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INTERPRETATION OF HOT-FILM ANEMOMETER RESPONSE IN A NON-ISOTHERMAL FIELD

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ABSTRACT

A new technique for interpretation of hot-film anemometer sensor response is described. This technique has been applied to simultaneous measurement of profiles of mean velocity, the three components of velocity vector fluctuation, and temperature fluctuation in non-isothermal pipe flow of water using multiple sensors[1]. Sensors operated in the constant temperature mode (CTA) respond to both mean and fluctuating velocity and temperature. The influence of mean temperature gradient on CTA sensor response was eliminated by appropriate adjustment of the sensors' operating resistances as the temperature gradient was traversed. The adjustments were derived from analysis of linearized CTA sensor response. A sensor operated as a resistance thermometer (CCA) responded to the mean temperature and temperature fluctuations and had negligible velocity response. Estimates of errors in the interpretation of responses are presented. Errors depend on the magnitude of the mean temperature gradient, sensors' coefficients of resistivity, and obedience to known cooling and yaw-sensitivity laws. Calculations are presented for uncoated 2-mil and 6-mil hot-film sensors. Examples of the application of this technique to the measurement of turbulence in water are presented.

INTRODUCTION

Hot-wire or hot-film anemometers operated in the conventional mode as heated sensors respond to variations of both the velocity and temperature of the surrounding fluid. These sensors have been widely applied to measurements of velocity in isothermal fluids; however, their application to non-isothermal flow fields has been limited because of the difficulty in separating the response to velocity and temperature.

Corrsin[1] developed the earliest reported technique for separating the response to velocity and temperature fluctuations. His technique, which was used by several investigators[2,3,4,5], required that the field of interest be traversed three times with the sensor(s) operated at three different overheat ratios. The technique did not allow measurement of the mean velocity or mean temperature. It is severely limited by the necessity to accurately relocate the sensor during each traverse and by the necessity of solving three simultaneous equations for the mean-square values of fluctuating velocity, temperature, and the velocity-temperature correlation. The latter limitation is particularly severe when making turbulence measurements in liquids wherein the range of possible overheats is limited. In such case the determinant of the coefficients of the three simultaneous equations becomes extremely small, thus decreasing the accuracy of their solution. In order to eliminate the necessity of making multiple traverses or operating the sensor(s) at several overheat ratios, simultaneous measurement of the velocity and temperature fluctuations in a small region may be made using more than one sensor in that region. Johnson[6,7] suggested this approach wherein a standard X-probe is supplemented by a third sensor which responds only to temperature. The principal limitation of this approach is that it requires not only equal velocity sensitivities of the X-probe sensors, as with any X-probe, but also equal temperature sensitivities. Equality of these sensitivities depends on matching the sensor physical characteristics which is usually possible with hot-wire sensors but is difficult with hot-film sensors.

Both of the above approaches were developed for analysis of the sensor operating voltages (bridge voltages) although the principles are equally applicable to analysis of linearized sensor responses. An extensive analysis of linearized response of sensors operated in a nonisothermal field has been given by Wiggins[8]. His technique is complicated by allowing the sensors to respond to variations in both mean velocity and mean temperature which necessitates velocity calibration at several ambient temperatures.

This paper presents a technique which allows simultaneous measurement of mean velocity, mean temperature, components of the velocity fluctuation, and the temperature fluctuation using either two, three, or four sensors[9]. Linearized signals of the sensors responding to both velocity and temperature are analyzed. The technique does not require multiple traverses or multiple sensor overheat ratios and furthermore provides a means of matching sensitivities of two or more sensors to velocity and temperature. Being able to avoid combining multiple experimental runs eliminates the requirement of exact repeatability of conditions and also relaxes requirements for excessively long run, drift free operation of earlier techniques. However, it, like all the other techniques reviewed, does not provide dynamic compensation which is necessary for transient turbulence studies. However, it does enable on-line data analysis.

SENSOR RESPONSE

The relationship between the bridge voltage, V, required to heat a sensor to a temperature, T_S, at operating resistance, R_S, and the loss of heat to the surrounding fluid is given by King's law modified to allow a choice of exponent, 1/m, on U_{eff} as:

$$V^2 = (A + B\, U_{eff}^{1/m})\,(T_S - T)\,R_S \qquad (1)$$

where A and B are functions of the sensor geometry and fluid properties, U_{eff} is the effective cooling velocity of the fluid seen by the sensor, and T is the temperature of the fluid in the region of the sensor. If the sensor is maintained at a constant operating temperature, T_S, hence constant resistance, R_S, and A and B are constant, the voltage will vary with changes in both the effective cooling velocity and the fluid temperature.

It is convenient to treat the bridge voltage electronically in order to obtain a signal which varies linearly with changes

*Present Address is Combustion Engineering, Inc., Windsor, Connecticut of the cooling velocity. The resulting signal is:

$$E = (V^2 - V_o^2)^m$$

$$= G \left\{A [(T_S - T)R_S - (T_{So} - \overline{T}_o)R_{So}] + B\, U_{eff}^{1/m} (T_S - T)R_S\right\}^m \quad (2)$$

where G is the gain of the linearizer, and the subscript o refers to reference conditions wherein U_{eff} is zero.

In the case of an isothermal flow field the term in square brackets in Eq. 2 (to be called the zero suppression term) is identically zero if the reference sensor operating temperature and fluid temperature are maintained at the same temperatures during the measurements. Equation 2 then reduces to:

$$E = G\, (B\, R_S \Delta T_S)^m\, U_{eff} \quad (3)$$

where ΔT_S is the sensor overtemperature defined as:

$$\Delta T_S = T_S - \overline{T} \quad (4)$$

The effective cooling velocity is well represented[9,10] by:

$$U_{eff} = U\, (\cos\theta)^{1/k} \quad (5)$$

where U is the velocity vector component in the mean flow direction, θ is the angle between the mean flow direction and the normal to the sensor, and k is a constant. Values of k for a single sensor probe and the three sensor probe were determined from a yaw calibration experiment conducted in a low speed, low background turbulence wind tunnel. For the single sensor, k = 1.24 showed good agreement between Eq. 5 and the experimental results for $0^\circ \leq |\theta| \leq 70^\circ$. For the three sensor probe the values of k were determine to be 1.26, 1.21 and 1.25 for the z, r and θ sensors, respectively. This calibration showed that the inclined wire responses were in agreement with Eq. 5 for variation in mean flow angles up to $\pm 30^\circ$ from the mean flow direction.

