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A study of a second-order predictive control system

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A STUDY OF A SECOND-ORDER PREDICTIVE CONTROL SYSTEM

BY

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A

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ABSTRACT

The purpose of this thesis is the study of a predictive control system by simulating it on an analog computer. Logic criteria for step reference inputs are derived by the phase-plane technique, and the corresponding logic network designed to bring the error and error rate of the controlled system toward zero in the least possible time.

The responses of the predictive control system for step, ramp, sinusoidal and exponential reference inputs are investigated with primary emphasis placed on the rise time and overshoot for several values of step inputs of the reference.

Finally, recommendations are made for further investigations in the areas which are deemed to be vital in helping to theorize and design a more general type of predictive control system.

ACKNOWLEDMENTS

The author wishes to express his gratitude to Dr. R. D. Chenoweth, Professor of Electrical Engineering, for providing valuable guidance and suggestions in the preparation of this thesis.

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LIST OF SYMBOLS

Symbol	Analytical or Physical Meaning of Symbol
$r(t)$ or r	Instantaneous value of the reference input
$\dot{r}(t)$ or $\dot{r}$	Instantaneous value of the rate of change of reference input.
$c(t)$ or c	Instantaneous value of the output of the controlled system.
$\dot{c}(t)$ or $\dot{c}$	Instantanous value of the rate of change of output of the controlled system.
$e(t)$ or e	Instantaneous value of the present error.
$\dot{e}(t)$ or $\dot{e}$	Instantaneous value of the present error rate.
$e_f(t)$ or e_f	Instantaneous value of the future error.
$\dot{e}_f(t)$ or $\dot{e}_f$	Instantaneous value of the future error rate
M	Polarized relay
F	Fast-time model relay
S	Sampling relay
R	Relay or a step input

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CHAPTER I

INTRODUCTION

The more recent problem in automatic control systems is not just to design a control system that gives an optimum response, but to design an optimizing control system. An optimizing control system is a system in which adjustments are made in operation of the controlled system to optimize its performance. An optimizing system is never exactly adjusted for optimum performance for any extended period, but a good control system will force the controlled elements to be near optimum adjustments at all times.

There are three kinds of optimizing control systems still in the infant stage of development: adaptive, exploratory and predictive control systems.

This study has been performed on a predictive control system which is roughly an analog to the processes of a human operator in manual and logical control of a system. The purpose of this investigation was the study of a predictive control system by simulating it on an analog computer. Logic criteria were developed with the help of error, e, and error rate, $\dot{e}$, phase-plane analysis to make the the desired adjustments in the preformance of the controlled system by forcing error and error rate toward zero in the least possible time. Though the logic criteria were designed primarily for step reference inputs, they were also used to study the response of the system to ramp, sinusoidal and exponential inputs. In addition the effect of large and small step reference inputs were investigated with respect to saturation of the controlled system.

This thesis considers the operation of a second order linear controlled system, having open loop poles at 0 and -1, the performance of which was optimized by a predictive control system. The system was simulated on the Electrical Engineering Department's Analog Computer and responses of the system to step, ramp, sinusoidal and exponential reference inputs were studied though the logic network was designed primarily for step reference inputs.

CHAPTER II

REVIEW OF LITERATURE

The predictive control is a recent development and nearly all literature is restricted to periodicals within the last six years. In 1955 Coales and Noton (1) described a relay-controlled servomechanism represented by an nth-order differential equation for which one and only one change-over at a unique time is necessary to bring error and error rate to zero in the least possible time. In a specific example they compared the responses obtained from a "bang-bang" control system using predicted change-over to that obtained from an orthodox linear but saturating servomechanism. In the end they concluded that the performance of the optimized "bang-bang" control is always better for all amplitudes of step, ramp and parabolic function inputs.

In 1958 Eckman and Lefkowitz (2) dealt with the application of digital computer control to a batch process. In this process the use of a predictor computer control along some simple operating path yielded the specific product, and the desired product specified a criterion of optimum performance.

A more general analytical approach to the use of model techniques was made by Eckman and Lefkowitz (8) in 1960. They made a general formulation of model techniques based on the fundamental ideas of linear closed loop systems.

The same year Phister (3) wrote a paper comparing predictive and exploratory control systems as applied to process industries. He described an exploratory control system as a control system in which intentional perturbations were made in the controlled system variable parameters in order to observe the effect on the objective; and the result was used to make further adjustments in the control variable parameters in order to get optimum response of the system.

He defined a predictive control system as a system in which measurements of uncontrollable variable parameters are substituted into the process equations, the controllable variable parameters are found from these equations so that the control objective is satisfied, and these computed adjustments are made on the process by the control system.

Comparing the merits of these two kinds of control systems he remarked that the predictive control could be employed in complex situations involving two or more controlled variable, and the exploratory control system would find use where only one or two variables are important.

In 1961 Chestnut, Sollecito and Troutman (4) presented a paper in which they described the application of predictive control to automatic landing of an aircraft. They showed a logic flow-chart and an analog mechanization of the control logic with inexact models. They added that when approximating a higher order system with a lower order model, the model may be made more effective when its most dominant time constant is proportional to the sum of the time constants in the higher order controlled system.

CHAPTER III

THE PREDICTIVE CONTROL SYSTEM

In general a predictive control system might be either a linear or nonlinear system, and the reference might be an arbitrary variable, f (t). Since a general predictive control system could be actuated by the future values of reference, output of the system and their derivatives it was required to predict these values. The reference input and its derivatives might be predicted by a fast-time predictor while the controlled output and its derivatives might be predicted by a fast-time model of the controlled system. On the basis of the predicted values of the reference and its derivatives, and the output of the controlled system and its derivatives, logic criteria for an nth order system might be developed by a phase-space analysis. According to Pontryagin's principle (34) for a controlled system of the nth-order, there must be (n-1) switching operations to force the error and all its derivatives to zero in the least possible time.

A. A Second-order Predictive Control System

In this study a second order controlled system having open loop poles at 0 and -1 was considered. The function of the predictive control system is outlined in the block diagram of Figure 1. There are three essential functioning parts of the system: controlled system, fast-time model and logic network.

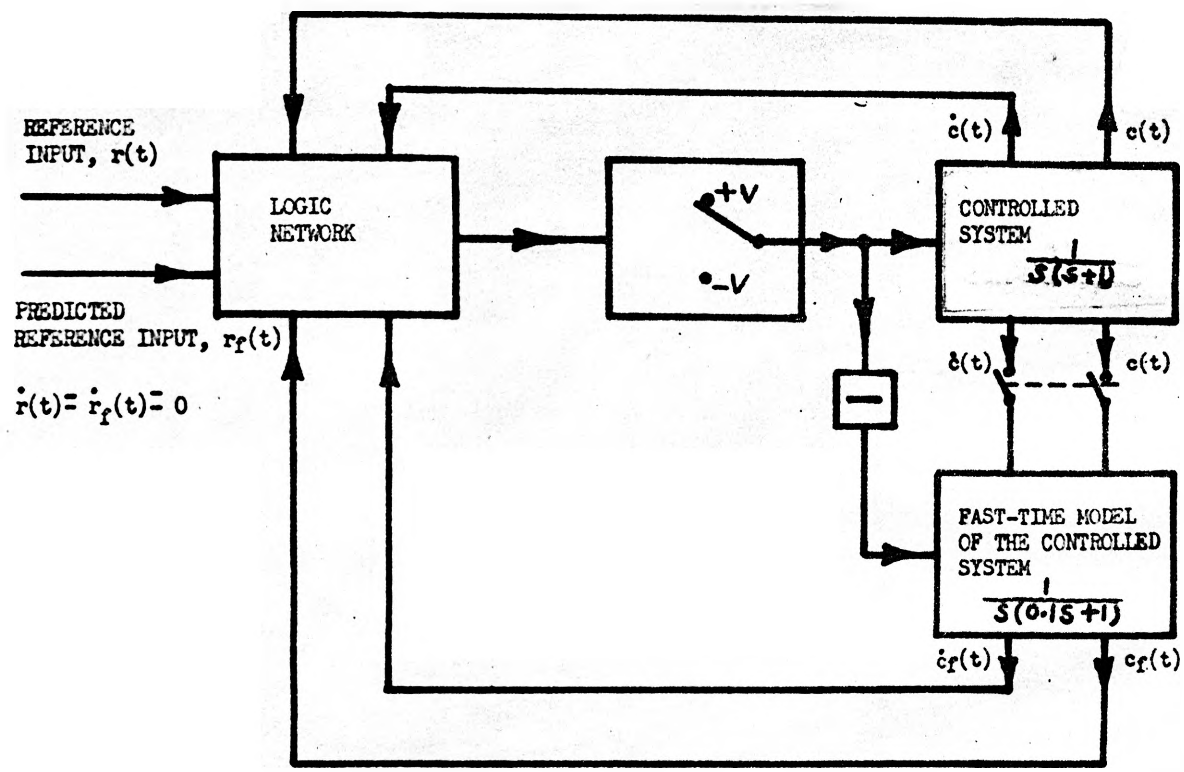


FIGURE 1. BLOCK DIAGRAM OF PREDICTIVE CONTROL SYSTEM.

The controlled system might be the actual devices to be controlled by the system. For this study the controlled system was assumed to have a transfer function consisting of the integrators and a time constant of one second.

The fast-time model was essentially the model of the controlled system having one-tenth of a second time constant; therefore it could predict quickly the future values of the controlled output and its derivative. Thus the output of the fast-time model provided the predicted output and its derivative for the logic network. It was necessary for the present values of the controlled system output and its derivative to be inserted as initial conditions on the fast-time model so that it would predict the future values of the output and its derivative successfully.

The logic network consisted of switching devices to control the power input to the controlled system in a manner to accomplish optimum response. This system had step, ramp, sinusoidal and exponential reference inputs although the logic network was designed on the basis of step reference inputs. Since the magnitude of a step does not change with time, it might be predicted that the magnitude of the function remained the same in the future. The fast-time model provided the predicted output and its derivative for the logic network; the step reference was also fed to the logic network, which after determining the future error, e_f, and its derivative, $\dot{e}_f$, switched the power input to the controlled system to force the error and error rate of the system toward zero in the least possible time. The power input to the controlled system was considered to be a constant, although in general it might be some function of time.

