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EXPERIMENTAL INVESTIGATION OF THE HYDRODYNAMIC STABILITY OF FLOW IN ROTATING PIPES USING THERMISTORS*

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ABSTRACT

The hydrodynamic stability of flow in a rotating pipe is investigated experimentally using thermistors. The experimental apparatus consists of a rotating pipe made of lucite, 3-1/4 inch inside diameter, six feet long, equipped with porous plugs at the ends to minimize recirculation. The working fluid is water and a range of axial and tangential Reynolds numbers up to 7,000 and 20,000, respectively, is covered. Thermistors were chosen for this study since their high electrical resistivity yields strong signals which permit the use of inexpensive and convenient electronics. They also have a high temperature coefficient of electrical resistance which minimizes the problem of noise due to brushes and slip rings. The intensity of the signal and the signal to noise ratio are better by an order of magnitude than those attainable with platinum film probes. The probes are rugged, inexpensive and are commercially available. The major problem encountered is their low frequency response. This, however, does not limit their usefulness in determining transition from laminar to turbulent flow regimes, as can be seen from the results of the present investigation, which were verified by visual diagnostic techniques utilizing dye streaks and hydrogen bubbles.

INTRODUCTION

Thermistors are oxide semi-conductors with high negative temperature coefficients of electrical resistance (usually some combination of di-valent and tri-valent oxides such as CuO, NiO on one side and Mn_2O_3 and Co_2O_3 on the other side). They have been studied in recent years as potential sensing elements in fluid flows. These studies were mainly concerned with the properties of the probes, their calibration and possible areas of use. Few publications however deal with the use of thermistors in fluid mechanics studies. J. Lumley[1], R. Lane et al.[2], and several other investigators made substantial contributions in this area. Pertinent information is also available from the major thermistor manufacturers.

Lumley[1] found that in water the spatial resolution and signal to noise ratio of thermistors exceeds those attainable with platinum film probes by an order of magnitude because of their relatively large electrical resistivity and high temperature coefficient of electrical resistance. On the other hand, thermistors have a lower density and smaller thermal conductivity than platinum, and thus have limited frequency response. Attempts are being made at present to overcome this limitation using film deposition. The probe usually consists of a thermistor bead mounted on a short support, with the bead being encapsulated in some insulating material, such as glass. The probes are rugged and their high electrical resistivity permits the use of simple electronics.

Lane et al.[2] investigated hot-thermistor anemometry for low velocity flows. A method for calibrating the probes is presented by them and the effect of water temperature is also discussed. They conclude that hot-thermistor anemometry may be used to measure steady-state and transient velocities in the range from 0.1 to 6 inches per second, and that individual probe calibration is necessary. The response to step velocity changes is also presented but no other transient measurements are reported.

Hot-thermistor anemometry was used in a recent investigation of the stability of Hagen-Poiseuille flow to non-axisymmetric disturbances by J. Fox et.al.[3]. The probes were used in a constant current mode to achieve a high sensitivity to the velocity fluctuations. The working fluid used in this investigation was water. The results obtained with hot-thermistor anemometry by these investigators appear to be satisfactory; however, they are planning further measurements using wedge-shaped film probes.

The present investigation is part of a study dealing with the effect of solid body rotation on the stability of flow in pipes. The rotation is produced by rotating the pipe about its axis. Rotating systems introduce an additional problem in the use of sensing elements like hot-films or hot-thermistors since the electric signals from the sensing elements must be transferred to a stationary frame of reference. This is usually accomplished by using brushes and slip rings. Since these introduce noise, filtering is usually required. In the case of thermistor probes, their large resistance makes filtering relatively easy and simplifies the construction of the brushes and slip rings.

Based on the factors discussed above, hot-thermistor anemometry appears to be well suited for the present investigation. The probes should be operated in a constant current mode, to achieve the highest possible sensitivity to velocity fluctuations and to reduce the cost of the electronic circuitry.