In a nonisothermal flow field the zero suppression term is not identically zero because of variation of the mean temperature, $\overline{T}$, as well as temperature fluctuations, t, about the mean. The influence of this term on the fluctuating component of the linearizer output voltage can be examined in a low-turbulence approximation by differentiating Eq. 2. Carrying out this differentiation after inserting Eq. 5:

$$e = G\, [A(R_S\Delta T_S - R_{So}\Delta T_{So}) + BR_S\Delta T_S\, \overline{U}^{1/m}\, (\cos\theta)^{1/km}]^{(m-1)} \quad (6)$$

$$\times\, BR_S\Delta T_S\, \overline{U}^{(1/m-1)}\, (\cos\theta)^{1/km}\, [u - \frac{\tan\theta}{k}\, v - \frac{m}{\Delta T_S}\, (1 + \frac{A}{B\, \overline{U}^{1/m}(\cos\theta)^{1/km}})\overline{U}t]$$

in which the differentials have been replaced by corresponding fluctuating quantities (i.e. e = dE, u = dU, v = Udθ and t = dT).

The zero suppression term in Eq. 6 will be eliminated if:

$$R_S\Delta T_S = R_{So}\Delta T_{So} \quad (7)$$

which physically means that the sensor's operating resistance, and correspondingly its temperature, must be changed as the mean temperature gradient is traversed. The requirement of Eq. 7 will be examined in more detail in Error Analysis. If the requirement of Eq. 7 is met, Eq. 6 becomes:

$$e = G\, s_U\, [u - \frac{\tan\theta}{k}\, v - \frac{m}{\Delta T_S}\, (1 + \frac{A}{B\overline{U}^{1/m}(\cos\theta)^{1/km}})\, \overline{U}\, t] \quad (8)$$

in which the velocity sensitivity, s_U, is given by:

$$s_U = (BR_S\, \Delta T_S)^m (\cos\theta)^{1/k} \quad (9)$$

which varies only if B or m vary with mean temperature of the fluid. This variation will be shown to be negligible. The temperature sensitivity of the sensor will change with variation of $m/\Delta T_S$, $A/B\overline{U}^{1/m}$ and $\overline{U}$. It will be shown that $m/\Delta T_S$ and A/B are nearly constant for many applications of interest; therefore, the temperature sensitivity depends only on $\overline{U}$.

The response to mean velocity is found in a low-turbulence approximation by substituting Eqs. 5 and 7 into Eq. 2 and averaging in time to get:

$$\overline{E} = G\, s_U\, \overline{U} \quad (10)$$

where s_U is given by Eq. 9. Note that if the conditions of Eq. 7 are met, the mean voltage output of the linearizer depends only on changes in mean velocity and does not depend on changes of mean temperature of the fluid. The linearizer output voltage is the sum of Eqs. 8 and 10.

If two sensors are operated at a point, one with its normal at an angle θ to the direction of $\overline{U}$ and the other perpendicular (θ = 0) to the direction of $\overline{U}$, the linearizer output voltages are, respectively:

$$E_1 = G_1\, (s_{U1}\, \overline{U} + s_{U1}\, u - s_{v1}\, v - s_{T1}\, t) \quad (11a)$$

$$E_2 = G_2\, (s_{U2}\, \overline{U} + s_{U2}\, u - s_{T2}\, t) \quad (11b)$$

where the sensitivities may be identified with reference to Eqs. 8 and 9. Thus, $s_{v1} = s_{U1}\frac{\tan\theta}{k}$ and $s_{T1} = s_{U1}\frac{m}{\Delta T_s}(1 + \frac{A}{B\overline{U}^{1/m}})\overline{u}$ for sensor 1, with similar expressions for sensor 2. When multiplied by the associated gain (e.g. G_1) they represent the slopes of the sensor's output voltage (e.g. E_1) vs changes in velocity and temperature. In practice these sensitivities are obtained directly from calibration experiments.

If voltage E_2 is subtracted from voltage E_1, the resulting voltage is directly proportional to v, the lateral component of velocity fluctuation, if $G_1 s_{U1} = G_2 s_{U2}$ and $G_1 s_{T1} = G_2 s_{T2}$. The first requirement is easily met by appropriate adjustment of the linearizer gains to obtain equal calibrations for response to $\overline{U}$. The second requirement is met if:

$$\frac{m_1}{\Delta T_{S1}}\, (1 + \frac{A_1}{B_1\, \overline{U}^{1/m_1}}) = \frac{m_2}{\Delta T_{S2}}\, (1 + \frac{A_2}{B_2\, \overline{U}^{1/m_2}}) \quad (12)$$

In many cases of practical interest $A/B\overline{U}^{1/m} << 1$; therefore, the requirement of Eq. 12 can be very nearly met by setting the sensor overtemperatures so that $m_1/\Delta T_{S1} = m_2/\Delta T_{S2}$ initially. In traversing the mean temperature gradient the requirement of Eq. 7 must be met for each sensor. However, this will not change the equality of Eq. 12 if both sensors have nearly equal electrical resistivities at a reference temperature and nearly equal coefficients of electrical resistivity. This requirement is met by commercially available sensors as demonstrated under Error Analysis. In cases where $A/B\overline{U}^{1/m} \geq 1$ the equality of Eq. 12 is met only if $A_1/B_1 = A_2/B_2$ and $m_1 = m_2$. These conditions are also very nearly met by commercially available sensors as shown under Error Analysis where consequences of not meeting these requirements are also assessed.

If a third sensor is operated as a constant current anemometer, CCA, at very low current, it will function as a resistance thermometer and one obtains the output voltage:

$$E_3 = -G_3 s_{T3}(\bar{T} + t) \tag{13}$$

where G_3 is an output amplifier gain and s_{T3} is the temperature sensitivity of the anemometer bridge voltage which is directly proportional to the operating current through the sensor[9].

The sensor current was selected in the range of a few milliamperes which for the film sensors and flows of this study provided an adequate sensitivity to temperature with very small difference in sensor temperature above that of the ambient fluid. Thus, there was negligible sensitivity to velocity. This was verified experimentally.

The temperature sensitivity of the CCA output is equal to that of the CTA linearizer output given in Eq. 8 if:

$$-G_3 s_{T3} = \left[\frac{m}{\Delta T_S}\left(1 + \frac{A}{B\,\bar{U}^{1/m}}\right)\right]\bar{E} \tag{14}$$

This equality can be satisfied as the mean temperature gradient is traversed by adjustment of G_3 or the CCA sensor operating current since all terms in the square bracket are known. If the temperature sensor is calibrated at a known gain and operating current, the temperature sensitivity is known for any other gain or sensor current. In the case where $A/B\bar{U}^{1/m} \ll 1$ and $m/\Delta T_S$ changes very little as the flow field is traversed, the CCA temperature sensitivity may be kept as a constant multiple of $\bar{E}$.

