06 Oct 1971

On Velocity Measurements in Non-Isothermal Turbulent Flows

R. Chevray

N. K. Tutu

ON VELOCITY MEASUREMENTS IN NON-ISOTHERMAL TURBULENT FLOWS

Rene Chevray and Narinder K. Tutu

Department of Mechanics
SUNY at Stony Brook, New York 11790

ABSTRACT

Turbulent velocity measurements in non-isothermal flows are conventionally performed by constant current hot-wire anemometry. The procedure involved however is cumbersome, the output signals are nonlinear and more critically a continuous signal of velocity fluctuations is not provided. The method described here utilizes two orthogonal wires situated a fraction of a millimeter apart. The upstream wire is operated in a low overheat, constant-current compensated mode thus providing a signal proportional to temperature while the down-stream wire, practically unaffected by the thermal wake of the first wire is operated in the constant temperature mode. Compensation for the effects of local temperature on the downstream wire is accomplished by the temperature signal obtained from the first wire. Variations of Kramers' law coefficients (both being functions of density and thermal conductivities) due to temperature variations are compensated through an analog circuit at the input of the logarithmic amplifiers of the linearizer.

By this method, linear and separate signals of the velocity and local temperature are thus obtained simultaneously and continuously. Advantages accruing from this method are readily adapted to cross-wire configuration thus permitting direct measurements of momentum and heat transfer in turbulent heated flows.

INTRODUCTION

Constant current hot-wire anemometry has been used routinely in the past to measure mean as well as fluctuating velocities in non-isothermal flows. It is well known that the output of such a system is not yet amenable to linearization and that no continuous signal of the velocity field alone can be obtained in this manner. Yet, a signal representative of the velocity alone is often required to perform spectral analysis of the fluctuating velocity for example or simply to compare the intermittency structures of the temperature and velocity fields in free turbulent shear flows.

Kramers' empirical relation (or a simplified equivalent for air, King's law) relating the heat transfer from cylinders to flow characteristics is usually used as a basis for hot-wire anemometry. Due to temperature variations, unfortunately, this equation becomes a functional of fluid properties. Variations of the relevant coefficients with temperature are given explicitly in the text for the case of water and the equations describing the compensation are presented for fluids in general. The underlying principle of compensation is then given and the associated probe arrangement described. A complete compensation circuit has been built and its testing for proper performance has been conducted in a controlled environment. Finally, measurements of the velocity in a heated jet of air have been made and are compared to results obtained by conventional means.

ANALYSIS

The Heat Transfer Equation

Thermal equilibrium conditions of a hot wire are described by Kramer's empirical equation[5] which expresses the heat transfer rate from circular cylinders held normal to a uniform stream as a function of flow parameters. This equation is usually written as:

$$I^2R = \{alk\, P_r^{0.2} + blk\, P_r^{0.33}Re^{0.5}\}(T-T_a) \qquad (1)$$

where a and b are constants and in which the fluid properties incorporated in the coefficients are to be evaluated at the "film temperature" $T_f = (T+T_a)/2$.

If the amplifier output of the constant temperature anemometer is E, it can be expressed as: E = RIc, where c is a constant depending both upon the bridge circuit and the overheat ratio. Also, if T_r is the reference temperature (the local fluid temperature at which the hot wire is calibrated for velocity measurements) so that:

$$(T-T_a) = (T-T_r)\{1-\theta/(T-T_r)\} \qquad (2)$$

in which $\theta = (T_a-T_r)$, then Equation 1 can be readily written in the familiar (albeit different from Kings' Law) form:

$$E^2 = (A + B\sqrt{U})\{1-\theta/(T-T_r)\} \qquad (3)$$

where

$$A = Rc^2(T-T_r)al\; c_p^{0.2}\;\mu^{0.2}\;k^{0.8}$$
$$B = Rc^2(T-T_r)bl\; c_p^{0.33}\;d^{0.5}\;\rho^{0.5}\;k^{0.67}\;\mu^{-0.17} \qquad (4)$$

It is therefore clear that A and B being functions of flow properties (and hence of the local fluid temperature) will, in addition to the factor $\{1-\theta/(T-T_r)\}$, be responsible for the change in the hot-wire response to velocity in non-isothermal flows.

Validity of Kramers' equation is still a question of some debate, indeed even the Reynold's number exponent is not universally accepted. Collis and Williams[1], for example, have suggested a temperature loading factor $(T_f/T_a)^{.17}$ which should multiply the right-hand side of Eq. 3. Although this seems justified on theoretical grounds as well, it should be noted that for the modest temperature range with which we are concerned here ($0\leq\theta\leq50^\circ C$), the corresponding change in the temperature loading factor is usually less than* 2%; and the difference in the exponents of Reynolds number in the two relations effectively further reduces this change. Although more thorough investigations need to be done, particularly on the Prandtl number influence in the heat transfer equation, Kramers' equation has been shown to give an acceptable enough relationship between the Nusselt number and the Prandtl number to warrant its use in this study.