ROTATING PIPE APPARATUS

The apparatus used in the present studies is shown in Figure 1. It was designed on the basis of the following specifications: a) conditions at the inlet and outlet of the rotating pipe must be controlled in order to obtain a flow of solid body rotation at the entrance and a flow field that is free of reversed flow along the entire length of the pipe; b) the pipe diameter must be sufficiently large to permit quantitative measurements in the flow field and to facilitate visual diagnostic approaches; c) a wide range of operating conditions must be possible; and, d) it should be possible to use different fluids as working media.

The test section of the apparatus consists of a 73-3/4 inch long lucite pipe of 3-1/4 inch inside diameter and 3-3/4 inch outside diameter (L/D = 23). Five stainless steel sleeves cemented to the outside of the pipe were machined to an outside diameter the center of which coincides with that of the inside diameter with a tolerance of ± 0.0015 inch. The pipe is supported by five self-aligning ball bearings through the sleeves. A pressure-tight settling chamber is provided at the pipe inlet while an open tank is used at the outlet. The downstream open tank is equipped with a drain system that can maintain a constant head in the tank at all axial Reynolds numbers. The drain, supply and connecting lines consist of one-inch diameter tygon tubing. The entire assembly is mounted on a 20 inch wide, 12 feet long, aluminum channel. With water as the working fluid, the apparatus can be operated at an axial Reynolds number N_{Rz} ranging from 0.0 to 12,000 within an accuracy of 0.5%, ($N_{Rz} = \frac{\bar{W}D}{\nu}$; where $\bar{W}$ is the is average axial velocity) and at a tangential Reynolds number ranging from 0 to 40,000 within an accuracy of 1.0% ($N_{R\theta} = VD/\nu = \omega D^2/2$ where V is the maximum tangential velocity).

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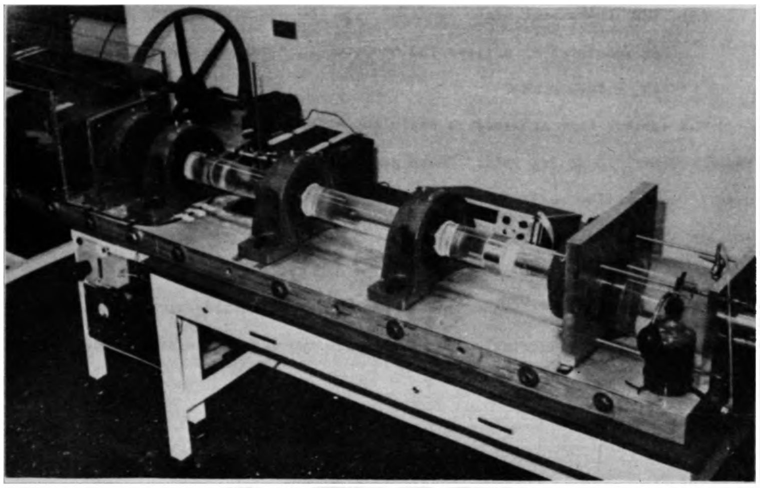


FIG. 1. ROTATING PIPE APPARATUS

The apparatus can be run in a single pass mode of operation with the water from the building supply line passing through the apparatus to a drain. Closed loop operation is also possible with the water being recirculated with the aid of a centrifugal pump. The flow rate through the apparatus is monitored at low flow rates by a rotameter and at high flow rates by a turbine-type flowmeter (Potter Aero. Corp. Model 5/8-5570). These flow-meters are capable of measuring the rate of flow over a range of Reynolds numbers from 80 to 14,500. The signals from the turbine-type flowmeter are registered using a Hewlett-Packard 5233L digital counter. The counter output is in kilocycles, readily yielding flow rates and axial Reynolds numbers.