If the fluctuating component of the CCA output voltage is subtracted from the fluctuating component of E_2 in Eq. 11b with Eq. 14 satisfied for these two sensors, the signal obtained depends only on u, the component of velocity fluctuation in the direction of $\bar{U}$.

The results obtained thus far are summarized as follows:

$$\bar{E}_1 = G_1 s_U \bar{U} \tag{15a}$$

$$\bar{E}_3 = -G_3 s_{T3} \bar{T} \tag{15b}$$

$$e_2 - e_3 = G_2 s_{U2} u \tag{15c}$$

$$E_1 - E_2 = -G_1 s_{v1} v \tag{15d}$$

$$e_3 = -G_3 s_{T3} t \tag{15e}$$

Thus, with three sensors, two operated as CTA's and one as a CCA, the mean velocity and temperature, two components of the velocity fluctuation, and the temperature fluctuation can be monitored. The signals of Eq. 15 can be treated either with analog or digital techniques.

The third component of velocity fluctuation, w, can be obtained by orienting a fourth sensor perpendicular to the first two velocity sensors, matching its sensitivities to those of sensor 2 as was done with sensor 1, and subtracting E_4, from E_1, similar to Eq. 15d. The principal limitation with operation of four sensors at once is the difficulty of construction of a probe with all sensors in a small enough region to represent a point measurement.

ERROR ANALYSIS

The derivation of the sensor responses involved several assumptions and requirements concerning the sensor characteristics and means of operation. These assumptions and requirements will be examined using measured hot-film sensor characteristics. The relationships of Eqs. 1 and 5 have been verified experimentally.

The equality of Eq. 7 requires that the operating resistance of the sensor be varied as a mean temperature gradient is traversed. The resistance of a sensor can be written as:

$$R_S = R_{So} + \alpha(T_S - T_{So}) \tag{16}$$

where R_S is the resistance at T_S, R_{So} is the resistance at T_{So}, and α is a sensor coefficient of electrical resistivity with units of ohms/degree. The standard definition of the coefficient of resistivity is α/R_{So}. The advantage of using the definition of Eq. 16 is that for any particular sensor it is independent of temperature; therefore, if the sensor resistance is known at any temperature it can be quickly found at any other temperature.

Combining Eqs. 7 and 16 and differentiating gives the required change in sensor operating resistance with change of the fluid mean temperature as:

$$\frac{dR_S}{d\bar{T}} = \frac{R_S^2\alpha}{R_S\Delta T_S\alpha + R_S^2} \tag{17}$$

The change is obviously a value less than the sensor's coefficient of resistivity.

The value of α measured for approximately five 6-mil hot-film sensors* and ten 2-mil hot-film sensors* was typically 0.0065 $\Omega/^{o}F$**. An operating resistance of 5.7Ω and overtemperature of 80°F were typically used for turbulence measurements in water with mean temperature of about 80°F. Putting these values in Eq. 17 gives $dR_S/d\bar{T} = 0.0060\ \Omega/F^{o}$.

Most anemometers allow changes in sensor operating resistance in units of 0.01Ω; therefore, the operating resistance to satisfy Eq. 7 can be met within 0.005Ω. Using this discrepancy, the previous values for R_S and ΔT_S, and a value of A/B of 0.25 $(ft/sec)^{1/2}$ which has been found typical for 2-mil and 6-mil hot-film sensors, the error in the response given in Eq. 2 due to the zero suppression term is less than 0.1 percent for velocities greater than 0.5 fps and less than 0.5 percent from 0.5 fps down to 0.2 fps. Thus, the requirement of Eq. 7 can be easily satisfied for most water flows of interest.

Equation 8 involves an assumption of low turbulence implicit in its derivation by differentiation. For convenience of illustration of the effects of the temperature fluctuations and the velocity-temperature correlation, this assumption will be examined for one-dimensional turbulence. The errors involved in the use of one-dimensional turbulence have been examined by numerous investigators including Hinze[11] and will not be discussed here.

Inserting the sum of mean and fluctuating values for E, U, and T in Eq. 2 gives:

$$\bar{E} + e = G\left[-AR_S t + BR_S(\bar{U} + u)^{1/m}(T_S - \bar{T} - t)\right]^m \tag{18}$$

The value of m is approximately 2 for most hot-wire and hot-film sensors; therefore, set m equal to 2 for convenience of illustration. Expanding Eq. 18, neglecting correlations above quadratics, and separating the mean and fluctuating responses gives:

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* All sensors referred to in this paper were manufactured by Thermo-Systems, Inc., St. Paul, Minnesota.

** All measurements referred to in this paper were performed during the experimental program described in Ref. 9.

$$\overline{E} = Gs_U\overline{U}\ \{1 + C_T{}^2 \frac{\overline{t^2}}{(\Delta T_S)^2} - (1 + C_T) \frac{\overline{ut}}{\overline{U}\ \Delta T_S}\ t + \ldots\} \qquad (19a)$$

$$e = Gs_U\ \{u - \frac{2\overline{U}}{\Delta T_S}\ C_T t + \frac{\overline{U}C_T{}^2}{(\Delta T_S)^2}\ (t^2 - \overline{t^2}) + \frac{1 + C_T}{\Delta T_S}\ (ut - \overline{ut}) + \ldots\} \qquad (19b)$$

where s_U is given in Eq. 9 with θ=0 and C_T is defined as:

$$C_T = 1 + \frac{A}{B\overline{U}^{1/2}} \qquad (20)$$

These relations enable the magnitude of errors to be determined. In the remainder of this section errors will be examined for a Reynolds number of 25,000 and a maximum wall heat flux of 3800 BTU/ft^2-hr for this flow resulting in a maximum temperature difference between the pipe wall and the bulk fluid temperatures. With an approach to the wall of 0.014 inches, these conditions provide an upper bound for error magnitudes in the study[9].

Comparing Eqs. 10 and 19a shows the error in measurement of mean velocity. Note that the effect of the velocity fluctuation intensities does not appear in the one-dimensional turbulence model. If A/B equals 0.25 $(ft/sec)^{1/2}$, the value of C_T is less than 1.5 for $\overline{U}$ > 0.25 fps. Measurements in water at closest approach to the wall of a heated 4-inch circular tube with local $\overline{U}$ = 0.26 fps and wall heat flux equal to 3800 Btu/ft^2-hr gave values for $\overline{t^2}$ and $\overline{ut}$ of 10.5$^oF^2$ and -0.0171 ft-^{o}F/sec, respectively. Using these values and ΔT_S = 80^{o}F the error in mean velocity measurement due to temperature fluctuation effects is +0.6 percent. This error is considerably less than that to be expected due to the velocity turbulence intensity[11].

