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TURBULENT INTERFACE DETECTOR USING A MULTIPLE ARRAY OF SINGLE HOT WIRES

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ABSTRACT

Intermittency circuits can be used to experimentally determine some of the properties of the turbulent interface that occur in turbulent shear flows. The signal $I(\underline{x}, t)$ is unity if turbulence is present at \underline{x} and zero otherwise. Using the signals obtained from detectors at one or more points, certain statistical measures of the interface position $Y(\underline{x}, t)$ can be determined. Also using a linear array of conventional single hot-wire probes, the position of the interface can be detected continuously to within some small error. The intent of this h-detector is similar to the wave-height detector used in studies of sea surfaces. The signal from the h-detector can be used directly to obtain a variety of important statistical measures of the shape of the interface. Measurements in a plane turbulent wall-jet are shown as an example of the application of an h-detector.

INTRODUCTION

A characteristic of free turbulent shear flows, such as wakes, jets and mixing layers, and semi-bounded flows, such as boundary layers and wall jets, is the existence of a relatively sharp interface between the turbulent fluid and the ambient fluid. At a fixed time the intensity of the turbulence does not decrease smoothly to zero from either the center line or a boundary to the outer edge of the flow, but there occurs a rather sharp discontinuity of the properties of the turbulent and the non-turbulent regions. In addition, the interface is a random convoluted surface moving with some convection velocity, and resembles a rough sea. The intermittent nature of the interface is readily observed by locating a hot-wire anemometer in the outer portions of an unbounded turbulent flow, and the signal will indicate periods of intense turbulent motions and periods of much less active motions.

The purposes of studying the topography of the random surface are:

1) The information is crucial to determining those statistical properties associated with the turbulent region only and the non-turbulent region only (conditional sampling). 2) The knowledge is basic to understanding the process of entrainment, whereby an ambient non-turbulent fluid enters the turbulent field. This is the heart of the process of mixing of fluids and also very fast reactions. 3) A description of the topographical features of the interface is necessary for the prediction of scattering and attenuation of electromagnetic and acoustic waves in regions of turbulence. 4) There is some indication that the large scale structure of the interface is correlated with the large eddies in the turbulence, hence a study of the interface can provide information on the large scale structure of the turbulence.

The boundary between the turbulent and the non-turbulent fluid is considered to be so thin that one can assign to it a definite location $Y = Y(\underline{x}, t)$, where \underline{x} is a two-dimensional vector (x, z) . Corrsin and Kistler¹ and Townsend⁹, using dimensional arguments and a model for the interface, estimated the thickness of the interface to be of the order of the Kolmogoroff length, $\delta_K \equiv (\nu^3/\epsilon)^{1/4}$, where ν is the kinematic viscosity of the fluid and ϵ is the local turbulent kinetic energy dissipation rate. There are neither strong theoretical arguments nor experimental measurements to support this result. However, all the experimental observations to date indicate that the thickness of the turbulent interface is thin enough so that its location can be specified without serious error.

Formally, the interface may be treated as a singular surface across which there is a jump in some property associated with the turbulence. The character-

istic parameter that is chosen to distinguish the turbulent from the non-turbulent fluid is the vorticity. The surface $Y(\underline{x}, t)$ is then assumed to separate vortical turbulent fluid from the irrotational ambient fluid. It should be noted that there are random fluctuations of velocity in the outer flow, as well as a mean motion due to entrainment, that are generated by the fluctuating random interface. These induced motions complicate somewhat the detection of the interface, but not in an insurmountable way. This effect is discussed later.

EXPERIMENTAL TECHNIQUE

The Intermittency Circuit

The basic feature of the experimental apparatus is the interface detector or intermittency circuit. The requirement is that the detector produce a signal $I(\underline{x}, t)$ that is "on" if there is turbulence at \underline{x} and "off" if there is no turbulence at \underline{x} . The discrimination between the turbulent and the non-turbulent fluid is made on the basis of the presence or absence of vorticity. With a single hot-wire probe, it is impossible to make this decision, and in practice, the determination is made on the basis that the signal "looks" turbulent or non-turbulent. Clearly this is arbitrary and the final intermittency signal is dependent on the operator's prejudice. However, there have been experiments performed using a vorticity probe¹, using smoke², and using a two-probe detector⁵ which obtained results that did not differ significantly from those obtained by a single probe.

In order to improve the difference between the turbulent and non-turbulent parts, the signal is usually sharpened by differentiation. This process amplifies the high frequency components of the signal relative to the low-frequency components. The motions that occur in the irrotational region are considered to be induced by the movement of the interface and are only indirectly related to the structure of the turbulence⁸. It is observed that the random irrotational fluctuations are of a lower frequency content than the fluctuations in the turbulence. Also the mean square fluctuations, $\overline{u_N^2}$, in the non-turbulent region can be a significant fraction of that which is obtained in the turbulent region, $\overline{u_T^2}$ ($\sim 1/3-1/2$ in a plane turbulent wall-jet). The discrimination between the turbulent and the non-turbulent signals generally cannot be made by only examining the fluctuating velocity signal without some form of enhancement of the high frequency components.