In order to allow steady rotation of the pipe over the range of tangential Reynolds numbers under investigation, a special drive mechanism has been devised which provides a range of speed reduction ratios from 400 to 1 to 13 to 1. It consists of a variable-speed 1-1/2 horsepower D.C. shunt-motor, a V-belt drive, a two-stage gear-belt type speed reducer, and a variable reduction-speed Vickers hydraulic transmission.

The rotational speed is indicated by a digital counter. The counter is connected to a D.C. circuit which is interrupted periodically by a microswitch actuated by one of the gear pulleys. The counter output is in kilocycles, readily yielding RPM's and tangential Reynolds numbers. At the higher rotational speeds a magnetic pickup was substituted for the microswitch.

An experimental study by A. Fejer et al.[4] revealed that the flow field in a rotating duct is composed of three regions: an inlet region in which reverse axial flow may occur near the duct wall, a central region in which the flow is relatively insensitive to axial position, and an outlet region where reverse axial flow may occur near the axis of the duct. These findings are strongly supported by the analytical results of Z. Lavan et al.[5].

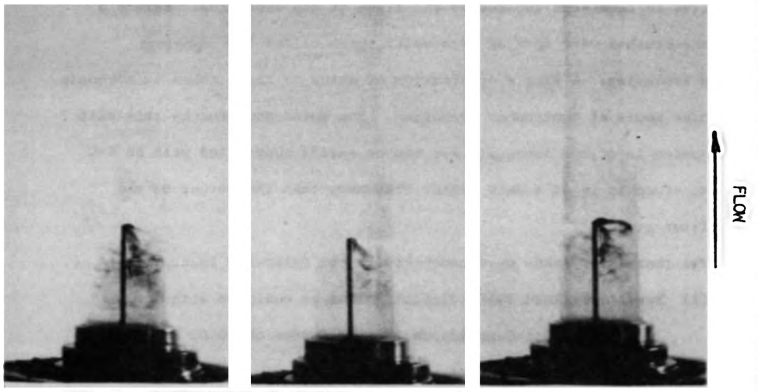


FIG. 2. PHOTOGRAPHS OF REVERSE FLOW IN ROTATING TUBE INLET REGION

Within the range of the present experiments ($0 < N_{Rz} < 7000$) the pressure drop across the porous plugs was found to be proportional to the square of the flow velocity and independent of the speed of rotation. The results of these measurements are shown in Figure 3. On the basis of these data, the ratio of the radial pressure differential in the pipe to the axial pressure drop across the plug can be expressed, for solid body rotation, in the form

$$\frac{\Delta P_r}{\Delta P_z} \doteq 0.004\Gamma^2$$

where ΔP_r is the radial pressure difference due to the rotation, and ΔP_z is the pressure drop due to the flow across a one-inch thick porous plug. From this relationship it is apparent that $\Delta P_r/\Delta P_z$ is of the order of 1/100 (or smaller) when the swirl ratio does not exceed the value of four. It was established experimentally that a plug of one-inch thickness imparted solid body rotation to the fluid if the swirl ratio did not exceed this value. At higher swirl ratios, back flow was observed through the outer portion of the porous plugs.

THERMISTOR MEASUREMENTS

The Hot-Thermistor System

The principle of operation of hot-thermistors is similar to that of other hot sensing elements. It is based on the change of the rate of heat transfer at the surface of the sensing element with the velocity of the fluid in which the element is immersed. Due to a temperature change, the sensing element undergoes a change in resistivity. This temperature dependence can be expressed in the form

$$R = R_\infty \exp(To/_T)$$

The reversed flow patterns at the ends of the pipe were also observed in the present study when a dye streak was introduced through a probe near