The error in the approximation of the response to velocity and temperature fluctuations given by Eq. 8 can be evaluated by squaring and averaging Eq. 19b. The result involves both odd, higher-order correlations and differences of even, higher-order correlations and multiples of even, lower-order correlations. The error can be evaluated numerically only if simplifying assumptions are made concerning the statistical nature of the turbulence.

In order for the velocity sensitivity given by Eq. 9 to be constant if the equality of Eq. 7 is met, the value of B must be constant. For a given sensor, B depends on the fluid properties as:

$$B \propto \frac{k}{\sqrt{\nu}}\ (Pr)^{0.33} \qquad (21)$$

where k is thermal conductivity, ν is kinematic viscosity, and Pr is Prandtl Number. Evaluation of the changes of B using Eq. 21 for water at 80^{o}F gives + 0.23%/^{o}F. Measured values for a 6-mil hot-film sensor were -0.053%/^{o}F change in $\overline{T}$ at constant ΔT_S and + 0.034%/^{o}F change in ΔT_S for constant $\overline{T}$. The value measured for a 2-mil hot-film sensor was -0.047%/^{o}F change in ΔT_S for constant $\overline{T}$. These values indicate that B can be considered constant for most applications in water. The temperature sensitivity in Eq. 8 varies with $\overline{T}$ through m/ΔT_S and A/B. The exponent, m, was measured for a 6-mil, hot-film sensor and decreased non-linearly from 1.96 to 1.68 when ΔT_S increased from 63^{o}F to 95^{o}F at constant $\overline{T}$ and had no variation when $\overline{T}$ was varied from 80^{o}F to 105^{o}F at constant ΔT_S. The exponent, m, for a 2-mil hot-film sensor decreased non-linearly from 1.97 to 1.86 when ΔT_S was increased from 50^{o}F to 86^{o}F at constant $\overline{T}$. The 2-mil sensor, therefore, appears to be less sensitive to operating conditions.

The variation of ΔT_S required to meet the requirement of Eq. 7 as $\overline{T}$ varies is found using Eq. 16 to be:

$$\frac{d\Delta T_S}{d\overline{T}} = \frac{1}{2}\ [\frac{R_{\overline{T}}}{(R^2_{\overline{T}} + 4\ \alpha\ R_S\Delta T_S)^{1/2}} -1] \qquad (22)$$

where $R_{\overline{T}}$ is the sensor resistance at $\overline{T}$. Using α = 0.0065 $\Omega/^o$F, R_S = 5.7Ω, ΔT_S = 80^{o}F, and $R_{\overline{T}}$ = 5.18Ω gives $d\Delta T_S/d\overline{T}$ = -0.084^{o}F/^{o}F which is about 0.1 percent decrease in ΔT_S per degree increase of $\overline{T}$.

The ratio A/B is given by Hinze[11] as:

$$A/B = 0.737\ (Pr)^{0.13}\ (\frac{d}{\nu})^{0.5} \qquad (23)$$

where d is the sensor diameter. Considering water at 80^{o}F and a 2-mil diameter, hot-film sensor operated at ΔT_S = 80^{o}F gives A/B = 0.121 $(ft/sec)^{1/2}$ which is about half of the values measured for 2-mil and 6-mil, hot-film sensors. Evaluation of the change of A/B at constant ΔT_S using Eq. 23 gives -0.0004 $(ft/sec)^{1/2}/^o$F for a 2-mil, hot-film sensor. Values measured for a 6-mil, hot-film sensor were +0.00155 $(ft/sec)^{1/2}/^o$F change of $\overline{T}$ at constant ΔT_S and +0.00025 $(ft/sec)^{1/2}/^o$F change in ΔT_S at constant $\overline{T}$. Values measured for three 2-mil, hot-film sensors ranged from +0.00066 $(ft/sec)^{1/2}/^o$F to +0.00174 $(ft/sec)^{1/2}/^o$F change in ΔT_S at constant $\overline{T}$.

The above evaluations may be combined to show the variation of the temperature sensitivity in Eq. 8 is less than +0.4%/^{o}F change of $\overline{T}$ for U > 0.1 fps if the equality of Eq. 7 is maintained. This means that the temperature sensitivity can be considered to be a constant for most applications. Also, the adjustment of the temperature sensitivity of a CCA sensor to equal that of a CTA sensor will depend only on $\overline{U}$ (see Eq. 14).

In order to satisfy the equality of Eq. 12 the ratio A/B and the exponent m must be equal for the two sensors. The values of A/B measured for five 2-mil, hot-film sensors ranged from 0.21 $(ft/sec)^{1/2}$ to 0.26 $(ft/sec)^{1/2}$ for the same operating conditions. The same sensors had three values of m = 1.90, one m = 1.71, and one m = 1.75.

The influence of a mismatch on the difference of Eqs. 11a and 11b is given by:

$$\frac{\overline{(e_1 - e_2)^2}}{s_u^2\ \overline{U}^2} = \frac{\overline{v^2}}{\overline{U}^2} + (mk)^2\ (\frac{A_2}{B_2\overline{U}^{1/m_2}} - \frac{A_1}{B_1\overline{U}^{1/m_1}})^2\ \frac{\overline{t^2}}{(\Delta T_S)^2}$$
$$-2mk\ (\frac{A_2}{B_2\overline{U}^{1/m_2}} - \frac{A_1}{B_1\overline{U}^{1/m_1}})\ \frac{\overline{vt}}{\overline{U}\Delta T_S} \qquad (24)$$

at the conditions previously cited in evaluation of Eq. 19a, $v'/\overline{U}$ = 0.06. The value of k has been measured to be 1.24. Using these values and those previously cited with the extremes of A/B and m given above, the three terms of Eq. 24 are:

$$\frac{\overline{(e_1 - e_2)^2}}{(s_U\overline{U})^2} = [360 + 4.04 - 25.8] \times 10^{-5} \qquad (25)$$

which shows that errors in measurement of $\overline{v^2}$ on the order of 6 percent are obtained.