The basic components of a single-probe intermittency circuit consist of:

1) signal enhancement, 2) squaring or absolute value, 3) level comparison and 4) smoothing or hold-time. Signal enhancement consists of sharpening the turbulent signal from a hot-wire anemometer to better distinguish the turbulent portions from the non-turbulent parts. Usually this consists of differentiation of the signal at least once. For the wall-jet flow, it was found that the second derivative of the signal allowed a much less controversial judgment to be made. The enhanced signal is then placed all on one side of a null level by means of squaring, rectification or absolute value.

The rectified signal will also contain spurious noise and there will be some fluctuations in the non-turbulent parts of the signal. A comparator or gate is then used which produces an "on" signal when the signal level is above some prescribed level and an "off" signal (zero) when the signal level is below that adjustable gate.

Since the "turbulent" signal passes through zero, there will be a spurious signal developed since there will be an "off" signal occasionally registered within a turbulent burst. These "drop-outs" can be removed by several tech-

niques. The rectified signal can be modified by either adding the same signal shifted slightly in phase² or equivalently adding the rectified second derivative to it. Another technique was introduced by Kovasznay, Kibens and Blackwelder⁵ that utilized a running averager to form a hold-time, τ_H , that was set to remove the drop-outs in an optimum way.

To eliminate the drop-outs, we have used a smoothing circuit consisting of an integrator with a variable time constant and a NAND gate. This latter device goes "on" when the input voltage exceeds a prescribed value and "off" if it drops below that level. Details of construction and operation are given by Kohan⁴. A schematic diagram of the operation is given in Figure 1.

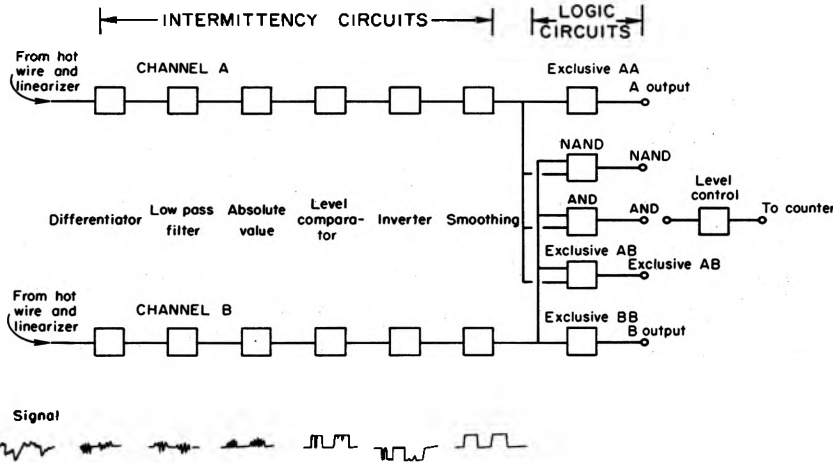


Figure 1 Schematic Diagram of the Intermittency Circuit

Logic Circuits

The output signals from two intermittency circuits: $I(\underline{x}, t)$ and $I'(\underline{x}+\underline{r}, t)$ can be processed by means of logic circuits to obtain the two-point correlation functions $\bar{I}(\underline{x}, t)I'(\underline{x}+\underline{r}, t)$. The logic circuits: a NAND gate, an AND gate, and an EXCLUSIVE AB gate can produce all the possible correlations. Each logic circuit accepts two signals (A and B) corresponding to the intermittency signal at two points, and the output O depends on whether A and B are high (h) or low (l). For example, using the NAND gate, if A=l, B=h then the output O=l; if A=h, B=l then O=l; if A=h, B=h then O=l; if A=l, B=l, then O=h. Therefore, using this circuit, γ_{00} can be measured, where γ_{00} is the probability that there is no turbulence at \underline{x} and no turbulence at $\underline{x}+\underline{r}$. In a similar way, other correlations can be measured.⁴

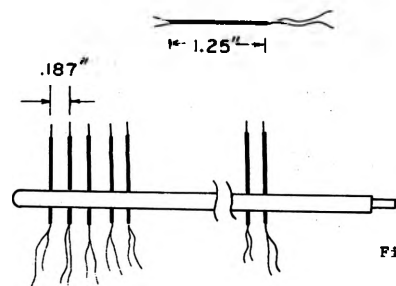


Figure 2 Schematic Diagram of h-detector Rake

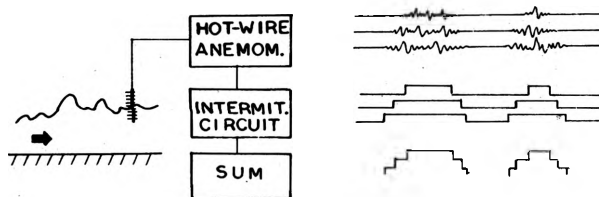


Figure 3 Technique for Generating the h-detector Signal

The h-Detector

By using a linear array of hot-wire anemometers, each of which detects the presence of turbulence, a signal can be generated that is proportional

to the position of the interface to within an error of the order of the spacing of the probes. Schematic diagrams of the hot-wire "rake" and the technique are shown in Figures 2 and 3. The basic idea is that an intermittency signal $I_j(\underline{x}, t)$ is generated by each probe ($j=1, 2, \dots, N$). The sum

$$\sum_{j=1}^N I_j(\underline{x}, t)$$

where N is the total number of probes, is proportional to the position of the interface $h(\underline{x}, t)$. If folding occurs, then this signal is proportional to the height of the turbulence that is present at \underline{x} , if the widths of all the folds were combined and added to the width of the main portion of the turbulence.