*It is only the change and not the absolute magnitude that matters. The exact value will depend upon the overheat ratio used.

The Principle of Temperature Compensation

In order to compensate the hot-wire output signal for fluid temperature variations, independent and simulataneous knowledge of the instantaneous fluid temperature is required. This is accomplished easily by using a two-wire probe as shown in Fig. 1.

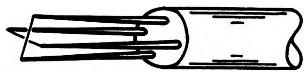

FIG. I PROBE CONFIGURATION

The two wires are orthogonal and are separated by a fraction of a millimeter. While the upstream wire is operated in a constant current compensated mode at a very low overheat, thus acting like a resistance thermometer and providing a signal proportional to the temperature difference, θ, the downstream wire, practically unaffected by the thermal wake of the first, is operated in the constant temperature mode. With such an arrangement, it is then possible to get a signal e, such that:

$$e = K\theta \qquad (5)$$

where, e is the output of the constant current anemometer and K is a calibration constant.

It is easy to see from Equation 3 that to eliminate the effect of factor $\{1-\theta/(T-T_r)\}$ on the hot-wire response, we need only divide the constant temperature anemometer output, E, by $\{1-\theta/(T-T_r)\}^{1/2}$. This will henceforth be referred to as the "main temperature compensation" because it, in general, constitutes a major part of the total temperature compensation. This operation is schematically shown in Fig. 2. Thus, after carrying out the main temperature compensation, we have a signal, V, given by:

$$V^2 = A + B\sqrt{U} \qquad (6)$$

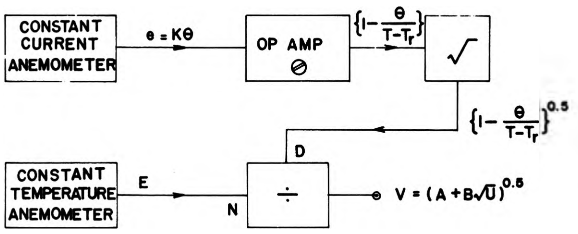

FIG. 2 MAIN COMPENSATION SCHEME

The square-rooter in the circuit shown in Fig. 2 can be omitted in situations for which small temperature changes are encountered and/or large overheat ratios are used (so that $\theta/(T-T_r) \leq 0.15$). The gain of the preceding operational amplifier should then be adjusted according to the following approximation:

$$\{1 - \theta/(T-T_r)\}^{0.5} \approx \{1 - 0.5\theta/(T-T_r)\}$$

Several schemes are available to accomplish this main compensation. Modification of the bridge is one of them. Although this was considered first in this study, it was abandoned since compensation for the variations of A and B must be performed by means of an external circuit and the main compensation can easily be incorporated in it.

Proper operation of any linearizer requires two separate controls for the coefficients A and B which are adjusted for particular hot-wire and circuit characteristics. The task of compensating for the variations in A and B is therefore

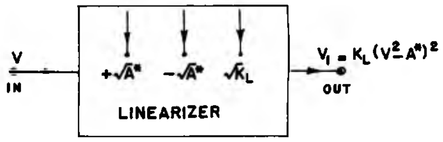

FIG. 3 COEFFICIENT CONTROLS

facilitated by access to these inputs. Such controls are illustrated schematically in Fig. 3 for a Disa type 55D10 linearizer which was available for this study. Feeding the output, V, of the divider in Fig. 2 to the linearizer, yields for its output a signal V_1 given by:

$$V_1 = K_L(V^2 - A^*)^2 \qquad (7)$$

where $\pm(A^*)^{0.5}$ and $K_L^{0.5}$ are the aforementioned internal fixed inputs to the logarithmic amplifiers of the linearizer. Substituting Eq. 6 in Eq. 7, we have:

$$V_1 = K_L(A + B\sqrt{U} - A^*)^2 \qquad (8)$$

It is clear from the above equation that if A* is held fixed, then the output, V_1, can be linearized only for a particular value of the local fluid temperature. On the other hand, if we choose to vary both A* and K_L in such a way that:

$$\sqrt{A^*} = \sqrt{A},\ \sqrt{K_L} = \sqrt{p}\ B^{-1} \qquad (9)$$

where p is a constant, then the output, V_1, not only becomes linear with velocity, but is also independent of the local fluid temperature; in this case it reduces to: $V_1 = pU$. Thus, to get a linearized velocity signal free of local temperature changes it is only necessary to generate functions $\pm A^{0.5}$ and B^{-1} and feed them to the inputs of the appropriate logarithmic amplifiers of the linearizer. Fig. 4 shows schematically how this compensation is carried out. The dashed lines show the original (uncompensated) connections of the linearizer, and the subscript r refers to quantities at the reference temperature T_r.