The error involved if the second term on the right side of Eq. 14 is ignored in matching the CCA and CTA temperature sensitivities is given by:

$$\frac{\overline{(e_2 - e_3)^2}}{(s_U \bar{U})^2} = \frac{\overline{u^2}}{\bar{U}^2} + \left(\frac{m A}{B\, \bar{U}^{1/m}}\right) \frac{\overline{t^2}}{(\Delta T_S)^2} - \frac{2\, m}{\bar{U}^{1/m}} \frac{A}{B} \frac{\overline{ut}}{\bar{U} \Delta T_S} \qquad (26)$$

At the conditions used to evaluate Eq. 19a, $\overline{u'}/\bar{U} = 0.15$. Using this value and the values for the other terms of Eq. 26 previously cited gives:

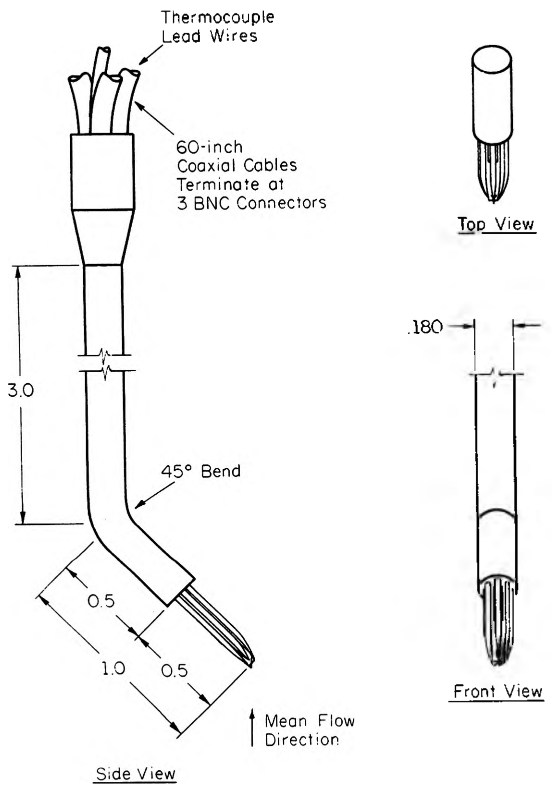
$$\frac{\overline{(e_2 - e_3)^2}}{(s_U \bar{U})^2} = [225 + 15.3 + 15.9] \times 10^{-4} \qquad (27)$$

which shows that errors of the order of 14 percent are obtained in the measurement of $\overline{u^2}$, indicating the desirability of applying corrections in the determination of this quantity.

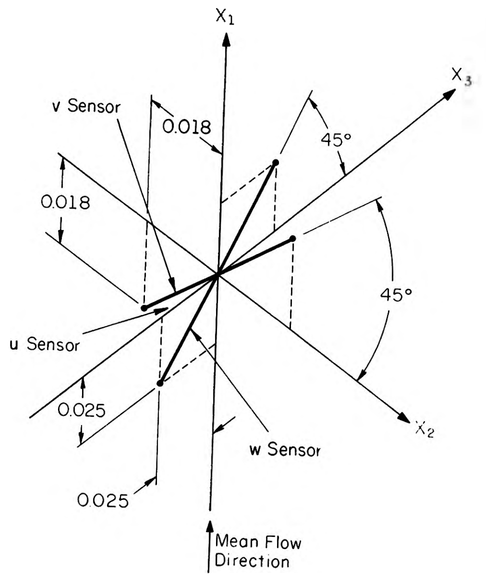
EXPERIMENTAL RESULTS

The technique described in this paper has been applied to measurements of the velocity and temperature fields in water flowing through an externally heated circular tube. The results presented in this section are all taken from Ref. 9 wherein may be found a complete description of the experimental program and facility as well as discussion of the data and data analysis techniques. All measurements were made in the region of fully-developed velocity and temperature characteristics. The fluid properties could be considered to be constant, i.e. the temperature behaved as a passive scalar, for nearly all measurements.

The measurements presented here were made using a probe which contained three 2-mil diameter, uncoated, cylindrical-film sensors. The overall probe geometry is shown in Fig. 1a, and the detailed sensor geometry is shown in Fig. 1b. The sensor and



(a)



(b)

Fig. 1 Three-sensor hot-film anemometer probe

support pin geometry were designed to minimize flow disturbances. Yaw tests showed no disturbances during rotation in the rz- and θz-planes for angles less than 34°. This angle corresponds to a local turbulence intensity of 56%; however, the highest intensities measured in the study were 15%. Therefore, the sensor supports offered no interference. The distance between adjacent sensors where they crossed was 0.003 ± 0.001 inch.

Measurements of the mean velocity profile outside of the buffer layer for several Reynolds Numbers, Re, and wall heat fluxes, q''_w (Btu/hr-ft^2), are presented in Fig. 2 with comparison to velocity distribution correlations given by Nikuradse[11] and

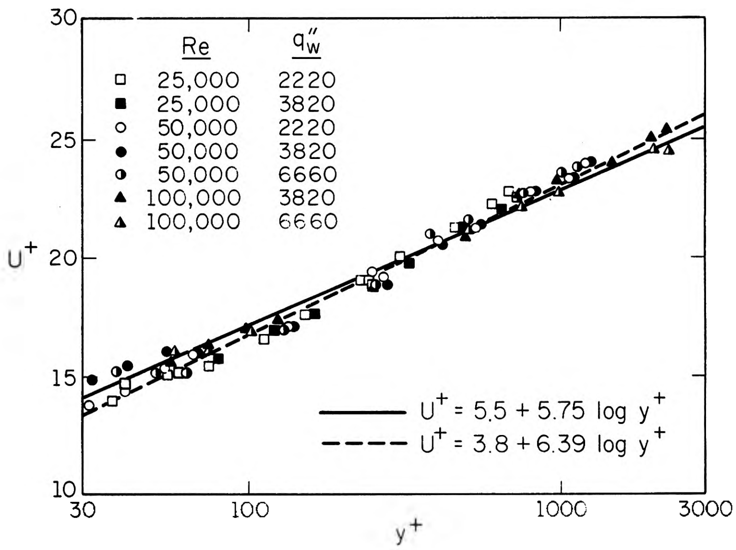


Fig. 2 Nonisothermal mean velocity profiles

Deissler[12] for isothermal flow. The difference between the wall temperature and the fluid mean temperature for the conditions shown ranged from 8.7°F to 29.3°F.

Complete discussion of this data as well as the rest of the data to be presented is contained in Ref. 9 and will not be included here. However, note that the data of Fig. 2 demonstrate the ability to measure the mean velocity distribution in a non-isothermal field with a hot-film anemometer if the equality of Eq. 7 is maintained. The scatter of the data in Fig. 2 is due to calibration drift of the sensor.

Fig. 3 shows the rms value of the axial component of velocity fluctuation (axial intensity) obtained using Eq. 15c. The curve designated 1210 - 20 indicates the distribution found using a single 2-mil, hot-film sensor in isothermal flow. Fig. 4 shows the rms value of the radial component of velocity fluctuation (radial intensity) obtained using Eq. 15d compared to that measured in isothermal flow.