B. <u>Logic Criteria for the Second order System with a Step Reference Input</u>

The logic network might be considered as the brain of a predictive control system. Considering error and error rate, logic criteria were developed by a phase-plane analysis.

In general there would be nine sets of error and error rate combinations, since either error or error rate might be positive, zero or negative. One of the sets would be zero error and zero error rate which would be a desirable combination for the optimum response of the system. Hence, there would be eight sets of error and error rate for which a suitable actuating power polarity must be provided to actuate the controlled system in order to force error and error rate toward zero.

Error, error rate, and the required power polarity combination for optimum response of the system is summarized in Table 1.

The tabulation of the eight sets of error, error rate, and power polarity was the outgrowth of the graphical analysis performed with the help of Figure 2 and Table 1.

An error might be defined as the difference between

POINTS AND RANGES BETWEEN POINTS (SEE FIGURE 2)	1	2	3	4	5	6	7	8
ERROR	0	−	−	−	0	+	+	+
ERROR RATE	−	−	0	+	+	+	0	−
POWER POLARITY	−	−	−	+	+	+	+	−

TABLE 1. SIGNS OF ERROR, ERROR RATE AND POWER POLARITY FOR OPTIMUM RESPONSE OF THE SYSTEM.

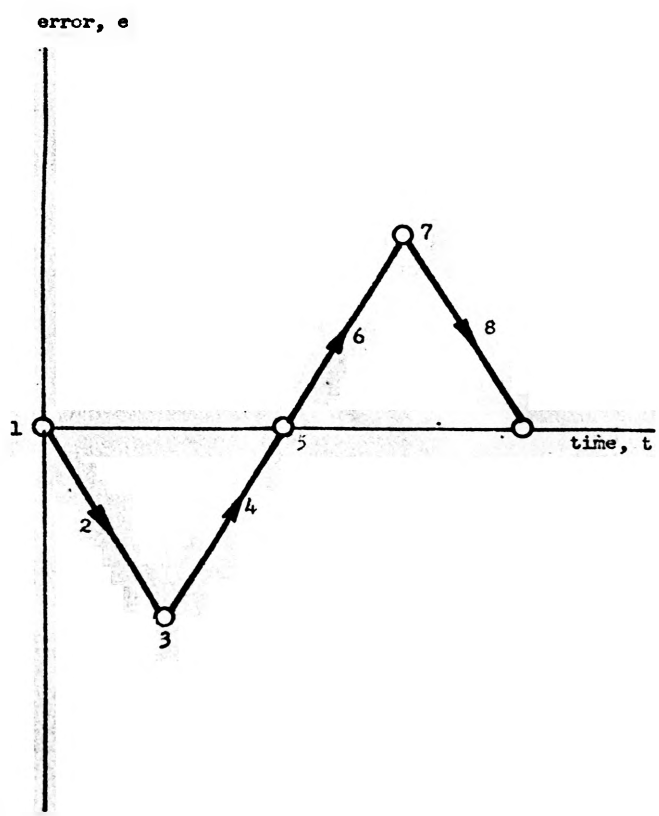


FIGURE 2. GRAPHICAL REPRESENTATION OF ERROR AND ERROR RATE.

the reference and the output of a system. Different combinations of error and error rate were represented graphically with the aid of Table 1 in Figure 2 by points and regions between point on the curve.

It is observed in Figure 2 and Table 1 that in the region between points 1 and 3 where the error and error rate as shown occur with a positive polarity actuating signal which will force the system toward a larger error; therefore a negative polarity signal must be applied to bring error and error rate toward zero. If the negative polarity signal to the controlled system continued any longer than was required it would force the system toward error and error rate as illustrated by the region from range 4 to point 7; hence, there must be power polarity reversal in the area of range 4 to compel error and error rate to zero. If the positive power polarity were maintained for a longer time than was required, error and error rate would extend beyond range 8 and would repeat the cycle; therefore, the power must be switched in the area of range 8 from positive polarity to negative to force error and error rate to zero.

In Table 1, the sign of error rate changed in the region when switching occured; the sign of the power polarity would be referred to the sign of the error in determing the logic criteria. Accordingly, from the table, it might be concluded that when error and error rate have opposite signs, switching must occur in such a manner that the polarity of the actuating power must be opposite to the sign of the error, and when the error and error rate have the same signs the power polarity applied to the controlled system must be of the same sign as the sign of the error. The fact that the derived logic criteria were incomplete led to the phase-plane analysis to locate the exact point where switching must take place to get the optimum response of the system. According to Pontryagin's principle there need to be one switching operatinn to bring the error and error rate to zero.

The phase-plane for the system under consideration was utilized to determine the proper switching instant. In order to determine the trajectories on the phase-plane, the output of the system may be written in the differential equation form as

$$\ddot{c} + \dot{c} = T \qquad (1)$$

where $\ddot{c}$ and $\dot{c}$ are second and first derivatives of the output of the controlled system respectively and T is a positive or negative constant.

$$c = R - e \qquad (2)$$

where R is the reference input.

Substituting Equation 2 in Equation 1 gives;

$$\ddot{e} + \dot{e} + T = \ddot{R} + \dot{R} \qquad (3)$$

Since R is considered to be constant in this case Equation 3 becomes;

$$\ddot{e} + \dot{e} + T = 0 \qquad (4)$$

Letting x = e, and y = $\dot{e}$; Equation 4 becomes;

$$\dot{y} + y + T = 0$$

This may be rewritten as

$$\frac{dy}{dt} + y + T = 0 \qquad (5)$$

Since dx = y dt, dt may be eliminated from the above equation and upon rearranging terms,

$$\frac{y\,dy}{T + y} = -\,dx \qquad (6)$$

Solving Equation 6, the equations for the phase-plane trajectories are obtained as

$$y - T \ln (T + y) + x = \text{constant} \qquad (7)$$

Replacing x and y in the above equation by e and $\dot{e}$ respectively,

$$\dot{e} - T \ln (T + \dot{e}) + e = \text{constant} \qquad (8)$$

This equation may be plotted on ande and $\dot{e}$ phase-plane for different values of the constant as shown in Figure 3. Considering the value of the constant in Equation 8 to be zero, the relation between e and $\dot{e}$ may be

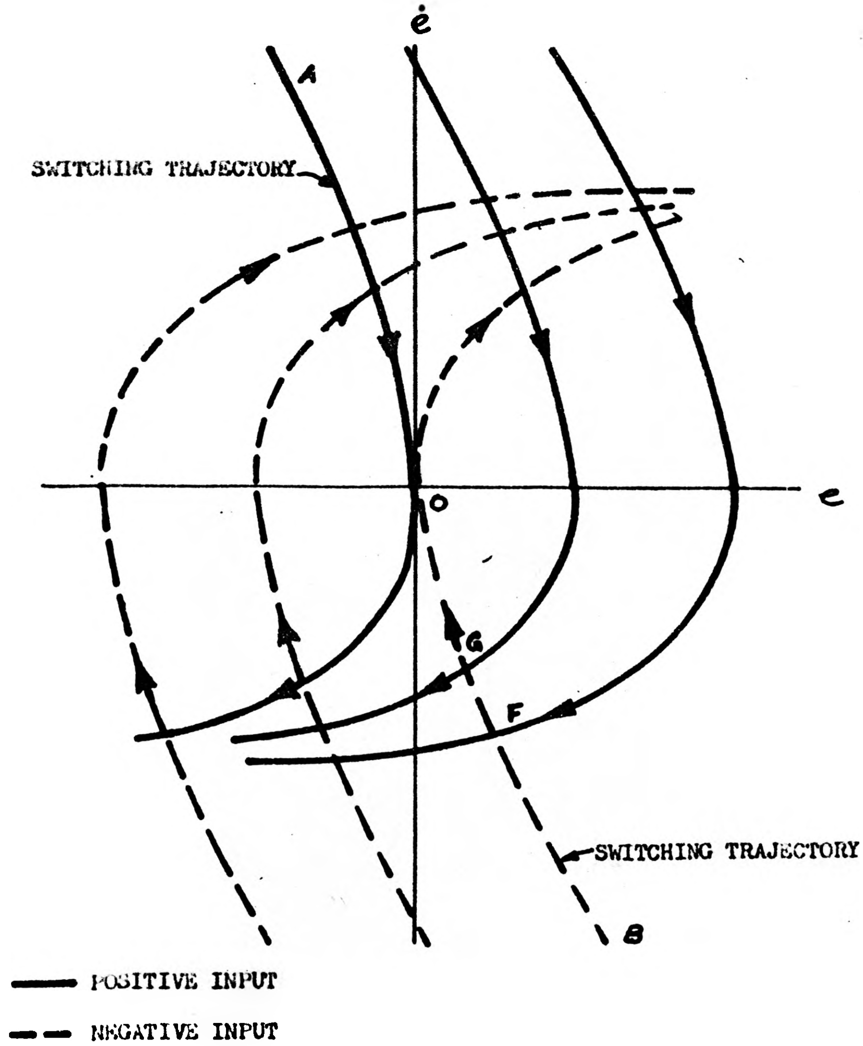


FIGURE 3. PHASE-PLANE DIAGRAM OF THE SYSTEM.

represented by the trajectories passing through the origin, which are called the switching trajectories because optimum response of the controlled system is accomplished by switching the power polarity applied to the system along these curves. AO and BO are the switching trajectories as illustrated in Figure 3.