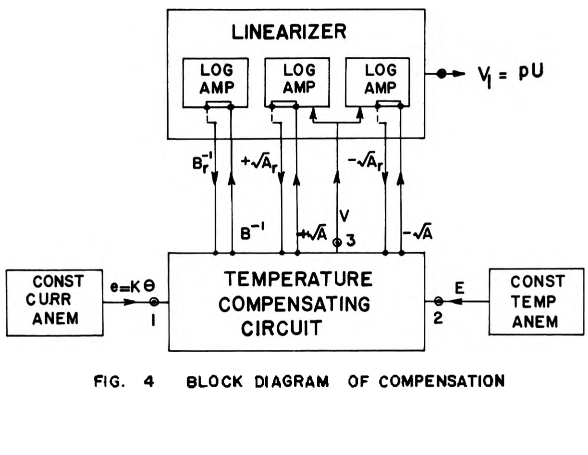

FIG. 4 BLOCK DIAGRAM OF COMPENSATION

Coefficient Variations for Water

For fixed wire characteristics, A and B being functions of fluid properties, will show variations with both the local fluid temperature and the nature of the fluid. Here, as an example, we shall give expressions for $A^{0.5}$ and B^{-1} as functions of local temperature change, θ, for water. Let the film temperature at reference be $(T_f)_r = (T+T_r)/2$. In the temperature range of interest ($0 \leq \theta \leq 50^\circ C$), c_p can reasonably be assumed constant for water; so that we have[†] from Eq. 4:

$$A/A_r = (\mu/\mu_r)^{0.2}\ (k/k_r)^{0.8}$$

$$B/B_r = (\rho/\rho_r)^{0.5}\ (k/k_r)^{0.67}\ (\mu/\mu_r)^{-0.17} \tag{10}$$

From the International Critical Tables[4], the thermal conductivity of water can be expressed as:

$$k = k_r[1+\varepsilon(T_f-(T_f)_r)] \tag{11}$$

where, $\varepsilon = 2.81 \times 10^{-3}/\{1 + 2.81 \times 10^{-3}[(T_f)_r-20]\}$ per °C. The conductivity ratios appearing in Eq. 10 were calculated using the above equation, whereas the density and viscosity values were taken from "Handbook of Chemistry and Physics"[3]. As can be seen from Figs. 5 and 6, which show the variation of $(A/A_r)^{0.5}$ and $(B/B_r)^{-1}$ with θ for water, for the temperature range under consideration $(A/A_r)^{0.5}$ and $(B/B_r)^{-1}$ can be taken as linear functions of θ without introducing any significant errors (the values in the figures show the maximum errors due to linearization).

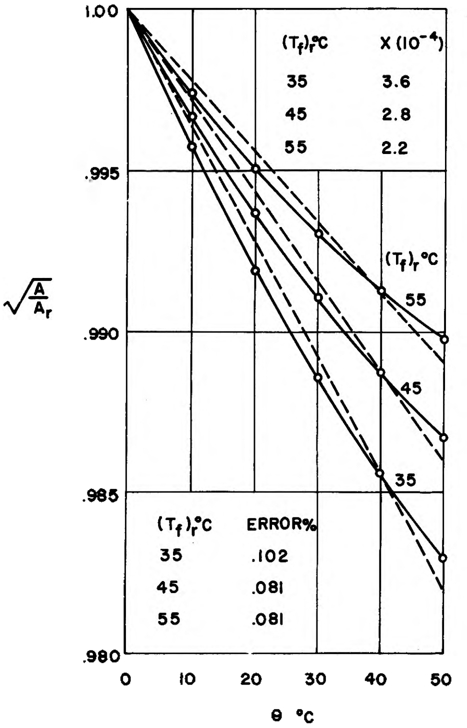

FIG. 5 VARIATIONS OF $(A/A_r)^{1/2}$ FOR WATER

Consequently, using Eq. 5, the representative curves can be expressed as:

$$\sqrt{A} = \sqrt{A_r}\ (1-X\theta) = \sqrt{A_r}\ (1-Xe/K)$$

$$B^{-1} = B_r^{-1}\ (1-Y\theta) = B_r^{-1}\ (1-Ye/K) \tag{12}$$

where X and Y are functions of the reference film temperature, $(T_f)_r$.[#] Some of these values are given in Figs. 5 and 6 and, since $(A/A_r)^{0.5}$ and $(B/B_r)^{-1}$ are only weakly dependent upon

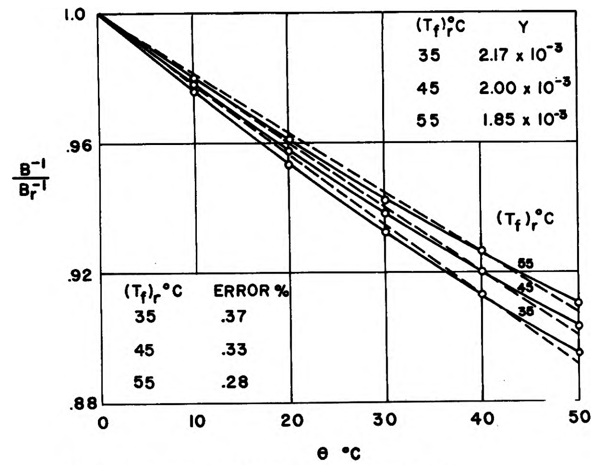

FIG. 6 VARIATIONS OF $(B/B_r)^{-1}$ FOR WATER

$(T_f)_r$, for other values in this range X and Y can readily be found by interpolation. At this point, it is interesting to note that the change in the coefficient $A^{0.5}$ for water is about an order of magnitude less than the corresponding change in B^{-1}.