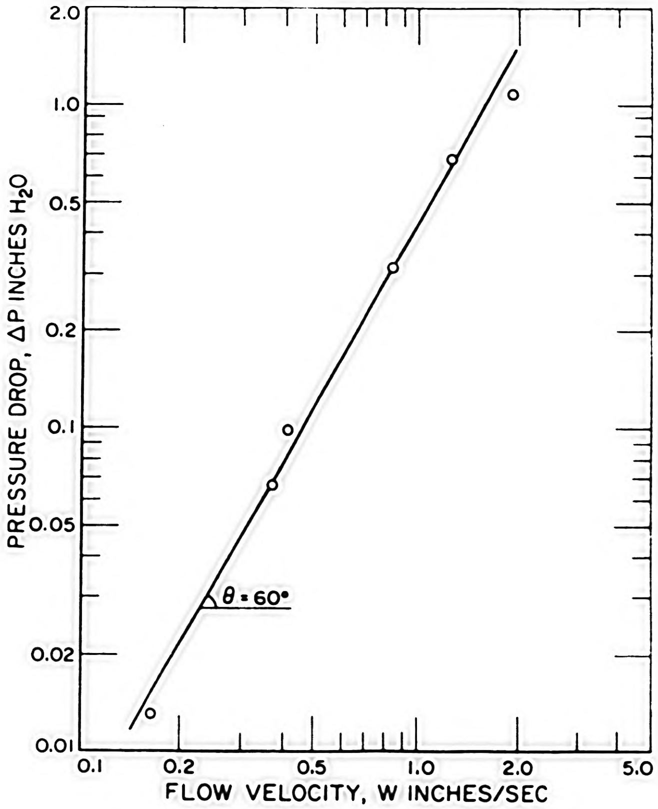


FIG. 3. PRESSURE DROP ACROSS ONE-INCH OF POROUS MATERIAL VERSUS FLOW VELOCITY.

the axis of the pipe, first in a streamwise direction a short distance downstream of the entrance and then in an upstream direction from a point downstream of the exit. As the pipe was rotated at relatively high swirl ratios ($\Gamma = N_{R\theta}/N_{Rz}$), the dye streak introduced at the entrance traveled a short distance along the axis and then, moving radially outward into the vicinity of the wall, reversed its direction. The dye streak introduced near the exit entered the pipe axially in the upstream direction and moved again to the vicinity of the wall, turning in this case into the direction of the flow. Fig. 2 shows some dye streaks observed during these experiments.

The end effects were eliminated when the pipe was equipped with porous plugs at both the inlet and outlet. These porous plugs are made of General Electric low-density (2-3% dense) "Nickel Foametal". They are 3-1/4 inch in diameter, the plug at the entrance being one-inch thick and the other being of a thickness of two inches. The plugs were matched to the inside diameter of the pipe and were cemented to it with RTV adhesive. The pore size of the material ranges from 0.020 inch to 0.100 inch and there are from 11 to 25 pores per lineal inch.

The upstream plug imparts solid body rotation to the fluid at the entrance. It can be arugued that if the axial pressure drop across that plug is large in comparison to the radial pressure gradient associated with the rotation, the flow downstream of the plug should be free of reverse axial components. Since the plug was of uniform thickness, the flow field downstream of it (up to some limiting swirl ratio) should approximate a combination of plug flow and solid body rotation. The limiting swirl ratio is governed primarily by the porosity of the plug.

where T_o is of the order of 2000°K to 5000°K while R_∞ ranges from 1 to 75,000 ohms.

The components of the constant current hot-thermistor anemometer used in the present study are shown in Figure 4. They include a 24-volt D.C. power supply (two 12-volt batteries connected in series), and a capacitance forming an R-C filter for the suppression of the noise due to the slip rings and brushes.

