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AN ANALYSIS OF SINGLE WIRE EARTH RETURN(SWER) SYSTEM

FOR RURAL ELECTRIFICATION

BY

BALWINDER SINGH SAMRA, 1945-

A THESIS

Presented to the Faculty of the Graduate School of the
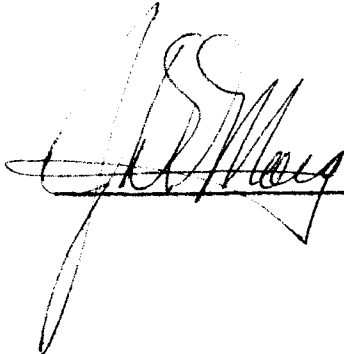
UNIVERSITY OF MISSOURI-ROLLA

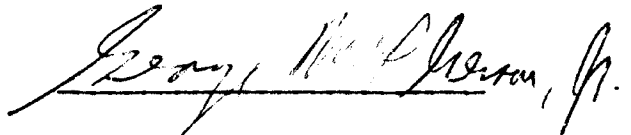In Partial Fulfillment of the Requirements for the Degree

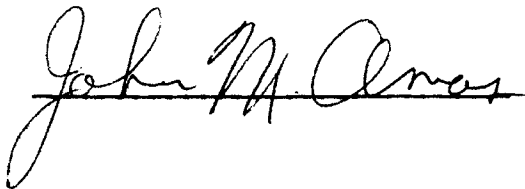MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

1972

Approved by

_____ (Advisor)

ABSTRACT

The primary objective of this investigation was to determine the general applicability of Single Phase Earth Return(SWER) system for rural electrification where the load density is low, for example electrification of farms and small villages in emerging nations. The limitations of SWER system of distribution are many and it is necessary to make judicious use of such transmission.

In this investigation the construction costs of transmission and distribution by SWER system is compared with single phase and three phase systems for loads of 25KVA and 50KVA. Phase converters are required to operate a three phase motor on a single phase supply. Three different types of static phase converters are discussed and their merits and costs are compared. Four different types of grounding systems are discussed in this study. The heating of earth electrodes due to continuous ground current flowing through the electrodes is investigated. Co-ordination of SWER systems with telecommunication lines is discussed in detail. Danger to humans and animals due to potential gradient at the surface of the earth in the vicinity of the grounding system caused by continuous ground current is also investigated in this study.

ACKNOWLEDGMENT

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The moral encouragement and valuable suggestions of Professor George McPherson, Jr., are highly appreciated. The author would like to thank Dr. J. M. Amos for his help and service as committee member. Thanks go to Mrs. Carol Rodman for typing the manuscript.

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I. INTRODUCTION

More than 85% of the population of India is in villages. The progress of the nation depends mainly on the progress of the rural population. A majority of the village population is engaged in agriculture and small percentage in small scale industries of a varied nature. To improve the rural economy, it is necessary to adopt modern methods of agriculture and mechanisation of small scale industries. The Govt of India is running a "Grow more food" campaign, thus giving the improvement of rural economy a high national priority. One of the ways this can be attained is by supplying electric power to the rural areas.

Rural India comprises a very large number of sparsely populated villages located at varying distances all over the country. The villages which have populations above 2000 are already electrified with three phase or single phase supply. Their number is only 10% of the total number of villages in India. The rest of the villages are 10 to 15 kms away from the present supply lines and have populations less than 2000. Most of the villages have a population of 500 or even less. The electrical load is mostly water pumps for irrigation and domestic lighting. In some villages there may be some processing industries based on agriculture products. The demand for electric power is mostly seasonal and that too with very low load factor except for domestic lighting. Hence the supplier of electric power has great difficulty in equating revenue and expenditure. To close the gap every attempt is made to develop load demand and to effect a reduction in capital, operation and maintenance expenditures, on rural

sub-stations and distribution lines. To supply an individual farm, whose average load is 5KVA, the expenditure by the electric supply authority is usually at least ten times the capital cost incurred in supplying a typical suburban home. In rural areas, therefore, it is essential, consistent with technical considerations, that every known economy of construction be employed.

There are three methods of rural electrification which can be considered:

a) Three phase system

b) Single phase system

c) Single phase single wire system (SWER)

The object of this project is to investigate the practicability of using a single line with ground return for small rural lines where the loads are isolated and their supply by a regular line is uneconomical. Most of the problems associated with SWER are discussed in the next sections together with possible solutions. Such systems have successfully been used in New Zealand and Australia where the load density is low for supply to isolated farms.

The system layout studied is shown in Fig. 1. It consists of one single phase isolating transformer with a transformation ratio of one to one. As the standard voltages of distribution are 11, 22, and 33KV for rural areas. The ground return system should work at 6.33KV, 12.7KV and 19.05KV for proper co-ordination with the existing system. The primary winding of the isolating transformer is connected to the existing lines. One terminal of the secondary winding of the isolating transformer is solidly earthed and the other terminal is connected
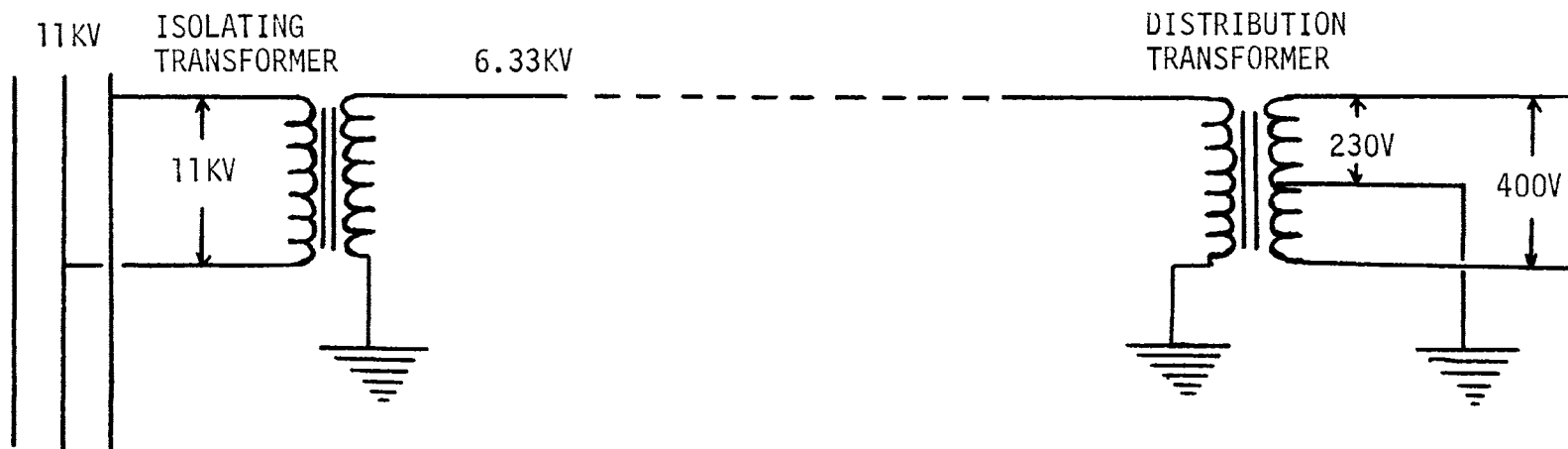
FIG. 1: SCHEMATIC DIAGRAM FOR SWER

to a single wire line. The same type of arrangement is employed at the distribution sub-station. The midpoint of the secondary winding of the distribution transformer is also earthed.

The earth is used as a return conductor in this case. The second method is not to use an isolating transformer, but to connect the 11KV line directly to the distribution transformer. The disadvantage of this system is that ground current has to travel a long way back to the main transformer which may be quite far. This is undesirable, due to interference in telecommunication systems as discussed in a later section. The most serious problem of SWER system is danger to humans and animals due to potential gradients near the vicinity of the earth electrode because the current flows through the ground continuously. The following points are discussed in detail in this investigation:

a) Economic consideration of transmission and distribution by SWER system and phase converter costs.

b) Technical considerations of SWER system such as earth conductivity, different types of electrodes, heating of electrode, co-ordination with communication lines and danger to humans and animals.

II. ECONOMIC CONSIDERATION OF SWER SYSTEM

A. TRANSMISSION AND DISTRIBUTION COSTS

On account of considerable reduction in capital cost effected by the use of single wire earth return systems, distribution of electric power to rural areas is a very attractive proposition.

The economics of ground return system may vary from country to country depending upon the nature and extent of load, labor and material costs. It should also be noted that the voltage of distribution adopted determines the capital cost. However, distribution voltages between 4.6KV and 13.2KV will not vary the capital cost very much, because the cost of line materials in this voltage range does not differ appreciably. But the amount of power that can be distributed rises considerably as higher voltage of distribution is adopted. In case of 33KV distribution, the cost of line materials and other associated equipment is twice that of 11KV line per km, but the amount of power that can be transmitted outweighs the cost differential.

The single wire earth return system (SWER) derives its economy from the fact that it employs earth as one conductor. This reduces the initial investment, not just equivalent to the cost of the conductor and its accessories only, but reduction occurs from lesser number of supports required, no cross-arms and reduced labor and transportation charges. The economics of the system depends on the length of the line and whether the isolating transformer has been installed in the system or not. It improves with the length of the line, but the isolating transformer, the use of which is not always necessary, has

an adverse effect on the cost which is more pronounced when the length of the line is not more than four or five kms.

The cost of the SWER system with 6.33KV lines, both with and without the isolating sub-station has been worked out with line lengths varing from one km to thirty km in steps of one km. Corresponding estimates have been prepared for the conventional single phase two wire and three phase systems also, on the basis of prevailing market rates of materials and labor. The loads assumed are 25KVA and 50KVA, i.e., ten isolated customers each having a load of 5KVA in the case of 50KVA load and five customers each having a load of 5KVA in the case of 25KVA load. It is assumed that each customer is supplied from a separate, pole mounted distribution transformer. ACSR conductor equivalent to 4AWG copper on wood supports is considered suitable for the line. The same conductor has been used in the estimates for the single phase two wire system. For three phase system the conductor size is 6AWG. The results are given in Fig. 2 and Fig. 3.

The percentage saving by using SWER as compared with three phase and single phase two wire system of distribution at 25KVA and 50KVA loads versus total line length are plotted. From Fig. 2, it can be seen that the percentage saving by using SWER, without an isolating transformer, varies with total length of distribution line and the total load delivered. Fig. 3 shows the percentage saving, with isolating transformer included. It can be seen that it is not economical to use SWER if the length of the total distribution line is less than five kms when compared with single phase two wire systems of distribution.
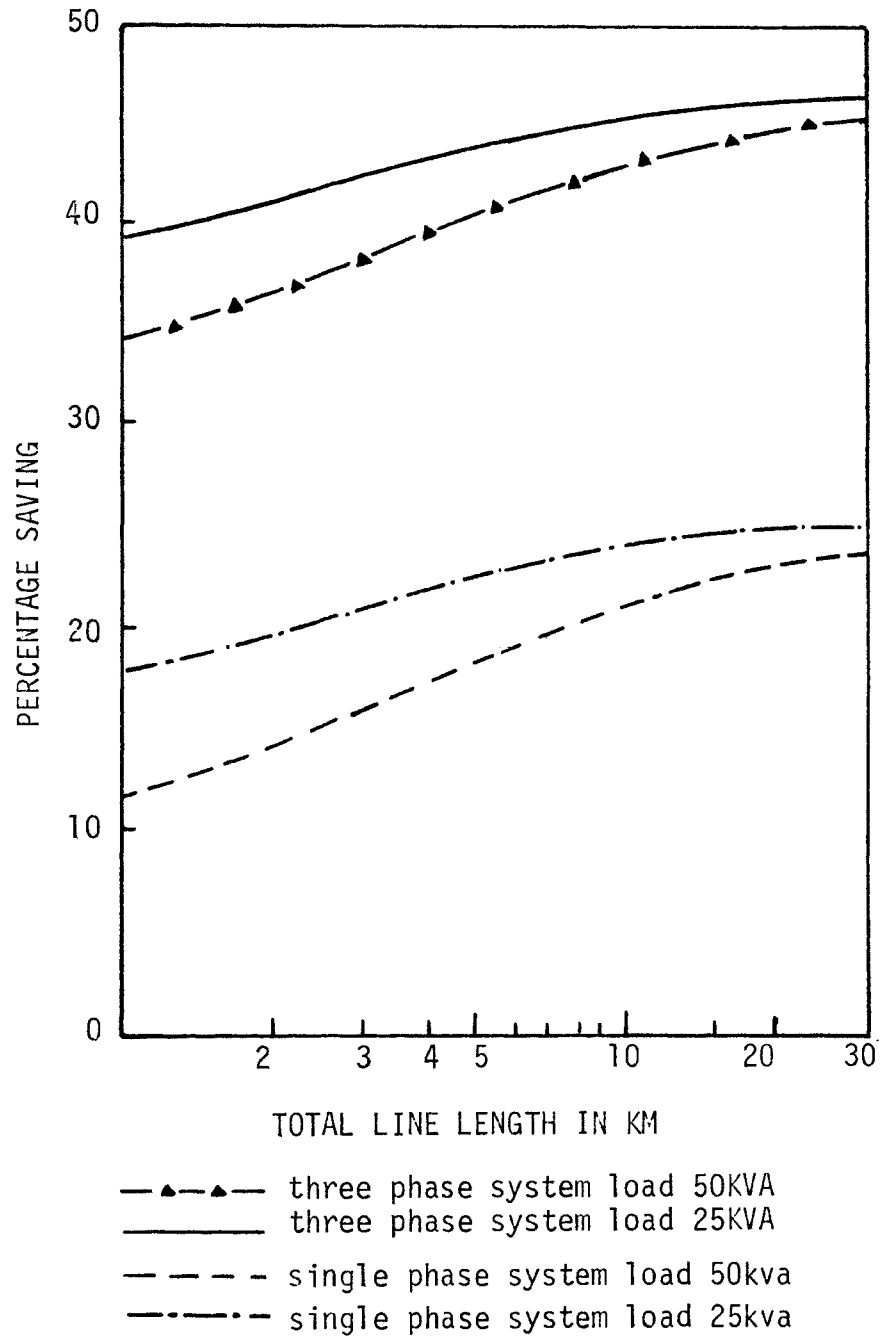
FIG. 2: PERCENTAGE SAVING OF SWER (WITHOUT ISOLATING TRANS-
FORMER) AS COMPARED WITH OTHER SCHEMES
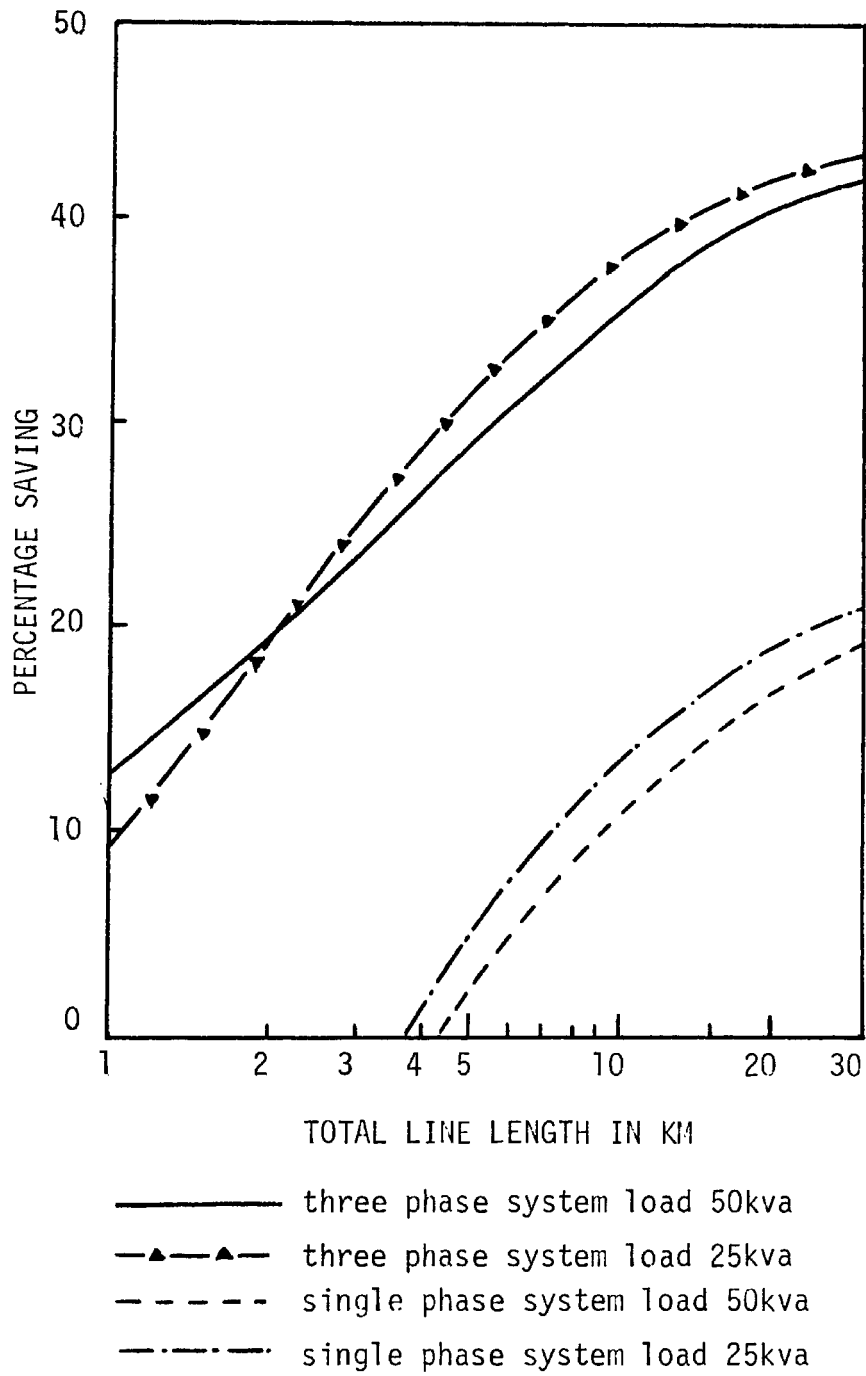
FIG. 3: PERCENTAGE SAVING OF SWER (WITH ISOLATING TRANSFORMER)
AS COMPARED WITH OTHER SCHEMES