Although Eq. 12 is given only for water, a similar relation can be obtained for other fluids. In other instances - as for the case of air - an analytical expression for these variations can be derived. In any case, it is important to note that the linearity in Eq. 12 is not essential for carrying out the temperature compensation. Strongly nonlinear expressions will simply result in complexity for the temperature compensating circuit.

Error Analysis

Linearization of $A^{0.5}$ and B^{-1} will inevitably result in errors and the corresponding expressions have been derived explicitly below.

a) Total Velocity: From Eqs. 9 and 12 we have:

$$A^* = A_r(1-X\theta)^2,\ K_L = pB_r^{-2}(1-Y\theta)^2 \tag{13}$$

Substituting Eq. 13 in Eq. 8, we get:

$$V_1 = pB_r^{-2}(1-Y\theta)^2\{A + B\sqrt{U} - A_r(1-X\theta)^2\}^2 \tag{14}$$

Now, since the calibration is done at the reference temperature, T_r, we have from Eq. 14, when θ = 0:

$$V_1 = pU \tag{15a}$$

and because the same calibration curve is to be used for velocity measurements at different fluid temperatures, the measured velocity U_m is given by:

$$U_m = V_1/p \tag{15b}$$

Combining Eqs. 14 and 15b, a relationship between the measured velocity, U_m, and the actual velocity, U, is obtained. The corresponding expression for the error, ε, is:

$$\varepsilon = (U_m-U)/U = \alpha^2/U + (\beta^2-1) + 2\alpha\beta/\sqrt{U} \tag{16}$$

where

$$\alpha = \frac{A_r}{B_r}\{\frac{A}{A_r} - (1-X\theta)^2\}(1-Y\theta)$$

$$\beta = \frac{B}{B_r}(1-Y\theta) \tag{17}$$

[#]The authors would like to thank one of the reviewers for pointing out that a departure from this variation of B^{-1} can be expected for different values of L/D as well as for specific sensor types. No systematic study of the influence of these parameters has been carried out.

[†]A_r and B_r refer to A and B when $T_a = T_r$. The properties with the subscript r are evaluated at $(T_f)_r$ whereas the non-subscripted properties are evaluated at T_f.

b) Velocity Fluctuations: Denoting the sensitivities to velocity and temperature fluctuations by s and t respectively, they become after introduction of V_1 from Eq. 14:

$$s = p\,\frac{B}{B_r}\,\frac{A_r}{B_r}\,\frac{(1-Y\theta)^2}{\sqrt{U}}\left\{\frac{A}{A_r} - (1-X\theta^2)\right\} + p\,\frac{B^2}{B_r^{\,2}}\,(1-Y\theta)^2 \qquad (18)$$

$$t = 2p(1-Y\theta)(-Y)\,Q^2 + 2p(1-Y\theta)^2\,Q\left\{\frac{1}{B_r}\frac{dA}{d\theta} + \frac{2A_r}{B_r}(1-X\theta)X + \frac{\sqrt{U}}{B_r}\frac{dB}{d\theta}\right\} \qquad (19)$$

where, $$Q = \frac{A_r}{B_r}\,\frac{A}{A_r} - \frac{A_r}{B_t}(1-X\theta)^2 + \frac{B}{B_r}\sqrt{U}$$

Ideally, of course, the sensitivities s and t should be independent of flow conditions, and as given by Eq. 15a should have values p and zero, respectively.

In the temperature range $0^\circ \le \theta \le 50^\circ C$ and for the velocity range $0.05 \le U \le 5$ m/sec., the worst case, absolute errors are shown in Table 1 for water.

TABLE 1 Errors introduced by linearization

$0^\circ C$	U, m/s	ε x 100	s/s_r	t/s_r
20	.05	.3	1.004	0.000
20	.5	.5	1.005	0.000
30	.05	.2	1.003	0.000
30	.5	.4	1.004	0.000
50	.05	-.4	.995	0.000
50	.5	-.6	.994	0.000

Clearly, the errors due to linearization of $A^{0.5}$ and B^{-1} for water are negligible (i.e., less than 1%). As can be seen from Fig. 5, the variation of $A^{0.5}$ with temperature is small. Even though this variation is typically one order of magnitude smaller than the corresponding one for B, close examination of the results in Table 2 justifies retention of compensation for A.

