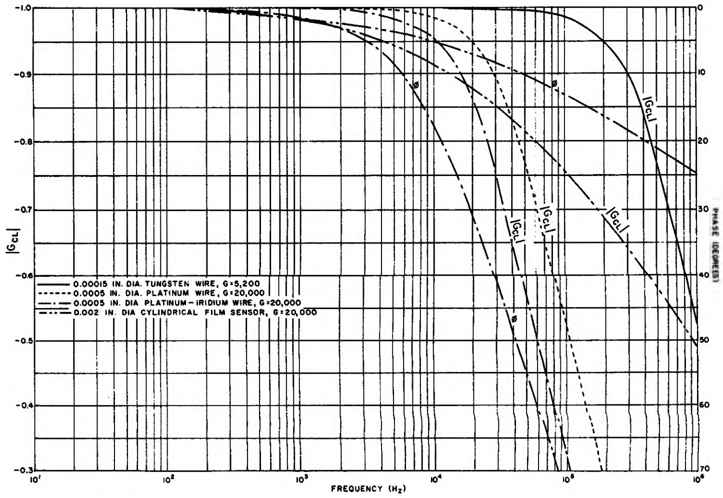
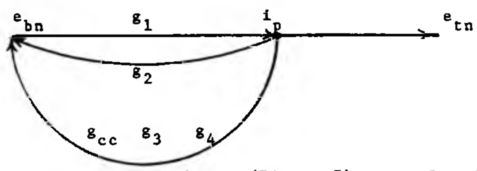


Figure 8 Amplitude Response of Anemometer System with Constant Temperature Compensation at Indicated Gain

With the constant current system, the parameter e_b/q permitted simple separation of amplifier characteristics from the sensor and bridge. The feedback system in the constant temperature system makes the possibility of separating these much less obvious. To examine this, the voltage change at the amplifier output, e_{tn}, due to a noise voltage at the amplifier input, e_{bn}, will be explored. This can then be compared with the output voltage, e_t, due to a change in heat transfer, q, between sensor and environment.

To examine the "noise" voltage, the signal flow diagram is:



Sensor current and bridge voltage (Figure 7) are related by the equation:

$$E_t = I_p\ (R_p + R_3) \tag{26}$$

Therefore:

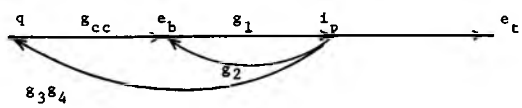
$$\frac{e_{tn}}{i_p} = R_p + R_3 \tag{27}$$

and

$$\frac{e_{tn}}{e_{bn}} = \frac{g_1\ (R_p + R_3)}{1 - g_1\ g_2 - g_1\ g_{cc}\ g_3\ g_4} \tag{28}$$

Equation 28 would, of course, be valid for any input voltage signal, e_b.

The change in output, e_t, due to a change in heat transfer between sensor and environment, q, is represented by:



The resulting equation is:

$$\frac{e_t}{q} = \frac{g_{cc}\ g_1\ (R_p + R_3)}{1 - g_1\ g_2 - g_{cc}\ g_1\ g_3\ g_4} \tag{29}$$

The signal-to-noise ratio at the output will be e_t/e_{tn}. Combining Eqs. 28 and 29 gives:

$$\frac{e_t}{e_{tn}} = \frac{g_{cc}\ q}{e_{bn}} \tag{30}$$

Since:

$$g_{cc} = e_b/q$$

then:

$$\frac{e_t}{e_{tn}} = \frac{e_b}{e_{bn}} \tag{31}$$

Therefore, defining the signal-to-noise ratio at the amplifier input terminals gives the same results as defining it at the amplifier output even though the system is in a closed loop. The term e_b/q in Eq. 20 then has the same significance for both the constant temperature system and the constant current system.

The results above indicate that for a given bridge, sensor, and amplifier, the signal to noise ratio is identical for both the constant temperature and constant current anemometer system. (Appendix III discusses the apparent discrepancy between this conclusion and sensitivity definitions in previous reports[12,13]).

4. Calculating Signal to Noise Ratio

From Eq. 20 the input signal to the amplifier can be determined. To obtain signal to noise ratio the noise source in anemometry must be considered. Freymuth[8] has done a rather complete analysis of noise sources. The total mean square noise from the bridge and the amplifier input is:

$$\overline{e_{bn}^2} = 4k\ T_o\ \Delta f\{R_{eq} + \frac{R_p\ [(1 + R_p/R_3)\ (1 + R_1/R_p) + \Delta T/T_o]}{(1 + R_p/R_3)^2}\} \tag{32}$$

If $R_3 >> R_p$, then Eq. 32 reduces to:

$$\overline{e_{bn}^2} = 4k\ T_o\ \Delta f\ [R_{eq} + R_1 + R_p\ (1 + \Delta T/T_o)] \tag{33}$$

Eq. 33 represents a maximum since the net effect of decreasing R_3 is to reduce the value of $\overline{e_{bn}^2}$. For the anemometer system, R_3 should be kept high because of its influence on the signal size, e_b.

If the bridge resistors can be ignored compared with the amplifier, then Eq. 33 reduces to:

$$\overline{e_{bn}^2} = 4k\ T_o\ \Delta f\ R_{eq} = \overline{e_a^2} \tag{34}$$

Most anemometer amplifiers now in use have an R_{eq} of 250 or above. In this case Eq. 34 is sufficient since most anemometer sensors operate at resistances of 20 ohms or less. The appearance of R_1 in Eq. 33 does indicate the desirability of a low bridge ratio, R_1/R_p, on the anemometer. The value of R_1 can be eliminated in an AC coupled, constant current system by not including the bridge.

On anemometers now available the value of R_{eq} is less than 200 ohms. In this case the noise caused by the resistors and sensor can be significant and should be considered, particularly for high resistance sensors.

To improve the amplifier noise, $\overline{e_a^2}$, a transformer coupling has been used to minimize noise by better impedance matching. Although this was useful with tube inputs, the low impedance of transistor devices makes transformer coupling much less attractive. Therefore, most modern anemometer systems are now direct coupled. Transformer coupling could be used on either constant temperature or constant current systems, but has traditionally been applied to constant current systems since they are normally AC coupled.

Eq. 20 gives the signal size for a given frequency of the input signal. Eq. 32 gives the noise for a certain bandwidth, Δf. To obtain the signal to noise ratio, one only needs to take the ratio of Eqs. 20 and 32.

For simplicity, one can consider Eqs. 23 and 34 only. Eq. 23 is useful since normally signal to noise considerations are only of concern at high frequencies. Eq. 34 is generally applicable because the bridge resistors and sensor make only a very small contribution to the total equivalent input noise. The signal to noise ratio is then:

$$\frac{\text{Signal}}{\text{Noise}} = \frac{\sqrt{\overline{e_b^2}}}{\sqrt{\overline{e_{bn}^2}}} = \frac{\lambda Z\ R_r\ I_p\ q}{\sqrt{\overline{e_a^2}}} \tag{35}$$

Eq. 35 is sufficient for most signal to noise ratio calculations.

For sensors in Tables II and III, the value of e_b/q has been calculated in terms of frequency and current using property values given in Table 1. If at 200 ft/sec, the current is an 0.00015-inch diameter tungsten wire is 65 ma, then at 100 KHz:

$$\sqrt{\overline{e_b^2}} = 2.04 \times 10^{-2}\ q \tag{36}$$

If the equivalent input noise of the amplifier is:

$$\sqrt{\overline{e_a^2}} = 1.5\ \sqrt{\Delta f} \times 10^{-9} \tag{37}$$

then with a bandpass filter set at 10 KHz, the signal to noise ratio will be:

$$\frac{\sqrt{\overline{e_b^2}}}{\sqrt{\overline{e_a^2}}} = 1.36 \times 10^{5}\ q \tag{38}$$

For a signal to noise ratio of one, $q = 7.35 \times 10^{-6}$ BTU/hour = 2.15×10^{-6} Watts. Converting this to a velocity fluctuation magnitude is considered in the next section.