The SWER system has been developed especially for use in sparsely
settled areas and its application is restricted to such areas where
Kw loading is about 2Kw/km and will not increase with time to cause
line currents in excess of eight amps. It is not recommended that use
be made of SWER system where the Kw loading will steadily increase and
double itself in eight or ten years. A normal SWER line is not easily
convertible to a three phase line. Attempts to provide for later con-
version will reduce considerably the economics of the SWER system, and
subsequent conversion could be difficult and costly.

The eight amps limit in the ground current is adopted due to
safety reasons and to reduce the interference in telecommunication
lines as discussed in a later section. With this limit a SWER system
with a voltage of 6.33KV can deliver 50KVA load and at 12.7KV a load
of 100KVA. Most of the villages in India are within 12kms from the
present 11kv, 22kv and 33kv distribution system. It is assumed that
the total length of the SWER system will not exceed thirty Kms.

B. PHASE CONVERTERS COSTS

A major disadvantage of the SWER system is the need to use single
phase motors. Generally single phase capacitor start and run type
motors are preferred. Single phase motors cost more than three phase
motors per horsepower, as shown in Fig. 4. Their performance is also
not as good as that of three phase motors. They take high starting
current and operate at low power factor. The demand for single phase
induction motors is in the range of very small and fractional horse-
power motors. They are seldom available for loads above 2H.P. Most
of the loads for irrigation pumping and other small scale industries
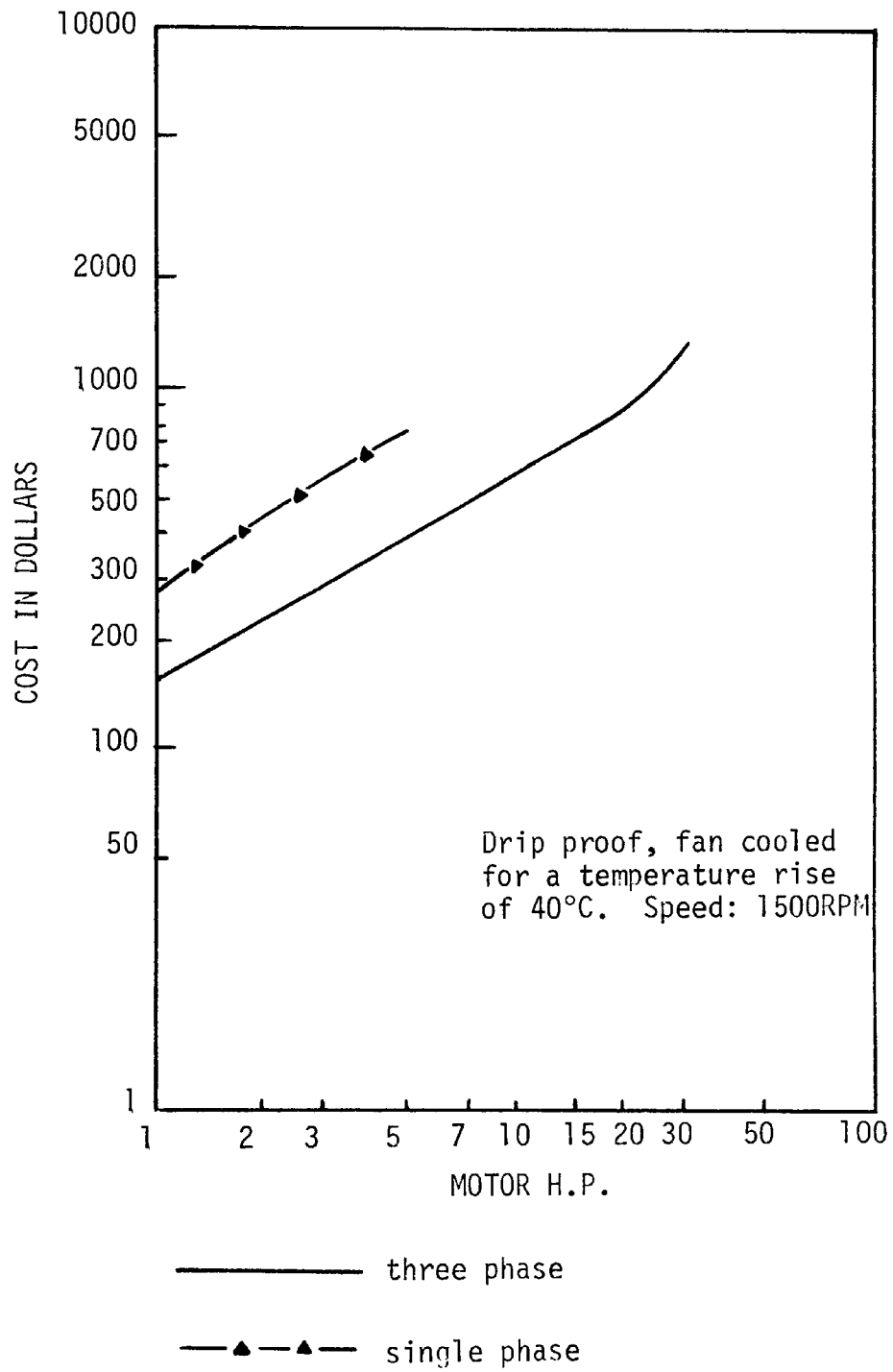
FIG. 4: COST OF SINGLE PHASE AND THREE PHASE INDUCTION MOTOR VERSUS MOTOR H.P.

based on agricultural products are between the range of 3 H.P. to 7.5 H.P. Repulsion-induction motors can supply such loads, but at much higher costs.

To operate a three phase motor on a single phase supply, phase converters are used. They are of two types, static and rotary. Static phase converters are used with individual three phase motors and rotary phase converters to operate a group of three phase motors. We will consider only static phase converters here.

A static phase converter is a device which supplies polyphase power to a load from a single phase source using only passive circuit elements. Ideally such equipment should produce balanced polyphase voltages at all loads. The losses in these elements should be kept to a minimum to obtain a high efficiency of conversion. The criterion for selecting the phase converter parameters is to give balanced voltages at the terminal of the load. One measure of the unbalance is the ratio of the negative sequence component of the current to the positive sequence component. The three schemes most commonly used are:

i) Capacitor only.

ii) Capacitor-reactor.

iii) Auto-transformer - Capacitor.

Analyses of the circuits and calculation of the circuit elements for each of the above schemes are given in Appendix A, B and C, respectively. It can be seen that, for a specified slip, the proper selection of the circuit elements will give balanced operation at full load except in the case of the capacitor only scheme. In the latter scheme perfect balance can not be obtained at full load. Experimental results by Huber[1] confirms this. He has concluded that the auto-

transformer capacitor scheme will give balanced operation at full load without exceeding the name plate value of line current of a 3∅ motor. But in the case of the capacitor only scheme, due to overheating caused by excessive current in one phase at full load, the motor has to be derated by 25% to keep the currents in all the three phases within the name plate value. Sharma[2] has concluded that the capacitor-reactor scheme gives balanced operation at full load and the current in all three phases does not exceed the name plate value. It is also concluded that starting torque varies from 175% to 200% of full load torque of the motor when the motor is used in conjunction with phase converters. The overall efficiency of the motor is reduced by 2% due to phase converters.

The cost of different types of static phase converters and the magnetic starter versus load are shown in Fig. 5. The cost of static phase converters plus the three phase motor (drip proof, fan cooled for 40°C temperature rise) versus motor H.P. are shown in Fig. 6. It can be seen that the capacitor only scheme together with three phase motor is still cheaper than the single phase motor of the same horse-power even when the three phase motor must be derated by 25%.
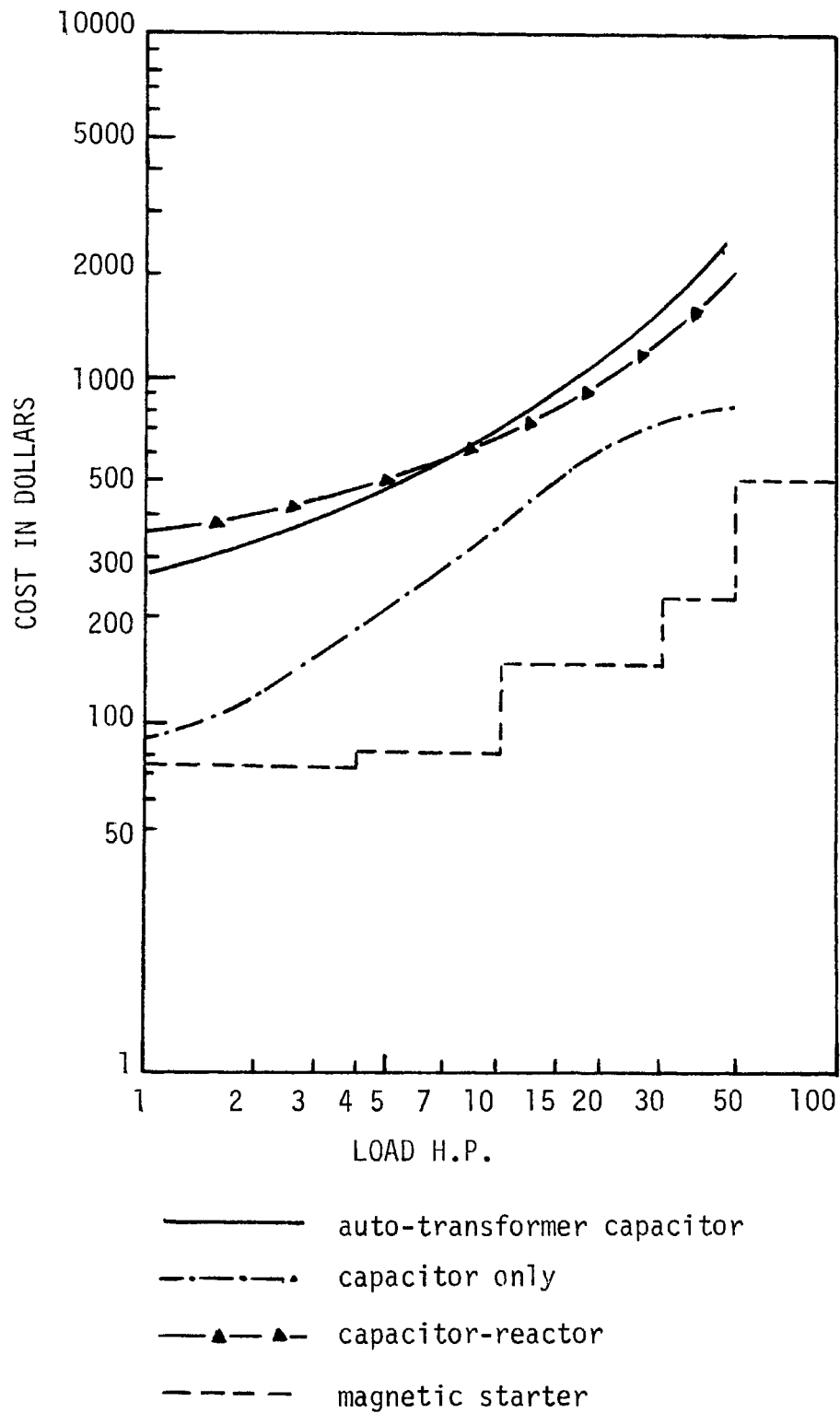
FIG. 5: COST OF PHASE CONVERTER AND MAGNETIC STARTER VERSUS LOAD H.P.
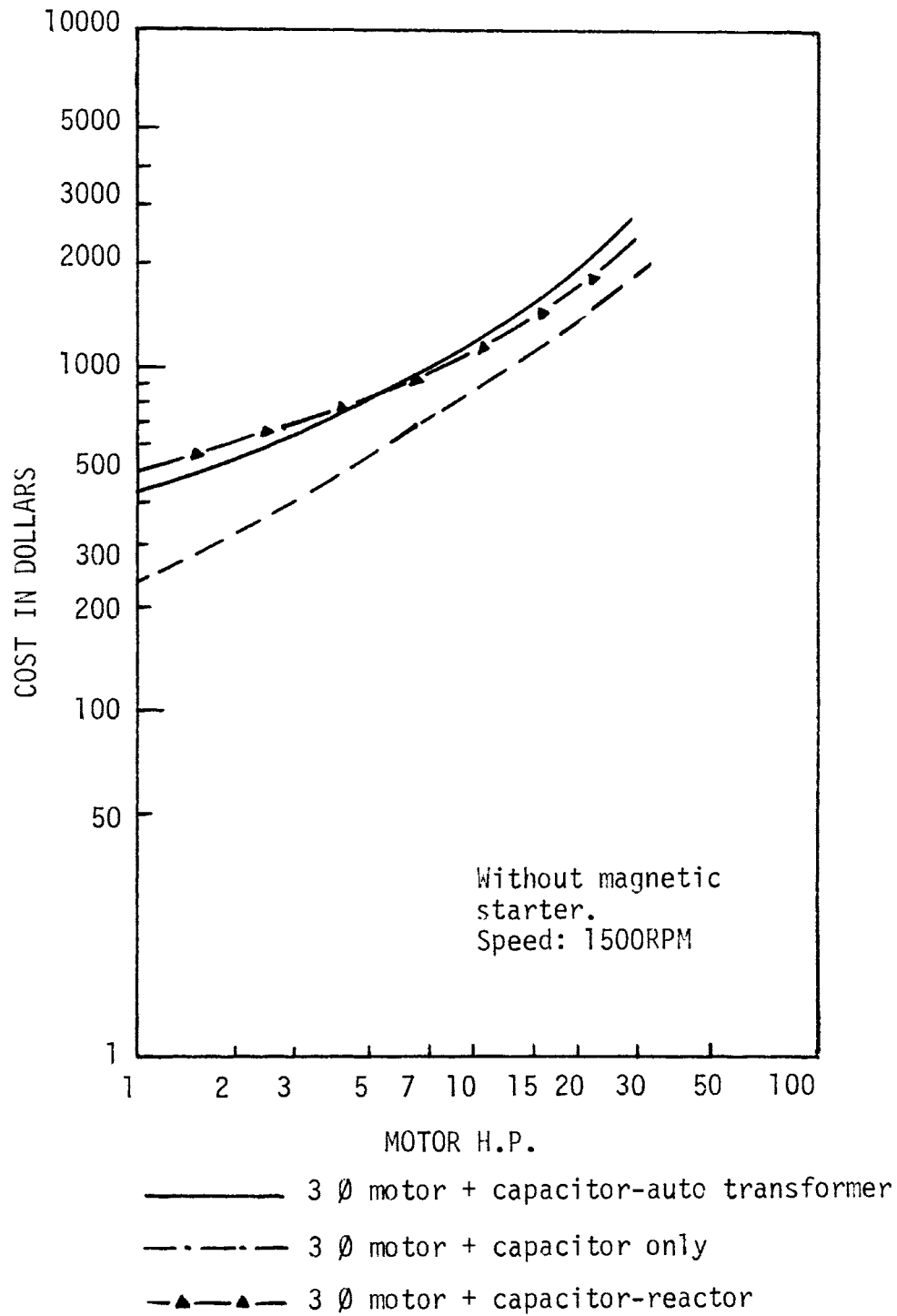
FIG. 6: COST OF PHASE CONVERTER PLUS THE COST OF THREE PHASE
INDUCTION MOTOR VERSUS MOTOR H.P.