It is observed from Table 1 that switching must take place somewhere in the region where error and error rate are of opposite signs, corresponding to the second and fourth quadrants of the phase-plane plot. The crossing points of different trajectories with trajectories for zero constant of Equation 8 in the second and fourth quadrants gives the points where switching must take place to get optimum response as shown in Figure 3, but there is no means to determine the switching points. Hence a fast-time model is proposed to implement the switching requirements by predicting the switching points. The fast-time model predicts the future values of error and error rate, and for this reason the predictor model is actuated by a signal of an opposite polarity to the actuating signal of the controlled system. Switching must take place at such a time that the controlled system trajectory passes through the origin to give optimum response as shown in Figure 4. If the switching took place late, the system would operate in a limit cycle as illustrated in Figure 5,

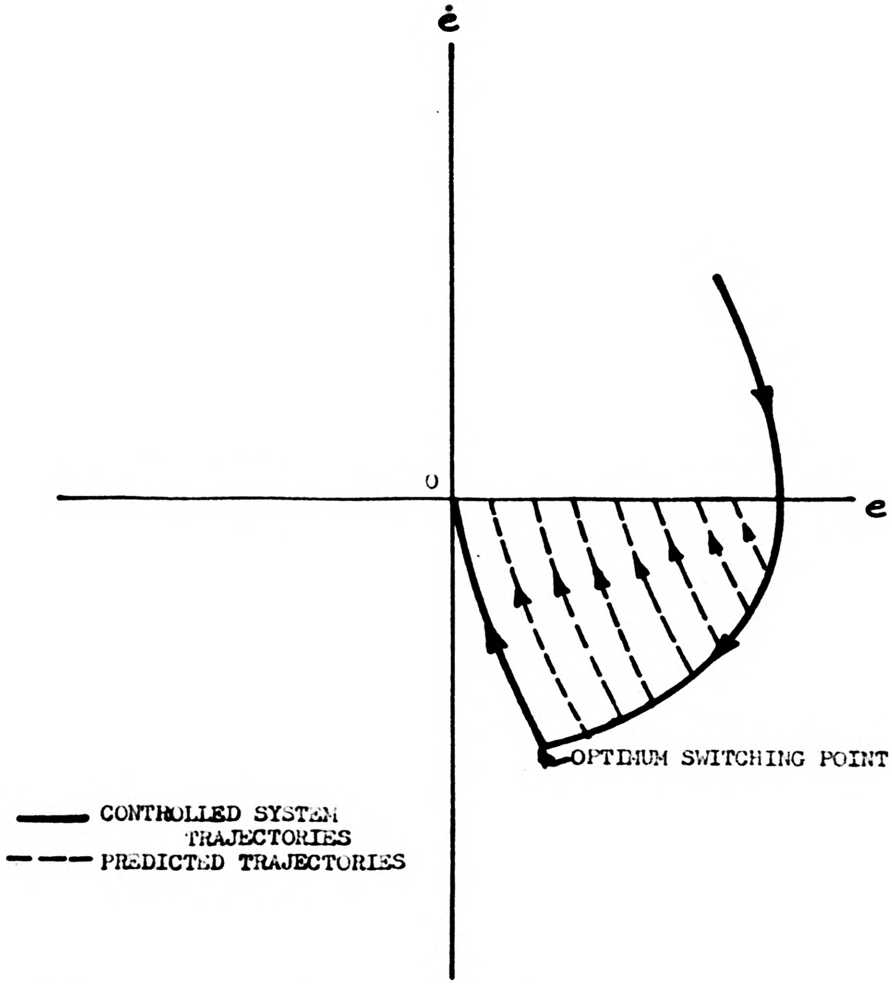


FIGURE 4. PHASE-PLANE TRAJECTORIES OF THE SYSTEM WITH THE FAST-TIME MODEL.

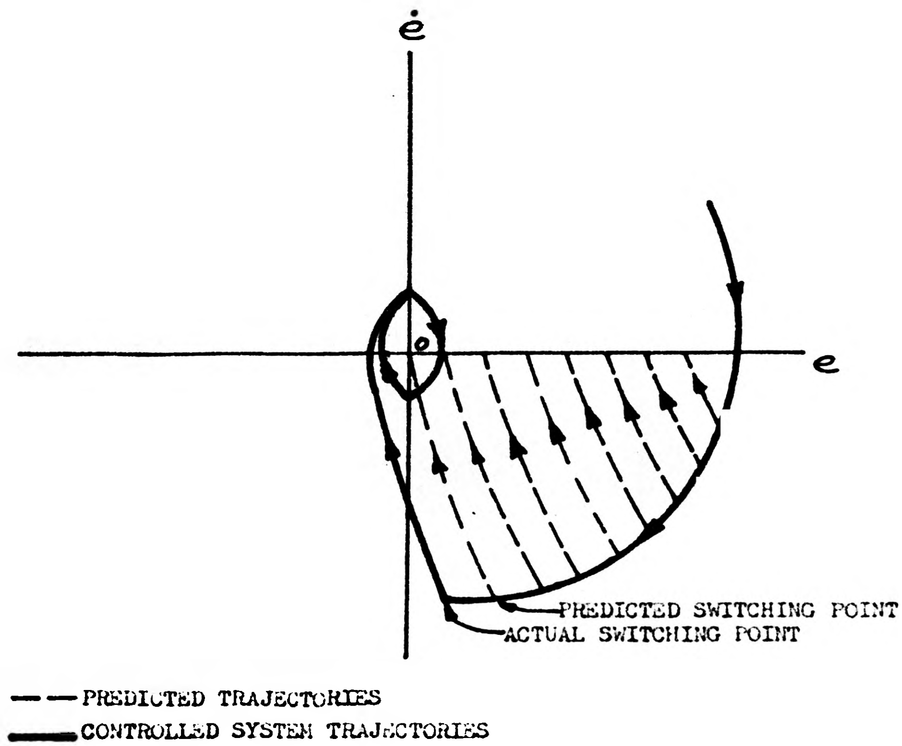


FIGURE 5. PHASE-PLANE OF THE SYSTEM SHOWING LIMIT CYCLE DUE TO DELAYED SWITCHING.

and the system response would have a relatively large steady-state ripple. It is noted from Figure 4 that the length of predicted trajectories varies from the time when prediction starts until the optimum switching takes place. For a large error the length of the predicted trajectory is smaller and it gradually increases until the optimum switching point occurs. Hence the prediction rate is higher for large errors than that for smaller ones. It should also be noted that the predicted switching trajectory is the largest one, therefore the prediction rate is smallest for it.

The complete logic criteria might be summarized as follows:

1. When error and error rate are of the same sign, the power polarity applied to the system must be of the same sign as the sign of error.

2. When error and error rate are of opposite signs, the sign of the power polarity to the controlled system must be the same as that of the future error, until switching takes place. Approximate time for switching the power polarity is determined with the help of the predictor model which is actuated by power of opposite polarity to that applied to the controlled system. The present values of error and error rate are inserted into the fast-time model as initial conditions and predictions

takes place until $\dot{e}_f = 0$ where prediction stops. If e_f is positive when the prediction stopped the next prediction begins in the same manner with present error and error rate as initial conditions and the prediction stops as before. Prediction continues in the same manner until both e_f and $\dot{e}_f$ are equal to zero, and at this point switching must take place in order to get the optimum response of the system. Meantime, the power polarity to the controlled system is held as it was when prediction started.

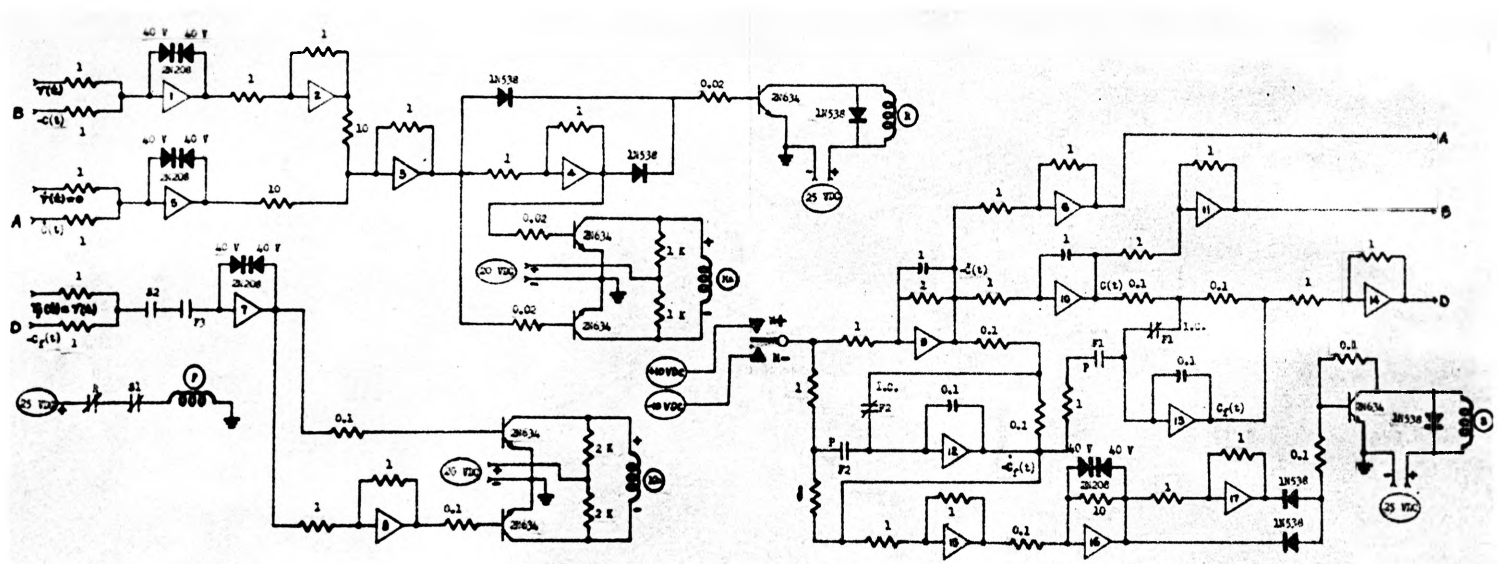
CHAPTER IV

ANALOG SIMULATION OF THE SYSTEM

The predictive control system outlined in Figure 1 was simulated on a Philbrick HKR Analog Computer as shown by the computer schematic diagram in Figure 6.

A polarized relay, M, with two indepedent driving coils was used to actuate the system and the fast-time model with a positive or a negative voltage. The controlled system and the predictor model were connected in such a manner that the actuating power polarity signal to the fast-time model was of opposite polarity to that of the controlled system to fulfill the logic criteria.

