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S. J. Kline

G. R. Offen

W. C. Reynolds

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G. R. Offen, S. J. Kline and W. C. Reynolds

Department of Mechanical Engineering
Stanford University
Stanford, California

ABSTRACT

The paper covers first a short review of the history of research on turbulent shear and second a description of current experiments which may lead to further understanding.

The first portion categorizes the kinds of data which have been taken and discusses what can be learned from each. It then summarizes what is firmly established concerning the nature of turbulent shear, mostly from work of the past decade. A description of the several interpretations of these data under theoretic study by current leading researchers is then given.

The second portion of the paper discusses the extraordinarily difficult problem of identifying and measuring the actual production of turbulence in a boundary layer. The difficulties arise from the fact that production is a partly-coherent, intermittent process buried in relatively high amplitude noise. The measurement problems are discussed and a potential solution for the measurement of turbulence production with adequate accuracy is proposed.

INTRODUCTION

The present paper is divided into two parts. The first part is a brief review of the history and current knowledge about the mechanisms which create turbulent shear. The second part is a discussion of current thinking concerning experiments which may lead to further increases in understanding these mechanisms.

The history of this problem is both long and complex, and recent papers by the same group have covered much of what will be presented. Consequently, the first part of the paper is brief and gives references to recent works where the reader can find more extensive treatments of various aspects of the problem.

The second part of the paper stems primarily from the work of the past decade, and involves identifying some of our past difficulties in the field of turbulence measurements more sharply as a necessary forerunner to improved measurements and understanding. It should be clearly understood that it reports work still in progress, but may also be useful in suggesting techniques which have other applications in fluid mechanics.

BRIEF HISTORY OF TURBULENT SHEAR RESEARCH

It is now nearly a century since O. Reynolds discovered the set of related phenomena we call turbulence, and derived the famous "Reynolds Equations" showing that the time-average effect of turbulent fluctuations is to increase the apparent, or effective,stresses in fluid motions. Unfortunately, the averaging process inherent in the derivation of the Reynolds equations causes a loss in information with the result that one is left with more unknowns than equations. This creates what is conventionally called the closure problem. During the century, successive waves of enthusiasm for various mathematical approaches to the closure problem have occurred, but it is only within the past ten years or so, that we have begun to unravel any of the physical details of the complex processes which create and maintain turbulent flow.

It is possible to categorize the developments of the past century in turbulence in many ways, but for the present purposes it is perhaps sufficient to identify four categories:

(i) The collection, and generalization via non-dimensional correlations, of data in cases of exemplary or engineering utility.

(ii) Attempts to create closure by use of data and/or ad hoc assumptions concerning defined parameters or terms in the governing equations.

(iii) Data and theories founded on the use of conventional statistical methods, most typically two-point space time correlations, spectra, and Fourier Transform theory.

(iv) Direct attempts to determine "structure" by experimentation.

Category (i) includes the familiar mean-profile and shear coefficient correlations for tubes, flat plates, etc. They are of great utility as base data for design and for checking theories. They are perhaps the firmest data we have. At the same time, one must keep in mind that they are all statistics, and in fact never exist. Instantaneous profiles, for example see Kim, et al[11], almost never correspond to the mean. They are as much a figment of our statistics as that elusive fellow, the average man.

Category (ii) exists primarily for the purpose of solving immediately pressing design problems. A number of "levels" of possible closure exist and they are of varying degrees of complexity and sophistication. Professor W. C. Reynolds has recently given an up-to-date discussion of the various schemes[20]. The work sets the various methods into clearer relations with each other and is an excellent starting point for classwork. A recent comparison of the success of various semi-empiric theories for prediction of two-dimensional incompressible boundary layers is also available from the Thermosciences Division at Stanford[14]. Similar work has recently been done for compressible flows (Langley Lab, NASA Conference, 1968) and for 3-dimensional boundary layers[23]; however, in these instances the data are much less complete.

Category (iii) stems primarily from the work of G. I. Taylor and T. Von Kármán in the 1930's, and the expansion of these ideas by G. K. Batchelor[1] for homogeneous turbulence and A. A. Townsend[21] for shear flows. These works are extensive and are not repeated here. What is essential to a number of comments that follow, however, is the foundations, the assumptions, on which these methods rest. Basically, they view turbulence as a series of eddies of various sizes. They assume further: the largest eddies are created by energy transferred from the mean flow; energy on the average then is passed to successively smaller eddies until the eddy size is reached where viscosity quickly damps the fluctuation; the eddies are all more or less of the same kind (or form). These ideas are all made very explicit in the monographs of Townsend and Batchelor. There are, however, some other assumptions that often are not made entirely explicit.

These are that the process is statistically stationary, and that representation of the eddies by sinusoidal decomposition, on the average, is illuminating. That is, sinusoids are a useful waveform basis set. We shall have more to say about these assumptions, particularly in the second part of the paper.