III. TECHNICAL CONSIDERATION OF SWER SYSTEM

A. EARTH AS A CONDUCTOR

1. SOIL RESISTIVITY

Soil resistivity depends upon the type of soil and, therefore, varies with distance as well as depth. If no specific measurements for a definite spot of ground are made, the figures of Table I may be used as an average resistivity through the ground.[3]

Table I: SOIL RESISTIVITY

Type of ground	Organic wet soil	Moist soil	Dry soil	Bedrock	Unit
Resis-tivity	10	10^2	10^3	10^4	ohm-meter

Further the resistivity is much lower below the subsoil water level than above it. In frozen soil, as in a surface layer in winter, it is particularly high. The resistivity of a certain soil also varies with moisture, temperature and salt content of the soil[3] as shown in Fig. 7. The above factors serve to indicate that, though earth may be considered as a conductor of practically unlimiting con-ducting capacity, yet its resistance is largely determined by its chemical ingredients and the amount of moisture present in it. That is why it varies from place to place and from time to time. It is therefore necessary to measure the resistivity of earth in the area where a SWER system is planned.
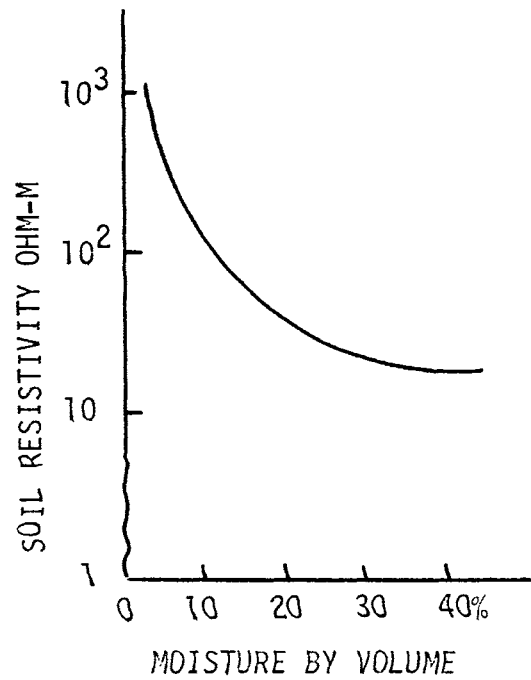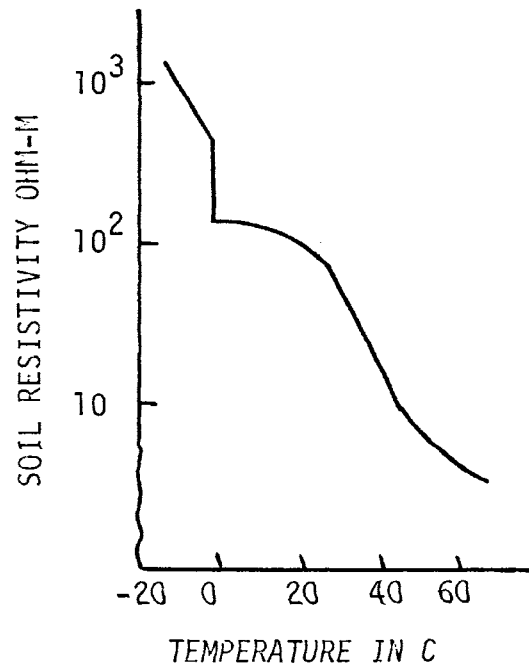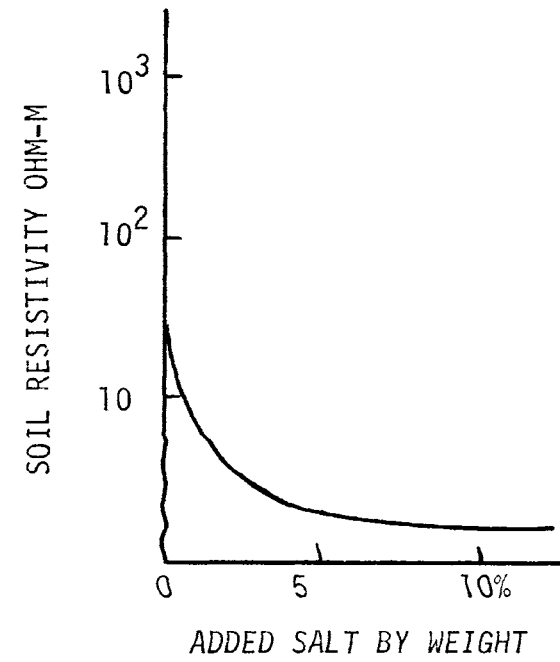
FIG. 7:   SOIL RESISTIVITY VERSUS MOISTURE, TEMPERATURE AND SALT CONTENT

2. DISTRIBUTION OF EARTH RETURN CURRENT IN THE GROUND

According to Rudenberg[3], the flow of earth return current between two earth electrodes occurs in the form of streamlines radiating in space from the electrode. This behavior is particularly true in case of DC, but for AC, the distribution is modified due to the inductive effect of the magnetic field of ground current, except very near the electrodes where the local resistance dominates. Elsewhere the distribution is so modified that the energy of the magnetic field of the ground current and hence its self inductance tend to be minimum. Hence the AC return current through the ground does not spread to as great a distance as does direct current but concentrates on paths in closer proximity to the overhead line conductor itself.

It has been proven by Rudenburg that the return currents in the ground spread with power transmission frequencies, as far as some kilometers, with audio transmission frequencies as far as some one hundred meters over the surface and into the depth of the earth.

3. RESISTANCE AND INDUCTANCE OF THE GROUND RETURN PATH

According to Rudenberg[3], the effective resistance of the ground return current path is given by

$$R = (\pi^2 \cdot f\ell) \times 10^{-7} \quad \text{ohms}$$

where

f = supply frequency Hz

ℓ = length of ground path in meters

It should be noted that the resistance of the earth path is dependent upon the frequency of current and length of path but is independent of

earth resistivity.

The inductance of the ground return path is

$$L = 2Ln \ \frac{562.8}{h} \ \left(\frac{\rho}{f}\right)^{1/2} \cdot 10^{-4} \quad H/km$$

where

h = height of conductor above ground in meters

ρ = resistivity of earth in ohm-meter

At power frequency (50Hz) when h is ten meters and earth resistivity if one hundred ohm-m, the value of inductance is 0.88mH/km or 0.28ohm/km. This is about half the value of the self inductance of the conductor as caused by the field lines in air above the ground and supplements this inductance. The inductance given by the above equation is proportional to length of line and depends on the height of the transmission line and to a lesser degree on resistivity of earth and frequency of the current.

Table II gives the return path's resistance in ohm/km for various frequencies in moist soil of resistivity 100 ohm-m. The same return resistance would be obtained by a fictitious copper wire of an equivalent diameter d as given in third line of the Table II. The self inductance of a fictitious conductor of diameter d and depth D under the surface, the ohmic and inductive effect of which may be taken equivalent to those of ground return path can be calculated. By comparison of the above equation with the self inductance of such an ideal return system, the equivalent depth can be calculated. The values for equivalent depths is shown in the last row of Table II. The resistivity of earth is assumed constant throughout, i.e., ρ = 100 ohm-m.

Frequency f	25	50	150	500	5000	Hz
Ground Path Resistance R	0.025	0.05	0.15	0.5	5.0	ohm/km
Diameter of Return Conductor d	3.01	2.13	1.23	0.68	0.21	cms
Depth of Return Plane D	1130	800	460	250	80	meters

TABLE II: GROUND PATH RESISTANCE AND DIAMETER OF COPPER CONDUCTOR HAVING THE
SAME RESISTANCE PER KM.

B. GROUNDING ELECTRODES AND GRIDS

For the transfer of power through earth, it is necessary to have ground electrodes at each end of the line, through which the current is conveyed to the earth. The resistance of the grounding arrangement constitutes the major part of the total resistance of the ground return path.

As a matter of fact, when we measure the earth resistance of an electrode by the usual method with an earth tester we actually measure:

    i)   The resistance of the electrode along with its connection.

    ii)   The resistance between the electrode and the ground in the immediate vicinity of the electrode. This is usually known as local resistance.

    iii)   The resistance of the soil in between the spike and the electrode.

Obviously item (i) is small and so also is (iii). It is rather the local resistance which accounts for most of the measured resistance. The main factor governing the local resistance are the resistivity of the surrounding earth and the nature of the electrode employed.

There are many types of earth electrodes used. Some of them are considered here. Schwarz[4] in 1954 developed analytical expressions for various grounding systems and compared the results with the experimental results with very close agreement.

1. DEEP DRIVEN ROD OR PIPE

The resistance of driven rods can be found by the equation given below:

$$R = \frac{\rho}{2\pi\ell} \; Ln \; \frac{2\ell}{d} \quad ohms$$

$$= 0.366 \; \frac{\rho}{\ell} \; log_{10} \; \frac{2\ell}{d} \quad ohms.$$

where

ρ = resistivity of earth in ohm-m.

ℓ = length of the electrode below ground in meters,

d = diameter of rod or external diameter of pipe in meters.

It can be seen from the above equation that the resistance decreases rapidly as the length of the electrode increases and also less rapidly as the diameter increases.

2. MULTIPLE ELECTRODES IN RECTANGULAR AND SQUARE PATTERN

It is sometimes necessary to drive a number of rods or pipes and connect them in multiple to obtain the desired low resistance of the grounding system. To obtain best results attention must be paid to the spacing of these multiple rods. It is found that 90% of the resistance to earth is situated within an area the radius of which is roughly equal to the length of one rod. The electrodes should thus be spaced far enough away from each other so as not to overlap the resistance area of their neighbors. The resistance of an array of rods can be calculated by the following equation:

$$R_{rb} = \frac{\rho}{2\pi n\ell_1} \left[Ln \; \frac{4\ell_1}{b} - 1 + \frac{2k_1\ell_1}{\sqrt{A}} \; (\sqrt{n}-1)^2 \right] ohms$$

where

n = number of rods or pipes in an area A square meters.

2b = diameter of rod or external diameter of pipe in meter.

ℓ_1 = length of each electrode below ground in meters.

k_1 = constant to be determined from Fig. 9(a).

ρ = resistivity of earth in ohm-m.

Fig. 8 shows how the resistance varies with the number of electrodes used and spacing between them.

3. GRIDS

Grid is a grounding system consisting of a conductor, forming a square or rectangular pattern, buried beneath the surface of earth. Resistance of a grid is given by

$$R_g = \frac{\rho}{\pi \ell}\left[Ln\left(\frac{2\ell}{\sqrt{dH}}\right) + k_1 \frac{\ell}{\sqrt{A}} - k_2 \right] \quad ohms$$

where

ℓ = length of grid conductor in meters.

d = diameter of grid conductor in meters.

H = depth of burial of grid conductor in meters below earth surface.

A = area of grid in square meters.

$k_1 \& k_2$ = constants to be determined from Fig. 9(a and b).

ρ = resistivity of earth in ohm-m.