The simulation of the controlled system was accomplished by a time delay and an integration, and that of the fast-time model was identical to the controlled system with two differences. First, the fast-time model was ten times faster than the actual system. The second difference was the introduction of initial conditions into the fast-time model from the actual system. The actual system simulation on the computer consisted of amplifiers 9 and 10 and that of the fast-time model consisted of amplifiers 12 and 13 in Figure 6. The time constant of the initial condition circuit was such that the fast-time model could be charged with the correct initial conditions quickly. Relay F energized from contacts on relays S and R was employed to transfer initial conditions from the controlled system to



Note: All relays are shown in their de-energized positions.

FIGURE 6. ANALOG COMPUTER DIAGRAM FOR PREDICTIVE CONTROL SYSTEM.

the fast-time model.

To satisfy the logic criteria determined previously, simulation was accomplished in such a way that the input power polarity to the controlled system and the predictor model would be controlled by the logical decisions to force error and error rate toward zero in the least possible time. This was achieved by satisfying the switching criteria established on the basis of the phase-plane analysis. The simulation of the logic network might be divided into two main parts: first, the part which was employed to compare the signs of error and error rate and the second part was the section which compared the signs of the future error and error rate. In the simulation amplifiers 1 through 6 were used to compare the signs of the error and error rate and amplifiers 7 and 16 compared the signs of the future error and error rate respectively. The outputs of these amplifiers in conjuction with relays R, S, M and F satisfied the logic criteria.

In general, starting with initial conditions on the controlled system, error and error rate might be at any point in the phase-plane plot. Assuming that there were no initial conditions imposed on the controlled system at the instant a step reference input was applied the magnitude of e would be maximum and positive and that of $\dot{e}$ zero. One of the coils of the polarized relay, Ma, was driven by the outputs of amplifiers 3 and 4, and the other one, Mb, was energized by the outputs of amplifiers 7 and 8. With e positive and $\dot{e}$ zero, the output of the summing amplifier 3 was -4 volts which energized relays R and Ma. The coil Ma of the polarized relay was oriented in such a manner that the relay actuated the controlled system with a positive polarity power. At the same time, R was energized, and S was de-energized; therefore, F was de-energized. Since S was de-energized, S1 was closed and S2 was opened. The result was F3 remained open, and F1 and F2 were in position to set initial conditions into the fast-time model.

The positive power polarity to the system resulted in negative $\dot{e}$ and positive e. The outputs of amplifiers 2 and 5 were +40 volts and -40 volts respectively due to Zener diodes, and the output of amplifier 3 was zero making R and Ma de-energized. Since R remained unactuated F was energized and switched F1 and F2 from initial condition positions to the prediction positions and prediction started. As soon as $\dot{c}_f$, which is equal to $\dot{e}_f$ in this case, became zero, S was actuated and this opened S1 and F was de-energized, so F1 and F2 returned to the initial condition positions and the cycle of prediction repeated. Thus when $\dot{e}_f$ is zero contacts, F3 and S2, will be closed simultaneously for about 2 milliseconds. In this whole operation, the polarity of the polarized relay remained positive, because e_f was still positive, but as soon as e_f became negative the polarity of the polarized relay switched to negative. Hence, the logic criteria developed for the fourth quadrant of the phase-plane of e and $\dot{e}$ is satisfied.

As soon as the polarized relay switched, both e and $\dot{e}$ became negative and the output of amplifier 3 was +8 volts which energized R and Ma. As soon as R was energized, F became de-energized which made F1 and F2 to return to initial condition positions and open F3 causing Mb to be de-energized. The polarity of the actuating power to the controlled system remained negative. Hence, the logic criteria established for the third quadrant of e and $\dot{e}$ phase-plane were fulfilled. At that instant, e and $\dot{e}$ were so small that F was not actuated due to its large time constant in comparison with the time required to force e toward zero. The system then starts functioning in a limit cycle as shown in Figures 7 and 8.

Before the simulation of the predictive control system was made on the computer the polarized relay and various other sensitive relays were tested to determine their pick-up times and the amount of power to energize them. It was found that their pick-up times were from almost instantaneous to 2.5 milliseconds and it required from 10 to 17 milliamperes of current to energize them. Considering the pick-up times of the relays, the time constant of the controlled system was chosen to be 1 second. Since the computer was capable of delivering 1 milliampere of current at ± 50 volts, transistor amplifiers were added to deliver enough power to drive the relays. Figures 9, 10 and 11 show respectively the analog computer and equipment used in the laboratory, the relay assembly in the computer circuit and the relay assembly.

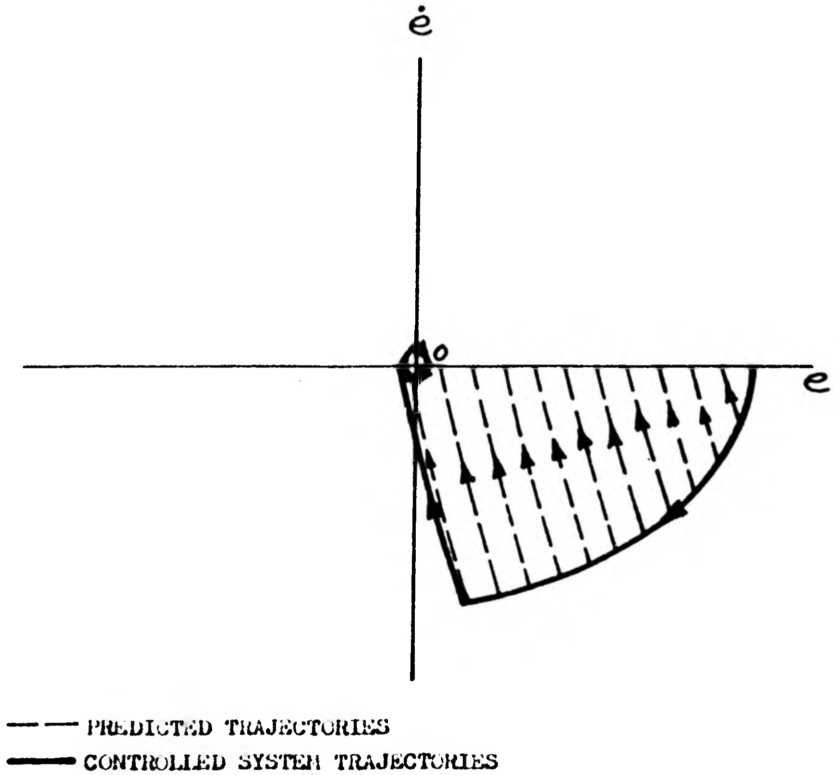


FIGURE 7. PHASE-PLANE DIAGRAM ILLUSTRATING THE ACTUAL PERFORMANCE OF THE PREDICTIVE CONTROL SYSTEM.

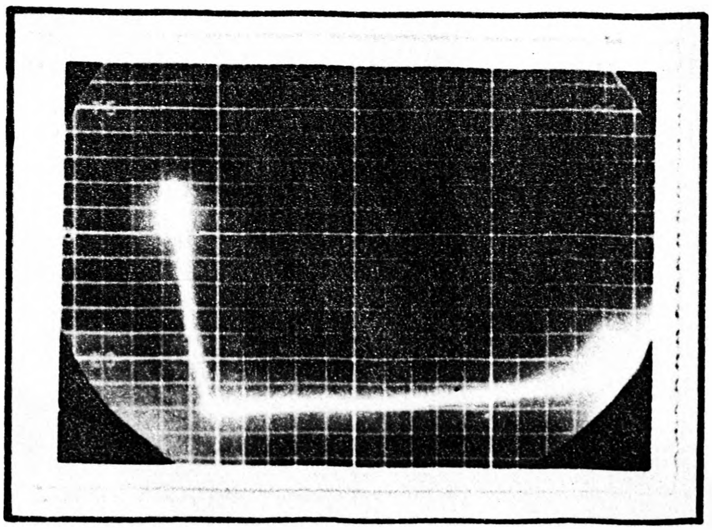


FIGURE 8. PHASE-PLANE PICTURE OF TRAJECTORIES.

