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ABSTRACT

All Laser Doppler Velocimeter (LDV) measurements are made by scattering light from small particles suspended in the flow. Further, the scattered light is collected from some finite region within the flow -- commonly called the "sample-volume." These two facts comprise the necessary starting point of an analysis of an LDV.

The received (heterodyned)E-field can be written:[1]

$$E(\underline{K},t) = \text{const.} \sum_n \exp i\underline{K} \cdot (\hat{r}_{\underline{n}}(t) + \underline{L}) \exp(-i\omega_o t) P(r_{\underline{n}}(t) + \underline{L}) \quad (1)$$

where $\hat{r}_n(t)$ is the time dependent position of the n^{th} particle relative to a fixed position vector $\underline{L}$, and $\underline{K}$ is the scattering vector. The last term is the amplitude of the signal received from the n^{th} particle.

The quantity of interest is the spectrum of the light detector output. We will confine ourselves to photomultipliers and their analogs. The output of the light detector will be denoted by i(t), the time dependent current.

The following relations are known:

1) The current out is proportional to the square of the incident E-field.
2) The spectrum of the output current is the Fourier transform of the autocorrelation of the photomultiplier current.

Symbolically:

1) $$i(t) = A|E^2| \quad (2)$$

2) $$I(\underline{K},\omega) = \int R_{ii}(\underline{K},t) \exp(i\omega t)\, dt \quad (3)$$

where $$R_{ii} = \langle i(0)\, i(t) \rangle \quad (4)$$

The brackets denote a time average. Stationarity is assumed. It can be shown that the autocorrelation of the heterodyne signal from the LDV can be written:[2]

$$R_{ii}(\underline{K},t) = \langle E(\underline{K},0)\, E^*(\underline{K},\tau) \rangle \quad (5)$$

Keeping in mind that E(K,t) is a function of the particle positions, we define a probability density function $G(\underline{\Delta r},t)d\Delta r$ which is the probability that a particle moves a distance $\underline{\Delta r}$ in time t with respect to the mean velocity $\overline{V}$. R_{ii} can be explicitly written in terms of G and P the amplitude weighting function:

$$R_{ii}(\underline{K},t) = \rho\text{const. Re} \int G(\underline{\Delta r},t) \exp[-iK \cdot \underline{\Delta r}] \int \exp[-i\underline{K} \cdot \underline{\overline{V}}t] \times P(r)P(\underline{r} + \underline{\Delta r} + \underline{\overline{V}}t)\, dr\, d\Delta r \quad (6)$$

The inner integral contains the influence of the optics on the spectrum and G( ) contains the influence of particle diffusion with respect to the mean flow.

For laminar flow and sufficiently large particles ($\geq$ 2000 Å):

$$G(\Delta r,t) = \delta(\Delta r) \quad (7)$$

and Eq. 6 becomes:

$$R_{ii}(\underline{K},t) = \rho \text{ const Re} \int \exp[i\underline{K} \cdot \underline{\overline{V}}t] P(\underline{r})P(\underline{r} + \underline{V}t)dr \quad (8)$$

In turbulent flow, the diffusion of the particles is caused by the turbulent fluctuations; thus G(Δr,t) is a function of the turbulent structure. For the particles normally used in LDV work, G may be taken to be the space-time correlation function for a fluid element.[3]

It can be shown that the length scale for R_{ii} is on the order of the inverse of the scattering vector - (10^{-4} - 10^{-5} cm.). This limits the time scale of the measurement to periods over which the particles move 10^{-4} -10^{-5} cm. with respect to the mean flow. This time scale is typically much shorter than the Lagrangian time scales that determine diffusion in a turbulent system. In this case, an excellent approximation to G is:

$$G(\underline{\Delta r},t) = \frac{1}{t} F\left(\frac{\Delta r}{t}\right) \quad (9)$$

where F(V') is the probability density function for a turbulent fluctuation. Under certain well-defined circumstances, F( ) can be obtained directly from the LDV spectrum. Finally, the moments of the spectrum can be computed in terms of the turbulent parameters. For instance:

$$\text{MEAN} = \underline{K} \cdot \underline{\overline{V}} \quad (10)$$

$$\text{VARIANCE} = \frac{\overline{V}^2}{4\sigma^2}\left[1 + \frac{\overline{v'^2}}{\overline{V}^2}(1 + 4K^2\sigma^2)\right] \quad (11)$$

The term σ is the characteristic length of the sample volume in the flow direction, and v' is the RMS fluctuation velocity.