The following comments apply to the above calculation procedure:

a) A bandpass filter should be used that has a band-pass of 10 percent or less of the signal frequency.

b) With a low pass filter set at approximately the frequency of interest, the calculated signal to noise ratio will be somewhat low compared to the actual.

c) If significant bandwidth above the frequency of interest is included, the calculated signal to noise ratio will be high compared to the actual.

The calculation is strictly valid only for the frequency being observed. To include a wide range of frequencies in the noise calculation, it is necessary to perform an integration[8]. However, for extracting maximum information from small signals, a good band-pass filter or spectrum analyzer should be used and the procedure is valid. When observing waveforms encompassing a wide range of frequencies, the above can be used to give a conservative estimate of the signal to noise ratio at the maximum frequency of interest.

5. Velocity Sensitivity

In most applications of anemometry, velocity is the parameter of interest. To convert all previous equations to velocity sensitivity, it is necessary to obtain the value of $q/v = g_v$ either from calibration curves or from heat transfer equations.

Although a number of equations (e.g., Ref. 10) have been proposed to relate velocity to heat transfer on a hot wire, to illustrate the procedure the equation generally referred to as "Kings Law" will be used.

$$Q = [A + B\ (V)^{1/2}]\ (T_s - T_e) \tag{39}$$

From Eq. 39:

$$q = \frac{B\ (T_s - T_e)}{2\ V^{1/2}}\ v \tag{40}$$

This relation can be used to substitute for q in Eqs. 20-23, 35, 36 and 38.

Calibration curves can also be used to obtain the relation between q and v. For example, if a calibration curve is plotted with I_p on the ordinate and velocity as the abscissa, then the

Table 1

MATERIAL PROPERTIES

QUARTZ:

DENSITY = ρ = 131 lbs/ft^3

SPECIFIC HEAT = C_p = 0.188 BTU/lb - °F

THERMAL CONDUCTIVITY = k = 0.798 BTU/Hr-ft-°F.

THERMAL DIFFUSIVITY = $\partial = \frac{k}{\rho C_p}$ = 0.034 ft^2/hr

PLATINUM:

DENSITY = ρ = 1334 lbs/ft^3

SPECIFIC HEAT = C_p = 0.0349 BTU/hr - °F

TEMPERATURE COEFFICIENT OF RESISTANCE, λ

FILM: 0.00145/°F.

WIRE: 0.00216/°F.

PLATINUM - 20% IRIDIUM:

DENSITY = ρ = 1350 lbs/ft^3

SPECIFIC HEAT = C_p = 0.0348 BTU/lb - °F.

TEMPERATURE COEFFICIENT OF RESISTANCE, λ = 0.00047/°F

TUNGSTEN:

DENSITY = ρ = 1170 lbs/ft^3

SPECIFIC HEAT = C_p = 0.0344 BTU/lb - °F.

TEMPERATURE COEFFICIENT OF RESISTANCE, λ = 0.0025/°F

slope at a given velocity is:

$$s\ (\text{slope}) = \frac{i_p}{v} \tag{41}$$

Since, from Eq. 18:

$$q = 2\ I_p\ R_p\ i_p \tag{42}$$

then:

$$q = 2\ I_p\ R_p\ s\ v \tag{43}$$

For example, if $s = 4.85 \times 10^{-5}$ ampere - sec/ft, and R_p = 14.4 ohms, $q = 9.08 \times 10^{-5}$ v. For a heat flux of 2.15×10^{-6} watts, v = 0.024 ft/sec. Therefore, at 200 feet/sec. a velocity change at 100 KHz of 0.024 ft/sec can be measured with a signal to noise ratio of one. This represents a 0.012 percent change in velocity.

In high frequency anemometry a basic assumption normally made is that high frequency changes follow the same basic calibration curve (or heat transfer equation) as low frequency or essentially steady state data. If this is not the case, then Eqs. 40 or 43 are not proper for use with high frequency data. For example, the effective area for heat transfer can vary with frequency when the conduction losses to the supports represent a high percentage of the total heat transfer. This occurs with non-cylindrical sensors in gases[9]. Another problem, particularly in liquids, is the possibility the boundary layer is not a pure thermal resistance. Again Eqs. 41 and 43 may need a frequency dependent term to represent high frequency data properly.

It should be emphasized that even a perfect constant temperature anemometer can do no better than measure q properly. If the relation between q and v is frequency dependent, for a "flat" response relative to velocity additional compensating circuitry would have to be added.

To examine how e_b varies with velocity for high frequencies, Eq. 40 can be substituted into Eq. 23 to give:

$$\frac{e_b}{v} = \frac{\lambda Z\ R_r\ I_p\ B(T_s - T_e)}{2\ V^{1/2}} \tag{44}$$

Using Eqs. 39 and 40 and neglecting the 'A' Term:

$$I_p \sim V^{1/4} \tag{45}$$

Therefore:

$$\frac{e_b}{V} \sim V^{-1/4} \tag{46}$$

and the velocity sensitivity decreases with velocity level. Of course, if turbulence intensity is the parameter of interest:

$$\frac{e_b}{v/V} \sim V^{3/4} \tag{47}$$

and sensitivity increases as velocity increases.

6. Temperature Sensitivity

In some anemometer applications temperature sensitivity becomes an important parameter. From Eq. 6:

$$q = h_o S_o t_e \tag{48}$$

This can now be substituted into Eqs. 20, 21, 22, 23, 35, 36, or 38 exactly as the velocity sensitivity was handled.

For example, to obtain a good estimate of the low frequency or "steady state" sensitivity we can use Eq. 22. Substituting in Eq. 48 gives:

$$\frac{e_b}{t_e} = \frac{h_o S_o}{I_p} \quad \left(\frac{R_p}{R_r} - 1\right) \tag{49}$$

However, using Eqs. 5 and 10:

$$\frac{e_b}{t_e} = \lambda I_p R_r \tag{50}$$

This corresponds exactly to Eq. 16 which, with the assumptions made in Eq. 22, is correct.

Similar calculations can be made for composition or, for that matter, combinations of variables. The accuracy of the results will depend largely on the accuracy of the equation or calibration data relating heat transfer to the variable of interest. Once this is determined, the signal amplitude and, thereby, the sensitivity of the bridge as a function of frequency can be readily calculated.

7. Conduction Losses to Supports

In all the above calculations, "end losses" are ignored. There is data in the literature on end losses, but the emphasis has been toward making a heat balance on the sensor. The effects of end losses on frequency response seems to be less well known. However, an article by Bellhouse and Schultz[9] for wedge shaped thin film sensors pointed out the strong influence of what could be called "end losses" or "side losses". This brings up the general problem of "end losses" as regards frequency response. An intuitive description follows.

In a steady state calibration, the entire sensor, including supporting structure, will always be at an equilibrium condition. Therefore, as velocity is changed a new temperature distribution can take place on the sensor if necessary for equilibrium. However, at frequencies where the entire probe cannot come to temperature equilibrium, this redistribution of temperature does not have time to take place. The result can be an error in amplitude response at high frequencies that is not shown by the calculations in this paper. Again, "end loss" effects are an appropriate way to describe them.