The data in category (iv) are of two distinct types: (a) measurements of statistical parameters directly suggested by the theoretic framework of category (iii) notably long-time averaged two-point-space-time correlation coefficients and spectra; (b) visual data providing instantaneous mean velocity and fluctuation profiles as a function of time. Data of type (b) are of recent origin and stem largely from the laboratories at Stanford and Ohio State[5] thus far. We shall examine some examples shortly. These data have the considerable advantage of providing an overall view of the structure and also of explicitly displaying the time sequences of events; they have the disadvantage of relatively large experimental uncertainties.

If we are concerned with understanding how turbulence is created and maintained, then categories (i) and (ii) above, despite their undoubted practical utility, provide essentially no information. Category (iii) initially was erected with the idea of providing such information, but over three decades of careful and sophisticated experimentation and theoretic development have yielded very little advance in either increased understanding or predictive capability. Some of the reasons for this will appear below. We turn then to some remarks about the visual data from category (iv) and what they tell us about turbulent shear.

Turbulent flows are classifiable into three broad groups: grid turbulence, bound shear flows, and free shear flows. Grid turbulence is old, nearly wornout turbulence; it has limitingly low values of turbulence production and shear. Hence, it is of little interest if we are concerned with turbulent shear. Most visual studies to date are for bound shear flows (boundary layers and channel flows), and hence we will limit ourselves to that class.

A summary of our ideas about turbulent shear, based primarily on the visual studies, but drawing also on other sources is given by Kline[15]; the detailed work is described by Kim,et al[10]. The discussion which follows is restricted to some summary remarks about what is known concerning turbulent shear in boundary layers.

1. The layer consists of an inner and outer portion with distinct and different scaling laws in both time and space. These two layers interact with each other in the processes of turbulence production. The older idea that the innermost layer, the viscous sublayer, is steady, two-dimensional and truly "laminar like" is incorrect; the layer has a definable three-dimensional structure and is time-dependent; it interacts with the outer layer[4,8,10,12,13]. Elements of fluid marked at y^+ values as low as 0.1 are found in the outer layer, at distances sufficiently far downstream.

2. Turbulence is not a single state. Rates of turbulence production can be varied,often downward to zero and upward by an order of magnitude,by any of a number of phenomena which can affect the "stability" of the flow via the boundary conditions, the constitutive equation, or the body forces. These twelve phenomena include:

<u>Boundary Conditions</u>

a. roughness
b. streamwise pressure gradient
c. wall curvature
d. blowing or suction
e. moving walls in a channel flow[22]

<u>Body Forces</u>

f. centrifugal
g. coreolis
h. MHD
i. EHD
j. energy release generated by reactions
k. bouyancy

<u>Constitutive Equations</u>

ℓ. polymer additives

3. Correlation coefficients of several kinds appear to have wave-like properties. In the inner layers of bound shear flows and near the edges of jets, data show that propagation speeds of correlation coefficients reach values in excess of several times the local flow speed. Cross-Spectral data of Morrison and Kronauer[18] exhibit clear "wave-like" bunching in the double spectral plane.

4. While the mean velocity profile in a flat-plate boundary layer is stable to not only very small, but also to finite amplitude perturbations, according to the best available theory, the observed instantaneous profiles often are unstable in form (see Fig. 1). What is more, these unstable profiles are often,

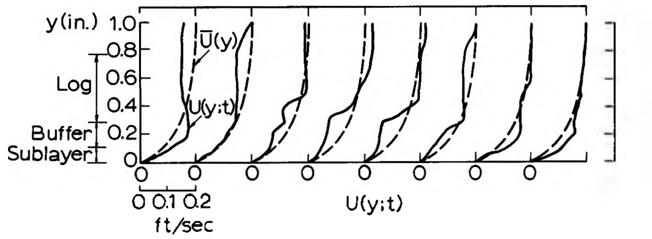

Figure 1 Instantaneous velocity profiles in flat-plate boundary layer; from Kim,et al. (Ref. 10): U_∞ = 0.25 ft/sec.

although not always, followed by what appears to be growth of an oscillating mode and then by "break-up" into smaller, more chaotic fluctuations. Motion pictures of these events are documented in Kim, et al.[11]. Moreover, Kim, et al.[10,11] have shown, for two flows studied,that essentially all the net turbulence is produced during these events, and that these events occupy <u>at most</u> only 40% of the total time. Thus turbulence production is intermittent, not quasi-continuous (see Fig. 2).

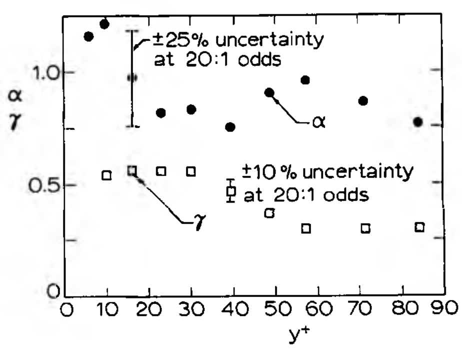

Figure 2 Fraction of turbulence produced during bursting time (α) and fraction of time occupied by bursts (γ) versus y+ in a flat-plate boundary layer; from Kim,et al. (Ref. 10).