4. COMBINATION OF ROD BEDS AND GRID

The resistance of the combination can be found by the equation
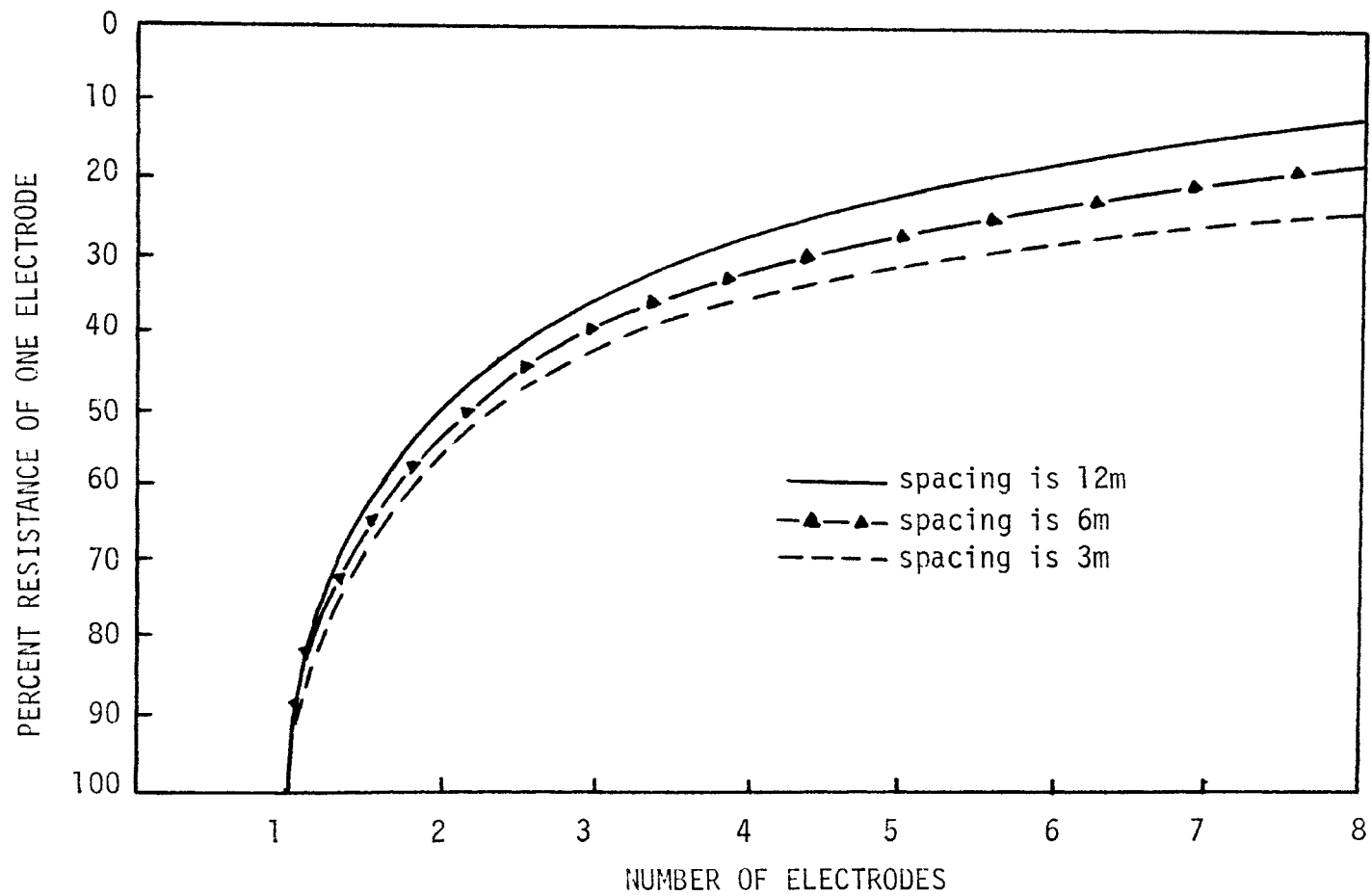
FIG. 8: RESISTANCE OF MULTIPLE ELECTRODES OF VARIOUS SPACING AND OF DEPTH 3m
(IN TERMS OF % RESISTANCE OF ELECTRODE) AS A FUNCTION OF THE NUMBER OF ELECTRODES
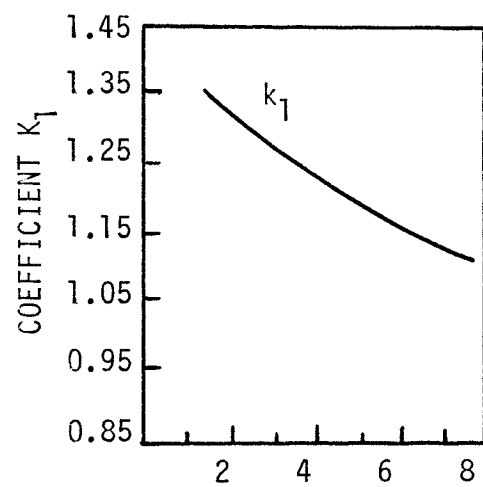
COEFFICIENT K_1

LENGTH TO WIDTH RATIO OF AREA A

(a)
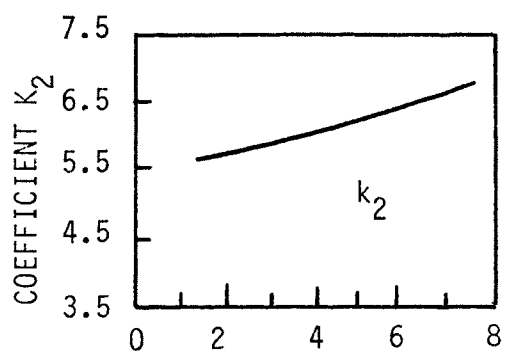
COEFFICIENT K_2

LENGTH TO WIDTH RATIO OF AREA A

(b)

FIG. 9: CURVES FOR COEFFICIENTS K_1 AND K_2

$$R = \frac{R_{rb} R_g - R_m^{\,2}}{R_{rb} + R_g - 2R_m} \quad ohms.$$

where R_m is given by the equation

$$R_m = \frac{\rho}{\pi \ell} \left[Ln \frac{2\ell}{\ell_1} + k_1 \frac{\ell}{\sqrt{A}} - k_2 + 1 \right] \quad ohms.$$

ℓ = length of grid conductor in meters

ℓ_1 = length of electrodes below ground in meters.

A = area covered by the combination in square meters.

$k_1 \& k_2$ = constants from Fig. 9(a and b).

Table III shows resistance of various electrodes for different values of earth resistivity. There are certain configurations of electrodes and grids which can not be handled by the above equations. For these configurations, model tests are made. A scale model of the electrode or grid or the combination of both is placed in a tank of electrolyte whose resistivity is known and resistance is measured by passing a current through the electrode. The potential gradient at the surface of the electrolyte is also measured.

C. HEATING OF GROUND ELECTRODES

Flow of current continuously through the ground electrode for supplying power to any load will heat up the electrode and the ground surrounding it. For calculating the temperature rise in the soil for a certain loading an equation is developed in Appendix D which is given as follows:

$$\theta = \frac{I^2 R^2}{2 \rho \lambda} \quad in \ degrees \ centigrade$$

NO	ELECTRODE CONFIGURATION	SOIL RESISTIVITY IN ohm-meters					
		100	200	300	400	500	1000
1	Single electrode vertical three meters long and 4cm in diameter	24.6	52.8	79.2	105.6	132.0	264.04
2	Single electrode vertical six meters long and 4cm in diameter	15.0	30.0	45.0	60.0	75.0	150.0
3	3 electrodes in triangular pattern (3m long and 4cm in dia) with a spacing of 6m between successive electrode	10.40	20.80	31.20	41.60	52.00	104.00
4	3 electrodes in triangular pattern (6m long and 4cm in dia) with a spacing of 6m between successive electrodes	6.7	13.4	20.1	26.8	33.5	67.0
5	4 electrodes in square form (3m long and 6cm in dia) with a spacing of 6m between the electrodes	7.20	14.40	21.60	28.80	36.00	72.00
6	4 electrodes in square form (6m long and 4cm dia) with spacing of 6m between electrodes	2.9	5.8	8.7	11.6	14.5	29.00
7	Square grounding grid of size 6x6m buried to a depth of 0.3m below earth surface. The dia of conductor is 7mm hot galvanized steel	7.1	14.2	21.3	28.4	35.5	71.0
8	Square grounding grid of size 10x10m buried to a depth of 0.3m below earth surface.	4.48	8.96	13.44	17.92	22.40	44.80
9	Square grounding grid of size 20x20m buried to a depth of 0.3m below earth surface.	2.78	5.56	8.34	11.12	13.90	27.80

TABLE III: RESISTANCE OF VARIOUS EARTH ELECTRODES

where

I = current flowing through electrode in Amps

R = resistance of ground connection in ohms.

λ = heat conductivity of soil in Watts/°C-m (taken as 1.2 W/°C-m)

θ = temperature rise in °C

ρ = resistivity of earth in ohm-m

If the temperature rise is such that its value at the electrode surface is 100°C, the moisture in the soil will be evaporated and a thin dry layer of soil will be formed around the electrode or possibly some vapor will be generated which will form a film between the electrode and the soil resulting in a large resistance increase and complete interruption of the circuit. For safety 60°C may be taken as the maximum permissible temperature rise.

The permissible current will be low for high resistivity and high for low resistivity. The permissible current for various resistivities are given in Fig. 10 both for isolated electrodes and for grids.

In the case of grids the temperature rise of the soil will be low because total current flowing into the ground is distributed throughout the grid conductors. So the maximum current that can flow through grids may be higher than that of isolated electrodes of the same resistance.

D. CO-ORDINATION WITH TELECOMMUNICATION CIRCUITS

Communication circuits running near the SWER will have induced voltages due to electrostatic and electromagnetic coupling. The electrostatically induced voltage in communication circuits due to SWER will be considerable as compared with single phase two wire line
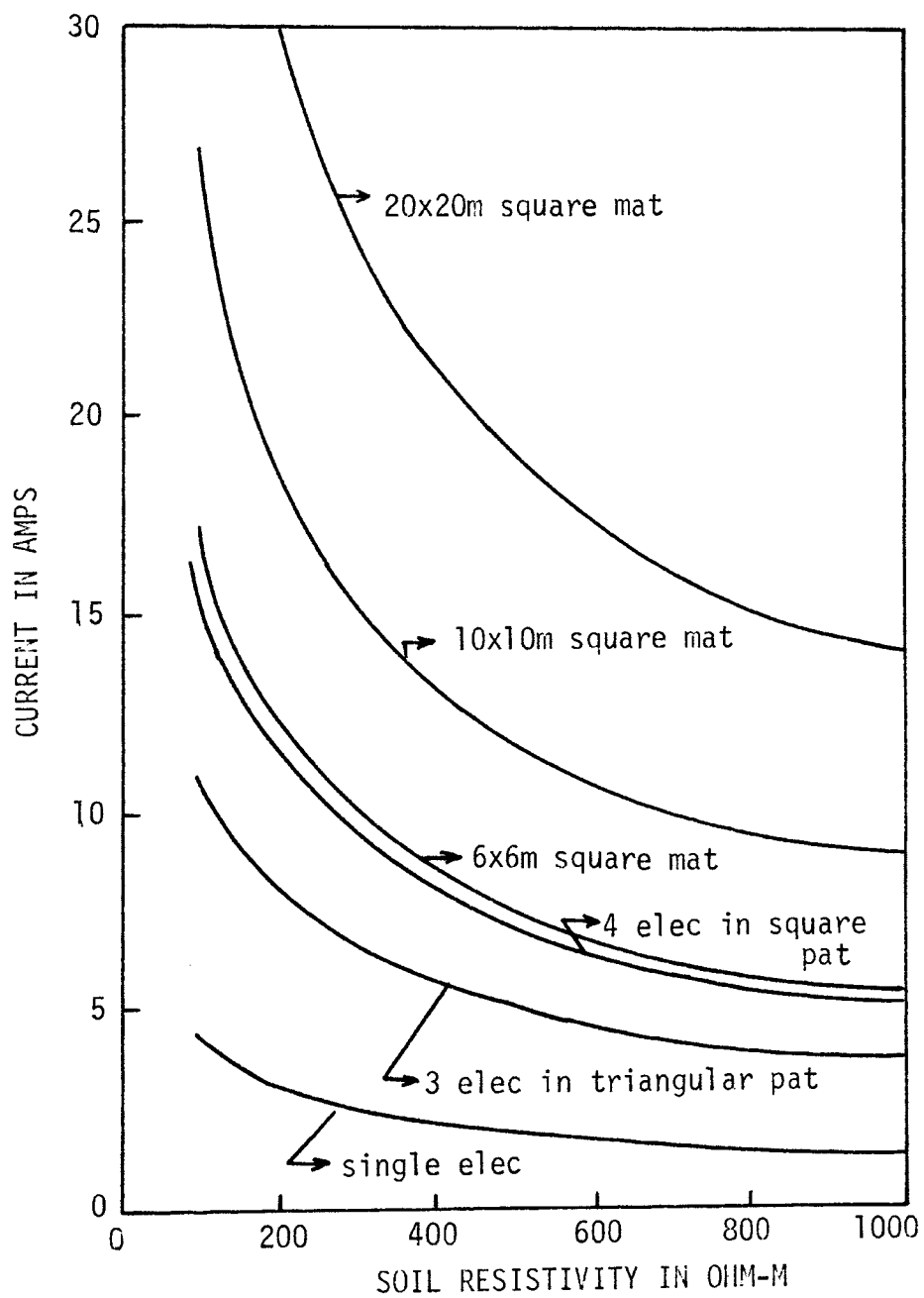
FIG. 10: PERMISSIBLE CURRENT VERSUS SOIL RESISTIVITY FOR TEMPERATURE RISE OF 60°C

or three phase systems, because the positive and negative charges are separated by the distance equal to twice the height of line supports. The derivation of the equation given below is shown in Appendix E.

$$\frac{e_s}{E} = \frac{2}{\ln \frac{4h}{d}} \times \frac{hk}{(a^2+h^2+k^2)}$$

where

h = height of SWER line above ground in meters and a conductor dia d meters.

k = height of communication line above ground in meters.

a = distance of horizontal separation in meters.

e_s = electrostatically induced e.m.f. in volts

E = working voltage of SWER line in volts.

The predominating cause for interference is mutual inductive coupling between the SWER system and the communication line. The magnetic flux produced from a current flowing in a loop is proportional to $I \cdot A / \ell$ where I is the current in the loop, A is area of loop and ℓ is the length of magnetic path. This magnetic flux links with communication lines and induces voltages in them. The loops encountered in high resistivity soil are larger than in low resistivity soil. The extent of the induced e.m.f. depends on the magnitude of the earth current, soil resistivity, the separation of SWER from the circuit and the length of parallelism of both lines. Required separation distances can be calculated. A formula was developed by Radley[5] in 1931

$$e_m = \omega \cdot M \cdot \ell \cdot I \qquad \text{volts}$$

where

M = mutual inductance in H/km

I = current flowing in the ground circuit in Amps

ℓ = length of parallelism in kms.