The shift of the radial intensity data at Re=50,000 and Re=100,000 is due to lack of yaw sensitivity and angle determination for the radial sensor; its angle was assumed to be 45°, which affects only the magnitude of sensitivity to the radial component of velocity fluctuations as shown in Eq. 8. The shift of data at Re=25,000 in Fig. 4, although partially due to the lack of known yaw sensitivity and angle, is also due to a change of water properties across the large temperature gradient.

The data of both Figs. 3 and 4 indicate successful separation of the response to velocity and temperature fluctuations.

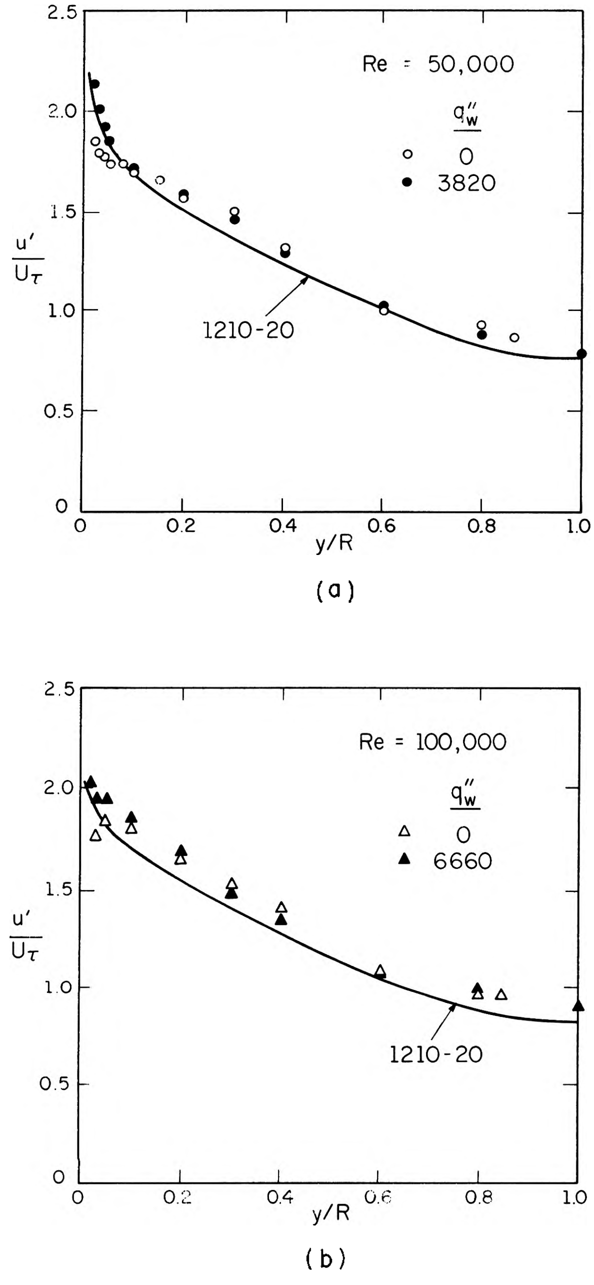


Fig. 3 Nonisothermal axial velocity intensity

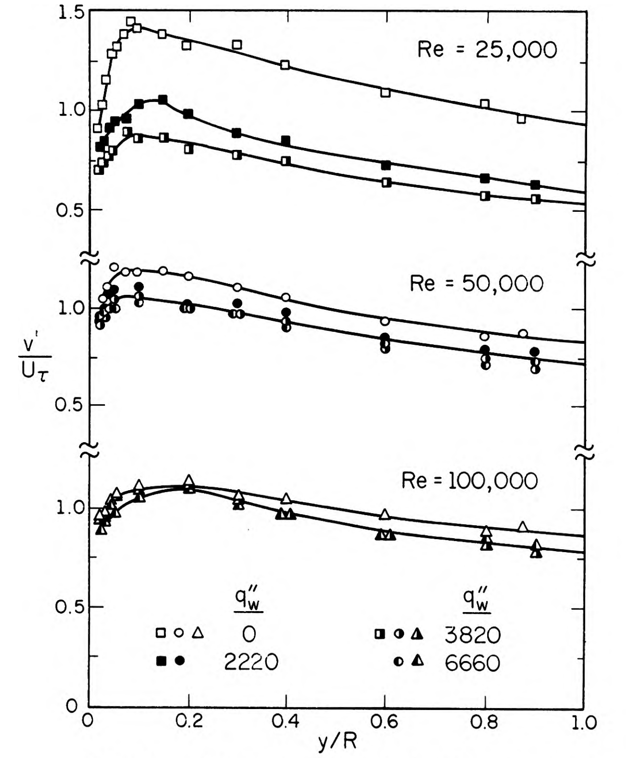


Fig. 4 Nonisothermal radial velocity intensity

Fig. 5 presents the temperature fluctuation intensity at several Re and q''_w measured using the sensor operated as a CCA with response given by Eq. 13. The data shift at Re=100,000 is considered to be due to a probe malfunction. Discussion of measured frequency response characteristics of the hot-film sensor operated in this manner is given in Ref. 9.