Narahari, et al.[19] have proposed a correlation for the average time between zones of high production (called bursts). Their data indicate this average time scales on outer layer variables in distinction to the wall-layer streak spacing which scales on inner layer variables. This further reinforces the importance of the interaction of the inner and outer layers in the production process.

The results in items 1 to 4 are all data, that is, direct observations of nature in some form. However, there are currently a number of different ideas concerning the meaning of these data and hence in what way they should be used in attempting to create further theoretic advance. A few remarks on the principal ideas involved follow.

Kim, et al.[10,11] have suggested that the turbulence production arises from a local intermittent instability which is part of a limit cycle of events in the boundary layer. In particular, they suggest that existing large disturbances cause the low-speed streaks near the wall to lift, that is migrate into the outer flow quite rapidly; this creates a local instability, which, in turn, causes creation of large fluctuations which subsequently break up into numerous more chaotic fluctuations. The large fluctuations create further streak-lifting, and so the process is maintained.

E. Mollo-Christensen in the 1971 Von Kármán lecture of the AIAA suggests that a more appropriate model is perhaps a series of interlocked non-linear feed-back loops arising from secondary and higher order instabilities.

M. Landahl of Stockholm postulates the existence of wave packets in turbulent shear flows. He believes that the phase speed of higher order disturbances frequently becomes equal to the phase speed of the lowest order disturbance and that when this happens, the two waves remain locked together, travelling with the same speed. According to Landahl the cumulative effect of these two disturbances causes a sudden break-up of the wave structure, and this break-up is directly related to the rapid production observed during "bursting".

It is still too early to evaluate the merits of these various ideas. However, certain other recent data and calculations are relevant.

The data of Hussain and Reynolds[9] show that none of the several earlier wave theories are adequate to predict the correct trends in turbulent shear flow even for very weak disturbances. The calculations of Ling and Reynolds[17] strongly suggest that no theory which omits the interactions of the large fluctuations with the background turbulence can fully describe turbulent shear flows.

The success of Lahey and Kline[16] in describing essentially all existing two-point space-time correlation data for a wide variety of correlations and many kinds of flow suggests that a two-part model for the velocity perturbation is appropriate (see examples in Figs. 3 and 4). The two parts used are respectively Markoff noise and a travelling wave with a stochastic jitter on the base wave number and a phase coefficient randomly distributed in time. Since several known physical processes could give agreement with this type of mathematical representation, further experiments are suggested to clarify the many remaining questions.

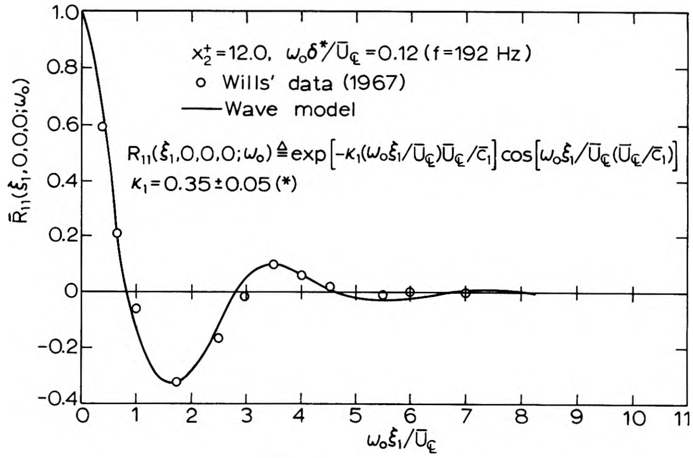

Figure 3 Comparison of two part model for space-time correlation ($\bar{R}_{ij}$) with jet data of Wills (Ref. 24). ω_o = frequency; ξ_1 = streamwise probe separation, $U_{\mathfrak{C}}$ = jet centerline speed; κ_1 = fitted parameter; from Lahey and Kline (Ref. 16). This case is an average "goodness" of fit. The symbol * means an adjusted (fitted) parameter; others are fixed.

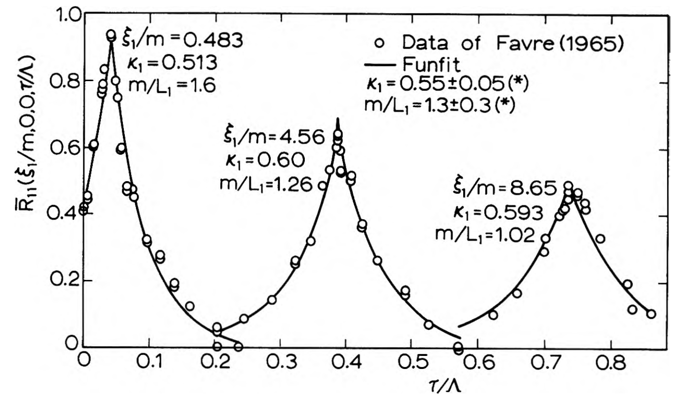

Figure 4 Comparison of two-part model for space-time correlation ($\bar{R}_{ij}$) with grid turbulence data of Favre, et al. (Ref. 25) for three streamwise separations (ξ_1/m); m/L_1 taken from data; K_1 fitted; Λ = macroscale; τ = time delay. (A case of particularly spectacular agreement.) The symbol * means an adjusted (fitted) parameter; others are fixed.