The wedge probe has particularly high "end losses"[9]. In a poor heat transfer medium such as most gases, the wedge will get hot some distance back from the area where the platinum film is. A slow change in velocity will cause the temperature distribution in the areas not covered by the platinum film to change. However, if the velocity change is rapid, only the "sensitive" portion will respond. Therefore, a rapid velocity change will have as little as half the amplitude of a slow change[9]. For the relatively large wedge studied, frequencies as low as 0.1 Hz are required to get velocity response that agrees with the steady state calibration curve.

For the hot wire or cylindrical film sensors, the conduction losses to the supporting structure take place only at the ends. Therefore, even in gases the amount of heat conducted to the support is a small fraction of the convective heat transfer directly from the "sensitive" areas unless the sensors are very short. One of the significant advantages of the cylindrical film sensors actually comes from the fact that the ceramic substrate is a very poor thermal conductor compared with the metals used for hot wires. Therefore, a film sensor can be built much shorter than a wire for the same conduction losses to the supports[2]. This, coupled with the generally higher frequency response of films compared with wires for a given diameter gives the cylindrical film sensor distinct advantages for many applications. Finally, short wires also have very low resistance, while the resistance of a film sensor can be maintained even though it is quite short.

Larger cylindrical sensors, both wires and films, do have special problems of their own related directly to diameter. For example, boundary layer separation and other real flow effects not only cause extraneous (sensor generated) signals, but may also contribute to frequency response errors due to thermal "lags" associated with the boundary layer in the separated region. Also, on a film, there may be a circumferential temperature redistribution that can cause frequency response errors. The extraneous signals due to separation (e.g., Von Karman vortices) are there and can easily be observed on large cylindrical sensors. The other effects mentioned have not, to our knowledge, been observed and identified and therefore may not be a problem.

8. Cooled-Film Sensors

Cooled-film sensors[2] are used in applications where internal cooling is required for:

a) Survival in high temperature environments.

b) Special characteristics available with cooling.

The cooled sensor is typically a 0.006-in. dia. tubular sensor connected so cooling fluid can be passed through the interior. Figure 9 shows the cross-section of a cooled cylindrical film sensor compared to an ordinary cylindrical film sensor and a wire sensor. The following will discuss some of the typical characteristics that are peculiar to cooled film sensors.

Figure 2 gives basic data for the tubular sensor, uncooled, with an inside to outside diameter ratio of 0.75. In addition, data is included for a cooled sensor with the assumption that the heat transfer coefficient for the cooling fluid is infinite. Details of the calculations are given in Ref. 2.

Figure 10 is a plot of e_b/q and ϕ versus frequency for an 0.006 inch-diameter cylindrical film rod, tube, and cooled tube. The tube configuration, as would be expected, only improves

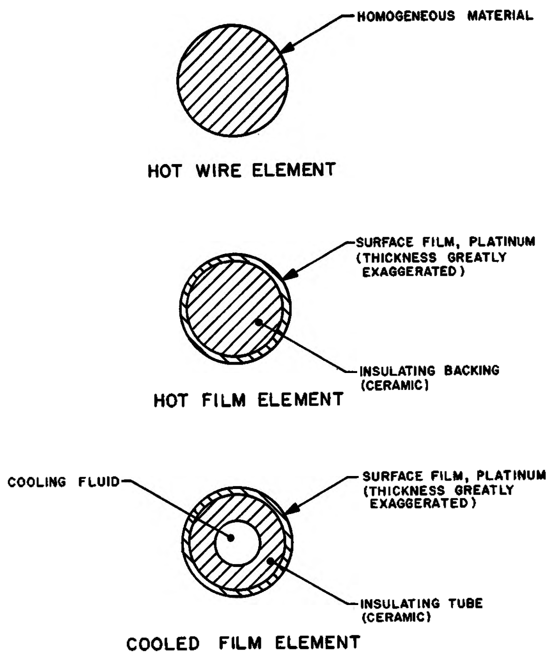


Figure 9 Cross-Sections of Sensor Types

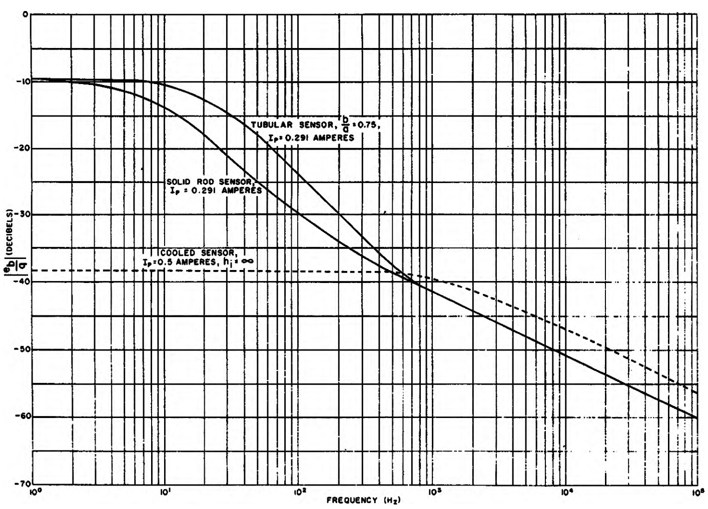


Figure 10 Comparison of 0.006 in. Dia. Cylindrical Film Sensors for V = 300 ft/sec

response near the "corner" frequency. Cooling the sensor degrades the "sensitivity" to low frequencies while improving the response at high frequencies. The decrease in low frequency sensitivity is frequency compensated by the electronic circuitry, but the signal-to-noise ratio is decreased.

The decrease in low frequency sensitivity is caused by the constraint of the cooling fluid. At high frequencies, the "thermal wave" does not penetrate to the cooling fluid. However, the increase in current due to the cooling does show up. This actually causes an improvement in the sensitivity, e_b/q, at high frequencies when compared with a solid rod. The cooled tube shown in Figure 10 has only moderate cooling, the current to the sensor, I_p, being increased from 0.291 ampere to 0.5 ampere.

This last characteristic is often useful even when the cooling is not required for survival in the environment. In shock tubes, for example, one of the characteristics of an uncooled sensor is the low current and, therefore, low e_b/q at zero velocity. When the shock front comes by it takes time for the sensor to respond even with constant temperature compensation. By cooling the sensor the current through the sensor, and therefore the response, can be increased even for zero flow conditions.

In using cooled sensors, there are many practical considerations involving the plumbing for the cooling, limited overheat ratios, cooling fluid turbulence, etc. However, the primary purpose here is to point out the theoretical advantages and disadvantages. These show up quite well on Figure 10.

9. Surface Coatings on Sensors

Coating the surface of anemometer sensors is often desirable. In electrically conducting fluids such as mercury an electrically insulating coating is a practical requirement. Even in water, as it is found in nature or obtained from the local supply, the electrical conductivity is high enough to make an insulating coating desirable. Finally, on film sensors in environments with severe contamination an insulating coating not only appears to reduce contamination but also serves to protect the metal film from erosion by particulate matter.

The improved response of a film sensor over a hot wire for a given diameter was largely due to the advantage of sensing the surface of the sensor only (since the resistive element is only on the surface). Once an insulating coating is added, the immediate question is its effect on frequency response. The question then becomes one of what the expected decrease in response will be for realizable insulating coatings.

To aid in the calculations, two assumptions about the coating are made:

1) The coating thickness is small compared to the significant dimensions of the sensor.
2) The side of the coating not exposed to the environment is maintained at constant temperature.

The first assumption is true for thin coatings and permits one to use one-dimensional equations. The second assumption is valid when the sensor is maintained at constant temperature by the electronic control circuitry. In other words, ideal electronic compensation is assumed.