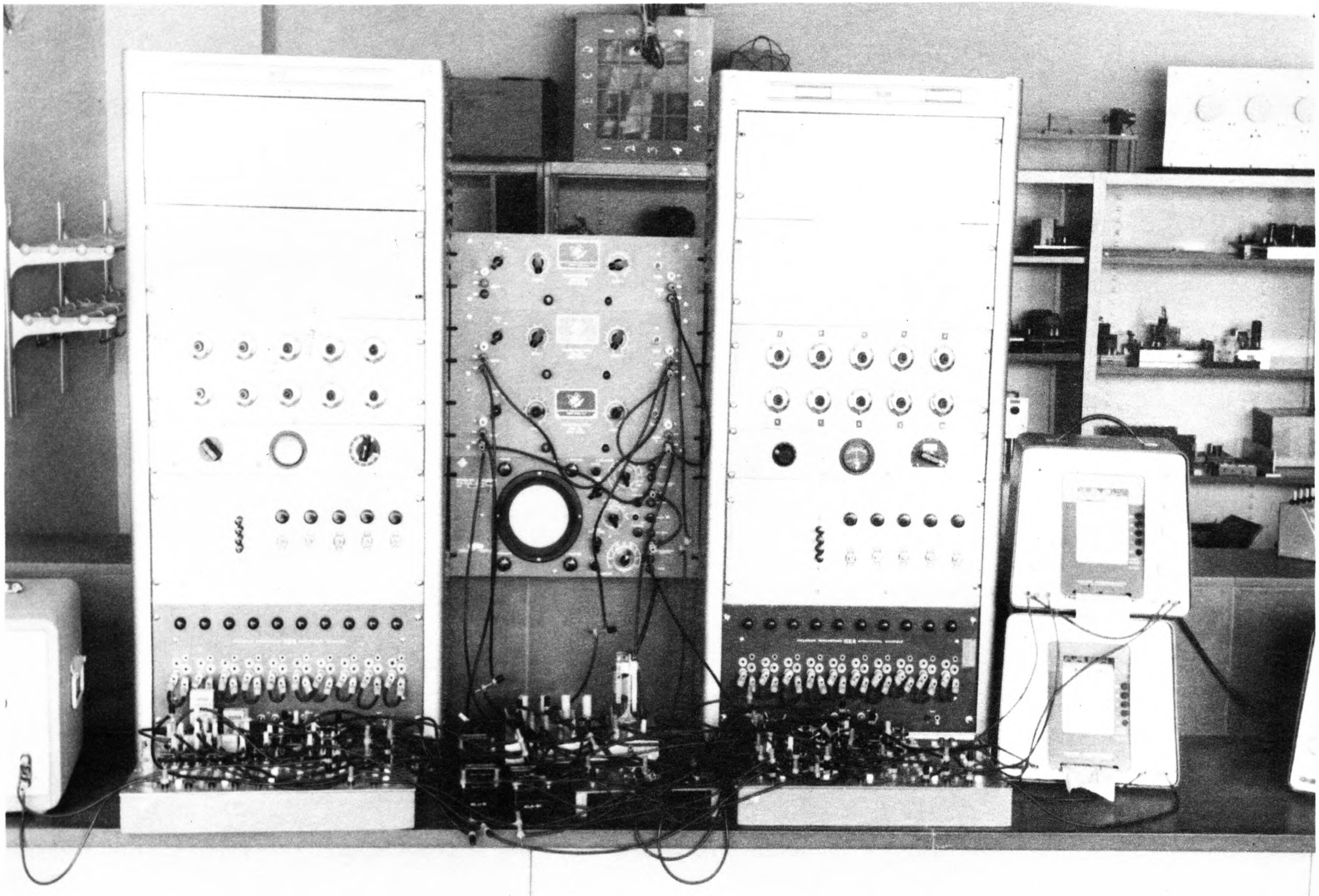


FIGURE 9. PHOTOGRAPH OF THE ANALOG COMPUTER AND EQUIPMENT USED IN THE LABORATORY.

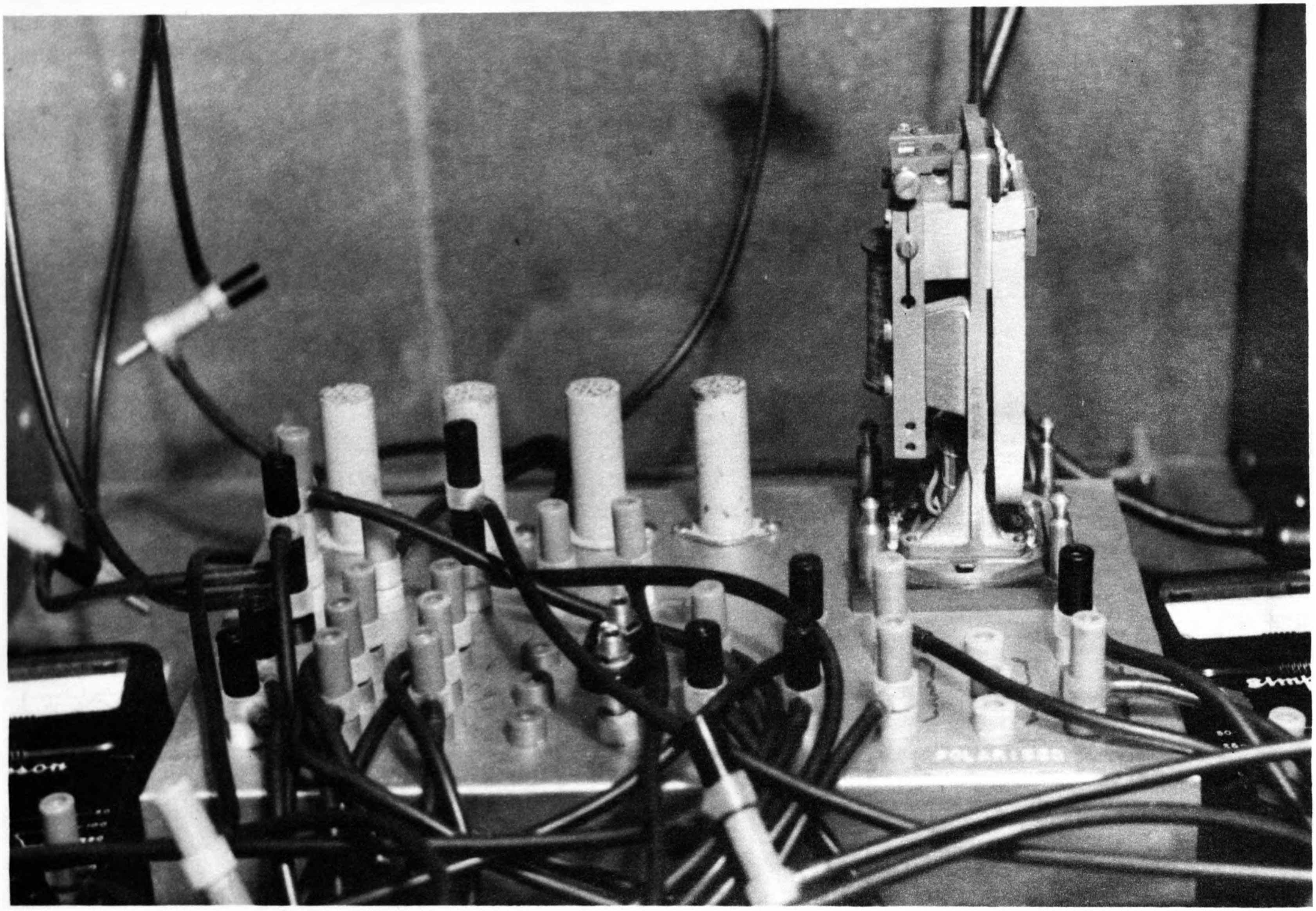


FIGURE 10. PHOTOGRAPH OF RELAY ASSEMBLY IN COMPUTER CIRCUIT.

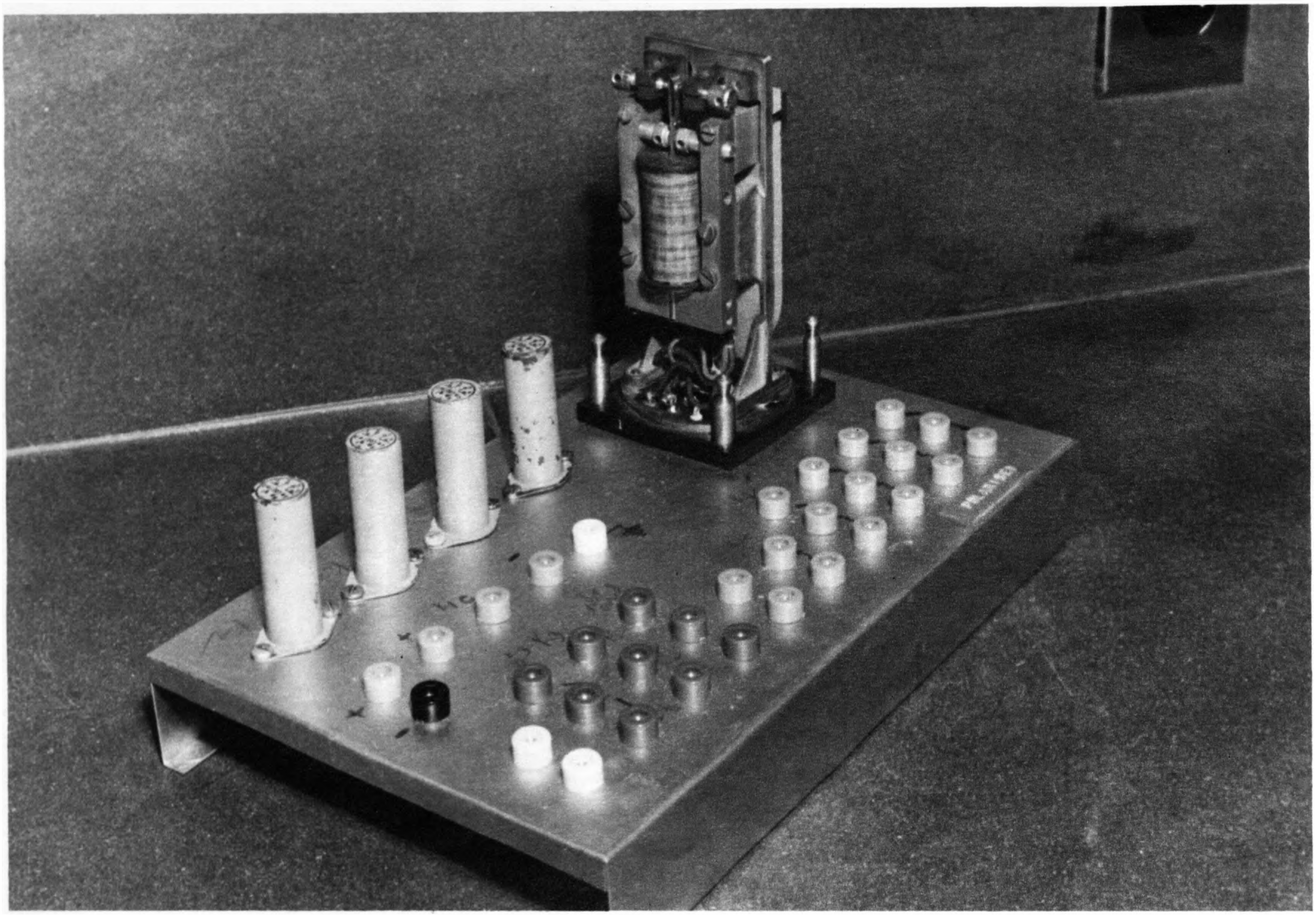


FIGURE 11. PHOTOGRAPH OF RELAY ASSEMBLY.

CHAPTER V

DISCUSSION OF RESULTS

The predictive control system with a logic network designed for step reference inputs was simulated on the analog computer. The responses were observed with Brush Recorders as shown in Figures 12, 13 and 14, and a study was performed comparing overshoot, response time, velocity saturation and limit cycle for different step reference inputs to the system both with and without the fast-time model. Another area that was briefly investigated was system response to ramp, sinusoidal and exponential reference inputs, although the simulation does not involve the rate of change of the reference.