TABLE 2 Errors introduced by linearization of B^{-1} and neglecting the variations of A

$0^\circ C$	U m/s	ε x 100	s/s_r	t/s_r
30	0.05	-2.6	.989	-0.000
50	0.05	-4.8	.972	-0.000
50	3.00	-1.2	.991	-0.003

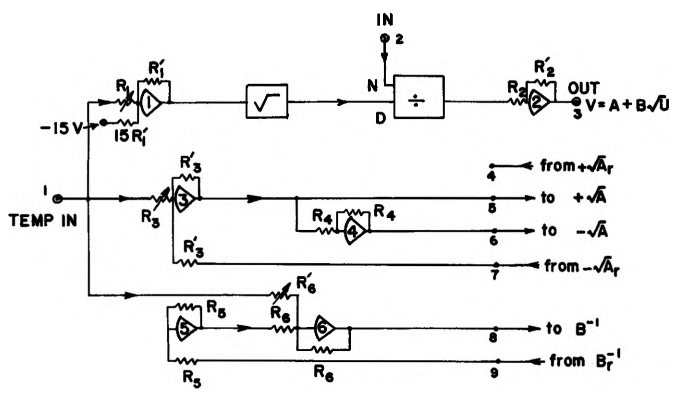

FIG. 7 TEMPERATURE COMPENSATING CIRCUIT

<u>The Temperature Compensating Circuit</u>

The complete circuit diagram of the temperature compensating circuit is given in Fig. 7 and some of its details are given in the Appendix. Essentially, it was designed assuming K > 0 (i.e., for a positive value of θ the temperature signal is positive). Should K, however, be negative an inverter must be added in the circuit to render it effectively positive. As such, the circuit diagram shown is applicable only for fluids, such as water, having positive values of X and Y. However, for fluids with negative values of X and/or Y, the number of basic components remaining unchanged, there is no difficulty in making the necessary modifications to the circuit. To minimize D.C. drift chopper stabilized amplifiers are the obvious choice, but presently available high quality FET input operational amplifiers proved satisfactory and were used in the circuit. As a final remark, points 7 and 9 of the circuit provide access to the values $A_r^{\,0.5}$ and $B_r^{\,-1}$ needed for proper adjustment of R_3 and R_6'.

RESULTS AND DISCUSSION

In order to retain maximum sensitivities for parameters involved, the primary concern in testing the proposed circuit was to eliminate extraneous influences. For lack of proper facilities water was ruled out to avoid possible contamination of the wire, while oils and liquid metals were not considered for practical reasons. A large heated axisymmetric jet of air with low exit turbulence level was available for this study and the compensating circuit was tested with this facility.

There exists for air a choice of heat transfer equations - Kramers' relation and King's law for instance. A comparative experimental evaluation of the two relations has been done and will be presented as part of a future report. Essentially, it was confirmed that within the limits of experimental errors in evaluation of hot-wire temperature, either relation yields results accurate enough for air. Consequently, the resulting equations describing the compensation have the same functional forms and the preference of one over the other, therefore, does not detract anything from the generality we wish to preserve here. For simplicity, the local temperature was used in conjunction with Kings' Law to evaluate the coefficients A and B for air, and in this connection, it is worthwhile to note that reasonable agreement already exists between these theoretical predictions and Corrsin's[2] experimental results.

It is a matter of some difficulty to determine the hot-wire temperature and hence the value of the temperature difference $(T-T_r)$, which is required to properly set the potentiometer, R_1 (Fig. 7). Since any error in this setting would result in either over or under compensation for the velocity signal, proper adjustment becomes critical. It is our experience that for a given type of wire, it is best to find this temperature difference by placing the hot wire in a heated flow and adjusting the potentiometer, R_1, until the linearized output indicates the correct velocity. Of course, this need be done only once for a given type of wire.

To test the temperature compensating circuit, mean velocity measurements were made at the exit of a heated jet of air both by using a Pitot tube and the temperature compensated hot wire. Ideally, it would be preferable to test the circuit by performing a series of calibrations at different fluid temperatures, but lacking continuous control of temperature this could not be done. Results obtained by making velocity measurements at different temperatures are tabulated in Table 3. Three runs were made on different days with different wires of the same material and same aspect ratio. To bring out the effectiveness of the compensating circuit, velocities were measured with compensation