Thus, we currently are planning two kinds of experiments. The first by Acharya and Reynolds will study the propagation of large amplitude disturbances in a turbulent channel flow in order to provide the basis for comparison with improved theories of a wave type. The second by Offen will be discussed below. It is concerned with methods for determining more details about the processes which occur during the intermittent periods of high turbulent shear and high turbulence production in boundary layers.

A RATIONAL APPROACH TO THE FILTERING PROBLEM IN A TURBULENT FLOW

1. Introduction

One of the bits of conventional wisdom among turbulence researchers is that the velocity trace appears to be very similar to the output from a suitably bandwidth-limited noise source. Much data has been collected which describes the distribution of the magnitude of some property, such as the energy of the fluctuations, among various values of a parameter called "frequency". Recent studies have gone further in this direction by attempting to find a relationship between some feature of the fluctuation and the associated frequencies. In particular, the velocity trace has been filtered, and the output record has been analyzed instead of the original trace. Neither the meaning of the word "frequency" nor the effect of the filter on the shape of the output have been seriously questioned in these investigations. However, it now appears necessary to have a clear understanding of what is meant by the statement that a fluctuation has a certain frequency, phase, and amplitude. Since filters are used to discriminate between fluctuations of different frequencies, a discussion of the concept of "frequency" is not complete unless one considers the manner in which the filter separates the various frequencies and the wave-form of the base frequency of the decomposition.

This question of definitions and filter characteristics will be treated from the frame of reference of someone who is searching for a set of waveforms among the velocity fluctuations which can be treated as unique, but are quite different from those fluctuations usually characterized as noise. It is hoped that this set of waveforms can be completely described by a small number of parameters, and can be related to the turbulent motion during certain visually observed events known as bursts.

Once the word "frequency" has been defined, the implications of the choice of a filter on the results will be discussed. At the conclusion of the paper a model of the velocity fluctuations in a low speed, water turbulent boundary layer will be proposed as a basis for the design of filters which are to be used in this problem and possibly other turbulent flows. The ideas to be presented here are currently being used at Stanford to design digital filters for research into the mechanisms of turbulent bursting.

2. The Meaning of Frequency in a Noisy Signal

There is no ambiguity in the definitions of frequency, amplitude, and phase when the signal is stationary and sinusoidal in form. However, when the signal consists mainly of noise, or a coherent part buried within some form of noise, the meaning of these parameters may not be clear. Several interpretations are possible, as outlined below, and each one may be the most useful for certain purposes. It will be seen that the main differences among the various interpretations is the length of time over which the signal is analyzed. Naturally this difference implies not only variations in the loss of information due to the averaging process but also separate techniques for each type of signal analysis.

"Eyeball" Frequency. It appears that the most appropriate way to specify a definition for these terms is to analyze the operation used to detect and isolate the fluctuations of differing frequencies. For example, when a person studies a trace, such as a velocity record measured in a turbulent shear flow, and attempts to specify the frequency of the fluctuation around a given instant of time, he usually looks for the time between successive peaks. This operation is equivalent to taking a one period average of variable length and neglecting changes in the DC level, the amplitude of the supposed sinusoid, and its phase. It is a valid operation, which could be automated, and, therefore, one can define an "eyeball frequency" as described above. This approach is very similar to the method that assigns a frequency to a noisy signal (assuming no DC) by counting zero crossings per unit time.

Persistent Frequency. One of the most common methods of decomposing a signal into its sinusoidal components is to take the Fourier Transform of the entire record. The resulting function in the frequency domain can be thought of as the source of the coefficients of a Fourier series representation of the entire original signal. This process yields an amplitude and phase angle for each "persistent frequency", and the fluctuation represented by this frequency is considered to contribute to the total signal for the entire length of the data record. If one were to pass the decomposed signal through an ideal narrow band-pass filter, the output would be a constant amplitude, constant phase sinusoid; that is the filter output would be identical to the signal from a good sinusoidal wave generator.

Temporary Frequency. If one divides the total data record into shorter intervals and then performs a Fourier Transform on each interval, the coefficients are those associated with a "temporary frequency". The interpretation is identical to the previous one for the "persistent frequency", but applied to the shorter interval. Thus slow variations with time are envisioned. That is, although the fluctuation represented by a "temporary frequency" is assumed to contribute to the total signal for the duration of this shorter interval, the contribution from each such fluctuation may vary between times which are far apart. For example, if a signal consists of intermittent pulses of a given frequency, f_o, superimposed on random background noise, the Fourier Transform is a smooth curve except for a spike at f_o if the transform is calculated over an interval containing a pulse. Transforms calculated over other intervals will not contain the spike.