A 20-probe rake was constructed using Thermo Systems Model 1276 subminiature probes and spaced 3/16-inch apart. The spacing was determined for the wall-jet configuration currently being studied such that the "active" portion of the boundary layer ($\bar{y} \pm 4\sigma$) was spanned by the rake. Each hot-wire anemometer circuit was built using integrated circuits and using constant temperature operation.

The output signal from the h-detector $d(\underline{x}, t)$ is discrete, corresponding to the number of probes that are "on". The statistics of the detector signal, $d(\underline{x}, t)$, will differ from $h(\underline{x}, t)$ since:

$$h(\underline{x}, t) = d(\underline{x}, t) + \epsilon(\underline{x}, t) \tag{1}$$

where $\epsilon(\underline{x}, t)$ is the error. The mean error $\bar{\epsilon} \leq s$ where s is the spacing of the probes. This error can be improved if we add to the detector signal the value I/2 where the "on" value of I was assumed to be unity. Therefore $\bar{y} = \bar{d} + y_0 \pm s/2$, where y_0 corresponds to the position of the rake. Clearly $\lim_{s \rightarrow 0} f_d(\eta) = f_h(\eta)$, i.e., the first order probability density function for s=0 the detector output tends to that for the continuous random signal $h(\underline{x}, t)$ as the spacing tends to zero.

Experimentally, the average value of \bar{y} obtained from the h-detector and the values obtained by single-probe devices differ only by a value of the order of the spacing error. For our experiments, this error corresponded to the scatter of the experimental data. Similar results have been obtained for the probability density function $f_d(\eta)$ obtained directly from the signal $d(\underline{x}, t)$ and compared to $f_h(\eta)$ obtained by the conventional traverse single-probe measurement of $\gamma(y)$ provided that the effect of folding or the multi-valued nature of the interface is considered.

The h-detector, or multi-probe linear array, is a workable system that produces a signal proportional to the position of the interface. There is a close similarity to the wave-height detectors used in studies of sea-surfaces. Clearly the difficulties that are encountered in its use are operational, in that a large number of separate circuits must be constructed and calibrated. However, the potential use of this assembly is quite large, since each individual signal from the probe may be processed by computer methods³ and elaborate statistical information on the turbulent velocity field obtained.

To obtain two-point statistics for $h(\underline{x}, t)$, two separate h-detectors are required. However, since this is not always desirable because of mutual interference of the h-detectors, it is noted that statistical quantities like the space-time correlation for h, i.e., $\mathcal{H}(\rho; \underline{x}; \tau)$, can be measured by using a single probe for I and the h-detector⁷. Typical plots of the space-time correlation function, $\mathcal{H}(-\xi, 0; \tau; x_0/d_0 = 359)$, for a turbulent wall-jet are shown in Figure 4. From these data, values of the convection velocity are obtained. The spatial correlation functions, $\mathcal{H}(\xi, 0; 0; x_0/d_0 = 359)$ and $\mathcal{H}(0, \zeta; 0; x_0/d_0 = 359)$, are shown in Figure 5. The interpretation and analysis of data of this type will not be considered here. Suffice it to say that the use of a Fourier transform or an orthogonal decomposition allows determination

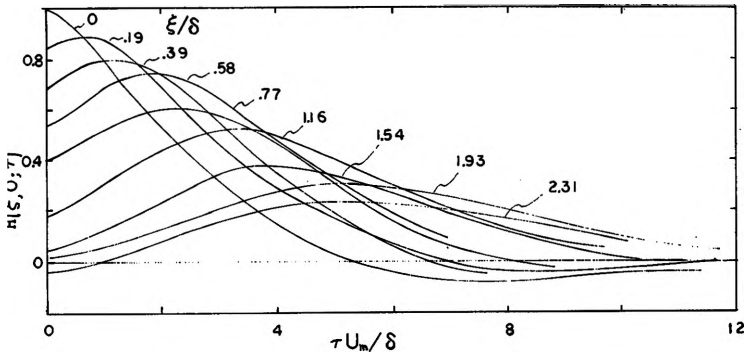


Figure 4 Space-time Correlation Curve for $h(\underline{x}, t)$ obtained in a Wall Jet Flow ($x_0/d_0 = 359$)

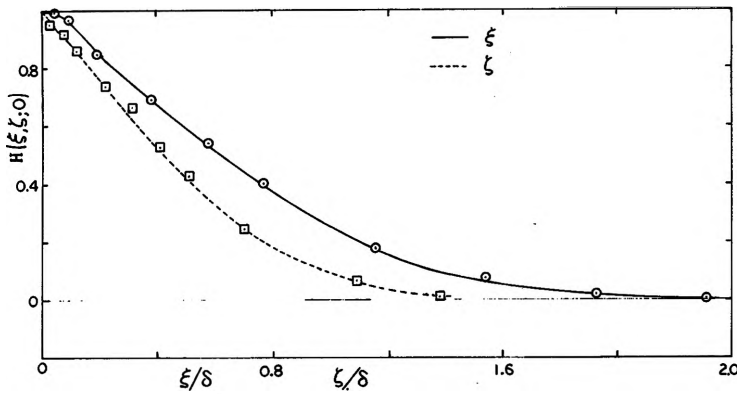


Figure 5 Spatial Correlation Function $H(\xi, \zeta; 0; x_0/d_0 = 359)$

of a preferred shape of a bulge or indentation. A model of the interface can be constructed of bulges that are randomly spaced on the interface and are convected with some velocity, U_c .