The governing differential equation, with boundary conditions, is:

$$\frac{\partial^2 t}{\partial x^2} - \frac{1}{\alpha}\frac{\partial t}{\partial \tau} = 0 \qquad (51)$$

$$q = q_{sm}\, e^{i\omega\tau} \quad \text{at } x = 0$$

$$t = 0 \quad \text{at } x = \ell$$

The constant temperature control point is selected as zero merely for convenience. Solving Eq. 51 for q gives:

$$q_x = q_{sm}\, e^{i\omega\tau}\ \frac{e^{-\lambda(\ell - x)} + e^{\lambda(\ell - x)}}{e^{-\lambda\ell} + e^{\lambda\ell}} \qquad (52)$$

$$\lambda = \sqrt{\frac{i\omega}{\alpha}}$$

The parameter of interest is the ratio of the heat transfer at $x = \ell$ to the heat transfer at $x = 0$. If this ratio is one, then the surface coating does not affect heat transfer to the sensitive surface and the sensor gives an accurate reading. Setting $x = \ell$ in Eq. 52 and dividing by q at $x = 0$ gives:

$$\frac{q_\ell}{q_o} = \frac{2}{e^{-\lambda\ell} + e^{\lambda\ell}} \qquad (53)$$

Rearranging Eq. 53 gives:

$$\frac{q_\ell}{q_o} = \frac{1}{\cos\mu\ell\,\cosh\mu\ell + i\,\sin\mu\ell\,\sinh\mu\ell} \qquad (54)$$

The amplitude and phase shift can then be written as:

$$\left|\frac{q_\ell}{q_o}\right| = \frac{1}{[(\cos\mu\ell\,\cosh\mu\ell)^2 + (\sin\mu\ell\quad\sinh\mu\ell)^2]^{1/2}} \qquad (55)$$

$$\phi = -\tan^{-1}\ \frac{\sin\mu\ell\quad\sinh\mu\ell}{\cos\mu\ell\quad\cosh\mu\ell} \qquad (56)$$

These are plotted on Figure 11 as a function of $\mu\ell$. Figure 12 shows the thickness of quartz and polyester required to reduce the amplitude response various percentages as a function of frequency.

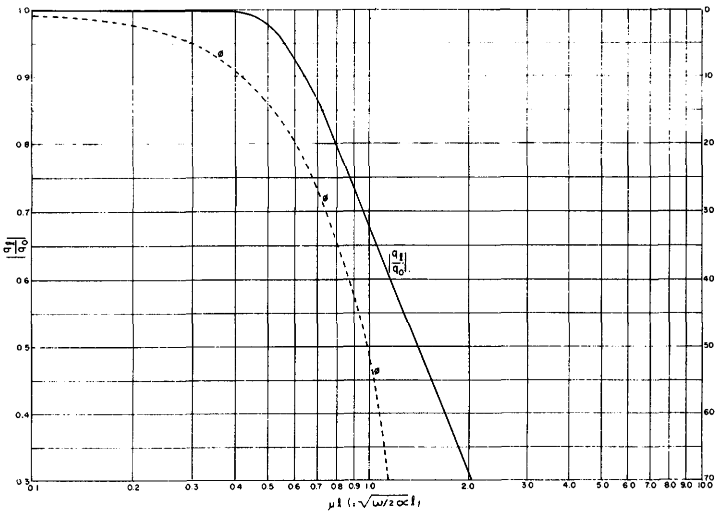


Figure 11 Effect of Thin Insulating Coatings on Film Sensors

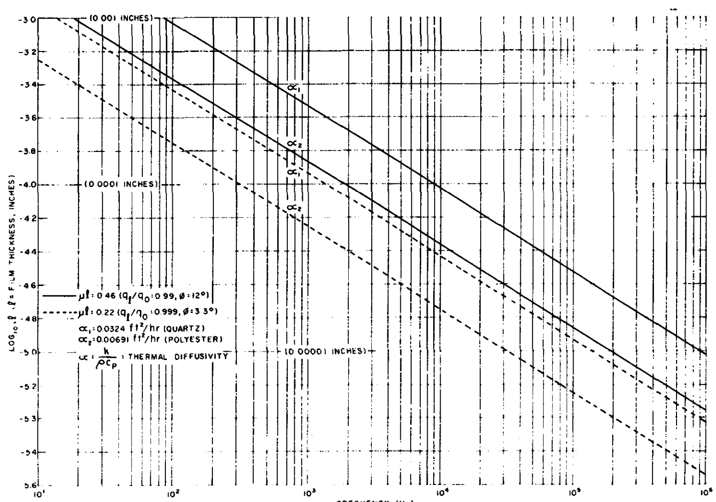


Figure 12 Influence of Coating Thickness on Frequency Response for Film Sensors

10. Steady State Analysis

The DC (or very low frequency) stability could also be described as the accuracy of the system. A large number of variables affect system accuracy in a given measurement situation. These include sensor contamination by the environment, amplifier stability, possible resistance shifts in the sensor or bridge resistors, and errors associated with calibration accuracy and its relevance to the measurement situation. In the following, only errors associated with the temperature sensitivity and long term stability of the amplifier and bridge resistors will be discussed since these define attainable accuracy with a given anemometer system independent of the sensor and external environment.

Amplifier specifications are generally given in terms of equivalent input characteristics. This makes the data independent of the particular gain established by the feedback resistors. For an anemometer, the amplifier impedances are low so the primary influences are equivalent voltage shifts at the input due to temperature and time. Also, changes in the resistance of bridge resistors due to temperature or time can be converted to equivalent input voltage. Therefore, the parameter of interest is the change in output voltage for a change in input voltage under closed loop conditions and for low frequencies. "Low frequencies" here include from zero up to the frequency where the sensor in use will no longer follow environment changes without compensation.

To establish the influence of amplifier drift or resistor changes, their effect on output voltage must be compared with that of a change in signal level. Substituting into Eq. 28 for the transfer functions and letting $K/GE_t \to 0$ and $Z \to \infty$ (steady state) gives:

$$\left.\frac{e_t}{e_b}\right)_{ss} = \frac{1}{2}\left[1 + \frac{1}{r_1}\right]\left[\frac{2r_1\ r_o - r_1 + 1}{r_o - 1}\right] \qquad (57)$$

$$r_o = R_p/R_r$$

$$r_1 = R_p/R_3$$

Therefore, the steady state ratio between output and input voltage is a function of the "overheat" and ratio of the sensor resistance to the resistor in series with the sensor.

Similarly, rearranging Eq. 29 and doing similar substitutions to calculate $e_t/q)_{ss}$ gives:

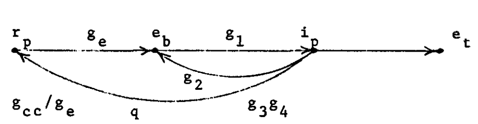
$$\left.\frac{e_t}{q}\right)_{ss} = \frac{1}{2I_p}\left[\frac{R_p + R_3}{R_p}\right] \qquad (58)$$

Combining Eqs. 58 and 40 gives:

$$\left.\frac{e_t}{v}\right)_{ss} = \frac{B\,(T_e - T_s)}{4I_p\ v^{1/2}}\quad\left[\frac{R_p + R_3}{R_p}\right] \qquad (59)$$

Eqs. 57 and 59 give the output voltage change for a change in input voltage or a change in velocity, respectively.