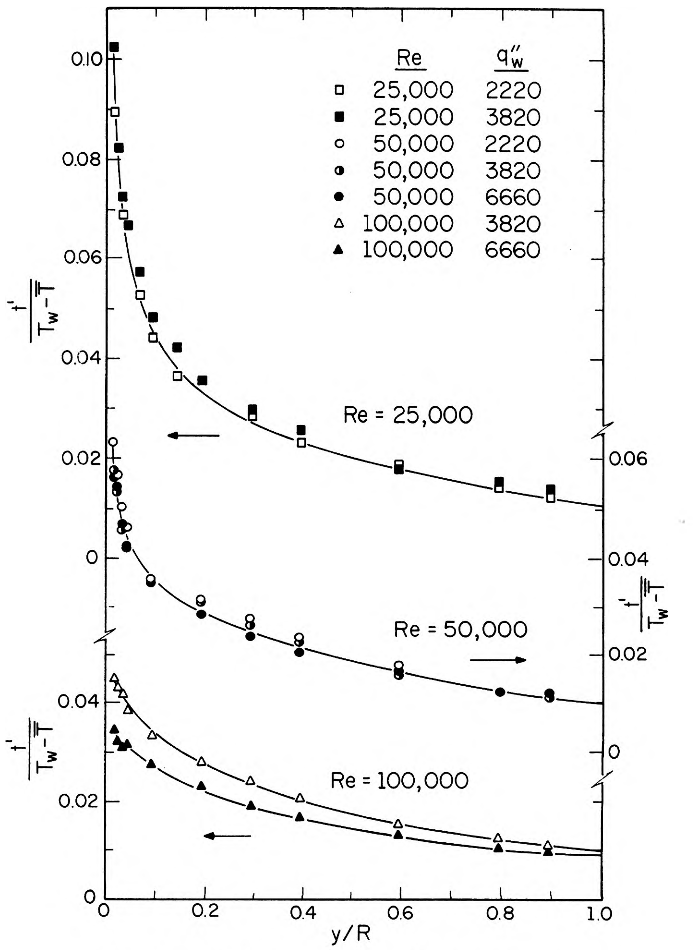


Fig. 5 Temperature fluctuatuion intensity

Figures 6 and 7 compare non-isothermal and isothermal measured and normalized power spectral densities of the axial and radial component velocity fluctuations, respectively, in a region of very large mean temperature and velocity gradients. The radial velocity component spectra show no apparent spectral shift due to the temperature field. However, the axial velocity component spectrum shows a slight increase in wave number for the non-isothermal conditions. This relatively small shift may be the result of less energy being transferred from the axial velocity component to the radial and azimuthal velocity components. This is consistent with a shifting of the viscous cut-off toward higher wavenumbers as the kinematic viscosity decreases with temperature increase.

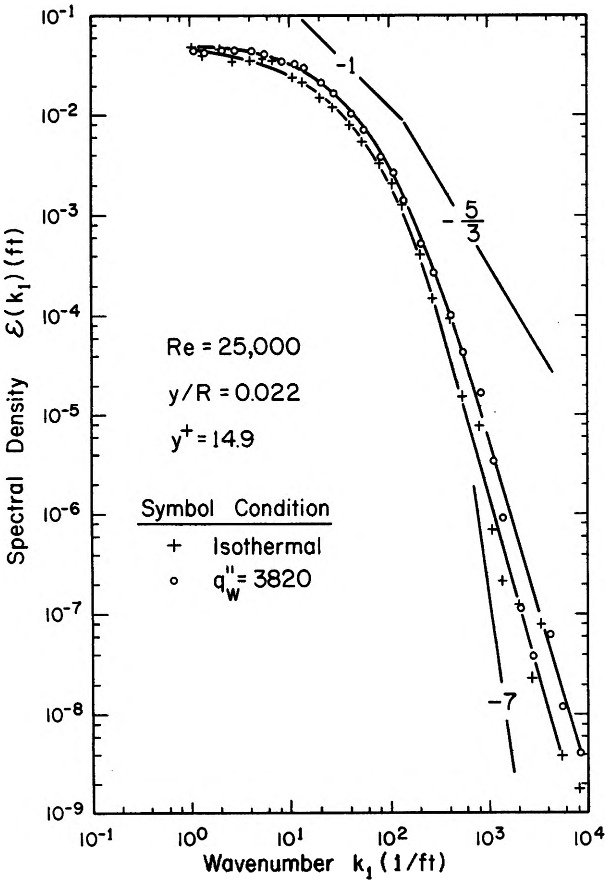


Fig. 6 Comparison of isothermal and non-isothermal measurements of normalized axial velocity power spectra

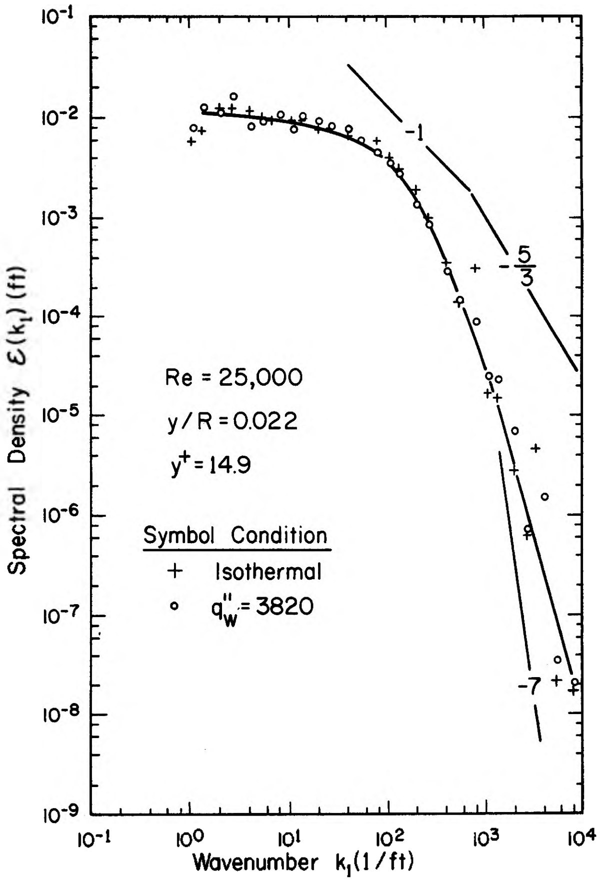


Fig. 7 Comparison of isothermal and non-isothermal measurements of normalized radial velocity power spectra

Figure 8 shows the corresponding power spectral density for the temperature fluctuations for the experimental condition of Figs. 7 and 8. It is noted that the spectral distribution follows more closely the axial velocity component spectrum, but at low wave numbers tends to be distributed between the axial and radial velocity component distributions. Therefore, it is suggested that the agreement shown between the isothermal and non-isothermal velocity spectral densities demonstrates the validity of the applied techniques for interpreting sensor response to velocity in non-isothermal flows.

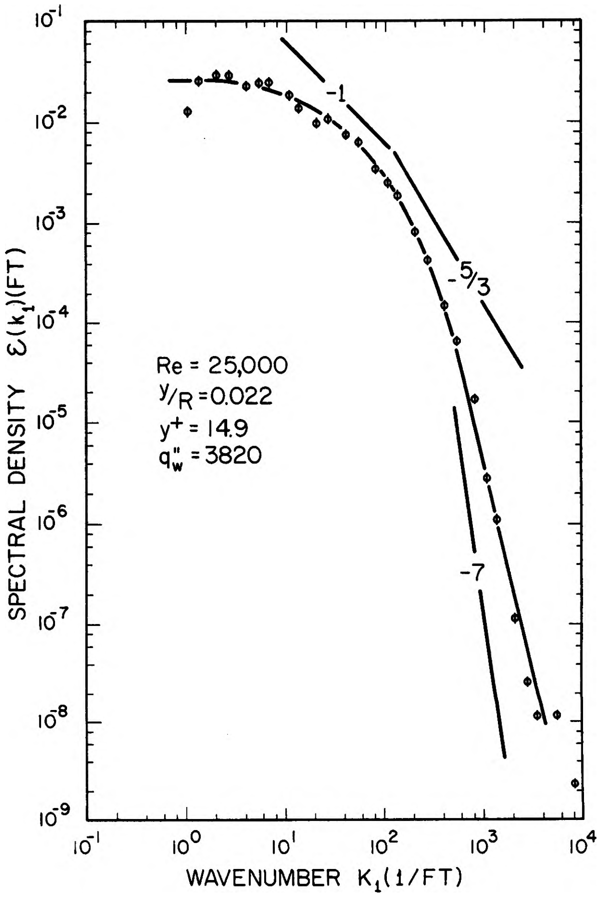


Fig. 8 Normalized power spectral density of temperature fluctuations

CONCLUSIONS

A technique for the operation and interpretation of the linearized response of hot-wire or hot-film anemometer sensors in a nonisothermal turbulence field has been presented, analyzed, and demonstrated to be successful. The basic principles of the technique are the satisfaction of the equalities given in Eqs. 7, 12, and 14 during operation of multiple sensors.