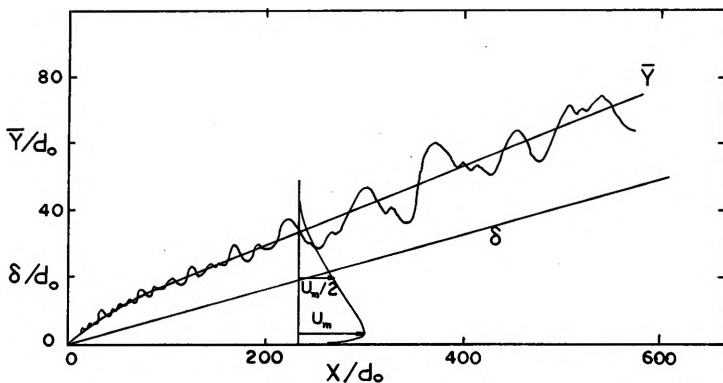


Figure 6 Typical turbulent Shear Flow - A Wall Jet

A SAMPLE SHEAR FLOW

Some of the characteristic properties of the interface will be described by considering a typical turbulent shear flow. Figure 6 is a schematic representation of a two-dimensional turbulent wall-jet. The boundary layer thickness, δ , (the point where $U = U_m/2$; U_m is the maximum velocity) grows linearly with downstream distance x (corrected for the virtual origin) from an exit slot of width, d_0 . Further, it is experimentally observed that other length scales such as \bar{Y} , the mean position of the interface, also increase linearly with x . The maximum velocity, U_m , varies as x^{-a} , where $a = 0.53$. The outer portion of the flow somewhat resembles a two-dimensional free jet.

The interface of any large Reynolds number shear flow that is unbounded in at least one direction has the following typical features:

- 1) The characteristic sizes of the bulges are relatively large. The

mean length of a turbulent bulge is of the order of δ . The ratio of the root-mean-square height of the bulge about the mean location to the boundary layer thickness, δ , is about 1/5. Therefore, the interface is essentially flat and may be considered as a ripple on a thick field of turbulence. There are exceptions to this generalization, e.g., boundary layers with strong adverse pressure gradients have corrugations that penetrate nearly to the wall.^{2,5}

- 2) The mean position of the interface occurs at roughly 3/4 the overall thickness of the turbulent field ($\delta_T = 2\delta$ for the wall jet).⁹
- 3) There is difference in the mean velocity, U_T , and other statistical measures in the turbulent portions and the non-turbulent portions of the flow. e.g., $\Delta U \equiv U_T - U_N \approx 0.05(U_m - U_{min})$, for "driven" flows such as conventional boundary layers and wakes, and for the wall jet, $\Delta U \approx 0.2 U_m (U_T/U_N \approx 2-3)$.
- 4) The convection velocity, U_c , of the bulges of the interface of the wall jet obtained from the space-time correlation of h roughly corresponds to the mean velocity at the mean position of the interface, \bar{Y} .
- 5) The probability density function for the position of the interface is reasonably Gaussian.
- 6) There is folding of the interface that occurs. However, the amount is small enough that for many purposes the interface may be considered as a single-valued function of position \underline{x} .

STATISTICAL MEASURES OF THE INTERFACE LOCATION

Provided that the interface can be considered as a single-valued function of position \underline{x} , then certain simple probability measures can be used to describe the random variable, $Y = Y(\underline{x}, t)$. Define:

$$h(\underline{x}, t) \equiv Y(\underline{x}, t) - \bar{Y}(\underline{x}), \quad (2)$$

where \bar{Y} is the mean location of the interface and it is assumed that the random variable is stationary in time. Further, for the two-dimensional flow considered here, \bar{Y} is independent of the transverse direction (z) and depends only on x , hence $\bar{Y} = \bar{Y}(x)$. A suitable detector located at a fixed position (\underline{x}) in the flow will indicate the presence of turbulence, and produce a signal (usually unity). During the times that the probe is immersed in non-turbulent fluid, the output of the detector will be zero. This binary random intermittent function $I(\underline{x}, t)$ is then defined as:

$$I(\underline{x}, t) = 1 \text{ (turbulence at } \underline{x}) \\ = 0 \text{ (no turbulence at } \underline{x}) \quad (3)$$

Intermittency Factor

The intermittency factor, γ , is defined as the fraction of the total time that the detector observes turbulent fluid, viz.:

$$\gamma(\underline{x}) = \bar{I}(\underline{x}, t) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T I(\underline{x}, t') dt' \quad (4)$$

The intermittency factor, γ , is typical of the data that can be obtained by using a single, fixed detector in the flow. Results of these measurements for the wall jet are shown in Figures 7 and 8. Of interest is the fact that a characteristic length associated with the interface, (\bar{Y}), does not scale with the downstream distance in the same way as the length scale of the boundary layer proper, (δ), until quite large distances downstream, ($x/d > 400$). In this sense, full self-preservation of the boundary layer is not achieved until distances greater than those indicated by observing conventional average velocities and measures of the turbulence such as u^2 , etc.