The third item is a change in one of the bridge resistors. Considering the sensor, the following signal flow diagram applies:



The transfer function is:

$$\frac{e_t}{r_p} = \frac{g_e\ g_1\ (R_p + R_3)}{1 - g_1g_2 - g_1g_{cc}g_3g_4} \qquad (60)$$

This is identical to e_t/e_b with the addition of g_e. Using Eqs. 57 and 13 then gives:

$$\frac{e_t}{r_p} = -\frac{I_p}{2}\left[\frac{2r_1r_o - r_1 + 1}{r - 1}\right] \qquad (61)$$

From Eqs. 57, 59 and 61, the effect of amplifier drift or bridge resistor changes on accuracy can be established.

In the following example, data for a tungsten wire (0.00015 inch dia.) and a platinum film (0.002 inch dia.) will be determined. Physical data on the sensors given in Tables I and II is used. In addition, let:

R_3 = 40 ohms

T_e = 20°C

T_s = 200°C

V = 300 ft/sec.

I_p(wire) = 0.059 amperes

I_p(film) = 0.128 amperes

The various output sensitivities are now as follows:

	Tungsten Wire 0.00015 in. Dia.	Platinum Film 0.002 in. Dia.
Velocity Sensitivity ($\frac{e_t}{v}$) (millivolts/ft/sec.)	2.16	5.28
Voltage Sensitivity ($\frac{e_t}{e_b}$) (DC voltage gain)	4.58	7.88
Resistance Sensitivity ($\frac{e_t}{r_p}$) (millivolts/milliohms)	0.186	0.380

These sensitivities can now be used to calculate errors.

If the amplifier has an equivalent input drift of 20 microvolts, for the tungsten wire this represents an output drift of 91.6 microvolts. This in turn represents a velocity "error" of 0.043 ft/sec. or 0.014 per cent. For the film, the equivalent figures are 157.6 microvolts, 0.030 ft/sec., and 0.01 per cent. This is certainly negligible for most measurements.

The operating resistance of the tungsten wire is 14.4 ohms (Table II). An 0.01 percent change in resistance would be 1.44 milliohms. This represents an output voltage change of 268 microvolts and an equivalent velocity change of 0.124 ft/sec (0.041 percent). Similar calculations for the film sensor (R_p = 9 ohms) gives 342 microvolts and 0.065 ft/sec. (0.022 percent). Therefore, the resistance stability of the sensor is very important for DC stability. It should be noted that the resistor, R_3, can give a similar effect. Therefore, stable resistors with low temperature coefficients must be used.

APPENDIX I

RULES FOR SIGNAL FLOW GRAPHS

The formal signal flow graph techniques permit writing the transfer function of the system without solving the algebraic relations. In the following the term forward path refers to a continuous path "from input to output along which no node is encountered more than once". A feedback loop or just loop is a continuous path "that forms a closed loop along which each node is encountered once per cycle". The gain of the system is then written as a ratio by using the following rules.

1) Numerator: The numerator of the equation is the sum of gain products of all forward paths, each multiplied by a factor. The factor is "the algebraic sum of all possible sets of loops which do not touch each other and do not touch that forward path."

2) Denominator: The denominator is the algebraic sum of "all possible sets of non-touching loops."

The algebraic sign is plus for an even number of loops and minus for an odd number of loops. The sum of gain products is generalized to include loops taken "none at the time" to be unity (plus). Also, when loops are taken one at the time (minus) the question of touching does not come up. A general proof of the validity of the above procedures is given by Mason[4].

WIRE MATERIAL	SENSOR DIMENSIONS DIAMETER inches	mm.	LENGTH inches	mm.	THERMAL IMPEDANCE, Z, in °F.-hour/BTU	NOMINAL REFERENCE RESISTANCE R_r, in ohms	NOMINAL OPERATING RESISTANCE R_p, in ohms	e_b/q volts-hour/BTU
1) Tungsten	0.00015	0.0038	0.050	1.25	$\frac{10^7}{4.65\ fi}$	8	14.4	$\frac{1}{\frac{3.145X10^{-5} fi}{I_p} + 8.26\ I_p}$
2) Tungsten	0.0002	0.005	0.050	1.25	$\frac{10^7}{8.28\ fi}$	4.5	6.3	$\frac{1}{\frac{8.50X10^{-5} fi}{I_p} + 7.07\ I_p}$
3) Platinum	0.0002	0.005	0.050	1.25	$\frac{10^7}{9.41\ fi}$	6	11.2	$\frac{1}{\frac{9.3X10^{-5} fi}{I_p} + 7.36\ I_p}$
4) Platinum	0.0005	0.0125	0.100	2.50	$\frac{10^5}{1.21\ fi}$	2	3.6	$\frac{1}{\frac{3.05X10^{-3} fi}{I_p} + 5.4\ I_p}$
5) Platinum-20% Iridium	0.0002	0.050	0.050	1.25	$\frac{10^7}{9.41\ fi}$	18	21	$\frac{1}{\frac{17X10^{-5} fi}{I_p} + 31.5\ I_p}$
6) Platinum-20% Iridium	0.0005	0.0125	0.100	2.50	$\frac{10^5}{1.21\ fi}$	6	7	$\frac{1}{\frac{5.03X10^{-3} fi}{I_p} + 25\ I_p}$

TABLE 2: THERMAL IMPEDANCE AND SENSIVITY (e_b/q) DATA FOR TYPICAL WIRES USED IN ANEMOMETRY.

APPENDIX II

CONSTANT TEMPERATURE SYSTEM

The following gives the derivation of the transfer functions g_1, g_2, g_3, and g_4 required to write the system transfer function (Eq. 25). The transfer function, $g_{cc} = e_b/q$, is derived in the text and represented by Eq. 20.

For a linear amplifier, the representative equation is (see Fig. 7)

$$E_t = K + E_b\, G \qquad \text{(II-1)}$$

where:

K = output voltage when input, E_b, is zero (equals "offset" of amplifier)

G = voltage gain of amplifier

Similarly, from Fig. 7:

$$E_t = I_p\,(R_p + R_3) \qquad \text{(II-2)}$$

Equating Eqs. (II-1) and (II-2) gives:

$$I_p = \frac{K}{R_p + R_3} + \frac{E_b\, G}{R_p + R_3} \qquad \text{(II-3)}$$

Therefore:

$$g_1 = \frac{i_p}{e_b} = \frac{G}{R_p + R_3} \qquad \text{(II-4)}$$

This is the transconductance of the amplifier.

The feedback, g_2, is present because a change in current, i_p, through the bridge can result in change in the bridge off-balance, e_b, if the bridge is not at balance. Using Eq.(II-3)gives:

$$\frac{E_b}{I_p} = \frac{R_p + R_3}{G} - \frac{K}{I_p G} \qquad \text{(II-5)}$$

Therefore, substituting for I_p from Eq. II-2:

$$g_2 = \frac{e_b}{i_p} = \frac{R_p + R_3}{G}\left(1 - \frac{K}{E_t}\right) \qquad \text{(II-6)}$$

If the bridge is at balance, $E_t = K$ and $g_2 = 0$.

The transfer function, g_3, represents the change in heat input to the probe due to a change of current in the probe. Since:

$$Q = I_p^2\, R_p$$

$$g_3 = \frac{q_o}{I_p} = 2\, I_p R_p$$

The final function, g_4, is one since an electrical heat input to the probe is equivalent to a heat input to the probe from the environment. The closed loop gain then is a comparison of the change in electrical heat input, q_o, to the change in heat input from the environment, q. In a perfect system they should be equal in magnitude and opposite in sign to exactly cancel.