A. Overshoot

In Figure 4 it was shown that when the error was large the prediction rate was faster than when the error approached zero. By the time the predicted trajectory reached zero the actual system trajectory moved ahead and the switching was delayed as shown in Figure 5, resulting in an overshoot of the output. Almost immediately following the initial overshoot the response of the system settled down into a limit cycle.

It is observed from Table 2 that for the system with the fast-time model there is a linear relationship between the magnitude of the step reference input and the absolute

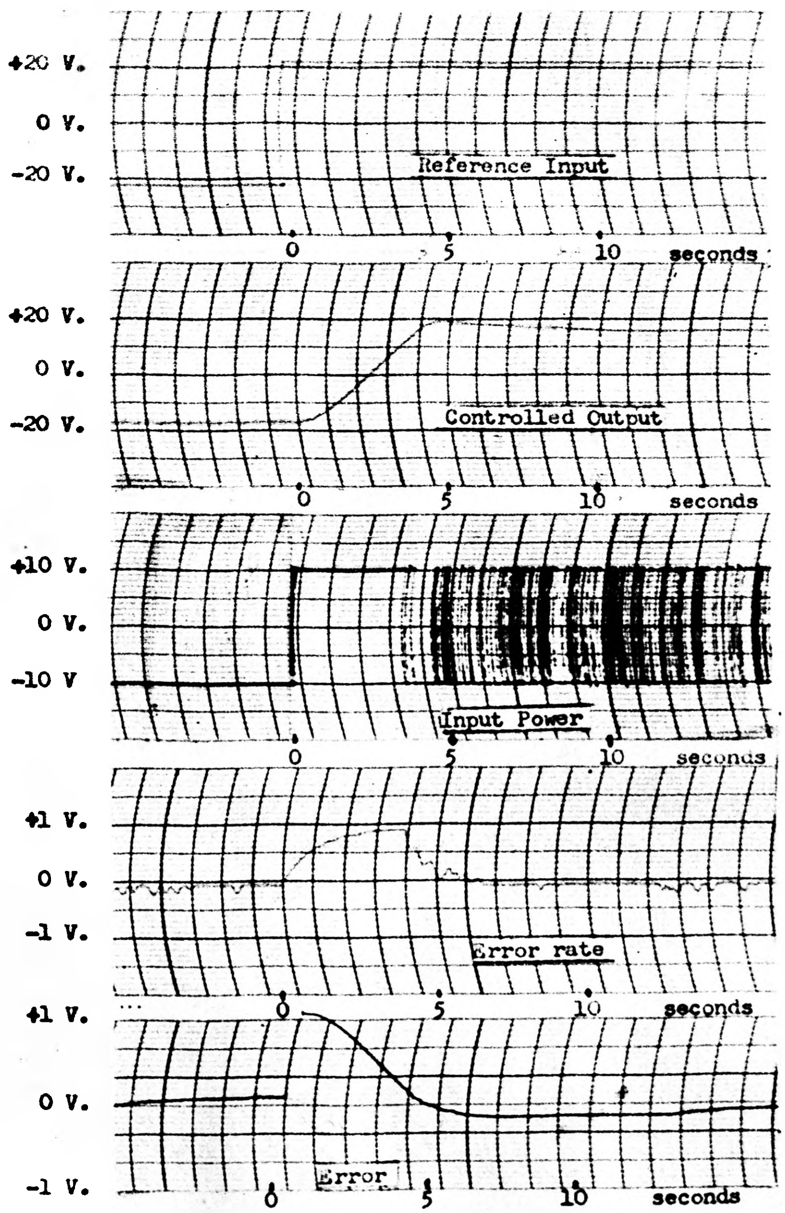


FIGURE 12. RESPONSE OF THE SYSTEM TO A STEP INPUT

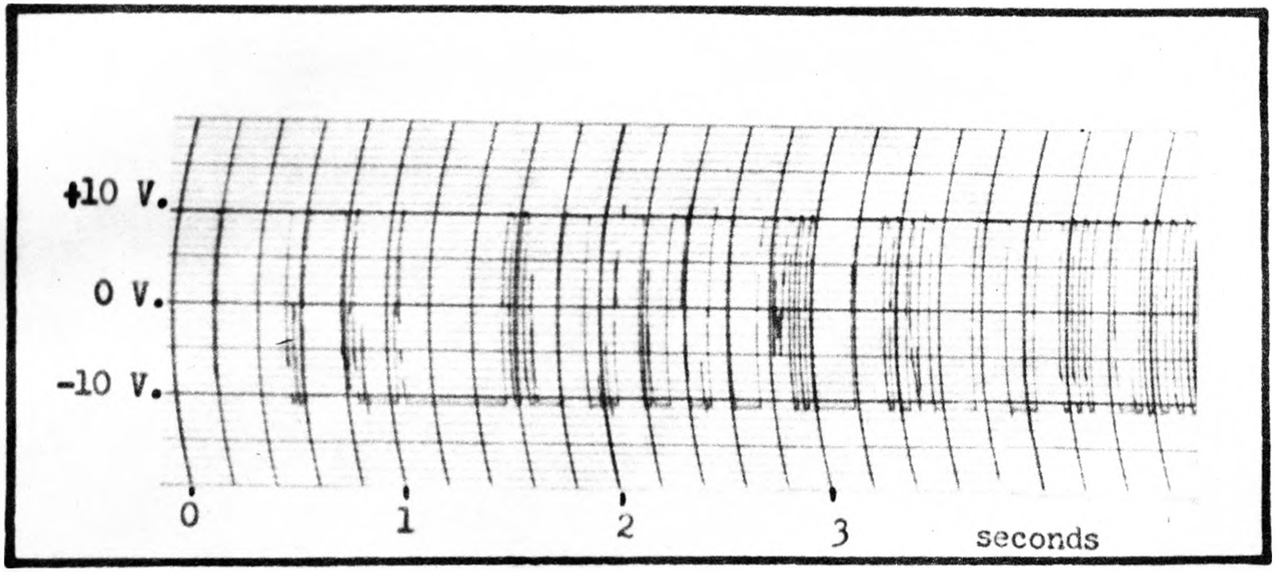


FIGURE 12a. THE FUNCTION OF THE POLARIZED RELAY ON EXTENDED TIME SCALE.

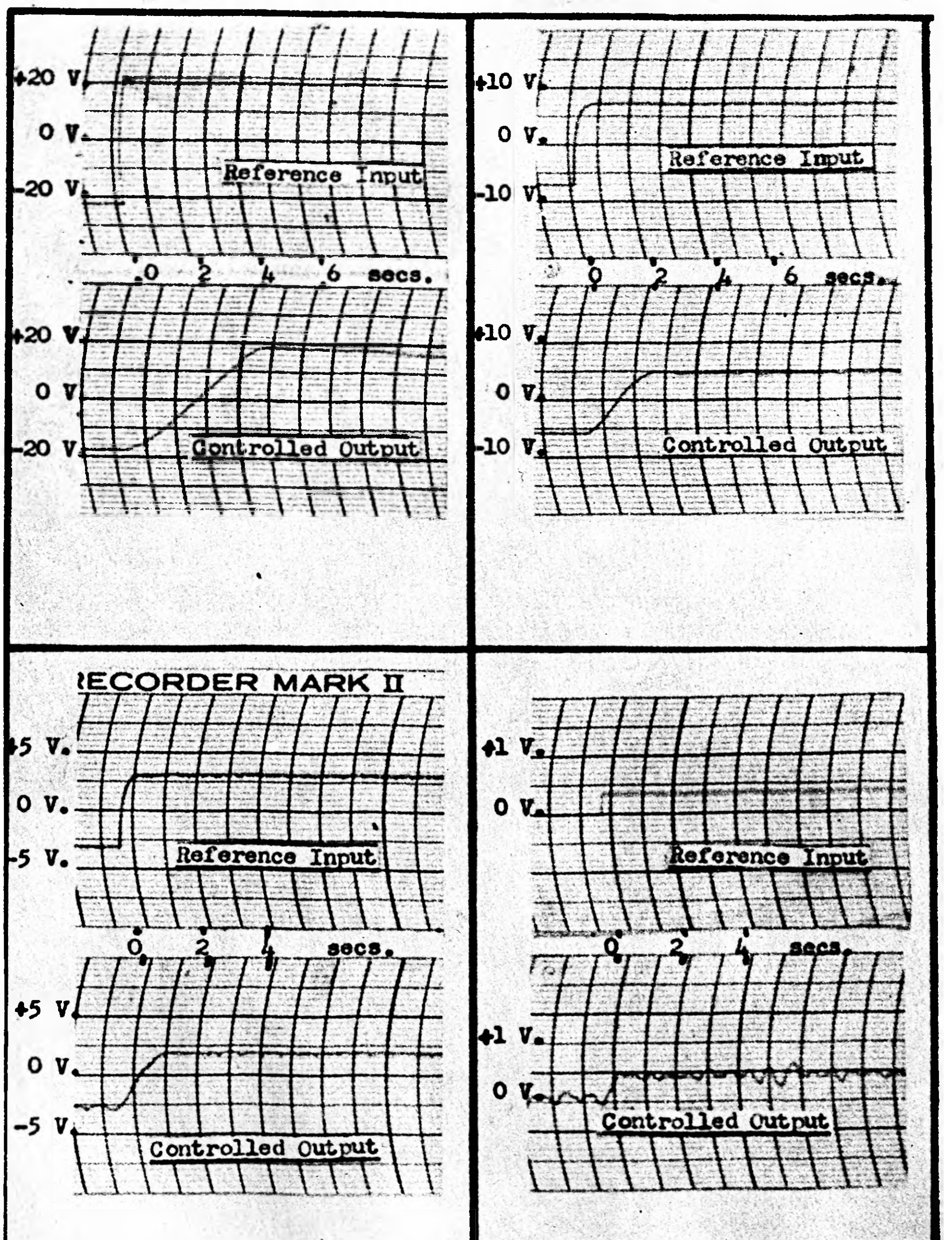


FIGURE 13. RESPONSE OF THE SYSTEM TO VARIOUS STEP INPUTS.