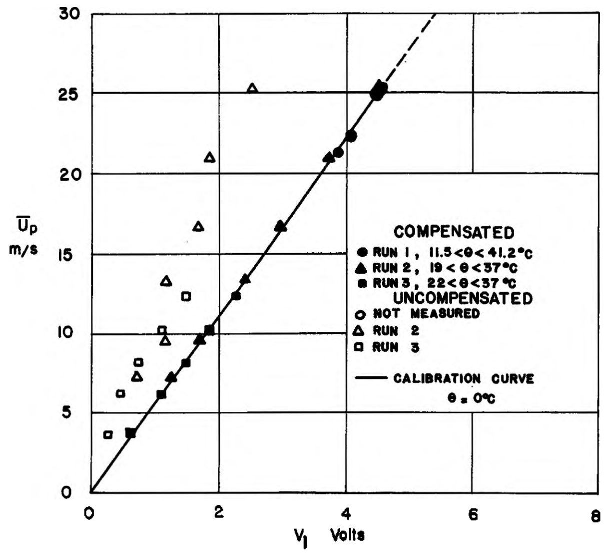

FIG. 8 RESULTS OF COMPENSATION

in and out of the circuit, thereby showing the errors involved when compensation is omitted. By suitable transformations, the calibration curves (at $\theta = 0$) for the three runs were reduced to a single one. This is shown by the solid line in Fig. 8; the points in this figure represent the modified linearizer output and the velocity measured by Pitot tube. This figure clearly shows that the use of the proposed temperature compensating circuit results in a linear, temperature compensated signal for velocity.

Although the circuit was tested for mean velocity only, since measurements were performed at the exit of the jet where the turbulence intensity is negligible, it can be concluded from Fig. 8 that the linearizer output is compensated for velocity fluctuations as well since this is limited only by the electronic properties of the devices used. The frequency response of the amplifiers used in this study (Analog Devices 146J) is 150 kHz whereas the frequency response (-3 dB) of the square

TABLE 3 Results of Temperature Compensation

θ°C	$\overline{U}_p$ m/s	$\overline{U}_c/\overline{U}_p$	$\overline{U}_u/\overline{U}_p$
	Run 1, June 21, 1971		
11.6	22.4	1.008	
25.1	25.1	1.001	
35.2	25.3	0.997	
41.2	21.4	1.003	
	Run 2, June 28		
32.3	25.3	0.983	0.551
36.4	21.0	0.986	0.492
30.3	16.7	0.983	0.553
34.6	13.4	0.990	0.489
19.6	9.4	0.981	0.669
24.9	7.2	0.960	0.551
	Run 3, July 16		
22.7	12.5	1.008	0.671
26.6	10.3	1.006	0.605
32.3	8.3	0.999	0.507
36.8	6.3	0.986	0.414
30.8	3.8	0.929	0.421

rooters and dividers (Analog Devices 426A) is better than 20 kHz and the 1% vector error phase shift in multiplier mode is 10 kHz. The conventional method (constant current anemometry using 3 different heating currents as described comprehensively by Corrsin[2]) of making such measurements being based on the same fundamental equations as used for the compensating circuit cannot be considered an independent test for the proposed method. In fact, such conventional measurements of turbulent intensities are bound to be erroneous due to non-linearity of the hot-wire response and additional assumptions needed to find the sensitivities to fluctuations. Apart from these errors there is no reason to expect the measurements by these two methods to be different. Nevertheless, mean velocity measurements were performed in the above mentioned heat jet (on different days), both by present and conventional methods. The results are plotted in Fig. 9, and the agreement is as good as could be expected for air. Although the analysis was carried out in details for water, all theoretical expressions remain to be verified experimentally. For each particular combination of liquid and sensor type, an experimental test should obviously be performed.

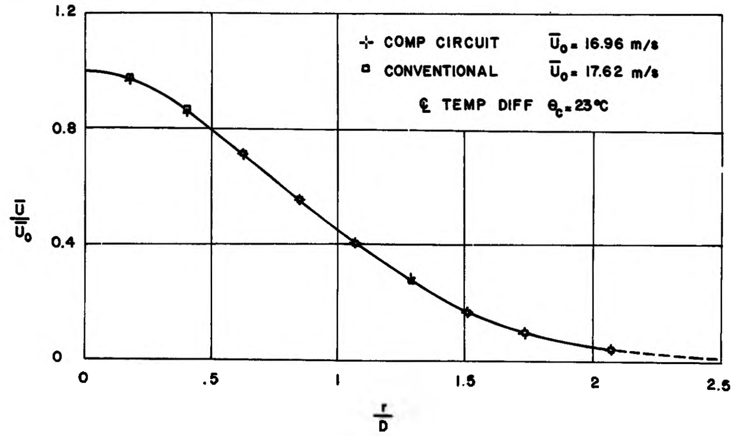

FIG. 9 MEAN VELOCITY IN HEATED JET X/D = 10

CONCLUSIONS

From Kramers' heat transfer equation, the relations describing the compensation have been presented. Particular values of the coefficients in this equation are a function of the fluid considered as well as of its temperature, but the functional form of the equations remains unchanged for any fluid.