APPENDIX III

SENSITIVITY AND SIGNAL TO NOISE COMPARISONS

In the literature[12,13] reference has been made to a difference in sensitivities between the constant current and constant temperature modes of operation. Here, sensitivity is defined as the voltage generated at the sensor due to a change in velocity past the sensor. The following equation is given to compare sensitivities of the two systems:

$$\frac{(e_b)_{cc}}{(e_b)_{cT}} = 2\,\frac{R_p - R_e}{R_e} \qquad \text{(III-1)}$$

$(e_b)_{cc}$ = voltage across sensor operated constant current

$(e_b)_{cT}$ = voltage across sensor operated constant temperature

This equation indicates the constant current system is more sensitive at high overheats and the constant temperature system more sensitive at low overheats ($3/2 < R_p/R_e$).

As sensitivity is defined, the result of Eq. III-1 is correct. However, the concern of this paper is signal to noise ratio. Since this defines the size signal that can be distinguished from background noise, this would seem to be the more important parameter. The difference in results between the sensitivity calculation of Eq. III-1 and the signal to noise ratio results comes from the effect of the feedback on the noise signal.

From Eq. 28, the value of output noise for a given input noise, e_{bn}, is (after substituting for the transfer functions from Appendix II):

$$\frac{e_{tn}}{e_{bn}} = \frac{R_p + R_3}{\dfrac{(R_p + R_3)K}{G\quad E_T} - \dfrac{2I_p^2\, R_p\, \lambda R_r\left[1 - \dfrac{R_p}{R_p + R_3}\right]}{\frac{1}{Z} + h_o S_o - \lambda R_r I_p^2\left(1 - \dfrac{2R_p}{R_p + R_3}\right)}} \qquad \text{(III-2)}$$

Since G is large, the first term in the denominator approaches zero. Adding the assumption of steady state ($Z \to \infty$) and that $R_3 >> R_p$, Eq. III-2 can be written:

$$\frac{e_{tn}}{e_{bn}} = \frac{R_p + R_3}{2R_p}\left[\frac{R_e}{R_p - R_e}\right] \qquad \text{(III-3)}$$

Since, from Eq. 27:

$$\frac{i_n}{e_{tn}} = \frac{1}{R_p - R_3}$$

Then

$$\frac{i_n}{e_{bn}} = \frac{1}{2\,R_p}\left[\frac{R_e}{R_p - R_e}\right] \qquad \text{(III-4)}$$

The apparent "noise" voltage across the sensor for a constant temperature system is now:

$$(e_n)_{ct} = i_n\, R_p = \frac{1}{2}\left[\frac{R_e}{R_p - R_e}\right] e_{bn} \qquad \text{(III-5)}$$

In a constant current system, the "noise" across the sensor is the same as the input noise since there is no feedback. In other words:

$$(e_n)_{cc} = e_{bn}$$

The ratio is now:

$$\frac{(e_n)_{cc}}{(e_n)_{ct}} = 2\,\frac{R_p - R_e}{R_e} \qquad \text{(III-6)}$$

Therefore for a given amplifier input noise, e_{bn}, the "apparent" noise across the sensor in the two cases has the same ratio as the sensitivities. In other words, the signal to noise ratio is the same for both the constant temperature and constant current systems.

APPENDIX IV

THERMAL IMPEDANCE OF FILM SENSORS

The governing equations for transient response of a film sensor are more complex than for the homogenous hot wire. In the hot wire, the assumption that the entire cross-section of the wire responds as a unit is usually made. This implies that there are no radial temperature gradients in the wire even when high frequency heat flux changes are present. In fact, this is not true. However, since the entire cross-section of the wire is "sensed" (is included as the resistive element), the hot wire does act very much like a simple thermal capacity[8]. Therefore, Eq. 2 will be used for wires with no further derivations.

On film sensors, only the surface is sensed rather than the entire cross-section. Therefore, the assumption of the sensor being a simple thermal capacity is not valid. However, in the region of the corner frequency and below, cylindrical film sensors do follow Eq. 2 very closely. The other extreme is the very high frequencies. Then the "thermal wave" penetrates the surface only a small distance and the film sensor behaves identical to a semi-infinite wall. Therefore, the semi-infinite wall is the limiting case at the high frequency end. First, the temperature distribution in a semi-infinite wall with a sinusoidally imposed surface temperature fluctuation will be calculated. From this, thermal impedance can then be determined.

Semi-Infinite Wall

The semi-infinite wall is perhaps the simplest and clearest example of a body on which a sinusoidal surface temperature drive can be applied. The differential equation is:

$$\frac{\partial^2 t}{\partial x^2} - \frac{1}{\alpha}\frac{\partial t}{\partial \tau} = 0 \qquad \text{(IV-I)}$$

where:

t = temperature, $^{\circ}F$

x = distance normal to surface, ft.

α = thermal diffusivity, ft^2/hr.

τ = time, hr.

The solution of this equation with the boundary conditions $t = t_{sm}\, e^{i\omega\tau}$ at $x = 0$, t is finite at $x = \infty$, is:

$$t = t_{sm}\, e^{-(1+i)\sqrt{\frac{\omega}{2\alpha}}\, x}\, e^{i\omega\tau} \qquad \text{(IV-2)}$$

Using the term $e^{i\omega\tau}$ rather than $\cos\omega\tau$ or $\sin\omega\tau$ as the driving force tends to simplify later calculations. It is understood that in the final equation only the real part is of interest. For example, writing Eq. IV-2 in trigonometric form gives:

$$\frac{t}{t_{sm}} = \exp\frac{-2\pi x}{\psi}\cos\left(\omega\tau - \frac{2\pi x}{\psi}\right) \qquad \text{(IV-3)}$$

$$\psi = \text{wavelength} = \frac{2\pi}{\sqrt{\frac{\omega}{2\alpha}}} \qquad \text{(IV-4)}$$

Eq. IV-3 is plotted in Fig. I-1. This gives a graphical representation of the necessary depth of the surface before it can be considered as a semi-infinite wall. As expected, the higher the frequency, the less this depth is.

Influence of Metal Film

The above calculations on the semi-infinite wall are for surface temperature oscillations on a homogeneous solid. The actual probes have a metallic film on an electrical insulator. If the probe is to be considered homogeneous, the influence of the metallic film on frequency response must be negligible.

The approximate thickness of the film can easily be determined by measurement of its surface area and electrical resistance. For present probe configurations, a thickness of 1500 Angstroms is the maximum attained. Letting the upper frequency limit be 500,000 cycles/second, the ratio of temperature amplitudes from one side of the film to the other (Eq.IV-3) is $\frac{t}{t_{sm}} = 0.9981$. This is assumed equal to unity in the following discussions.

Thermal Impedance

It is advantageous to introduce the idea of thermal impedance for determining the transient response of probes[1]. It is simply defined as:

$$Z = \frac{t_s}{q} \qquad \text{(IV-5)}$$

where:

t_s = change in surface temperature, $^{\circ}F$

q = change in heat flux from surface to probe interior, BTU/hr.

Thermal impedance is a relation between temperature and heat transfer just as electrical impedance is a relation between voltage and current. The larger the impedance, the greater the temperature potential that is needed to drive a given "flow" of heat. The heat flux from surface to interior is:

$$q = -k_g \int \left.\frac{\partial t}{\partial x}\right|_{x=0} dS_o \qquad \text{(IV-6)}$$

where:

k_g = thermal conductivity of the wall, rod, or tube, BTU/ft-$^{\circ}$F-hr.

S_o = surface area, ft^2

For the semi-infinite solid, utilizing Eq. IV-2 for t,

$$q = k_g\, S_o\, t_{sm}\,(1+i)\sqrt{\frac{\omega}{2\alpha}}\; e^{i\omega\tau} \qquad \text{(IV-7)}$$

and

$$Z = \frac{1}{k_g\, S_o\sqrt{\frac{\omega}{2\alpha}}\,(1+i)} \qquad \text{(IV-8)}$$

Eq. IV-8 then represents the thermal impedance for the semi-infinite wall. More important, it accurately represents the thermal impedance of all film sensors at high frequency.