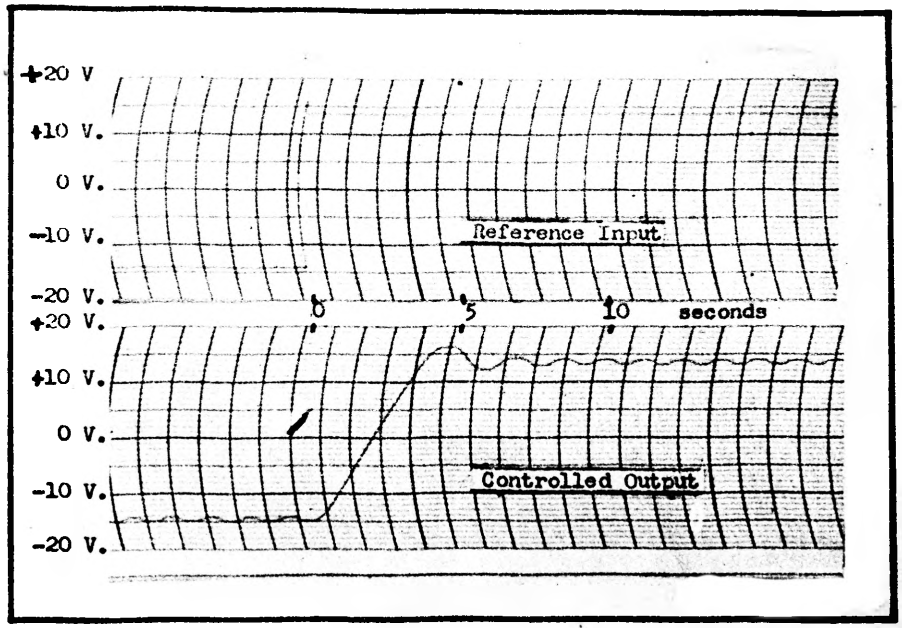


FIGURE 14. RESPONSE OF THE SYSTEM TO A STEP INPUT WITHOUT THE FAST-TIME MODEL.

SYSTEM WITH THE FAST-TIME MODEL	MAGNITUDE OF STEP REFERENCE INPUTS	OVERSHOOT		SETTLING TIME IN SECONDS	RESPONSE TIME IN SECONDS
		ABSOLUTE	PERCENTAGE		
	44 V	2.00 V	4.5 %	5.5	4.0
	11 V	0.50 V	4.5 %	3.0	1.4
	6 V	0.25 V	4.1 %	1.2	1.0
SYSTEM WITHOUT FAST-TIME MODEL	28 V	2.50 V	8.9 %	6.5	3.5

TABLE 2. RESPONSE OF THE SYSTEM WITH AND WITHOUT FAST-TIME MODEL AND ±10 VOLTS FORCING SIGNAL FOR VARIOUS STEP REFERENCE INPUTS.

value of overshoot, resulting in about a 4.5 per cent of overshoot in each case. The reason for this might be explained with the help of Figure 16. The magnitude of the overshoot depends on the amount of time it takes the fast-time model to approach the origin from the optimum switching point. Since it takes a finite amount of time for the fast-time model to approach the origin from the optimum switching point, the actual system moves ahead in the meantime and thus the switching of the actual system is delayed.

In Figure 16 the different trajectories are the result of different magnitudes of step inputs. The larger the step input, the larger is the initial error. The trajectory crossing the e-axis at D represents a larger step reference input than those crossing at B and A. On the phase-plane plot trajectories are almost parallel to one another and the delay in switching is proportional to the magnitude of reference inputs. Since the actual switching trajectories are parallel to one another, they divide the error axis in proportional to their respective delayed switching times. Therefore the amount of the overshoot is proportional to the magnitude of the step reference input and has identical value on the percetage-wise basis.

Without the fast-time model the switching is delayed still longer as shown in Figures 14 and 15, resulting in a comparatively large overshoot as shown in Table 2. There

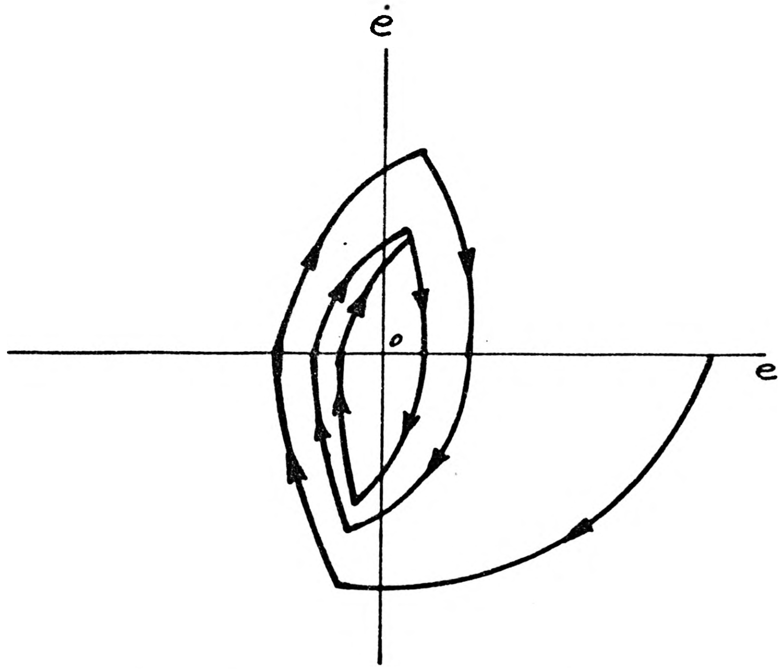


FIGURE 15. PHASE-PLANE DIAGRAM SHOWING THE PERFORMANCE OF THE SYSTEM WITHOUT THE FAST-TIME MODEL WITH LARGE LIMIT CYCLE.

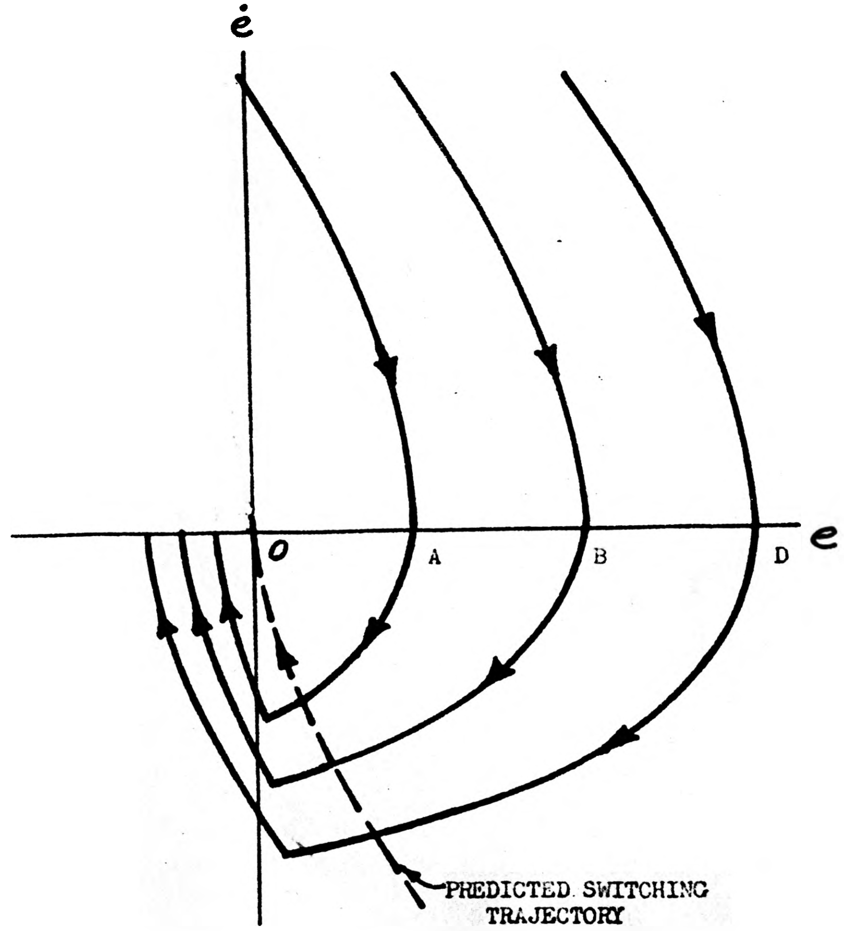


FIGURE 16. PHASE-PLANE DIAGRAM SHOWING DELAYED SWITCHING FOR DIFFERENT TRAJECTORIES.

is no way to predict the optimum switching point without the fast-time model, the system is a "bang-bang" system. In this case switching should take place when the system trajectory crosses the negative error axis, but due to the time constant of the switching relay the switching is still delayed as shown in Figure 15, and this results in a large overshoot.

B. Settling time

Another important concept that was studied is the time required for the system to damp out transient variations. Theoretically the time required is infinite if all variations are to disappear. Hence for practical purposes it is specified that the transient is over when the error has been reduced from its initial value to some minimum value which is not thereafter exceeded. The time required to reduce the error to 2 per cent of its initial value is called "settling time".

From Table 2, it is observed that the settling time for the system varies with the amount of overshoot because under the same conditions of operation the time required to force the overshoot to a minimum value would depend on the magnitude of the initial error. Therefore a larger settling time was required for larger overshoot.

C. Response time

Response time is the time required for the output of a system to cross the desired reference level for the first time as shown in Figure 17. From Table 2 it is observed that there is a relation between the magnitude of the forcing signal, the magnitude of the step reference input and the time constant of the controlled system. This is illustrated by assuming that if W is the amount of work required to force the system output to its response level by a voltage signal of V volts in time t seconds, a relation among W, V and t is shown as $W \propto V^2 t$, provided the controlled system involves pure intergration only. Hence if the forcing voltage is doubled, 2 V, the time required to force the output of the system to the same reference level is reduced to $\frac{1}{4}$ of the time. For an 11 volts reference and 10 volts forcing signal it took about the same time as the response time of the linear controlled system. But for a 44 volts reference and 10 volts forcing signal the response time was about four times more than the response time of the controlled system, and for a 6 volts reference with the same forcing signal the response time was about 0.6 times of the response time of the linear system.