Successive Temporary Frequency. The technique mentioned above can be extended by taking successive Fourier Transforms of the signal over a given interval, which is short relative to the total length of the data record, and moving the starting time of each transform by a fraction of the interval. Since the squared magnitude of a transform is known as a power spectrum, the squared magnitude of such successive short interval transforms are known as chrono spectra. A bank of conventional analog band-pass filters would operate on the data in a similar manner, and, therefore, the output trace from an analog filter, which we can call a "filtered chronology", is essentially an indication of the fluctuation of a given "temporary frequency". It is important to recognize that the instantaneous amplitude of the "filtered chronology" actually represents the magnitude of a fluctuation which has retained its periodic structure for some time. One must not be misled into thinking that the filter gives an "instantaneous frequency" just because it has an output at every instant of time.

However, before we discuss the "instantaneous frequency," we should note that both chronospectra and "filtered chronologies" are subsets of the amplitude-time-frequency space. Every signal can be considered as a surface in this space. Chronospectra are the result of cutting this space with planes that are parallel to the amplitude frequency axis with each plane intersecting the third axis at a different time. The average of all such planes is the Fourier Transform of the entire data record. "Filtered chronologies" are generated by cutting the space with planes that are parallel to the amplitude-time axis and intersecting the other axis at various frequencies. The average of all these planes is the original time record of the signal.

Instantaneous Frequency. An "instantaneous frequency" can be defined as the frequency of a sinusoid that "fits" the signal at a given instant of time. This definition is best explained by the use of an example. Let a signal, u(t), be represented by the following co-sinusoid during a very short interval of time:

$$u(t) = m + A\cos(\omega t+\theta)$$

If one can assume that m, A, ω, and θ are constant over this short time span, then one can solve for ω and the other three variables at any given time, t_o, by using u(t) and its first three derivatives. That is:

$$\omega = \sqrt{-\dddot{u}/\dot{u}}$$

$$\theta = \tan^{-1}\left(\frac{\omega\dot{u}}{\ddot{u}}\right) - \omega t_o$$

$$A = -\frac{\dot{u}}{\omega\sin(\omega t_o+\theta)}$$

$$m = u - A\cos(\omega t_o+\theta)$$

where dots above u represent differentiation with respect to time. Actually even this result does not give a true "instantaneous frequency" because one must average over four times the sampling interval in order to be able to calculate the third derivative.

One can certainly question both the utility and the meaning of this "instantaneous frequency". Is it valid to talk about the presence of a periodic component in a noisy record if such a fluctuation doesn't persist for some time? If one believes that it must be in evidence during a time interval, how long must this interval be and how much variation in amplitude and phase does one accept? In other words, is it meaningful to associate fluctuations from a random signal with a frequency? What is the significance of the difference between the output from a narrow band-pass filter and the result one would expect to get from an instantaneous type of analysis? As a result of the averaging effect, the filtered signal is a continuous signal which only passes through zero, but rarely stays there very long. On the other hand the "instantaneous frequency" is probably a discontinuous record with frequent, significant gaps (zero values). Because of questions like these, it is felt that the discussion of an "instantaneous frequency" is useful.

In order to gain additional insight into the meaning of the word frequency, it is instructive to explore the instantaneous idea one step further. Consider the case of variable amplitude and phase; for simplicity let there be no DC component. That is:

$$u(t) = A(t)\cos[\omega t+\theta(t)]$$

Furthermore, assume that $A(t) = a\cos\alpha t$ and $\theta(t) = \beta T$, where α and β are small relative to ω. This is the case of slowly varying amplitude and phase of a Fourier component and represents what one might expect to find in "real life" over a moderate interval of time. After using several trigonometric identities, but without using the assumptions of α and $\beta \ll \omega$, one can rewrite the above expression for u(t) as follows:

$$u(t) = \frac{a}{2}(\cos\omega_s t + \cos\omega_d t)$$

where $\omega_s = \omega + \beta + \alpha$ and $\omega_d = \omega + \beta - \alpha$

Thus, the original fixed-frequency cosine with varying amplitude and phase has become the sum of two cosines, each with a frequency different from the original one. If the rates of the amplitude and phase variation are truly small relative to the basic frequency, then both ω_s and ω_d are approximately equal to ω, and the original expression for u(t) becomes:

$$u(t) \cong a\cos\omega t$$

This discussion can be summarized by stating that one important part of <u>the problem of generating a filtered time record is the quest for an averaging time which is larger than $\frac{2\pi}{\omega}$ but smaller than $\frac{2\pi}{\alpha}$ and $\frac{2\pi}{\beta}$.</u>

<u>"Befrency"</u>. One additional frequency, which will be called "befrency", is defined as the reciprocal of the period of a signal of arbitrary waveform. This concept will be discussed in more detail shortly.