The physical probe arrangement together with the method of compensation is described and the complete analog circuit to accomplish this compensation has been built for air. The unit has been tested in a controlled environment and the corresponding results are well within the range expected from the analysis of errors introduced by linearization of the coefficients. While such linearization is not necessary, it has been made in this study (for a moderate temperature range of $0 \leq \theta \leq 50^\circ C$) to keep the circuit to maximum simplicity consistent with adequate compensation. Should, in a particular case, linearization be found inadequate, it can always be circumvented at the expense of complexity in the circuit. For simplicity of presentation the analysis has been presented for heated flows ($\theta \geq 0$), but it is obvious that the same compensating circuit will work adequately in the temperature range $f \leq \theta \leq g$ ($g \geq 0$, $f \leq 0$) provided the constants X and Y are chosen accordingly.

The system has adequate frequency response. The requirements here are not too demanding (typically from d.c. up to about 10 kHz) so that only d.c. drift has been paid particular attention.

Depending upon the flow velocity, a slight phase shift is introduced between the two signals by the physical distance separating the two wires. This corresponds typically to a delay of the order of 0.1 millisecond in a flow velocity of 10 meters/second. In types of measurements requiring precision control of the phase shift, a delay line should be introduced in the circuit to compensate for the mean delay associated with the probe configuration. This was not required and therefore not done in the present stage of this study. The method as described, is directly applicable in making direct measurements of Reynolds stresses and heat transfer in heated flows as well as in investigating the structure of the fluctuating velocity field alone.

APPENDIX

The potentiometers R_1, R_3 and R_6' (see Fig. 7) are to be adjusted for the following gains:

$$\frac{R_1'}{R_1} = \frac{1}{K(T-T_r)},\quad \frac{R_3'}{R_3} = \frac{\sqrt{A_r}\,X}{K},\quad \frac{R_6}{R_6'} = \frac{B_r^{-1}Y}{K}$$

The signs of the outputs of the "square-rooter" and "divider" are based on the characteristics of "Analog-Devices Model 426A Multiplier/Divider" which was used for the present study. The resistance ratio R_2'/R_2 will obviously depend upon the normalization factors of the square-rooter and divider, which in our case reduces to $1/\sqrt{10}$. Units numbered 1 through 6 are FET input operational amplifiers.

ACKNOWLEDGMENTS

The support of the Research Foundation of the State University of New York through Grant-in-aid No. 31-7161A and a Faculty Research Fellowship was essential in starting this study which is presently supported by the National Science Foundation under Grant GK 30479. The financial assistance of both sponsors is gratefully acknowledged.

REFERENCES

1. Collis, D. C. and Williams, M. J., "Two Dimensional Convection from Heated Wires at Low Reynolds Numbers", J. Fluid Mech., 6, 357-384 (1959).
2. Corrsin, Stanley, "Extended Applications of the Hot-Wire Anemometer" NACA TN No. 1864, 1949.
3. Handbook of Chemistry and Physics, 44th edition, The Chemical Rubber Publishing Co. (1962).
4. International Critical Tables, Vol. 5, p. 227, McGraw Hill Book Co., (1929).
5. Kramers, H., "Heat Transfer from Spheres to Flowing Media" Physica, 12, 61-80 (1946).

SYMBOLS

c_p	specific heat at constant pressure of fluid at T_f
D	initial diameter of round jet
d	diameter of the hot wire
E	output of the constant temperature anemometer
e	output of the constant current anemometer, $e = K\theta$
f	arbitrary negative constant
g	arbitrary positive constant
I	instantaneous probe current for the constant temperature hot-wire
K	calibration constant of the constant current anemometer
k	thermal conductivity of fluid at T_f
l	length of the hot wire
P_r	Prandtl number of the fluid at T_f
s	velocity fluctuations sensitivity, $s = \partial V_1/\partial U$
R	hot-wire operating resistance
r	distance from the center line of the round heated jet
Re	Reynolds number based on hot-wire diameter and evaluated at T_f
T	operating temperature of the hot wire
t	temperature fluctuations sensitivity, $t = \partial V_1/\partial\theta$
T_a	instantaneous local temperature of fluid
T_r	the local fluid temperature at which the hot wire is calibrated
T_f	film temperature = $(T+T_a)/2$
U	instantaneous total velocity
U_m	measured instantaneous total velocity
$\overline{U}_0$	mean center line velocity in the round jet
$\overline{U}_p$	mean velocity measured by Pitot tube
$\overline{U}_c$	mean velocity measured with compensated hot wire
$\overline{U}_u$	mean velocity measured with uncompensated hot wire
V_1	the output of the linearizer
X	$-d(A/A_r)^{0.5}/d\theta$
Y	$-d(B/B_r)^{-1}/d\theta$
ϵ	Error due to linearization in the measurement of total velocity
θ	instantaneous change in the local fluid temperature (T_a-T_r)
μ	absolute viscosity of fluid at T_f
ρ	density of fluid at T_f

Subscript r, except in case of T_r, refers to quantities evaluated at reference conditions.

DISCUSSION

H. M. NAGIB (Illinois Institute of Technology): What did you mean by the conventional method? Did you mean that you tried to keep the overheat or the temperature difference constant?