$\omega = 2\pi f$ where f is the frequency of supply in Hz.

e_m = electromagnetically induced e.m.f. in volts.

M can be obtained from Fig. 11 for a particular earth resistivity and separation. The directives of the International Consultive Committee on telephones (C.C.I.F.) which prescribes that the total voltage induced, i.e., both electromagnetic and electrostatic, in telephone lines because of the presence of a power line shall not exceed 60 Volts under normal conditions and 430 Volts under fault conditions. These limits call for a certain distance of separation between SWER and communication line.

In Fig. 12, variation in electrostatic and electromagnetic induced voltages with separation are shown. The graphs are drawn for a SWER line operating at 6.33KV and at 8Amps full load. In the case of electromagnetic induction for earth resistivities of 10,000, 1000 and 100 ohm-m the induced voltages are plotted. If the length of parallelism of the lines is known the safe separation distance between the lines can be calculated from Fig. 12. For example if the length of parallelism is 20kms, the earth resistivity is 100ohm-m then a safe separation distance about 100m will keep the induced voltages below 60 Volts.

A closer approach than 100m is permitted when absolutely necessary such as deviation towards the telephone line for special reasons as to enable it to reach a consumer's premises which might be situated close to a telephone line or to avoid major obstructions such
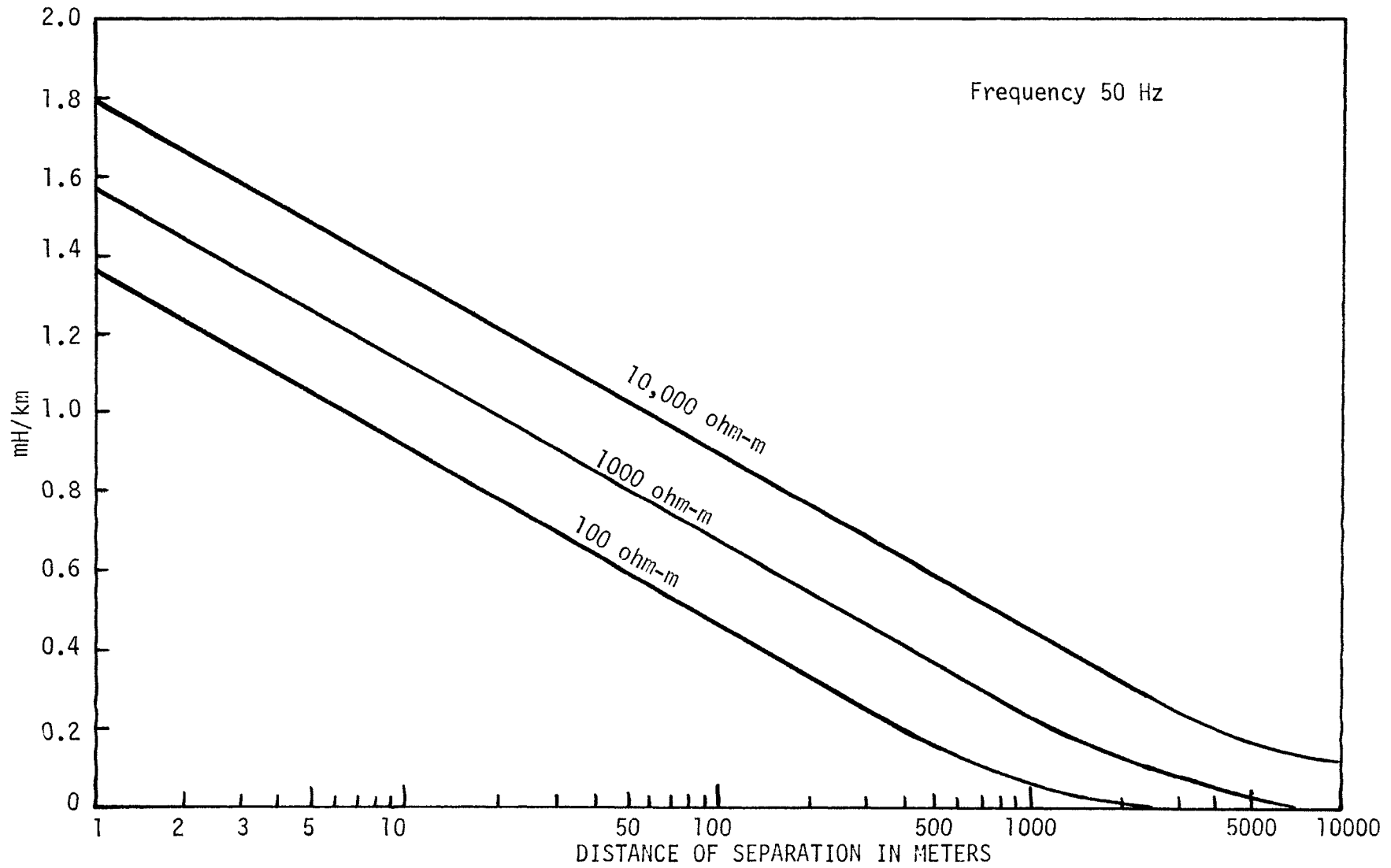
FIG. 11: MUTUAL INDUCTANCE BETWEEN SWER LINE AND TELEPHONE LINE VERSUS DISTANCE OF SEPARATION
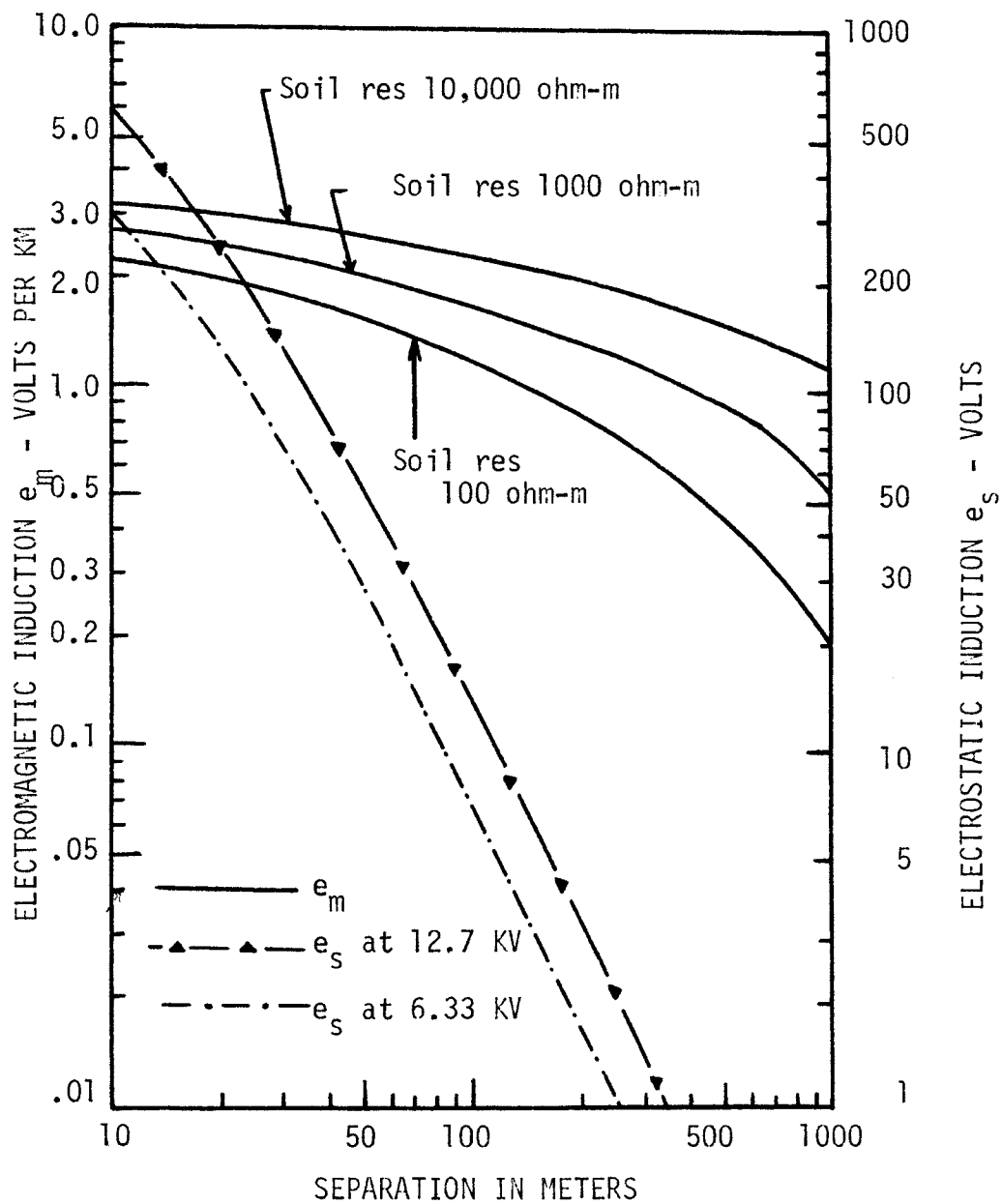
FIG. 12: ELECTROSTATIC AND ELECTROMAGNETIC INDUCTION VERSUS
SEPARATION BETWEEN SWER LINE AND TELEPHONE LINE

as clumps of trees or groups of buildings. However, provision should

be made that the effect of such an approach must be offset by greater

separation over other portions of the same continuous length of

parallel in order that the induction into the telephone line shall be

no greater than if the separation over the whole length of parallelism

had been uniform.

Interference(noise) in communication lines is also produced by

corona effects and harmonics in power lines. Corona noise can be

reduced by careful design of the line hardware. According to Robert-

son[6], the reactance of the earth return path is a function of fre-

quency. The harmonic currents which are usually associated with low

power factor magnetizing currents are confined to a path in earth

nearly beneath the overhead conductor than the fundamental frequency

currents, as shown in Fig. 13. It has been determined by the above

author that charging currents of SWER line at 11KV is only 0.03 amp/

mile. These charging currents can be offset by magnetizing currents

of the transformer (a 5 to 10KVA transformer has a magnetizing current

of about 0.015amps). So charging currents presents no major problem

of co-ordination with communication lines.

E. SAFETY

One of the natural effects of current flow through an earth

electrode is that a voltage gradient is present on the surface of the

earth in the immediate neighborhood of the electrode. The fundamental

reason for this potential gradient is that the current of the electrode

is not concentrated at one point, but is distributed over the soil in

its vicinity.
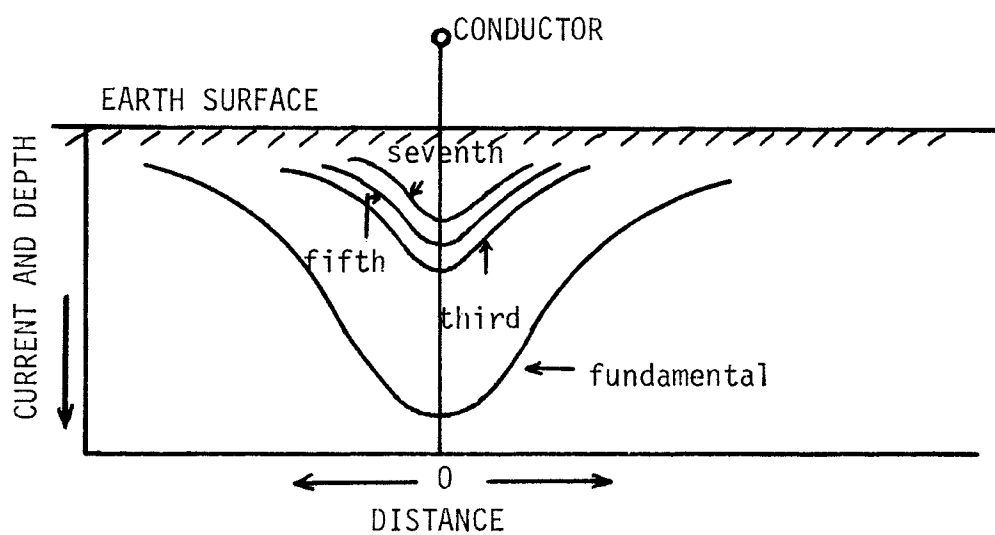
FIG. 13: HARMONIC CURRENT DISTRIBUTION UNDER SWER POWER
LINE

1. STEP VOLTAGE

The term step voltage means the potential difference which exists between foot to foot contact when a person takes a stride near a substation when current flows through earth electrode.

If a current of I amps flows through an electrode (the electrode is assumed to be hemispherical) then the current density at a distance x meters is given by

$$J = \frac{I}{2\pi x^2} \rightarrow \frac{I}{2\pi x^2} \qquad A/m^2$$

then electric field strength at the surface of the earth will be

$$E_r = \rho J \qquad volts/m \qquad (\rho = \text{resistivity of earth ohm-m})$$

$$= \frac{\rho I}{2\pi x^2} \qquad volts/m$$

if a man or animal is walking near the electrode, a voltage which may cause serious damage may be impressed. Let w meters be the width of the step.

The step potential can be calculated as follows

$$E_s = -\int_{x+w}^{x} E_r \cdot dx = \int_{x}^{x+w} E_r \cdot dx = \frac{\rho I}{2\pi} \int_{x}^{x+w} \frac{dx}{x^2} \qquad volts.$$

therefore

$$E_s = \frac{\rho I}{2\pi} \frac{w}{x(x+w)} \qquad volts$$

From the above equation it can be seen that the step voltage depends on the ground current, the distance from the electrode, the resistivity of the ground and on the step width.

Actually the danger to living creatures is due to the current passing through their bodies rather than on the magnitude of the

voltage shunted. This current depends upon the resistance of the body itself. The current resulting from step voltage enters the body via one foot and passes out through the other. The body resistance is not easy to determine, it varies from person to person, i.e., from 500 ohms to few thousands ohms. According to AIEE Report #80 it can be taken as 1000 ohms for the human body for purposes of calculation.

Then we can write that the step voltage

$$E_{step} = (R_b + 2R_f) I_b \qquad volts.$$

where

R_f = the resistance of ground just beneath the feet in ohms.

R_b = the resistance of the human body in ohms.

I_b = the safe body current = 10mA = 0.01 Amps.

To calculate R_f, the feet are considered to be equivalent to a hemispherical electrode whose radius is b meters at the surface of the earth. Then

$$R_f = \frac{\rho_s}{2\pi b} \qquad ohms \qquad\qquad I.$$

where

ρ_s = the resistivity of earth beneath the feet in ohm-m.

The equivalent foot radius b is taken as 10cm, so

$$2R_f = \frac{\rho_s}{\pi \cdot 0.1} = 3.2 \, \rho_s \qquad ohms.$$

Therefore

$$E_{step} = (1000 + 3.2 \, \rho_s) \, 0.01 \qquad volts.$$

If we take ρ_s = 100 ohm-m then the step voltage is 13.2 volts. This can be considered as the maximum voltage which a man can withstand without permanent injury to his body.