MODEL IMPLICATIONS IN CHOICE OF FILTER

Each of the definitions of frequency which have just been presented was derived by analyzing the operation used to detect the presence of a fluctuation at that frequency. Such an operation is called filtering. Consider for a moment the application of these ideas to a real problem. Clearly, one would not attempt to find an isolated fluctuation of a given frequency within a noisy signal by taking a Fourier Transform of a long data record. It should also be clear by now that if one did use a long-time average Fourier Transform filter, one would analyze the physical process incorrectly. The above mentioned isolated pulse would be lost in the Transform of the noise. This loss of an intermittent spike is apparently what happens in long-time averaged turbulence correlation functions (see, for example, Kline, et al.,[14] and Lahey and Kline[16]). Thus we return to the comment about the quest for an appropriate averaging time and the realization that this comment points very directly to the following ideas. <u>The choice of filtering technique must be appropriate for the model one has of the fluctuating quantity being studied. Conversely, every choice of a filter technique implies a model</u>.

1. <u>Model Based on Duration of Coherent Structure</u>.

The simplest difference between various fluctuation models is based on averaging time. In the frame of reference of the model, the distinction is described by the length of time one expects to find a coherent, or nearly coherent, signal superimposed on the background noise. A further difference is the rate of change from a coherent signal at one frequency to a second coherent signal at another frequency, or back to noise. Random noise (assumed to be of infinite band-width) is a subset of this class of models with zero duration of coherent signal and, hence, infinite rate of change between signal structures.

As an example, let us compare the output from a filtered noise generator and the signal from a hot wire placed near the wall in a turbulent boundary layer. The differences can be demonstrated dramatically by listening to the two signals.

When played through a speaker, the output of the noise generator sounds even, or smooth, compared to the turbulence, which crackles and pops intermittently. Suitable adjustment of the filter causes both signals to have the same power spectra, but they certainly sound quite different. This great difference serves to remind us that power spectra do not define a unique source; different sources not infrequently give the same spectra because information about phase and amplitude distribution are lost in the squaring and averaging processes required to generate a spectrum.

2. Linear Versus Non-Linear Models.

Almost all filters are used in a manner which implies that the original signal is due to the algebraic sum of a set of fundamental signals. That is, the assumption of linearity is basic to most standard filters, analog or digital. From the viewpoint of the model, this linear characteristic of the filter implies that the fluctuating phenomenon (e.g., turbulent velocity) is the result of the superposition of a set of phenomena which are more orderly than the resulting phenomenon. A well-known example of such a combination is an FM signal, which is the linear sum of a carrier wave plus a message and, therefore, belongs to the large class of signals which can rationally be treated by linear devices.

A significant question is whether the use of a conventional filter is appropriate in a study of turbulent bursting. Since the purpose of a filter is to separate a coherent signal from background noise, the use of such a linear filter implies that the total velocity signal is the result of a simple superposition of a coherent motion on top of background noise. However, it has been suggested that bursting involves non-linear interactions between the fluid which has been lifted away from the wall (the part which is postulated to behave in a coherent manner for a short period of time) and the fluid which is at some distance from the wall (the noise-like part). Thus there may be a contradiction between the model of the flow and the preconceptions under which the filter is operated.

3. Waveforms and "Befrency".

Many years ago Fourier[6] showed that any function which exists for a finite length of time can be reproduced by the sum of a set of suitably weighted sinusoids. However, he never claimed that such a decomposition would yield the most useful information about a given signal. Essentially all the standard filter techniques assume that valid information can be obtained about the physical process by studying the various sinusoidal components separately. This is a meaningful statement if one knows that a given waveform is present, for then one can calculate the spectrum of that waveform. Whenever one sees such a pattern in the frequency domain representation of a signal, one can deduce the shape of the postulated waveform. Although the use of the standard sinusoidal decomposition would not confuse the investigator in an orderly situation, as long as he started with the awareness of the existence of a particular waveform, the results would still be much clearer and easier to understand if the decomposition were presented in terms of the postulated waveform.* When the situation is less orderly, as seems to be the case with velocity fluctuations in a turbulent shear flow, the sinusoidal decomposition may never lead one to suspect the presence of a non-trigonometric waveform because the results would contain significant coefficients for many harmonics - i.e., they probably will blur over a possible unique structure.

An example of a waveform which may be applicable to turbulent boundary layer studies, and particularly to attempted decompositions of the Reynolds stress fluctuations, is a skewed triangular wave. The rise time during each cycle is significantly shorter than the decay time. Let us use the term "befrency" to designate the reciprocal of the period of such an oscillation, as measured at its base - the time axis.** Then the decomposition would be in terms of skewed triangular waveforms whose "befrencies" are multiples of the primary "befrency". If, in fact, the Reynolds stress fluctuations oscillate in a skewed triangular manner, then such a decomposition would show large contributions to spectra at only one, or at most a few, "befrencies," whereas with a standard trigonometric decomposition large contributions would be indicated for many frequencies. As just mentioned above, the higher harmonics displayed in the standard presentation could hide the existence of a burst signature. The two most important mathematical restrictions on the choice of the waveform are that its various harmonics obey orthogonality conditions, at least with respect to some specified weighting function, and that the set of waveforms be a complete set.