TUTU: No, the conventional method consists of operating the hot wire at three different overheats. One uses the constant current method and uses three different currents. One has three simultaneous equations to solve for the turbulence intensity.

NAGIB: What if you just try to use another sensor, a larger sensor, to change the overheat ratio or the temperature difference instead of a constant resistance on the other side of the bridge?

TUTU: By that method you can't compensate for the changes in A and B. In order to do so you have to build a separate circuit outside of the bridge. One does not gain anything by doing the main compensation on the bridge.

NAGIB: Can the opposite be done? That is can you get the velocity signal without any temperature influence on it?

TUTU: It is not practical. One would have to operate the wire at a very high temperature. Due to the limits of oxidation and things like that it is not possible to operate the wire at a sufficiently high temperature where the signal would only depend on the velocity.

T. HANRATTY (University of Illinois): What sort of temperature differences limit the method?

TUTU: It is possible to do a theoretical analysis to find out what sort of errors one gets if one does not include the compensation for A and B. For air, if one does only the main compensation and does not compensate for the variation of A and B, we found that in measuring mean velocity one can expect errors up to 50%. We are talking about temperature variations of up to 50°C in the fluid temperature.

NAGIB: How about if you have temperature variations of the order of 5°C?

TUTU: If you have temperature variations of about 5°C, the error is not very large, maybe about 5%.

L. FINGERSON (Thermo-Systems,Inc.): I just want to comment that you can't take into account A and B independently, but if you compensate right on the bridge, which you call the normal technique, you don't have to have the temperature coefficient so the two sensors match. In other words you can partially compensate for the effect of B and at the higher velocities the effect of A should be fairly small. You can partially compensate for the effect of B by not using matched temperature coefficients on the two sides of the bridge. Wouldn't that be correct?

TUTU: I don't know. One has to investigate it further because it depends on the fluid. For water, A varies very little and B varies very much but if you use it for air then the question is the reverse, because the coefficient B is very small and A varies very much and the main error comes from the coefficient A.

FINGERSON: This would be at low velocities, but A is not multiplied by the velocity, so as you go up in velocity this helps you out. This is the way we happen to do it, which you probably suspected by my bringing it up. But we also ran a number of calibration curves in both air and water and you are right, it works better in air than water. But you can compensate for DC changes over quite a wide range in temperatures quite accurately by doing it experimentally, but not for high frequencies. We are not saying that we understand the heat transfer, we are just doing it.

R. HUMPHREY (Disa): What is the frequency range of interest in your particular application?

TUTU: We would like to go up to 5000Hz, at the most.

GOLDSCHMIDT: Your last slide shows that the conventional methods seem to work and your compensation technique appears primarily as a convenience. Would you be willing to comment on the difference between your compensation circuit and those provided by Disa and Thermo-Systems and furthermore, what has been gained over the fluctuation-mode diagrams of years ago?

TUTU: The compensation which I know Disa provides has a very large time constant on the other side of the bridge. A resistance on the bridge may greatly reduce frequency response. Although one might compensate for the mean, he might not do so for the temperature fluctuations as well.

As far as your second question there is one very distinct advantage of our network. With the conventional method it is not possible to get a signal representative of the velocity alone. One can only measure mean quantities like $\overline{u^2}$ and correlations like $\overline{\theta u}$. One cannot do any spectrum analysis because one does not have a continuous signal directly proportional to velocity, which is needed for the purpose.

FINGERSON: How do you account for the fact that as the velocity changes your compensation must also change in the compensated constant current anemometer? It would seem that you should also take your velocity output and feed back to your constant current anemometer to change your compensation.

TUTU: It is true that the setting of the compensation is a function of the flow conditions. But if the turbulent intensity is not very high then the change in the time constant probably is not large. At each different point you have to use a new time constant setting, but at a given point then, because the turbulence intensity is not very large, one need not give a feedback from the velocity.

FINGERSON: So you have an automatic compensation system for the large temperature fluctuations, but not if you have large velocity fluctuations. It seems to me that the system then works well if you have large temperature fluctuations associated with small velocity fluctuations.

TUTU: Yes, it depends on how small is small. For velocity intensities up to maybe 15%, I should expect that there would not be a very great change in the time constant setting.

C. REED (Purdue University): The bulk of your paper deals with problems at which the velocity is rather high and I am wondering if you could comment on this situation. If you have a heated wire or a heated plate in which you have very low convection velocities but very high temperature fluctuations and temperature changes can you apply your method to measuring the velocity profile above a heated wire? And what kind of problems would you run into in that case?

TUTU: The accuracy of the compensating circuit depends on the heat transfer relation that one uses. It is obvious that for very small velocities where one is not sure of the heat transfer equations, the method will not do very well. For air, for example, with velocities as low as about 4 meters per second, we found errors of about 6%. For velocities less than 3 meters per second in air one can expect large errors. The principle of compensation is general, but limitations are set by the heat transfer relations one uses. Given an accurate heat transfer relation in a particular range it is always possible to compensate for it.