Cylindrical Film Response

Calculations for the solid cylinder are still tractable and have therefore been carried out in some detail. The cylinder is assumed infinite in length, end losses being considered separately. The basic differential equation and the appropriate boundary conditions are as follows:

$$\frac{\partial t}{\partial \tau} = \alpha\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right)$$

$$t = t_{sm}\, e^{i\omega\tau} \text{ at } r = a \qquad \text{(IV-9)}$$

$$t \text{ is finite at } r = 0$$

where:

r = radial distance from centerline, ft.

a = radius of cylinder, ft.

The solution of this equation after putting in the boundary conditions is:

$$t = t_{sm} \left[\frac{J_o(i^{3/2}\sqrt{\frac{\omega}{\alpha}}\, r)}{J_o(i^{3/2}\sqrt{\frac{\omega}{\alpha}}\, a)}\right] e^{i\omega\tau} \quad \text{(IV-10)}$$

$J_o(\ )$ = zero order Bessel function of the first kind

Let:

$$J_o(i^{3/2}\sqrt{\tfrac{\omega}{\alpha}}\, r) = \mathrm{ber}_o(\sqrt{\tfrac{\omega}{\alpha}}\, r) + i\, \mathrm{bei}_o(\sqrt{\tfrac{\omega}{\alpha}}\, r) = \mu_r + i\, \nu_r$$

Then, using this definition and Eq. I-6:

$$q = S_o t_{sm} t_g \sqrt{\frac{\omega}{\alpha}} \; \frac{(\mu_a \mu_a' + \nu_a \nu_a') + i(\mu_a \nu_a' - \nu_a \mu_a')}{\mu_a^2 + \nu_a^2} e^{i\omega\tau} \quad \text{(IV-11)}$$

where primes indicate the derivative with respect to r, and the subscript, a, refers to the value $r = a$.

From Eq. IV-5:

$$Z = \frac{1}{S_o k_g \sqrt{\frac{\omega}{\alpha}} \left(\frac{\mu_a \mu_a' + \nu_a \nu_a}{\mu_a^2 + \nu_a^2} + \frac{\mu_a \nu_a' - \nu_a \mu_a'}{\mu_a^2 + \nu_a^2}\, i\right)} \quad \text{(IV-12)}$$

Comparing Eq. IV-12 with Eq. 3 gives:

$$C = \frac{\mu_a \mu_a' + \nu_a \nu_a'}{\mu_a^2 + \nu_a^2} \quad \text{(IV-13)}$$

$$D = \frac{\mu_a \nu_a' - \nu_a \mu_a'}{\mu_a^2 + \nu_a^2} \quad \text{(IV-14)}$$

These are the functions shown in Fig. 2 for the solid cylindrical sensors. Both C and D approach the value $1/\sqrt{2}$ as $\omega \to \infty$, making the response approach that of a semi-infinite wall.

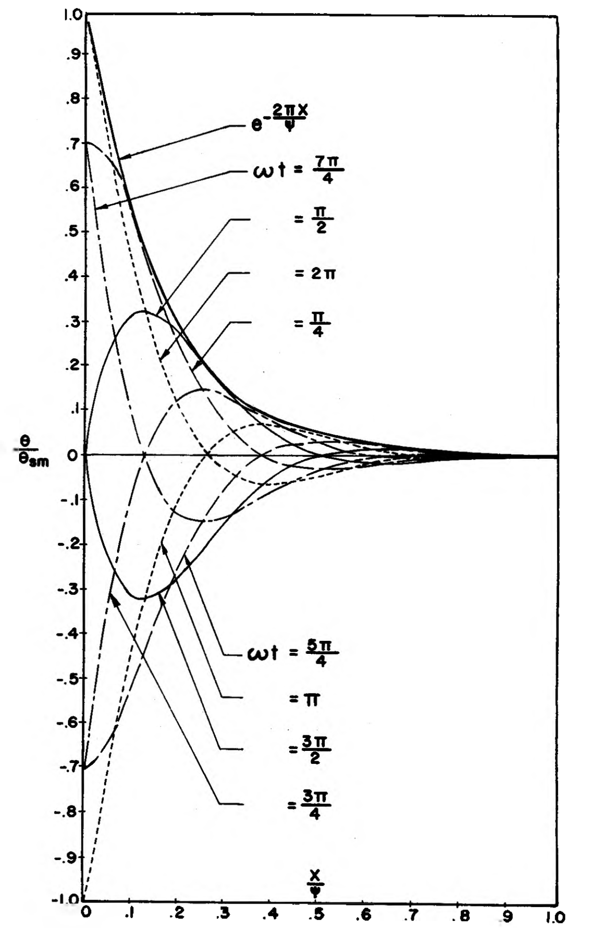


Figure IV-I Temperature Profiles in Harmonic Heat Waves in Semi-Infinite Solid

	DIAMETER		LENGTH	DIAMETER		LENGTH	DIAMETER		LENGTH
	inches 0.001, mm 0.025		inches 0.020, mm 0.5	inches 0.002, mm 0.05		inches 0.040, mm 1.0	inches 0.006, mm 0.15		inches 0.080, mm 2.0
$a\sqrt{\frac{\omega}{\alpha}}$	f	M	N	f	M	N	f	M	N
<0.347	<100	→ 0	→ 5.05×10^{-6}	<25	→ 0	→ 4.06×10^{-5}	<2.78	→ 0	→ 7.4×10^{-4}
0.347	10^2	0.187×10^{-4}	4.87×10^{-4}	25	0.374×10^{-4}	9.66×10^{-4}	2.78	0.748×10^{-4}	1.932×10^{-3}
0.550	250	0.731×10^{-4}	1.23×10^{-3}	625	1.462×10^{-4}	2.46×10^{-3}	6.94	2.924×10^{-4}	4.92×10^{-3}
0.850	600	3.41×10^{-4}	2.86×10^{-3}	150	6.82×10^{-3}	5.72×10^{-3}	16.7	1.364×10^{-3}	1.44×10^{-2}
1.10	10^3	8.32×10^{-4}	5×10^{-3}	250	1.664×10^{-3}	1.0×10^{-2}	27.8	3.33×10^{-3}	2×10^{-2}
1.39	1.6×10^3	1.71×10^{-3}	7.5×10^{-3}	400	3.4×10^{-3}	1.5×10^{-2}	44.5	6.8×10^{-3}	3×10^{-2}
1.90	3.0×10^3	4.83×10^{-3}	1.16×10^{-3}	750	9.66×10^{-3}	2.32×10^{-2}	83.4	1.93×10^{-2}	4.64×10^{-2}
2.43	4.9×10^3	9.24×10^{-3}	1.61×10^{-2}	1225	1.85×10^{-2}	3.22×10^{-2}	136.0	3.70×10^{-2}	6.44×10^{-2}
3.47	10^4	1.65×10^{-2}	2.16×10^{-2}	2.5×10^3	3.3×10^{-2}	4.32×10^{-2}	278.0	6.6×10^{-2}	8.64×10^{-2}
11.10	10^5	6.65×10^{-2}	6.65×10^{-2}	2.5×10^4	0.133	0.133	2.78×10^3	0.266	0.266
34.7	10^6	0.206	0.206	2.5×10^5	0.432	0.432	2.78×10^4	0.824	0.824
>34.7	$>10^5$	$2.06 \times 10^{-4}\sqrt{f}$	$2.06 \times 10^{-4}\sqrt{f}$	$>2.5 \times 10^4$	$8.41 \times 10^{-4}\sqrt{f}$	$8.41 \times 10^{-4}\sqrt{f}$	$>2.78 \times 10^3$	$5.08 \times 10^{-3}\sqrt{f}$	$5.08 \times 10^{-3}\sqrt{f}$

THERMAL IMPEDENCE

$$Z = \frac{1}{M + iN} \left(\frac{{}^{o}F - Hr}{BTU}\right)$$

SENSITIVITY (R_r = 6 ohms, R_p = 9 ohms)

$$e_b/q = \frac{1}{9.93\, Ip + \frac{140.5}{Ip}(M + iN)} \left(\frac{\text{Volts-Hr}}{BTU}\right)$$

TABLE 3: THERMAL IMPEDENCE AND SENSITIVITY (e_b/q) DATA FOR THERMO-SYSTEMS CYLINDRICAL FILM SENSORS.