In the sub-station area ρ_s, the resistivity of earth just beneath the feet can be increased by using crushed rock bed surfacing, which has a resistivity of 3000 ohm-m even under wet condition, of depth 4 inches to 6 inches. In case of animals, step voltage is the potential difference shunted between fore and hind legs. The step voltage for animals is of high value since the distance shunted between fore and hind legs could be twice as great as man. It is difficult to calculate the step voltage for different animals due to absence of knowledge regarding their body resistance. According to Robertson[6] experiments were carried out to ascertain the voltage gradient which would cause discomfort to animals. Tests were made on a cow, a bullock and some lambs and the bullock was the most susceptible, showing signs of discomfort at a gradient of 4.0 volts per foot distance.

2. TOUCH VOLTAGE

The touch voltage is the potential difference shunted between one hand and the feet. In general it is the potential difference between the ground electrode and points on the ground at one meter distance from the electrode. The path of current due to the touch voltage involves such vital organs as heart and lungs. Let

R_f = resistance of ground just beneath the feet in ohm.

R_b = body resistance in ohms (1000 ohms).

I_b = safe body current in amps = 0.01 amps.

From equation I

$$R_f = \frac{\rho_s}{2\pi b} = \frac{\rho_s}{2\pi \cdot 0.1} = 1.5 \, \rho_s \text{ ohms}$$

therefore

$$E_{touch} = (R_b + R_f) I_b \qquad \text{volts.}$$

$$E_{touch} = (1000 + 1.5 \, \rho_s) \, 0.01 \qquad \text{volts.}$$

taking $\rho_s = 100$ ohm-m

$$E_{touch} = 11.5 \text{ volts} = 11.5 \text{ volts}$$

The resistance between the hand and the electrode is neglected here.

3. REDUCING THE POTENTIAL GRADIENT AT THE SURFACE OF EARTH

To keep both step and touch voltage within the safe limits, it is necessary to reduce the potential gradient at the surface of the earth. This can be accomplished by reducing the resistance of the electrode to as low a value as possible and if necessary, by reducing the resistivity of the earth by chemical treatment where it is very high. Analytical expressions for the resistance of different types of electrodes and grids are given in Section B. To find the potential gradient analytically is almost impossible and probably inaccurate. There are two methods which are used commonly, one is to run actual experiments under simulated conditions and other is to employ model tests.

The author has run several actual experiments on different types of electrodes for SWER. After careful consideration the two schemes selected were:

a) Combination of four pipes of external diameter of 4cm, spaced
   6 meters from each other in a square pattern and depth of 6 meters.

All the pipes were connected to each other by a 7mm diameter galvanized steel wire, buried 0.3m under the ground surface.

b) Two pipes spaced 6 meters from each other, length of 6 meters and external diameter of 4 cm.

The soil resistivity was 100 ohms-m. The resistance of both these arrangements were measured as 2.3 ohms for scheme (a) and 9.6 ohms for scheme (b). The calculated values for these arrangements are 2.2 ohms and 9.8 ohms respectively. A current of 8 amps was passed through the first electrode arrangement (a) and a current of 1 amp was passed through the second (b) electrode arrangement. The potential was measured between the electrode and the selected points on the surface of the earth in the vicinity of the electrode (6 inch long spikes were driven into the ground at these points) with the help of a high impedence VTVM. The results are plotted in Fig. 14 and Fig. 15.

It is clear that both the E_{touch} and E_{step} are greater near the electrode. In both cases it is found that they are within safe limits. In Fig. 15 it can be seen that even when one pipe is disconnected from the other due to some reason the E_{touch} and E_{step} voltage will not exceed the safe limits.

It is recommended that the potential at the earth electrode and the earth lead should not exceed 20 volts with full load current flowing through the electrode. The earth lead connecting the electrode to the neutral terminal of the transformer should also be insulated.

The voltages are measured experimentally as stated earlier by probe spikes driven 6 inches into the ground. Indications are that
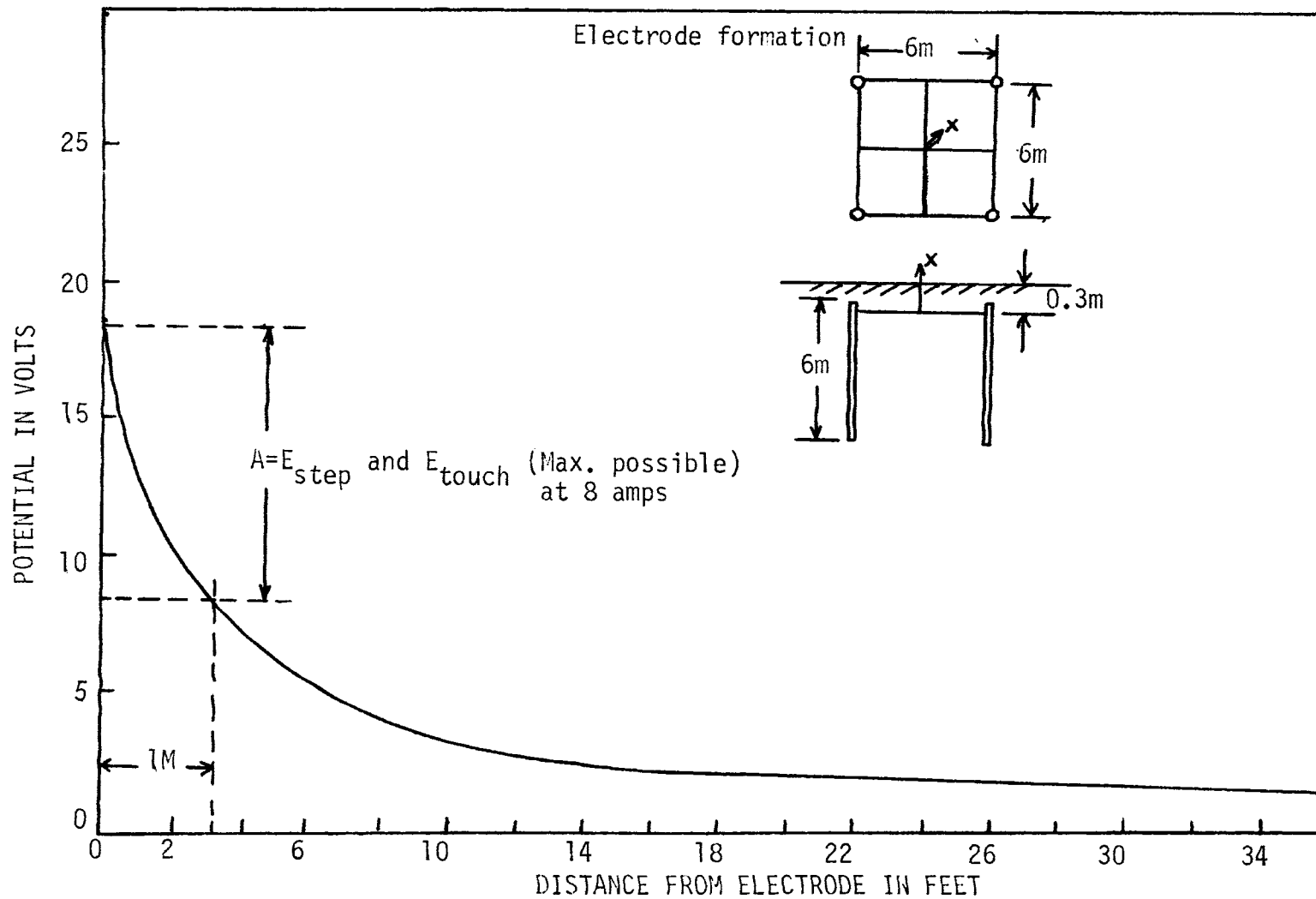
FIG. 14: POTENTIAL GRADIENT IN THE VICINITY OF THE ELECTRODE OF SCHEME (a)
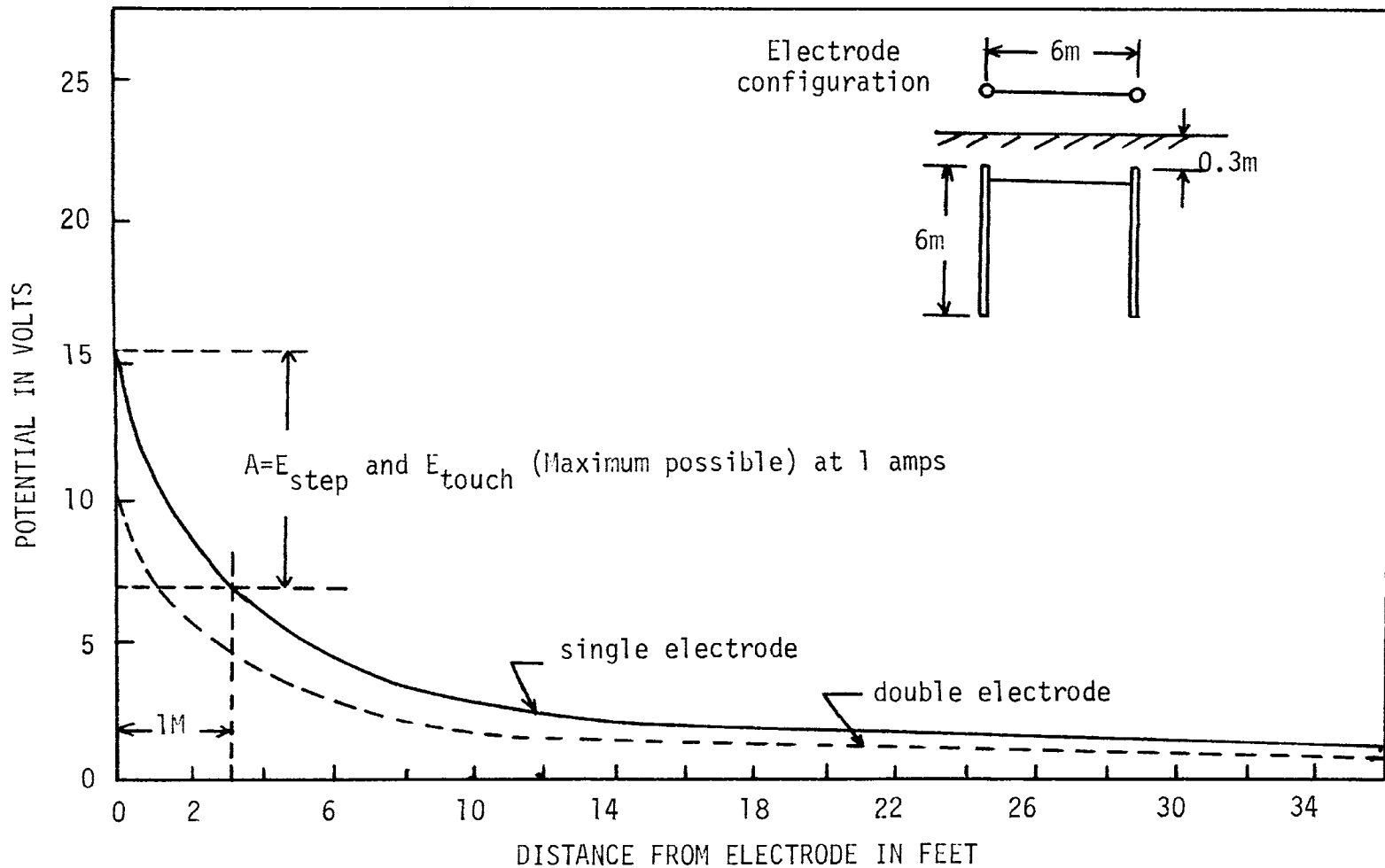
FIG. 15: POTENTIAL GRADIENT IN THE VICINITY OF THE ELECTRODE OF SCHEME (b)

the actual surface gradients are considerably lower than gradients measured by 6 inch deep probe spikes. This effect is assisted by our practice of driving the earth electrode into the ground until the tops are one foot below ground level. Because of this and the 20 volts limits, it is considered that the possibility of danger from potential gradient on ground is negligible.

Scheme (a) is suitable for isolating transformer sub-stations and scheme (b) is suitable for pole mounted distribution sub-stations. In both the schemes the earth electrodes should be connected to the neutral of the transformer with two separate earth leads placed away from each other to reduce the danger of disconnection of the electrodes accidentally.

IV. CONCLUSIONS AND RECOMMENDATIONS

The results of this investigation indicate the following:

a) Rural electrification may be done with the help of SWER systems where the load concentration is less than 2Kw/Km and there is no possibility that the load will increase appreciably in the very near future. If the total length of the distribution line is less than five kms, then the SWER system of distribution with an isolating transformer, is not economical (Fig. 3) and a single phase two wire distribution scheme should be used. As the total length of the distribution line increases the saving in construction costs by using an SWER system also increases as compared with other schemes.

b) The limitation of using three phase motor on a single phase supply can be eliminated by using static phase converters in conjunction with the three phase motor. In many cases the cost of a single phase induction motor is greater than or equal to the cost of a three phase induction motor of the same horsepower including the cost of a capacitor type phase converter and in some cases even when the three phase induction motor is 25% derated. But the cost of a three phase induction motor including auto-transformer capacitor or capacitor-reactor type phase converter may be higher than the cost of a single phase induction motor. Better performance of these two types of phase converters outweighs the extra cost incurred.

c) By adopting grounding grids of suitable sizes, the SWER system can be used in the areas of high soil resistivity. The resistivity of the soil in the vicinity of the grounding system, if

necessary can also be reduced by chemical treatment of soil such as adding of salt.

d) The touch and step voltages can be reduced considerably by properly designing the grounding system and can be kept within safe limits. The ground current at peak load should be limited to eight amperes to reduce the cost of grounding systems and to keep the temperature rise of the grounding system within safe limits.

e) There is no major problem in running SWER system parallel to telecommunication lines if a safe distance of separation is kept between the two lines and the ground current of the SWER system is limited to eight amperes. However, the effect of ground currents on railway signalling equipment needs further investigation.

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VITA

Balwinder Singh Samra was born on July 2, 1945, in Mardan, India. He received his high school education from Khalsa High School, Samrai-Jandiala, Punjab, India. He has received his college education from The College, in Swindon, England; Indian Engineering Institute, in New Delhi, India; G. N. Engineering College, in Ludhiana, India; and Asia Engineering Institute, in Ludhiana, Punjab, India. He received his associate degree in Electrical Engineering from Institution of Engineers(India), in Calcutta, India, in May, 1969.

He was employed by Punjab State Electricity Board as Junior Engineer in the Muktsar Operation Division until July, 1970.

He has been enrolled in the Graduate School of the University of Missouri-Rolla since January, 1971, and has been a candidate for the Master of Science degree in Electrical Engineering.

He is a registered Professional Engineer in England and in India.