DUAL DOMAIN PERSPECTIVE

Filters are generally described by their characteristics in the frequency domain. With this in mind, the statement that the choice of a filter implies a model of the fluctuations can be separated into two components: (1) the specification of a filter also means the specification of its characteristics in the time domain, and (2) these latter attributes show most clearly how the output from a filter is related to the input. This brings us to the following important point: whenever one designs or analyzes a filter, one must consider its behavior in both the time and the frequency domain. It is well to recall that multiplication in one domain implies convolution in the other domain. The most important relationships between filter characteristics in the two domains are discussed below (see, for example, Bracewell[3] or Gold and Rader[7]). The first two items are mainly applicable to digital filters.

(1) Square truncation of an infinite record in the time domain produces ripples in the frequency domain. That is, the Fourier Transform of the time record will contain bumps which would not be present if the record had not been truncated. Therefore, the output of a filter whose impulse response has been truncated is affected at any given frequency by the frequency content of the input at adjacent frequencies.

(2) Truncation in the time domain by the use of a smooth "window" will still cause ripples in the frequency domain, but they will be much smaller than those caused by square truncation. However, this improvement is at the expense of frequency selectivity. Such a trade-off condition is always present in filter design problems.

*Note that one must analyze the Fourier Transform of the signal, not the power spectrum, so that one does not lose phase information.

**The word "befrency" was generated by borrowing an idea on word transformations from Bogart et al.[2] They use the word "quefrency" for the independent variable of a "cepstrum," which is the (direct) Fourier Transform of the log of a power spectrum. That is, "quefrency" is the Fourier Transform of frequency.

(3) The better the frequency selectivity, the greater the ringing in the time domain. This may not be a problem for long-time averages, but it can be intolerable if one is either trying to do waveform identification by inspection of "filtered chronologies" or trying to trace a certain motion, such as bursting in turbulence, in the frequency-time space. A good example of the problems that can be caused by ringing is shown in Fig. 5. The upper trace is a time record of uv(t) as measured by a hot-wire x-probe near the wall in a turbulent boundary layer. The lower record is the output from a narrow band-pass filter whose input is the uv(t) signal. Note that the spikes in uv(t) become something like a damped sinusoid in the filtered trace.

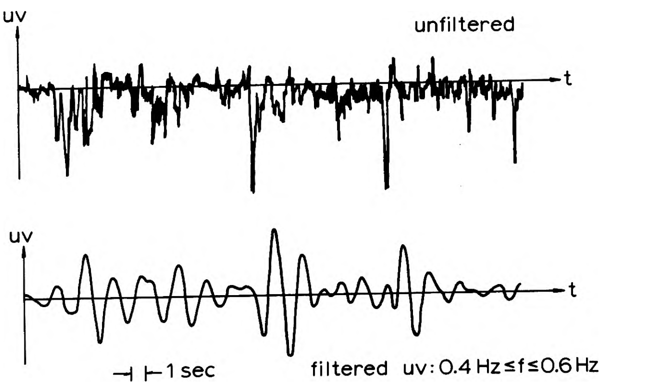

Figure 5 Effect of ringing in filters on record of uv product.

(4) A narrow function in one domain transforms into a wide function in the other domain. This relationship is the basis of an important limit on band-pass filtering: <u>the product of the bandwidth and the minimum data record length must be at least unity</u>. In physical terms this says, a band-pass filter cannot detect any changes which occur faster than one period of the bandwidth frequency.

OTHER EFFECTS OF FILTERING

Several other effects of a filter on a signal should be mentioned briefly.

(1) The probability density function of a "filtered chronology", in all but "pathological" cases, is more nearly a normal distribution than that of the input signal.

(2) The autocorrelation of the output of a filter is equal to the convolution of the autocorrelation of the input and that of the filter impulse response function. That is, if the filter operation is described, in the usual manner, by:

$$o(t) = h(t)*i(t)$$

where i(t) represents the input, o(t) the output, h(t) the filter transfer function, and * means convolution, then:

$$R_{oo}(\tau) = R_{hh}(\tau)*R_{ii}(\tau)$$

Note that if the input is random white noise, then $R_{oo}(\tau) = R_{hh}(\tau)$. As is to be expected, the effect of R_{hh} on R_{oo} also becomes more significant as the filter bandwidth is decreased. It should be kept in mind, however, that one does not want R_{oo} to be identical to R_{ii}; in fact, one has inserted the filter in order to make R_{oo} different from R_{ii}. But it is necessary to insure that the difference is due to the rejection of unwanted elements of the signal and not just to the response of the filter to noise. Again the need to design the filter system with a model of the fluctuating process in mind is apparent.