SYMBOLS

a radius of sensor

C, D function of $a\sqrt{\omega/\alpha}$ (Fig. 2)

C_p specific heat of wire material

C_T thermal capacity = $\rho C_p d_o S_o/4$

d_o wire diameter

$\overline{e_a^2}$ equivalent input noise of amplifier

e_b change in voltage at point A of Fig. 4

e_{bn} change in voltage at amplifier input

E_F "Fixed" voltage at point B on bridge

E_t voltage on top of bridge

e_t change in voltage on top of bridge

e_{tn} change in voltage at amplifier output

f frequency, Hz

G voltage gain of amplifier

G_{CL} closed loop gain of system

G_{OL} open loop gain of system

g_a Z = transfer function between heat trnasfer to sensor and sensor temperature

g_b transfer function between sensor temperature and heat transfer to sensor

g_c, g_e transfer functions

g_d transfer function

g_t transfer function between heat transfer to sensor and environment temperature

i $\sqrt{-1}$

I_p current in sensor

i_p change in current of sensor

K output voltage when input, E_b, equals zero

k Boltzmann constant

k_s thermal conductivity for substrate material

m change in fluid composition

Q heat flux

q change in heat flux from environment to sensor surface

q_{sm} amplitude of heat flow changes on surface of insulating coating

R_e sensor resistance at environment temperature

R_{eq} equivalent noise resistance

R_p operating resistance of sensor

R_r resistance at reference temperature, T_r

R_1 bridge resistor opposite sensor (Figure 7)

R_3 bridge resistor in series with sensor

r_p change in resistance of sensor

S_o wire surface area

s slope of velocity - current curve

T_o absolute temperature of bridge resistors

T_r reference temperature

T_e environment temperature

T_s sensor temperature

t_e change in the environment temperature

t_s change in temperature of the temperature sensitive

V velocity

v change in fluid velocity

Z thermal impedance

α thermal diffusivity of substrate material = $k/\alpha\ C_p$

Δf frequency range

ΔT temperature difference between sensor and bridge resistors

ℓ length of sensor

λ temperature coefficient of resistance of sensor

ρ density of wire material

ω frequency in radians per unit of time

Subscripts

ss steady state

REFERENCES

1. Ling, S. L., Measurement of Flow Characteristics by the Hot-Film Technique, Ph.D. Thesis, University of Iowa, Iowa City, June (1955).

2. Fingerson, L. M., A Heat Flux Probe for Measurements in High Temperature Gases, Ph.D. Thesis, University of Minnesota, Minneapolis, 1961.

3. Mason, S. J., "Feedback Theory - Some Properties of Signal Flow Graphs," Proc. Inst. Radio Eng., 44, 1144 (1953).

4. Mason, S. J., "Feedback Theory - Further Properties of Signal Flow Graphs," Proc. Inst. Radio Eng., 44, 920 (1956).

5. Lighthill, J. J., "The Response of Laminar Skin Friction and Heat Transfer to Fluctuations in Stream Velocity", Proc. Roy. Soc. (London), 224A. 1-23(1954).

6. Lowell, H. H., and Patton, N., "Response of Homogeneous and Two-Material Laminated Cylinders to Sinusoidal Environment Temperature Change, with Applications to Hot-Wire Anemometry and Thermocouple Pyrometry", NACA TN 3514, September (1955).

7. Freymuth, P., "Feedback Control Theory for Constant Temperature Hot Wire Anemometers", Rev. Sci. Instr., 38,677-681 (1967).

8. Freymuth, P., "Noise in Hot-Wire Anemometers", Rev. Sci. Instr. 39, 550-557 (1968).

9. Bellhouse, B. J., and Schultz, D. L., "The Determination of Fluctuating Velocity in Air with Heated Thin Film Gauges", J. Fluid Mech., 29, 289-295 (1967).

10. Collis, D. C., and Williams, M. J., "Two-Dimensional Convection from Heated Wires at Low Reynolds Numbers," J. Fluid Mech., 6, 357 (1959).

11. Freymuth, P., "Nonlinear Control Theory for Constant-Temperature Hot-Wire Anemometers", Rev. Sci. Instr., 40, 258-262 (1969).

12. Kovasznay, L. S. G., "Calibration and Measurement in Turbulence Research by the Hot-Wire Method", NASA TM 1130, (1947).

13. Hinze, J. O.,Turbulence, An Introduction to its Mechanism and Theory, McGraw Hill (1959).

DISCUSSION

T. HANRATTY (University of Illinois): My question is directed mainly towards working in liquids with hot-film probes but it might be applicable to other types of probes. Do you have to take into account the capacitance of the thermal boundary layer? The effect it would have on your analysis would be that the heat transfer coefficient would be a function of frequency.

FINGERSON: I am assuming the heat transfer is simply the thermal conductance, and I didn't include any effects of thermal capacity of the boundary layer. Particularly in liquids, there is the uncertainty of what happens in high frequency response due to the non-steady state boundary layers around the sensors, not only the thermal capacity, but it seems as if there should be some sort of an influence on the frequency response as stationary vortices are generated behind the cylinder. I skirted that problem entirely by defining Q as heat transfer to the sensor surface, and avoided the question of what you are measuring. But nevertheless in liquid measurements this is a real problem and I think it is one of the things that needs to be studied.

R. HUMPHREY (DISA): There was a paper, maybe only an internal report, at NASA-Lewis about 15 years ago by Shepard where he did evaluate this thermal boundary layer capacitance in relation to frequency response. He came to the conclusion that for liquids it did not come into effect until frequencies above 1000 Hz, and for air above 50,000 Hz.

V. W. GOLDSCHMIDT (Purdue University): If I understand correctly, one of the conclusions you come up with is that the sensitivity of a constant current and a constant temperature anemometer are the same. Hinze's analysis of sensitivity to velocity fluctuations in both a constant current and a constant temperature anemometer shows that constant current may be twice as sensitive as the constant temperature (depending on overheat). Can you clarify this discrepancy?

FINGERSON: If you have the same bridge resistors in the two systems looking at the same band width as far as the signal to noise ratio, the two systems are equivalent. The relations you refer to define a sensitivity of the voltage changes across the bridge to velocity changes past the sensor. My contention is that signal to noise ratio is the important parameter as it defines the minimum velocity change that can be measured. This conclusion is in agreement with Freymuth.

T. HOULIHAN (Naval Post Graduate School): I see that some of the elements that enter into the noise considerations are the resistors in the other legs of the bridge. Would you care to comment on John Lumley's half-bridge anemometer with regard to this problem?

FINGERSON: I believe that the difficulty with the half-brige is that you still have a resistor, going from your sensor into the amplifier because it's an inverting amplifier and this adds to the noise. With a straight bridge, you also have a resistor in series with the sensor, and so they are equivalent.