The reason that the responses of the predictive control system did not follow the $W \propto V^2 t$ relation is due to the time constant of the controlled system. Figure 17 shows that the response for each reference input follows the same slope while the system is in velocity saturation. Hence due to velocity saturation the response time for different magnitudes of reference did not exactly follow the above

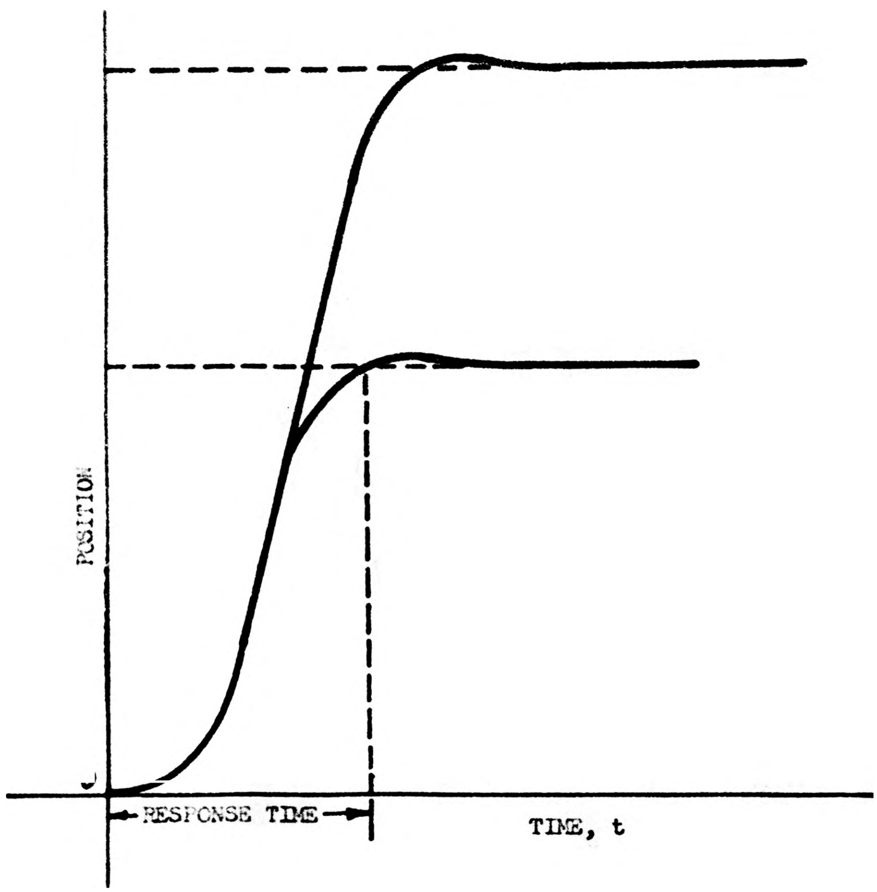


FIGURE 17. RESPONSES TO STEP INPUTS.

relation derived for a system involving pure integration.

The response time for the system without the fast-time model was about the same as that of the predictive control system with a similar reference input and the same forcing voltage signal because the effect of the addition of the fast-time model was to eliminate large overshoot by more precise switching and it had little effect on the response time.

D. Limit cycle

It is observed from the phase-plane picture, Figure 8, that there was no limit cycle of appreciable magnitude for the predictive control system but limit cycle became prominent when the fast-time model was taken out of the system as shown in Figure 15. One of the factors controlling the magnitude of limit cycle is the frequency of optimum switching. The switching in case of the predictive control system was faster than that without the fast-time model, hence the limit cycle in the later case was of larger magnitude than that of the former.

E. System response to the inputs other than step inputs

The next area that was briefly investigated is when the reference input was changed from a step to ramp, sinusoidal and exponential. It is noted from Figure 18 that the response did not follow exactly the reference where there was an appreciable rate of change in its magnitude because the logic network was designed on the basis of step reference inputs only. Therefore the system response follows as if the reference is a series of stair step inputs as shown in Figure 19. This effect may be observed in the actual responses in Figure 18.

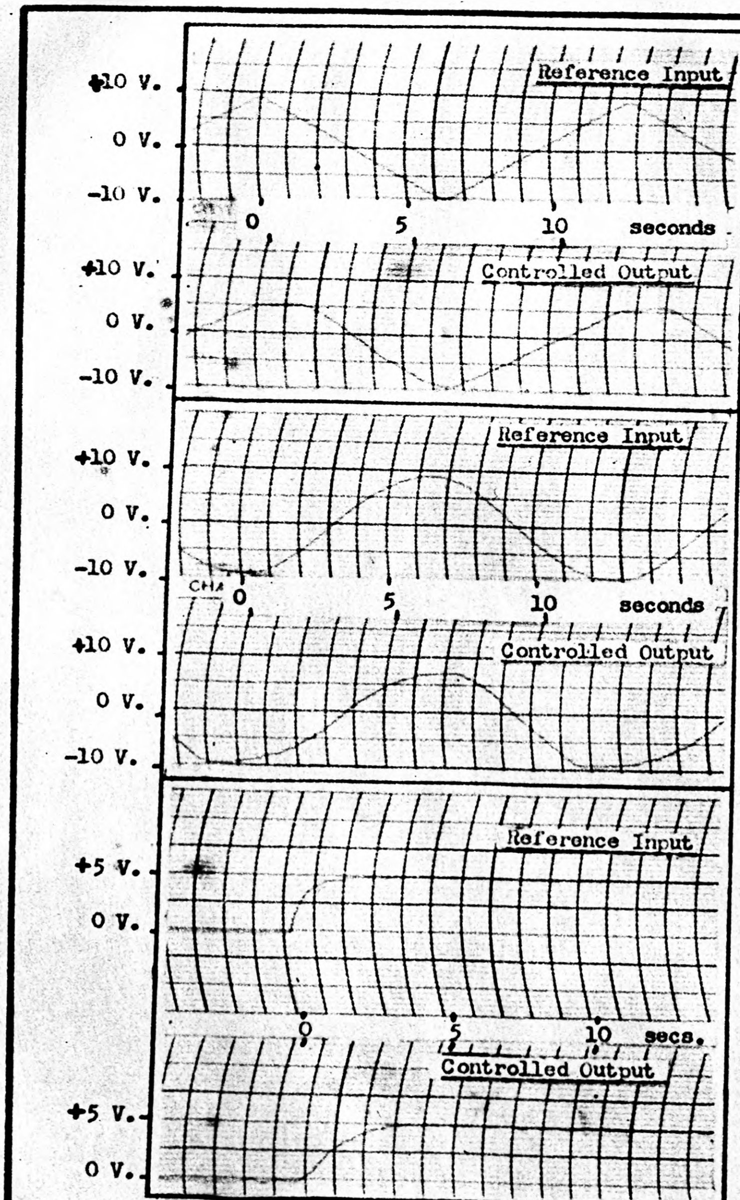


FIGURE 18. RESPONSE OF THE SYSTEM TO RAMP, SINUSOIDAL AND EXPONENTIAL INPUTS.

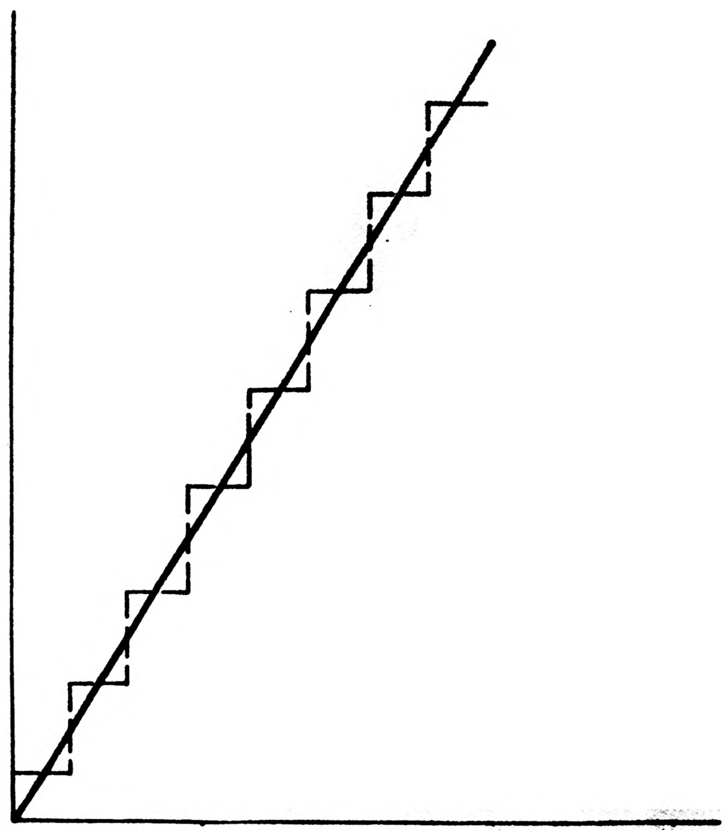


FIGURE 19. RAMP FUNCTION INPUT AND ITS APPROXIMATION BY A SERIES OF STAIR STEP INPUTS.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The analog simulation of the predictive control system enabled the study of the response of the system to various step inputs with a particular emphasis on overshoot, settling time, response time and limit cycle both with and without the fast-time model. The following conclusions can be drawn from the study.

This study might be helpful to a servo designer to develop a more refined control system by implementation of Pontryagin's optimum switching principle to a "bang-bang" servo.

Without a fast-time model the system would operate with a large overshoot and a large limit cycle. Hence the predictor model was essential to reduce the overshoot and ripples.

For large reference inputs velocity saturation occurred in the response of the system.

The logic network designed on the basis of step reference inputs will result in an optimum response of the system for reference inputs having no appreciable rates of change in their magnitudes.

The following recommendations for further investigation are made to study a more general type of predictive control system:

To develop logic criteria on a phase-space basis for implementation of predictive control systems of third and higher order systems.

To extend the present simulation to a study of the second order system with complex poles, including a study of ramp, exponential and sinusoidal excitations and load disturbances on the controlled system.

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