REFERENCES

1. Edwards, R. V., Angus, J. C., French, M. J., and Dunning, J. W., Jr. "Spectral Analysis of the Signal from the Laser Doppler Flowmeter: Time Independent Systems," J. Appl. Phys., 42, 837-850 (1971).
2. Cummins, H. Z. and Swinney, H. L., in Progress in Optics, Vol. 8, edited by E. Wolf, North Holland Publishing Co., Amsterdam, Netherlands, 1970.
3. Hinze, J. O., Turbulence, P. 352-364, McGraw-Hill, New York, 1959.

DISCUSSION

V. GOLDSCHMIDT (Purdue University): I believe you said that there were two different time scales. You said that if you used too small a sample volume you may get in trouble because of the averaging procedure. Can't you just substitute this space-volume averaging by taking a number of samples and doing time averaging on those?

EDWARDS: No! You will always have part of your spectrum determined by finite transit time broadening no matter how long or in which manner you average. The finite transit time effect is inversely proportional to the sample volume size so you can get into trouble with too small a spot size. A ten or fifty micron spot size would get you away from most of the finite sample volume problems.

GOLDSCHMIDT: We keep on saying if we compare laser measurements with the hot wire we have shown that the laser works. But we have no standard yet for the hot wire to know whether or not the hot wire works. Could it be that they are both consistently wrong?

Experiments reported elsewhere (see for instance Goldschmidt, Householder, Ahmadi and Chuang, "Turbulent Diffusion of Small Particles Suspended in Turbulent Jets", Paper 4-2, International Symposium on Two-Phase Systems, August 29 - September 2, 1971, Technion City) have suggested that small particles (in order of the turbulent scale) may have turbulent transport coefficient as much as 6 times larger than the momentum transport coefficient. These results put serious doubt on whether certain ranges of seeding particle sizes are representative of the motion of the fluid. What can we propose for experiments to measure slip velocity of the seeding particles used in laser anemometry?

EDWARDS: What the flow meter sees is determined by the scattering centers and you hope that the scattering centers are acting exactly like the fluid. In some situations the particles don't track the fluid flow exactly. Fortunately, we have at Case a group that makes very uniform size particles anywhere from about 500 Angstroms to about a micron in size or even larger. So what we plan to do is to load our system with scattering centers of increasing size, starting with the very smallest and going to the largest things we can get in there. We will then carefully examine the effect of particle size on the spectra.

D. McLAUGHLIN (Oklahoma State University): You said that you did not think you could get turbulent Lagrangian time scales, because of some limitations.

EDWARDS: By taking the spectrum of the output current of the photomultiplier directly, I don't think you can get Lagrangian time scales. If you take the output of the photomultiplier and FM detect it, then you can get Lagrangian time scales. In fact the next paper, to be presented here, has lots of pictures of turbulent velocity spectra. So the inherent limitation is in how you treat your signal when you get it out of your photomultiplier.

McLAUGHLIN: Your system was a reference scatter system. And you had estimates of probe volumes or sample volumes. There's no way you're going to get that small a probe volume with the reference scatter system.

EDWARDS: We have a paper on laminar flow (J. Appl. Phys., February, 1971) on this subject. What we do in all cases is calibrate the sample volumes. In other words we run the system in laminar flow and get out values for the sample volumes at that size. You have to be a little careful. Remember you have to intersect two beams to get a sample volume. If you don't align carefully you can get very small sample volumes. So the calculations are one thing but if you don't align the system, your sample volumes are really going to be smaller than you calculate them to be.