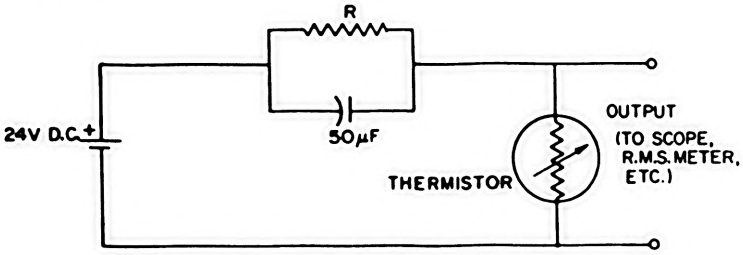


FIG. 4. CONSTANT CURRENT HOT-THERMISTOR ANEMOMETER UNIT.

Four different types of thermistor probes were used; they are shown in Figure 5 and their characteristics are:

(1) GE thermistor (Cat. 81B 202) with the thermistor bead of 0.043-inch maximum diameter encased in an insulating capsule. (R_o = 2000 ohms)

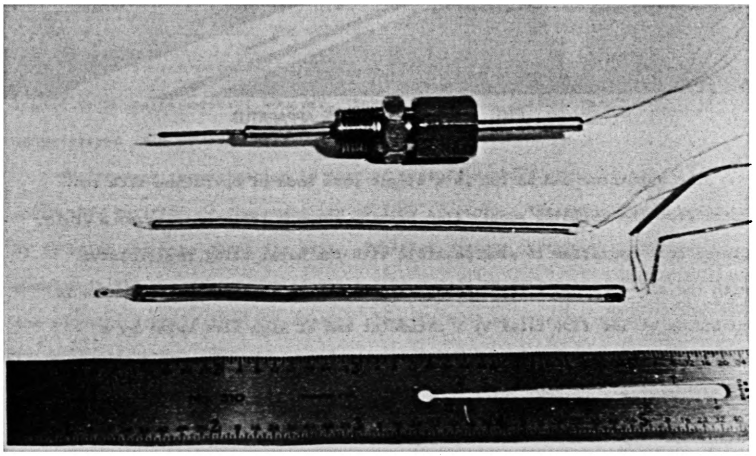
(2) GE thermistor (Cat. 81G 202) with the thermistor bead encapsulated in a glass rod of 0.100-inch diameter (R_o = 2000 ohms)

(3) VECO thermistor (Cat. P32A129) with the thermistor bead encapsulated in a glass rod of 0.060-inch maximum diameter. (R_o = 2000 ohms)

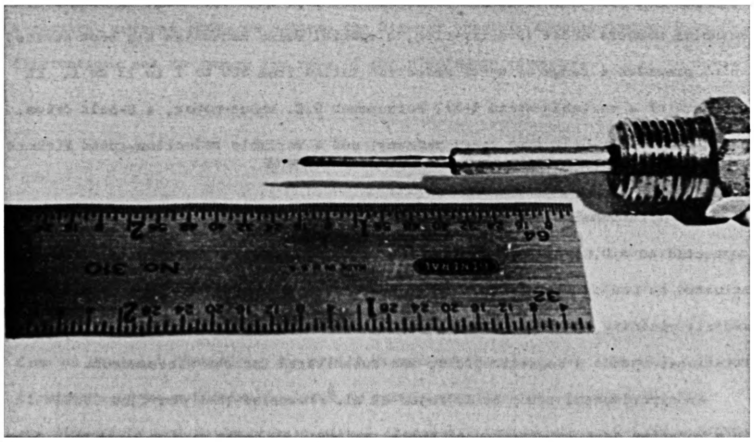
(4) VECO thermistor (Cat. AZ31A70) with the thermistor bead encapsulated in a glass rod of 0.020-inch maximum diameter. (R_o = 1000 ohms)

The sensors were attached to stainless steel tubing of various diameters ranging from 1/16 to 1/8 inch. These probes were then inserted into the pipe through teflon seals and the electrical leads from the sensors were soldered to the slip rings.