(3) The impulse response function of a series of elemental filters is usually different from the response function of any of the components. Consider, for example, the use of two simple RC circuits in tandem, and let both have the same time response function, $h_i(t) = \exp(-t/T)$. Since the mathematical representation of a cascade of filters is given by the convolution of the impulse response of each filter, the impulse response function of the combination is given by:

$$h(t) = h_i(t) * h_i(t) = t \exp(-t/T)$$

for positive times. Note that h(t) has quite a different character from the impulse response of the individual RC circuit (see Fig. 6).

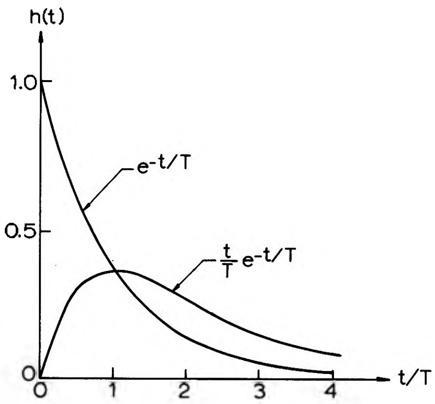

Figure 6 Comparison of response of an RC circuit ($e^{-t/T}$) and two RC circuits in series ($\frac{t}{T} e^{-t/T}$).

A PROPOSED MODEL OF THE FLUCTUATIONS IN A TURBULENT BOUNDARY LAYER

The following model is proposed to represent the velocity fluctuations observed near the wall in a turbulent boundary layer. This model is based partly on the dissertation by R. Lahey,[16] partly on experience with "filtered chronologies" of the streamwise fluctuating velocity component, and partly on the limitations imposed by filter theory. Since it makes no sense to suggest a filter transfer function which cannot be duplicated by any filter, digital or analog, the filter characteristics must be considered. Therefore, it is more accurate to call the proposed model a hypothesis instead. This model is to be used in a first attempt to detect bursts from a set of "filtered chronologies". If this approach is not successful, the consequences of waveform and non-linearity will be considered. As mentioned earlier, the use of a filter to find information which may help to describe the turbulence process is equivalent to the suggestion of a model of the process.

The fluctuations are represented by Markoff noise with a "coherent structure" that appears at random intervals superimposed on the background noise. This supposed structure consists of a sinusoidal oscillation whose frequency is not constant, but rather exhibits a jitter about a center frequency, f_o, within the following band:

$$\frac{2}{3}f_o << f << \frac{4}{3}f_o$$

For our experiment in a low-speed water flow, 0.2fps << U_∞ << 1.0fps, reasonable values of f_o are 2 Hz and 8 Hz, based on the results of previous investigators. Furthermore, this structure persists for about four cycles, using f_o as the basis. The amplitude of these oscillations remains approximately constant, and is of the same order of magnitude as the noise.

Since the purpose of this exercise is mainly to search for the presence of coherence in turbulence, a ripple of 5% is quite acceptable in both the pass-band and the stop-band, and the minimum attenuation in the stop-band need not be greater than -26 db, which is equivalent to 5% of the pass-band gain.

A plot of a filter transfer function which satisfies the above specifications is given in Fig. 7. It is flat near the center frequency and passes through the half-power points at the extremities of the postulated jitter band. Beyond these points the roll-off is approximately -26 db/octave. The transfer function is smooth over the whole band and reaches an attenuation of -26 db at frequencies which are 50% further from the center frequency than the jitter band. The bandwidth is wide enough

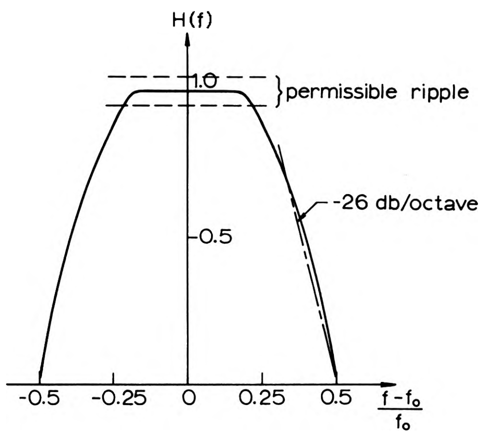

Figure 7 A proposed filter for studying the characteristics of turbulent bursts.

that the duration of the filter impulse response function is only as long as four periods of a fluctuation whose frequency is f_o. An attempt will be made to design a filter with zero phase shift, but it will very likely be necessary to accept a linear phase vs. frequency relationship. The linear dependency yields a constant time shift for the fluctuations at all frequencies. The proposed filter can be used directly on u(t) and v(t), but signal conditioning will be required for uv(t) prior to filtering in order to remove the impulsive nature of this signal.

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DISCUSSION

The discussion of this paper is included after the Johnson-Saylor paper.