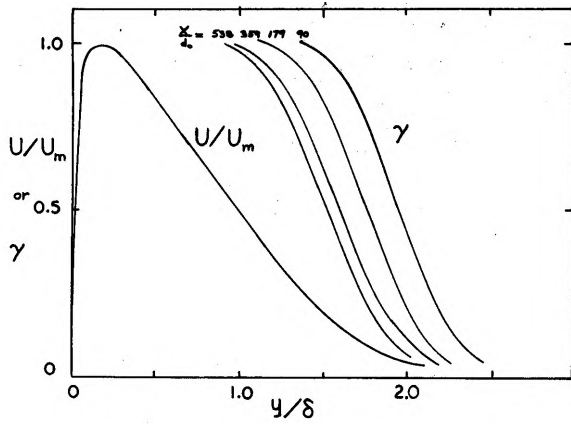


Figure 7 Velocity Profile and Intermittency Factor in a Plane Wall Jet

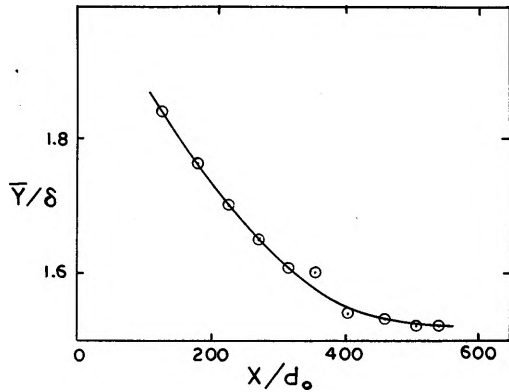


Figure 8 Mean Position of the Interface \bar{Y} versus Downstream Distance in a Plane Wall Jet

Other information can be obtained from a single detector of $I(\underline{x}, t)$. An estimate of the length scale of the turbulent bulges in the flow direction can be obtained by measuring the frequency of occurrence of $I(\underline{x}, t)$, which gives the rate that turbulent bulges sweep past the point \underline{x} . Define the mean frequency of occurrence of $I(\underline{x}, t)$, or the average number of times that $I(\underline{x}, t)$ rises from 0 to 1 per unit time as f_T . Further, since γ is the fraction of the total time that the flow is turbulent at the point \underline{x} , then the average time, θ_T , required for the passage of turbulence past the point \underline{x} is given by:

$$\theta_T \equiv \gamma / f_T \quad (5)$$

If it is possible to define a suitable convection velocity, U_c , for the advancement of the turbulent interface, then the average length of turbulent bulge may be estimated by:

$$\ell_T = \theta_T U_c \quad (6)$$

provided that the flow is essentially planar and the growth of the bulges is not excessive in the x -direction. Shown in Figure 9 is a plot of $f_T \delta / U_m$

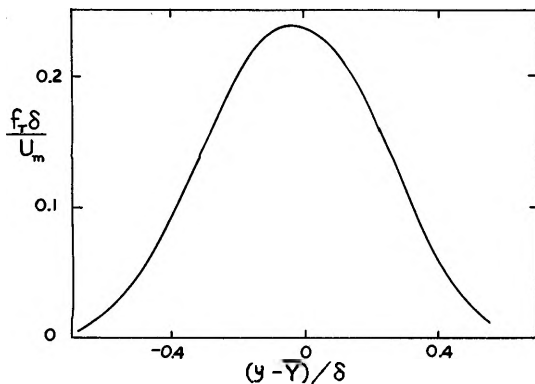


Figure 9 Dimensionless Frequency of Occurrence of Bulges

versus the dimensionless distance about the mean position of the interface, (\bar{Y}) . The maximum frequency occurs roughly at \bar{Y} . For the conditions shown in Figure 9, $(f_T)_{\max} = 15$ Hz.

Convection Velocity

The convection velocity of the interface can be formally defined in several ways. A rigorous method is by using space-time correlation functions for $h(\underline{x}, t)$, i.e., $\overline{h(\underline{x}, t) h(\underline{x}+\underline{\rho}, t+\tau)} \equiv H(\underline{\rho}, \tau; \underline{x})$ where $\underline{\rho} = (\xi, \zeta)$. Now the space-time correlation coefficient is defined by:

$$\overline{H(\underline{\rho}; \tau; \underline{x})} = \frac{\overline{h(\underline{x}, t) h(\underline{x}+\underline{\rho}, t+\tau)}}{[\overline{h^2(\underline{x}, t)}]^{1/2} [\overline{h^2(\underline{x}+\underline{\rho}, t)}]^{1/2}} \quad (7)$$

The convection velocity, U_{cT} , can be defined by the slope of the points of

$\left. \frac{\partial \overline{H(\xi, 0; \tau)}}{\partial \xi} \right|_{\tau \text{ fixed}}$. Also the convection velocity, $U_{c\xi}$, is defined by the slope of the points $\left. \frac{\partial \overline{H(\xi, 0; \tau)}}{\partial \tau} \right|_{\xi \text{ fixed}}$. The convection velocity, U_{cT} , appears

to have a better physical interpretation since it corresponds to the motion of the peak or maximum of the correlation curve, hence follows an "eddy" downstream. Measurements of U_{cT} obtained in this way indicate that for the wall jet, $U_{cT} \approx 1/5 U_m \approx 1.2 U_T$.