The slip rings and brushes are of simple construction. The slip rings consist of 1/2-inch wide brass bands cemented to the outside of the pipe with the ends joined by soldering. The brushes are two 1/2-inch wide ribbons made of brass screening. The ribbons are wrapped around the slip rings and connected to terminals located on the frame of the apparatus. Before a test the brushes were sprayed with water which filled the openings of the screening. A single application of water to the brushes is adequate for three hours of continuous operation. The noise produced by this slip ring system is of low intensity and can be easily suppressed with an R-C filter, since it is of a much higher frequency than the output of the thermistor probe.



A. PROBE 4, 1 AND 2 (FROM TOP TO BOTTOM)



B. PROBE 4 MOUNTED INSIDE TEFLON SEAL

FIG. 5. THERMISTOR PROBES

The thermistor leads were connected to the following instruments:

(1) Hewlett-Packard 3440A digital voltmeter equipped with a 344A D.C. multi-function unit; it measures the D.C. component of the output.

(2) DISA type 55D35 RMS unit measuring the root-mean-square

of the A.C. component of the output.

(3) Tektronix type 502A dual-beam oscilloscope monitoring the output.

(4) Tektronix type 564 storage oscilloscope equipped with type 3A3 dual trace differential amplifier unit and a type 2B67 time base unit; it stores the output for photographic recording.

(5) General Radio type 1564-A Sound and Vibration analyzer to analyze the frequency of the output.

A typical calibration curve for the probes is shown in Figure 6. The calibration curves are based on the photographs of hydrogen bubble streak lines produced by a fine wire located next to the thermistor probe being calibrated in the downstream open tank. Probe 1 was the first probe to be used. It was found that the output of that probe was strongly dependent on temperature and that the A.C. component of the output distorted the value of the D.C. component. Since a drift in the temperature of water is unavoidable when the system is operating in a recirculating mode with energy being supplied continuously to the system by the circulating pump, it was decided to use less sensitive probes for the mean velocity measurements. Probes 2 and 3 meet this requirement. They have a relatively low frequency response due to their size and therefore the distortion of the D.C. output is reduced. In addition, it was decided to use a higher resistance in the anemometer circuit with these probes (12 times larger than with Probe 1). Since the resistor is in series with the probe, small changes in the flow temperature, and hence probe resistance, are suppressed. Also, to compensate for the higher resistance of the circuit (48K ohm), the batteries were replaced with a 500 volts D.C. power supply (0.01% ripple). Calibration of Probes 2 and 3 showed that the revised system performed in a satisfactory manner. Flow field measurements made after these modifications are discussed in the following section.

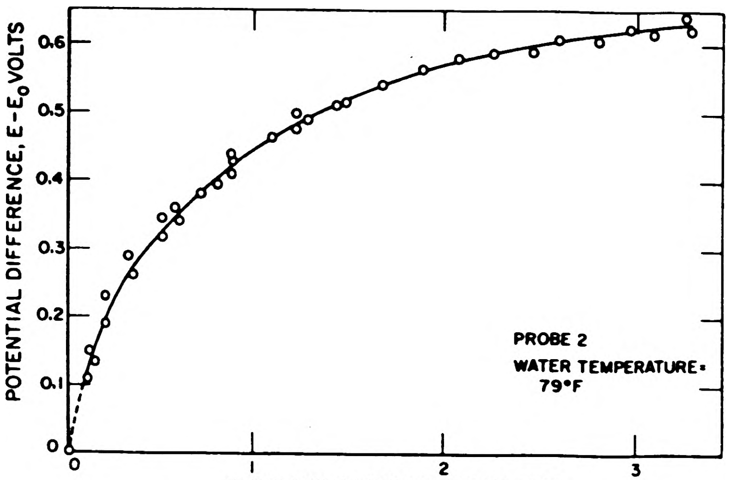


FIG. TYPICAL CALIBRATION CURVE FOR A THERMISTOR PROBE.

For the stability measurements Probe 4 was used since maximum possible frequency response is desirable in that case. This probe is the smallest commercially available, insulated probe. The size of the resistor (R) is limited here by the thermistor temperature which must not exceed the boiling point of the fluid at zero flow velocity.