The errors involved in the technique have been analyzed specifically for hot-film sensors operated in water; however, a similar analysis could be easily applied to well-characterized hot-wire sensors operated in air.

The data presented should provide confidence in the technique, particularly if one considers the difficulties inherent to making even isothermal turbulence measurements in water.

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SYMBOLS

A,B	sensor characteristics in Eq. 1
C_T	variable defined in Eq. 20
d	sensor diameter
e	fluctuating component of linearizer output voltage
E	total linearizer output voltage
$\overline{E}$	mean component of linearizer output voltage
G	output amplifier or linearizer gain
k	yaw sensitivity exponent in Eq. 5
k_1	wavenumber
m	cooling law exponent in Eq. 1
Pr	Prandtl Number
q''_w	wall heat flux
R	tube radius
R_S	sensor electrical resistance
Re	Reynolds Number
s	signal sensitivity
t	fluid temperature fluctuation
t'	rms value of t
T	instantaneous fluid temperature
$\overline{T}$	mean fluid temperature
T_S	sensor temperature
T_w	wall temperature
ΔT_S	sensor overtemperature defined in Eq. 4
u	fluctuating component of velocity in mean flow direction
u'	rms value of u
U	instantaneous component of velocity in mean flow direction
$\overline{U}$	mean velocity
U^+	nondimensional velocity, $\overline{U}/U_\tau$
U_τ	shear velocity
v	lateral component of velocity fluctuation normal to wall
v'	rms value of v
V	anemometer bridge voltage
w	lateral component of velocity fluctuation parallel to wall
y	distance from tube wall
y^+	nondimensional distance from tube wall, yU_τ/ν
α	sensor coefficient of electrical resistivity
$\varepsilon(k_1)$	one-dimensional power spectral density
θ	sensor angle
ν	fluid kinematic viscosity

<u>Subscripts</u>

o	reference condition
S	sensor
T	temperature
$\overline{T}$	fluid mean temperature
U	velocity in mean flow direction
v	lateral component of velocity
1	associated with sensor 1
2	associated with sensor 2
3	associated with sensor 3
4	associated with sensor 4

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DISCUSSION

J. WAY (Illinois Institute of Technology): How much time is involved in setting your circuit at each point if you are interested in getting the exact setting?

JONES: Not very much. It is a matter of adjusting settings on the anemometer dials since one knows what resistance he wants from knowing what the ΔT is. The mean temperature is known from the 3rd circuit; as long as one knows α, the adjustment is direct. The method is fairly rapid but it is not dynamic. It takes about the same amount of time as moving the sensor accurately to the next radial location. It is not a real hindrance, and it is nothing like using the technique of solving simultaneous equations to separate the velocity and temperature signals.

T. HOULIHAN (Naval Post Graduate School): What overheat ratios are utilized on each temperature and velocity probe?

JONES: Nominally 1.1 for the velocity sensors and no overheat for the temperature sensing channel. The velocity in the center of the pipe is a few feet per second at a Reynolds number of 25,000. This is one of the other difficulties in going in to water; you don't have the freedom of overheat setting that you do in air. This is one of the reasons we went directly away from the multiple overheat technique.

GOLDSCHMIDT: I would like to discuss a curve that was shown to us at the Euromec (Hot-Wire) Conference this last spring at Prague. This was by Hans Bruun of ISVR, Southampton. He conveniently groups data of many probes by considering $BU^N = E^2 - E_o^2$ and forcing B to be a constant at a constant temperature. His plot thus compares N as a function of U, ranging from about 1 meter per second to about 100 meters per second. The N coefficient seemed to decrease gradually from a value of about 0.7 to one of about 0.3. Would you comment on whether it tells us anything as to the constancy of the N or is this a fudge factor which results from making B constant. (H. Bruun's work now available in J. Phys. E: J. Scient. Instr., 4, 815-820 (1971) Ed.).

JONES: I must admit I don't really know the answer to it because what we were examining was A over B. The N value was determined from calibration. But, of course, we weren't dealing with anything like two orders of magnitude spread in velocity. We are talking about at most one order of magnitude as we approach the wall. We can't get down to a tenth of the velocity as we approach the wall with the multiple sensor probe. Thus, I can't really say. We have considered in our analysis, of course, that the N we evaluate for the flow field application is a constant. We were more inclined to consider the N to be the constant property and examine the behavior of B. However, either case is open to the experimenter, as indeed is allowing all parameters to vary.

H. H. SOGIN (Tulane University): We have attempted to develop a so-called generalized King's Law for the cone hot-film sensor and we have carried out tests in the range of velocity from about a half a centimeter per second up to about something like 200 centimeters per second. This was done by Goodman in his Ph.D. thesis. We have looked at the effects of free convection by orienting the sensor in different positions with respect to the gravitational field. If you limit the range of the Reynolds number of the sensor, then of course you can get one value. If you take the full range of the Reynolds number, you can get another. This is not new, of course. The value of N over the full range of Reynolds number that we had was something like 0.3. What is not established, is what is a good correlation. If you attempt to find a correlation from the viewpoint of the method of least squares, then one finds that the dispersion is enormously sensitive to small changes in the exponent N. I really don't know what would be a good or an acceptable dispersion. This is one of the things that I would like to learn here; what is an acceptable King's Law correlation?

S. KLINE (Stanford University): Morrow (Ph.D., Stanford) found that, in fact, you can't fit some kinds of metals, particularly nickel in his case, with a single exponent. You need two terms at least to get a reasonable fit to the universal heat transfer curve. He found further that if you have films, and I think the Disa and Thermo-Systems people agree with this, then you don't have the base metal properties so you still have to calibrate every probe anyway. You can't look them up. So that I agree with you that there isn't any universal N. It is a function of Reynolds number and it is also a function of the individual probe. And so you are stuck with calibrating the probes if you want to get reasonably accurate results.