Surface Flatness

A measure of the flatness of the interface is given by the ratio of σ / ℓ_T and is shown in Figure 10 for the wall jet at various downstream positions from the nozzle. The quantity, ℓ_T , was obtained at $y = \bar{Y}$ from the frequency, f_T , and the convection velocity, $U_c = U_m / 5$. The curve indicates that the indentations and bulges increase with downstream distance. Of note is the fact that the average length, ℓ_T , remains essentially constant with downstream distance.

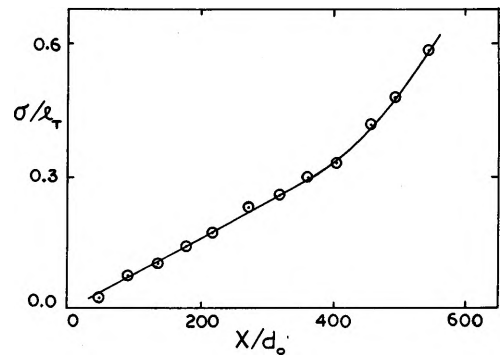


Figure 10 Flatness of the Surface: Plot of σ / ℓ_T versus Downstream Distance

Folding

In order to interpret experiments and also establish appropriate theories, it is necessary to determine the degree of folding of the interface. By folding, we mean the multi-valued nature of the interface function, $Y = Y(\underline{x}, t)$. A fold is said to occur if there exists non-turbulent fluid between a region of turbulent fluid and the fully turbulent region, or center line of the flow, or wall in the case of a wall jet. It is emphasized that the turbulent interface is assumed to be contiguous; there are no isolated regions of turbulent fluid in the non-turbulent region and vice versa. We do state that the interface can become highly convoluted and folds over itself in both the transverse and longitudinal directions. In fact a one-dimensional slice through the turbulent region would have the form shown schematically in Figure 11. The apparent holes and isolated turbulent spots are due to the convoluted nature of the interface.

Measures of Folding

One measure of folding is to locate two turbulence detectors one above the other and count the time that the outer probe indicates no turbulence while

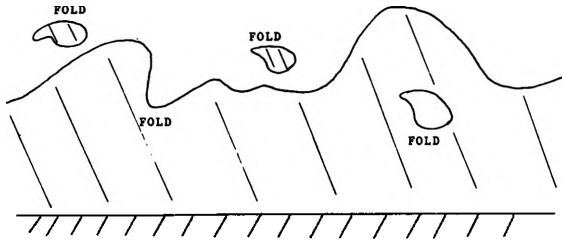


Figure 11 Schematic Diagram of Folding of Interface

at the same time the inner probe indicates turbulence. This technique may be readily performed using logic circuits.⁴ In fact, the folding time, τ_f , depends on location and probe spacing. In a wall jet, it was found that $\max \lim_{T \rightarrow \infty} (\tau_f/T)$ was about 0.05. Note that if several folds occur simultaneously, the two-probe system will detect only a single fold and hence will give low values.

Using the h-detector, which obtains the height of the interface directly, more elaborate statistics on the folds can be obtained. Further, some of the statistics of multi-valued functions such as the "expected-valuedness" can be obtained.⁶

Surface Slopes

There are a variety of methods for determining the average slope of the interface, e.g., from the spatial correlation function of $h(\underline{x}, t)$, from the autocorrelation of $h(\underline{x}, t)$ by using a convection velocity, from the assumed normal probability density of $h(\underline{x}, t)$ and $\partial h/\partial x$, and from the average lengths of the bulges.

1.-From the Spatial Correlation Function

Using Taylor's expansion for $h(x+\xi, z; t)$:

$$1 - \mathcal{H}(\xi, 0; 0; x) \approx \frac{1}{2} \left(\frac{\partial h}{\partial x} \right)^2 \frac{\xi^2}{h^2} \quad (8)$$

If $\frac{\delta}{h} \frac{\partial h^2}{\partial x} \ll 1$ and $\frac{\delta^2}{h^2} \frac{\partial^2 h^2}{\partial x^2} \ll 1$, then from a plot of $1 - \mathcal{H}(\xi, 0; 0; x)$ versus

ξ^2 the mean square slope can be computed. In the ζ -direction, a somewhat larger value of the slope was obtained. Thus, on the average, the slopes of the bulges are greater in the downstream direction than in the transverse direction.