APPENDICES

APPENDIX A

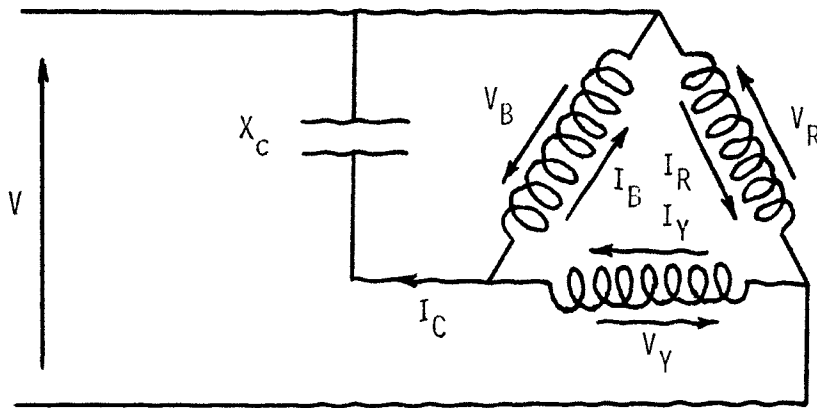The diagram for the capacitor type phase converter is given below:

FIG. 16: Capacitor type phase converter

Let I_1 and I_2 be the positive and negative sequence currents per phase of the induction motor. Z_1 and Z_2 are the positive and negative sequence impedences of the motor per phase at a specified slip. For a symmetrical motor, i.e., when $Z_B = Z_R = Z_Y$, then $I_R + I_Y + I_B = 0$, which means there are no zero sequence currents. Now

$$I_R = I_1 + I_2$$

$$I_Y = a^2 I_1 + a I_2$$

$$I_B = a I_1 + a^2 I_2.$$

Also $V = V_R = I_1 Z_1 + I_2 Z_2$. Therefore, $V_Y = a^2 I_1 Z_1 + a I_2 Z_2$ and $V_B = a I_1 Z_1 + a^2 I_2 Z_2$. Also $-V_B = (I_B - I_Y)Z_c$ and $I_c = -I_B + I_Y$.

$$V = -(V_B + V_Y) \quad \text{or} \quad V = -V_B - V_Y$$

Putting the values of V_B and V_Y in above equation we get

$$V = -(a^2 I_1 Z_1 + a I_2 Z_2) - (-I_B + I_Y)Z_C$$

Putting the values of I_B and I_Y in terms of positive and negative sequence currents and rearranging we get

$$V = I_1(aZ_C - a^2 Z_1 - a^2 Z_C) + I_2(-aZ_2 + a^2 Z_C - aZ_C)$$

Substituting the value of $V = V_R = I_1 Z_1 + I_2 Z_2$ in above equation and rearranging we get

$$I_1(a^2 Z_1 + a^2 Z_C + Z_1 - aZ_C) = I_2(a^2 Z_C - aZ_2 - aZ_C - Z_2).$$

For balanced conditions $I_2 = 0$. Therefore

$$a^2 Z_1 + a^2 Z_C + Z_1 - aZ_C = 0$$

$$Z_1(a^2+1) + Z_C(a^2-a) = 0$$

or

$$Z_C = \frac{-Z_1(1+a^2)}{(a^2-a)} \quad \text{ohms}$$

$$Z_C = Z_1 \frac{(1+a^2)}{(a-a^2)}$$

Now we know that $a-a^2 = j\sqrt{3} = \sqrt{3}\underline{/90^\circ}$ and $1+a^2 = -a = -1\underline{/120^\circ}$. Then

$$Z_C = -\frac{Z_1\underline{/120^\circ-90^\circ}}{\sqrt{3}}$$

$$= -\frac{Z_1\underline{/30^\circ}}{\sqrt{3}} = \frac{Z_1\underline{/-150^\circ}}{\sqrt{3}}$$

Expressing Z_1 also in polar co-ordinates, i.e., $Z_1 = Z_1\underline{/\theta}$ where θ is the power factor angle of the motor. Then

$$Z_C = \frac{Z_1\underline{/\theta-150^\circ}}{\sqrt{3}}$$

Also $Z_c = -jX_c$. Therefore,

$$-jX_c = \frac{Z_1 \angle \theta - 150°}{\sqrt{3}}$$

Expanding the right hand side further we get

$$-jX_c\sqrt{3} = Z_1[\cos(\theta-150°) + j\sin(\theta-150°)] \qquad \text{(A-1)}$$

Equating imaginary parts of equation (A-1)

$$Z_1 \cdot \sin(\theta-150°) = -\sqrt{3}\ X_c$$

Now

$$\sin(\theta-150°) = \sin\theta \cdot \cos(180°-30°) - \cos\theta \cdot \sin(180°-30°)$$

$$= -\sin\theta \cdot \cos30° - \cos\theta \cdot \sin30°$$

$$= -[\sin\theta \cdot \cos30° + \cos\theta \cdot \sin30°]$$

$$= -[\sin(\theta+30°)]$$

Therefore

$$|X_c| = \frac{|Z_1|}{\sqrt{3}} \sin(\theta+30°) \qquad \text{ohms.}$$

Equating real parts of equation (A-1)

$$Z_1 \cdot \cos(\theta-150°) = 0$$

$$-Z_1 \cdot \cos(\theta+30°) = 0$$

This condition is satisfied when $\theta = 60°$. This shows that complete balance in phase currents can be obtained when the power factor angle is 60°.

Capacitor KVAR Calculations:

Consider the capacitor lossless. We know $|X_c| = \dfrac{|Z_1|}{\sqrt{3}} \sin(\theta+30°)$

or

$$|Y_c| = \frac{\sqrt{3}\ |Y_1|}{\sin(\theta+30°)}$$

Multiplying both sides by $V_B{}^2$

$$|Y_c|\ |V_B|^2 = \frac{\sqrt{3}\ |Y_1|\ |V_B|^2}{\sin(\theta+30°)}$$

Now

$$KVAR = \frac{KVA\ of\ motor}{\sqrt{3}\cdot\sin(\theta+30°)}$$

APPENDIX B

The diagram for the capacitor-reactor type phase converter is given below:
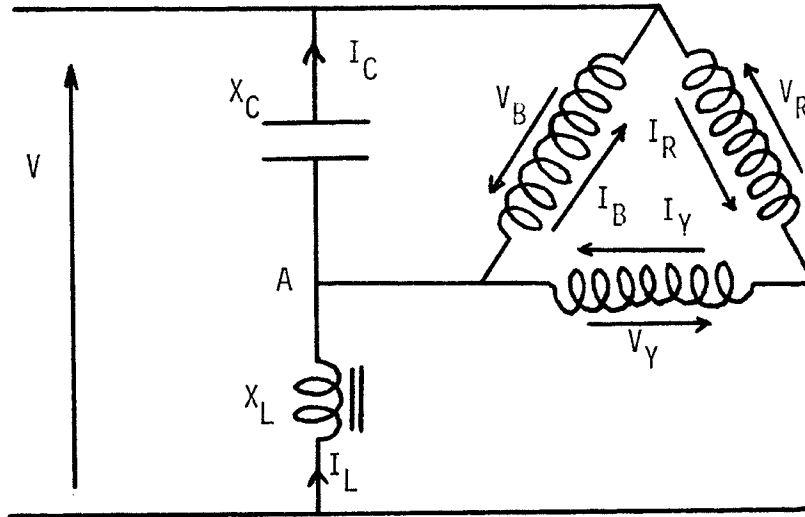
FIG. 17:   Capacitor-reactor type phase converter

From the above diagram $V = V_R = -V_B - V_Y$. In a symmetrical motor, i.e., when $Z_R = Z_Y = Z_B$, $I_R + I_B + I_Y = 0$, implying no zero sequence current. Let I_1 and I_2 be the positive and negative sequence currents per phase of the induction motor. Let Z_1 and Z_2 be the positive and negative sequence impedences of the motor per phase at a specified slip. Therefore

$$I_R = I_1 + I_2$$

$$I_Y = a^2 I_1 + a I_2$$

$$I_B = a I_1 + a^2 I_2$$

Similarly

$$V_R = I_1 Z_1 + I_2 Z_2$$

$$V_B = aI_1Z_1 + a^2I_2Z_2$$

$$V_Y = a^2I_1Z_1 + aI_2Z_2$$

At point A

$$I_L + I_Y = I_B + I_C \qquad\qquad (B-1)$$

We also know that $V_Y = Z_L I_L$. So $Z_L I_L = a^2I_1Z_1 + aI_2Z_2$. Therefore:

$$I_L = (a^2I_1Z_1 + aI_2Z_2)\cdot 1/Z_L$$

Similarly:

$$V_B = I_C Z_C$$

$$aI_1Z_1 + a^2I_2Z_2 = I_C Z_C$$

Therefore:

$$I_C = (aI_1Z_1 + a^2I_2Z_2)\cdot 1/Z_C$$

Putting these values in equation (B-1),

$$\frac{a^2I_1Z_1}{Z_L} + \frac{aI_2Z_2}{Z_L} + a^2I_1 + aI_2 = aI_1 + a^2I_2 + \frac{aI_1Z_1}{Z_C} + \frac{a^2I_2Z_2}{Z_C}$$

and rearranging

$$I_1\left[\frac{a^2Z_1}{Z_L} + \frac{aZ_1}{Z_C} + a^2 - a\right] = I_2\left[\frac{a^2Z_2}{Z_C} + \frac{aZ_2}{Z_L} + a^2 - a\right]$$

for balanced conditions $I_2 = 0$. Therefore

$$\frac{a^2Z_1}{Z_L} + \frac{aZ_1}{Z_C} + a^2 - a = 0$$

$$\frac{1}{Z_1}(a^2 - a) + \frac{a^2}{Z_L} - \frac{a}{Z_C} = 0$$

or

$$(a^2 - a) Y_1 + a^2 Y_L - a Y_C = 0 \qquad (B-2)$$

Where:

$$Y_1 = G_1 - jB_1$$

$$Y_L = -jB_L$$

$$Y_C = +jB_C$$

G is the conductance, B is the susceptance, the + sign indicates capacitive susceptance, and - sign inductive susceptance.

Also we know that $a^2 - a = -j\sqrt{3}$, $a^2 = -0.5 -j\,0.866$, and $a = -\,0.5 + j\,0.866$. Putting the values in equation (B-2) we get

$$-j\sqrt{3}\,(G_1 - jB_1) + (-0.5 - j0.866)(-jB_L) - (-0.5 + j0.866)jB_C = 0$$

or

$$(-\sqrt{3}B_1 - 0.866B_L + 0.866B_C) + j(-\sqrt{3}G_1 + 0.5B_L + 0.5B_C) = 0$$

Equating real and imaginary parts we get the following two equations.

$$-\,B_1 - 0.5B_L + 0.5B_C = 0 \qquad (B-3)$$

$$-\,\sqrt{3}G_1 + 0.5B_L + 0.5B_C = 0 \qquad (B-4)$$

Adding (B-3) and (B-4) we get

$$-\,\sqrt{3}G_1 - B_1 + B_C = 0$$

or

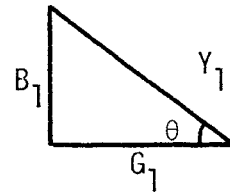$$B_C = \sqrt{3}G_1 + B_1 \qquad \text{mho}$$

Subtracting (B-4) from (B-3) we get (these are absolute values)

$$B_L = \sqrt{3}G_1 - B_1 \qquad \text{mho}$$

Now

$$G_1 = Y_1 \cdot \cos\theta$$

$$B_1 = Y_1 \cdot \sin\theta$$

Therefore

$$B_C = Y_1 \ (\sqrt{3}\cdot\cos\theta + \sin\theta)$$

Dividing and multiplying by two, the right hand side of the above equation

$$B_C = 2Y_1(\sqrt{3}/2 \ \cos\theta + 1/2 \ \sin\theta)$$

$$= 2Y_1(\cos 30° \ \cos\theta + \sin 30° \ \sin \theta)$$

$$Y_C = 2Y_1 \ \cos(\theta-30°) \tag{B-5}$$

Similarly

$$Y_L = B_L = 2 \ Y_1 \ \cos(\theta+30°) \tag{B-6}$$

Capacitor and Reactor KVA Calculations:

Multiplying both sides of equation (B-6) by V_Y^2

$$Y_L \ V_Y^2 = 2 \ Y_1 \ V_Y^2 \ \cos(\theta+30°)$$

We know

$$3 \ Y_1 \ V_Y^2 = \text{KVA of motor}$$

$$Y_L \ V_Y^2 = \text{Inductive KVA the reactor}$$

Therefore (considering the reactor lossless)

$$\text{KVAR (REACTOR)} = \frac{2}{3} \ \cos(\theta+30°). \quad (\text{KVA of motor})$$

similarly multiplying (B-5) on both sides by V_B^2

$$Y_C \ V_B^2 = 2 \ Y_1 \ V_B^2 \ \cos \ (\theta-30°)$$

Now

$$Y_C \, V_B{}^2 = \text{KVA of capacitor}$$

When the capacitor is lossless = KVAR of capacitor. So

$$\text{KVAR (CAPACITOR)} = \frac{2}{3} \cos(\theta - 30°) \text{ (KVA of motor)}$$

APPENDIX C

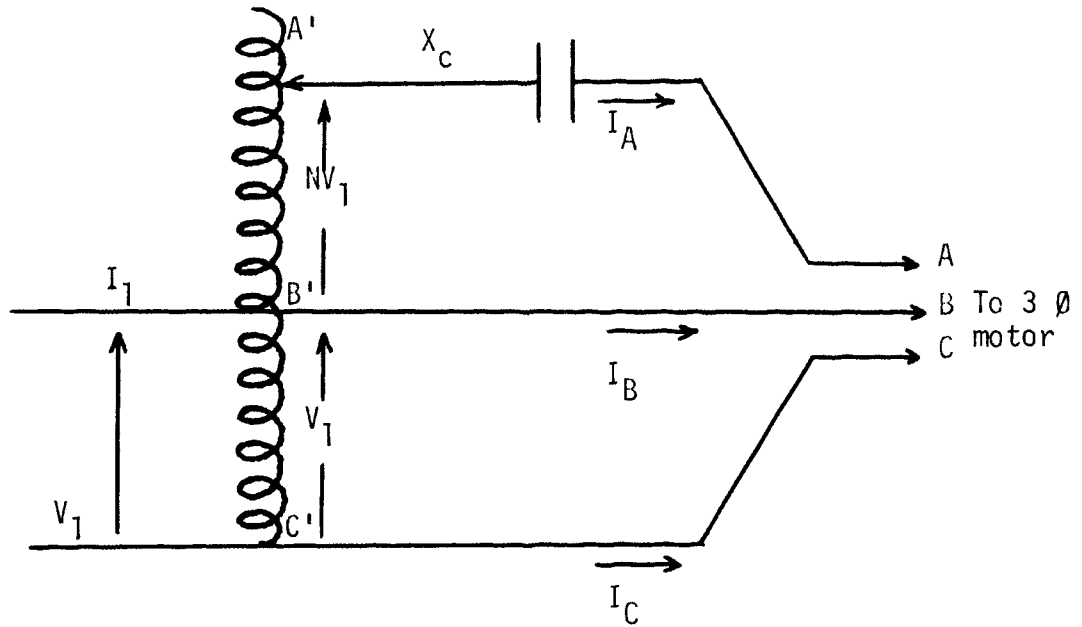The diagram for the auto-transformer capacitor phase converter scheme is shown below:

FIG. 18:   Auto-transformer capacitor type phase converter

Now from the figure

$$V_{A'B'} = V_{A'B} = NV_1$$

$$V_{B'C'} = V_{BC} = V_1$$

$$V_{C'A'} = V_{CA'} = -(V_1 + NV_1)$$

$$= -(1+N)V_1$$

Where N is

$$N = \frac{V_{A'B'}}{V_{BC}} = \frac{V_{A'B}}{V_1}$$

The positive, negative and zero sequence voltages are given by

$$V_{A'B_1} = 1/3(V_{ab} + aV_{bc} + a^2V_{ca}) \qquad\qquad (C-1)$$

$$V_{A'B_2} = 1/3(V_{ab} + a^2V_{bc} + aV_{ca}) \qquad\qquad (C-2)$$

$$V_{A'B_0} = 1/3(V_{ab} + V_{bc} + V_{ca}) \qquad\qquad (C-3)$$

Where V_{ab}, V_{bc} and V_{ca} are voltage phasors. Putting the values

$$V_{ab} = V_{A'B} = NV_1$$

$$V_{bc} = V_{BC} = V_1$$

$$V_{ca} = -(1+N)V_1$$

in equation (C-1) we get

$$V_{A'B_1} = 1/3(NV_1 + aV_1 - a^2V_1 - a^2NV_1)$$

$$= 1/3(a - a^2 - a^2N + N)\ V_1$$

$$= 1/3[(a-a^2) + N(1-a^2)]\ V_1$$

$$= \frac{V_1}{\sqrt{3}}\ [j + N(0.866+j0.5)]$$

$$= \frac{V_1}{\sqrt{3}}\ (1\underline{/90°} + N\underline{/30°}) \qquad\qquad (C-4)$$

Similarly equation (C-2) becomes

$$V_{A'B_2} = 1/3(N + a^2 - a - aN)\ V_1$$

$$= 1/3\ [(a^2-a) + N(1-a)]\ V_1$$

$$= \frac{V_1}{\sqrt{3}}\ [-j + N(0.866 - j0.5)]$$

$$= \frac{V_1}{\sqrt{3}} \left(1\underline{/-90°} + N\underline{/-30°} \right) \qquad\qquad (C\text{-}5)$$

and equation (C-3) becomes

$$V_{A'B_0} = 1/3(NV_1 + V_1 - V_1 - NV_1)$$

$$= 0$$

since the sum of the three currents must be zero, the zero sequence component of the current is also zero.

The equivalent circuit of an induction motor is usually expressed in terms of the equivalent wye circuit,[7] with the applied voltage being the line to neutral voltage. Expressing the component voltages of equation (C-4) and (C-5) in terms of line to neutral voltages gives

$$V_{A'n_1} = \frac{V_{A'B_1}\ \underline{/-30°}}{\sqrt{3}}$$

$$= \frac{V_1}{3} \left(N\underline{/0°} + 1\underline{/60°} \right)$$

$$V_{A'n_2} = \frac{V_{A'B_2}\underline{/30°}}{\sqrt{3}}$$

$$= \frac{V_1}{3} \left(N\underline{/0°} + 1\underline{/-60°} \right)$$

Next the unsymmetrical line impedences are resolved in sequence components

$$Z_{A0} = 1/3 \left(Z_A + Z_B + Z_C \right)$$

$$Z_{A1} = 1/3 \left(Z_A + aZ_B + a^2 Z_C \right)$$

$$Z_{A2} = 1/3 \left(Z_A + a^2 Z_B + aZ_C \right)$$

Where Z_A, Z_B and Z_C are line impedences in the phase converter circuit. Also

$$Z_B = Z_C = 0$$

Thus

$$Z_{A0} = Z_{A1} = Z_{A2} = 1/3 \ (Z_A)$$

The equation of the circuit can now be written[7] as follows

$$V_{A'n_1} = I_{a1} \ (Z_{A1} + Z_{m1}) + I_{a2} \ Z_{A2} \qquad \text{(C-6)}$$

$$V_{A'n_2} = I_{a1} \ Z_{A1} + I_{a2}(Z_{A2} + Z_{m2}) \qquad \text{(C-7)}$$

Where I_{a1} and I_{a2} are positive and negative sequence components of current I_A. Z_{m1} and Z_{m2} are positive and negative sequence impedence per phase of the induction motor at a specified slip. Putting the values of $V_{A'n_1}$, $V_{A'n_2}$, Z_{A1} and Z_{A2} in equations (C-6) and (C-7)

$$\frac{V_1}{3} \ (N\underline{/0°} + 1\underline{/60°}) = I_{a1}\left(\frac{Z_A}{3} + Z_{m1}\right) + I_{a2} \ \frac{Z_A}{3} \qquad \text{(C-8)}$$

$$\frac{V_1}{3} \ (N\underline{/0°} + 1\underline{/-60°}) = I_{a1} \ \frac{Z_A}{3} + I_{a2} \ (\frac{Z_A}{3} + Z_{m2}) \qquad \text{(C-9)}$$

Solving the above equations for I_{a1} and I_{a2}

$$I_{a1} = \frac{V_1}{3} \left[\frac{Z_{m2}(N\underline{/0°}+1\underline{/60°}) + (Z_A\underline{/90°}) \ 1/\sqrt{3}}{Z_{m1}Z_{m2} + \frac{Z_A}{3} \ (Z_{m1}+Z_{m2})} \right]$$

$$I_{a2} = \frac{V_1}{3} \left[\frac{Z_{m1}(N\underline{/0°}+1\underline{/-60°}) - (Z_A\underline{/90°}) \ 1/\sqrt{3}}{Z_{m1}Z_{m2} + \frac{Z_A}{3} \ (Z_{m1}+Z_{m2})} \right]$$

For balanced operation of three phase motors the negative sequence current I_{a2} should be zero. Therefore

$$Z_{m1} \underline{/\theta}_m (N\underline{/0°}+1\underline{/-60°}) - \frac{Z_A\underline{/90°}}{\sqrt{3}} = 0 \qquad (C-10)$$

where θ_m is the power factor angle.

Z_A may be any type of impedence but to minimize losses it is usually a capacitor for inductive loads, i.e., $Z_A = -jX_c$. Putting this value of Z_A in (C-10) and expanding it we get

$$NZ_{m1} \underline{/\theta}_m-90° + Z_{m1} \underline{/\theta}_m-150° = \frac{Z_A}{\sqrt{3}} = -j \frac{X_c}{\sqrt{3}} \qquad (C-11)$$

Equating the real parts of equation (C-11)

$$N = - \frac{\cos(\theta_m-150°)}{\cos(\theta_m-90°)}$$

$$= \frac{\cos(\theta_m+30°)}{\sin \theta_m}$$

Equating imaginary parts of equation (C-11) and replacing the value of N by the above calculated value we get

$$X_c = \frac{3}{2} \frac{|Z_{m1}|}{\sin \theta_m}$$

At unity power factor $\theta_m = 0°$ and $\sin \theta_m = 0$ both N and X_c tend to infinity. This scheme is therefore unsuitable for very high power factor loads.

Capacitor KVA Calculations:

We know

$$X_c = \frac{3}{2} \frac{Z_{m1}}{\sin \theta_m}$$

multiplying both sides by I_A^2

$$X_c \, I_A^2 = \frac{3 \, Z_{m1} \, I_A^2}{2 \, \sin \theta_m}$$

KVA of capacitor $= \dfrac{\text{KVA of motor}}{2 \, \sin \theta_m}$. If the capacitor is lossless

$$\text{KVAR} = \frac{\text{KVA of motor}}{2 \, \sin \theta_m}$$

Transformer KVA Calculations:

KVA rating of A'B' portion of the winding is

$$= NV_1 \, I_A \cdot 10^3$$

$$= \frac{N\sqrt{3}}{\sqrt{3}} \, V_1 I_A \cdot 10^3$$

$$= \frac{N}{\sqrt{3}} \cdot \text{KVA of motor}$$

Total KVA of transformer $= \dfrac{2N}{\sqrt{3}} \cdot \text{KVA of motor.}$

APPENDIX D

HEATING OF GROUND ELECTRODE

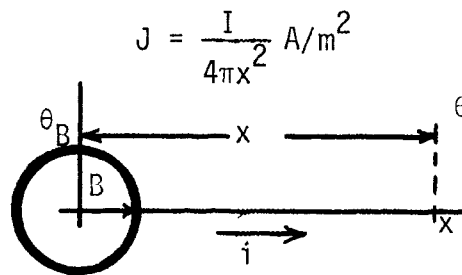The current density about a spherical electrode of radius B in the ground varies as a function of distance x,

$$J = \frac{I}{4\pi x^2} \; A/m^2$$

FIG. 19: Spherical electrode

The amount of heat generated is ρJ^2 Watts/m^3 (where ρ is the resistivity of soil in ohm-m) and the heat is partially stored in the volume elements of the ground, which has an average specific heat $\gamma = 17 \times 10^6$ W/°C-m^3, partly conducted from higher to lower temperature within the ground, the average heat conductivity being $\lambda = 1.2$ W/°C-m. The differential equation of the radial heat conduction about a sphere[8] is

$$\gamma \frac{d\theta}{dt} - \frac{\lambda}{x} \frac{d^2(x\theta)}{dx^2} = \rho J^2$$

It is difficult to solve this equation since J varies with x. For continuous ground current, the time derivative vanishes and the differential equation becomes

$$\lambda \frac{d^2(x\theta)}{dx^2} + \frac{\rho}{x^3} \left(\frac{I}{4\pi}\right)^2 = 0 \; .$$

With two simple integrations, the solution for temperature distribution over a distance x becomes

$$\theta = \frac{\rho}{\lambda} \left(\frac{I}{4\pi}\right)^2 \frac{1}{x} \left[\frac{1}{B} - \frac{1}{2x}\right] \quad °C$$

Then maximum earth temperature at the electrode will be when x = B, i.e.,

$$\theta_B = \frac{1}{2} \cdot \frac{\rho}{\lambda} \left(\frac{I}{4\pi B}\right)^2$$

and depends only on two constants ρ and λ and current density. The resistance of the spherical electrode of diameter 2B meters (buried in earth) is

$$R = \frac{\rho}{4\pi B} \quad \text{ohms}$$

then

$$I = 4\pi B \sqrt{\frac{2\lambda\theta}{\rho}} = 1/R \sqrt{2\rho\lambda\theta} \quad \text{amps}$$

the voltage of ground electrode is

$$E = IR = \sqrt{2\rho\lambda\theta} \quad \text{volts.}$$

APPENDIX E

ELECTROSTATICALLY INDUCED E.M.F.

The field of an infinite line having a ρ_L charge per unit length and at a distance r meters away from it is

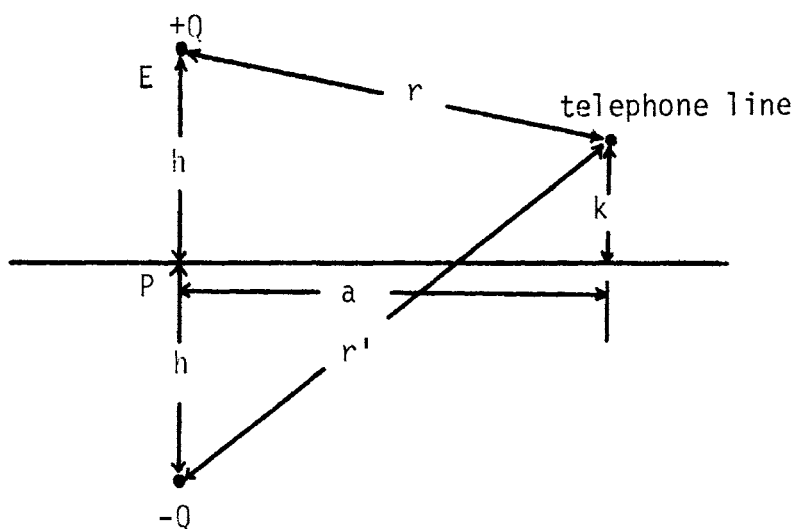$$E_r = \frac{\rho_L}{2\pi\varepsilon r} \; V/m$$

FIG. 20: Electrostatically induced e.m.f. in telephone line

Potential induced in telephone line due to +Q

$$V_+ = \int_r^h E_r \cdot dr$$

This is the potential with respect to the origin, i.e., point P(ground)

$$V_+ = \frac{\rho_L}{2\pi\varepsilon} \; \ln \left[\frac{h}{r}\right] \; \text{volts}$$

Similarly due to -Q the induced potential is

$$V_- = - \frac{\rho_L}{2\pi\epsilon} \ Ln \ \frac{h}{r'} \quad \text{volts}$$

Therefore the total potential difference between the origin and telephone line is

$$V = V_+ + V_- = \frac{\rho_L}{2\pi\epsilon} \left[Ln \ \frac{h}{r} - Ln \ \frac{h}{r'} \right]$$

$$e_s = \frac{\rho_L}{2\pi\epsilon} \ Ln \ \frac{r'}{r} \quad \text{Volts} \quad\quad (E-1)$$

Since the origin is the ground therefore the potential is with respect to ground.

The voltage E to ground of the high voltage SWER line of height h meters and diameter d meters is as below, where the electric field in this case is also the same, i.e., at a distance of r meters

$$E_r = \frac{\rho_L}{2\pi\epsilon r} \quad V/m$$

then

$$E = - \int_{2h}^{d/2} E_r \ dr = \frac{\rho_L}{2\pi\epsilon} \int_{d/2}^{2h} \frac{dr}{r} \quad \text{volts}$$

$$= \frac{\rho_L}{2\pi\epsilon} \ Ln \ \frac{4h}{d} \quad \text{volts} \quad\quad (E-2)$$

Dividing (E-1) by (E-2) we get

$$\frac{e_s}{E} = \frac{Ln \ \frac{r'}{r}}{Ln \ \frac{4h}{d}}$$

Now $r^2 = a^2 - (h-k)^2 = a^2 + h^2 + k^2 - 2hk$ and

$$r'^2 = a^2 + (h+k)^2 = a^2 + h^2 + k^2 + 2hk$$

Therefore

$$r'^2 = r^2 + 4hk$$

and

$$\text{Ln } \frac{r'}{r} = \frac{1}{2} \text{Ln} \left[\frac{r'}{r}\right]^2 = \frac{1}{2} \text{Ln} \left[1 + \frac{4hk}{r^2}\right]$$

for separations which are quite large as compared with height h

$$\text{Ln } \frac{r'}{r} \simeq \frac{2hk}{r^2+2hk} = \frac{2hk}{a^2+h^2+k^2}$$

this yields

$$\frac{e_s}{E} = \frac{2}{\text{Ln } \frac{4h}{d}} \times \frac{hk}{a^2+h^2+k^2}$$