2.-From the Autocorrelation Function

We can write:

$$\overline{h(\underline{x}, t) h'(\underline{x}; t+\tau)} = \overline{h^2(\underline{x}, t)} + \frac{\partial \overline{h^2}}{\partial t} \frac{\tau}{2} + \frac{\partial^2 \overline{h^2}}{\partial t^2} \frac{\tau^2}{2} - \left(\frac{\partial h}{\partial t} \right)^2 \frac{\tau^2}{2} + \dots \quad (9)$$

Since the function is stationary,

$$\mathcal{H}(0; \tau; \underline{x}) = 1 - \left(\frac{\partial h}{\partial t} \right)^2 \frac{\tau^2}{2h^2} \quad (10)$$

Now, assuming a frozen pattern moving with a convection velocity, U_c :

$$1 - \mathcal{H}(0; \tau; \underline{x}) = \frac{1}{2} \frac{(\tau U_c)^2}{h^2} \left(\frac{\partial h}{\partial x} \right)^2 \quad (11)$$

From the measurement of the autocorrelation coefficient, $\left(\frac{\partial h}{\partial x} \right)^2$ can be estimated.

3.-From the Average Length of the Bulge

An average slope of the interface can be defined from the variation of the mean length of the bulges of the interface with respect to distance from the boundary, i.e., l_T versus y . The slope $\approx 2 \left[\frac{2l_T}{2y} \right]^{-1}$ can be determined at some characteristic value of y , e.g., where $\gamma(x, y) = \frac{1}{2}$. For the wall jet, the slope was computed to be 1.5 at $y = \bar{Y}$.

4.-From the Probability Density Function

Assuming that $h(\underline{x}, t)$ and $\frac{\partial h(\underline{x}, t)}{\partial x}$ are both normally distributed and independent, then Corrsin and Kistler¹ have shown that:

$$\frac{1}{l_T + l_N} = \frac{1}{2\pi} \left[\overline{\left(\frac{\partial h}{\partial x} \right)^2} / h^2 \right]^{1/2} \exp \left[-\frac{y - \bar{Y}}{\sigma} \right]^2 \quad (12)$$

Note that for $y = \bar{Y}$:

$$\left(\frac{2\pi f_T \delta}{U_c} \right) \left(\frac{\sigma}{\delta} \right) = \left(\frac{\partial h}{\partial x} \right)^2 / h^2 \quad (13)$$

where f_T is the frequency of the $I(\underline{x}, t)$ pulses at $y = \bar{Y}$, and U_c is the convection velocity. For the wall jet, using this method the mean square slope was computed to be 0.98.

SUMMARY

Measurements of the surface topography of the interface of unbounded turbulent shear flows provides important information concerning the entrainment process and overall structure of the turbulent shear flow. Data on the shape of the surface can be obtained by using intermittency detectors located at different points in the field. Some of the techniques for obtaining one- and two-point probability functions were described. In addition, by using a linear-array of probes, e.g. 20, an interface detector was constructed that produced a signal proportional to the position of the interface to within an error dependent on the spacing of the probes. Using this device, more detailed information on the random interface of a plane turbulent wall jet was obtained. Additional measurements are in progress in our lab on the wall jet and also other turbulent shear flows.

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LIST OF SYMBOLS

d_0	Nozzle diameter
$d(\underline{x}, t)$	Signal from h-detector
f	Frequency of occurrence of turbulence at \underline{x}
$f_h(\eta)$	Probability density function of h
$h(\underline{x}, t)$	Height of interface relative to its mean position
$\mathcal{H}(\underline{\rho}; \underline{x}, \tau)$	Spatial correlation function of $h(\underline{x}, t)$
$I(\underline{x}, t)$	Intermittency function: $I(\underline{x}, t) = 0$ if turbulence at \underline{x} , zero otherwise
l_T	Average length of a "bulge" in the interface
l_K	Kolmogoroff length
$\frac{u_N^2}{h^2}$	Mean-square fluctuating velocity in non-turbulent fluid
$\frac{u_T^2}{h^2}$	Mean-square fluctuating velocity in turbulent fluid
U_c	Convection velocity of interface
U_m	Maximum velocity
U_N	Mean velocity in non-turbulent part of flow
U_T	Mean velocity in turbulent part of flow
x	Downstream distance
y	Distance from wall
Y	Position of interface relative to a boundary or center-line
γ	Intermittency factor
δ	Boundary layer thickness (for wall-jet, $\delta = y$ where $U = U_m/2$)
δ_T	Total boundary layer thickness
ϵ	Turbulent energy dissipation

γ_{00}	Probability there is no turbulence at x and no turbulence at $x+\tau$
$\chi(\rho; \chi; \tau)$	Space-time correlation coefficient of $h(\chi, t)$
θ_T	Average time for passage of a bulge
ν	Kinematic viscosity
σ	Root-mean-square of $h(\chi, t)$
τ	Time delay
τ_f	Folding time
χ	Position in the x-z plane

REFERENCES

1. Corrsin, S., and Kistler, A. L., "Free-stream Boundaries of Turbulent Flows", NACA Rept. 1244 (1955).
2. Fiedler, H., and Head, M. R., "Intermittency Measurements in the Turbulent Boundary Layer", J. Fluid Mech., 25, 719 (1966).
3. Kaplan, R. E., and Laufer, J., "The Intermittently Turbulent Region of the Boundary Layer", presented at the 12th International Congress of Applied Mechanics, Stanford University, 1968.
4. Kohan, S. M., "Some Studies of the Intermittent Region and the Wall Region of a Two-Dimensional Plane Wall-Jet", Ph.D. Thesis, Department of Chemical Engineering, Stanford University, Stanford, Calif., 1968.
5. Kovaszny, L. S. G., Kibens, V., and Blackwelder, R. F., "Large-Scale Motion in the Intermittent Region of a Turbulent Boundary Layer", J. Fluid Mech., 41, 283-325 (1970).
6. Lumley, J. L., "On Multiple-Valued Random Functions", J. Math. Phys. 5, 1198-1203 (1964).
7. Paizis, S., personal communication (1971).
8. Phillips, O. M., "The Irrotational Motion Outside a Free Turbulent Boundary Layer", Proc. Camb. Phil. Soc., 51, 220 (1955).
9. Townsend, A. A., "The Mechanism of Entrainment in Free Turbulent Flows", J. Fluid Mech., 26, 689 (1966).

DISCUSSION

T. H. HODGSON (Syracuse University): Wouldn't you say that with the advent of the mini-computer the time is not far off for you to produce complete 3-dimensional pictures of the interface using your logic circuitry?

SCHWARZ: The use of the computer would ultimately be the best way to go if you would like to do it that way. I don't like computers personally, because they discharge information at such an enormous rate you are inundated and you often don't quite go after certain features you would like to exploit. I prefer to pick on a particular idea, pursue it, and learn something about it. Now, I believe that Prof. John Laufer (University Southern California) has a set-up from which the output of a rake of hot-wire anemometers can be statistically processed to obtain a large amount of data. I have not seen any of the information yet. I think it involves an enormous effort initially to get the thing going. If you are willing to take the time and the patience that may be the way to do it.

S. KLINE (Stanford University): Professor John Laufer is getting an output and so is Prof. Willmarth of The University of Michigan.

SCHWARZ: I think we are talking about degree of effort and not so much of total effort. In other words, if you set up your whole procedure on a basis of processing the data, it requires total effort. You can do this partially or to any degree that you want. I personally don't subscribe to the total degree.

D. D. PAPAIOU (Purdue University): I have two questions to ask. You said that there are induced fluctuations in the potential flow which you can detect with your probe and I would like to know how you distinguish between what you call induced fluctuation and the fluctuation which is in the wake. I also want to ask you what do you think is the mechanism which really induced those fluctuations in the potential flow?

SCHWARZ: To answer the second part first, there is a theory by O. M. Phillips which considers the irrotational motions which are generated outside the interface as being those generated by a fairly rigid wall which is rippling with the turbulent characteristics. That theory then is relatively simple, the motions outside are irrotational and the solutions are Laplacian (with a random forcing function). The random forcing function is the turbulent field. A feature of that theory is that either the mean square (or the root mean square) of the fluctuations varies as the 4th power of distance from the wall. That has been confirmed by several people. There are also some features about the spectrum of the irrotational motions which seem to be confirmed also and although there are other features which haven't been completely confirmed the theory seems reasonable. The answer to the first part comes from my statement about the operator's bias. When one looks at a hot-wire signal he might see something that is very "scratchy" and then suddenly is "smooth" and then something that is very scratchy again. So one can sit down and say, "that's turbulent," and then one can say, "now it is not." There is a compromise to within some span. A multiple array of probes, may help because if you look, for example, at the signal at one probe and at the probe below it, then you can tell if the turbulence or the thing that you see in one probe is in fact turbulence, or whether it is something which is being induced by the bulge right below it.

V. W. GOLDSCHMIDT (Purdue University): You made reference to the possibility of folding of the interface. Would you comment on any evidence or lack of evidence for this?

SCHWARZ: There are two. One is visual observations, such as the films by Fiedler and Head who have pumped smoke into a turbulent boundary layer. If you observe these you notice that the large scale motions that billow have a tendency to carry over downstream so there is a degree of folding. The second is measurements. The γ_{ij} , two point correlation functions, sometimes show a signal for the top probe and no signal for the bottom probe. In other words, you see turbulence at a probe farther from the wall and no turbulence at a probe closer to the wall. This is then an indication of folding. Measurements of this type indicate that the folding occurs less than about 5% of the time in the turbulent boundary layer we referred to. I think it may be a bit less than that in a conventional turbulent boundary layer.

GOLDSCHMIDT: I believe it is in order to add to this review by noting that there seems to be quite a bit of interest developing in measurements at the interface. Probably one of the most active groups is the one at Johns Hopkins. I believe that in addition to Prof. Schwarz, Prof. Kovaszny, following the work of Prof. Stanley Corrsin is trying to measure intermittency in a heated boundary layer. Prof. Val Kibens at The University of Michigan is looking at a heated wake. Professors John Laufer and R. Kaplan at The University of Southern California are also working at the interface trying to couple its motion to conditions in the free stream. Ron Blackwelder has also joined that team. In Lyons (France) Dr. Jean Mathieu and Dr. G. Comte-Bellot took measurements of the interface in a jet. I also believe Dr. I. Wygnanski (now in Israel) and Dr. Fiedler (in Berlin) are also continuing some of the work they did while at Boeing. At Purdue University, we are now trying to take interface measurements in a plane heated jet, while Prof. R. Chevray at S.U.N.Y., Stony Brook is studying a circular jet.