The frequency response of the probes was determined with an octave band analyzer based on their output in turbulent flow. Even though limited in range, the response was deemed sufficient for the purposes of this investigation, i.e., for indicating transition from the laminar to the turbulent flow regimes.

The cut-off frequency for Probe 1 is approximately five cps, while it is approximately ten cps for Probe 4. The response of the other probes is almost an order of magnitude lower. These findings are in agreement with the calculated characteristics based on probe properties.

Velocity Surveys

A thermistor probe mounted radially in a pipe which is rotating around its axis while a swirling flow is passing through does not sense the tangential velocity of the flow if the fluid is in solid body rotation. And, in that case, the probe output remains unaltered by changes of the tangential Reynolds number as long as the axial velocity distribution is unaffected by these changes. Since within the range of the present investigation the flow is expected to be in solid body rotation in the rotating pipe, this simple technique was used for the exploration of the axial flow field.

Two procedures were used in these tests: first, the axial Reynolds number was kept at various constant values while the tangential Reynolds number was increased; then, the tangential Reynolds number was kept constant and the axial Reynolds number was changed. The thermistor output was found to be independent of the manner in which the tests were run, and for a given set of operating conditions the signal was invariant with time as long as the swirl ratio did not exceed the value of four and the flow was laminar. As mentioned earlier, this limiting swirl ratio reflects the condition that for solid body rotation of the fluid, the radial pressure gradient in the pipe must be considerably smaller than the pressure drop across the porous plugs. Velocity distributions were measured over a range of Reynolds number from 500 to 7000 near the downstream end of the pipe when the pipe was stationary. It was found that the dimensionless axial velocity profile was essentially independent of Reynolds number within the laminar regime and also within the turbulent regime while a significant change in the profile shape occurred during transition (Fig. 7). These axial velocity profiles were found to be independent of the tangential Reynolds number up to the limiting swirl ratio of four. These results agree with the findings of Talbot^6 and Laven et al.^{5}.

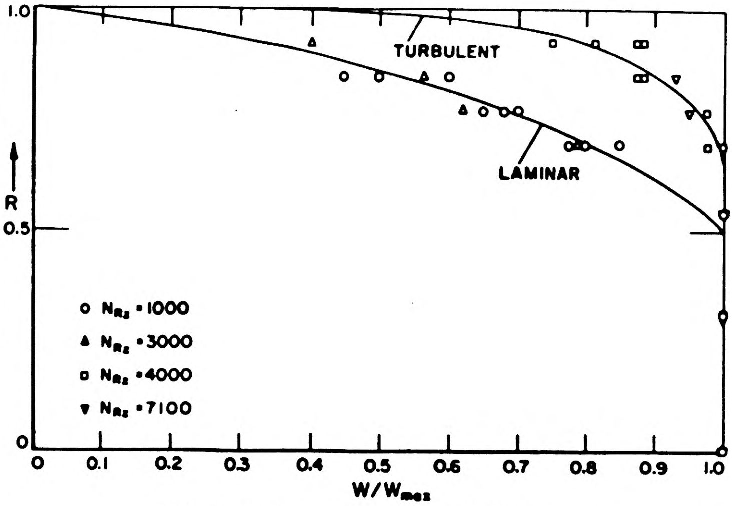


FIG. 7. LAMINAR AND TURBULENT MEAN AXIAL VELOCITY PROFILES.

Stability Measurements

Since the spectra of velocity fluctuations in turbulent flows cover a wide range of frequencies, transition from the laminar to the turbulent flow regime can be detected even with sensing elements having limited frequency response. And since the energy content of the fluctuations in the lower frequency range is usually large, probes responding to signals of low frequency are adequate. In the present study, thermistor Probes 1 and 4 were used for the stability measurements. Their maximum frequency response is approximately five cps and ten cps, respectively.

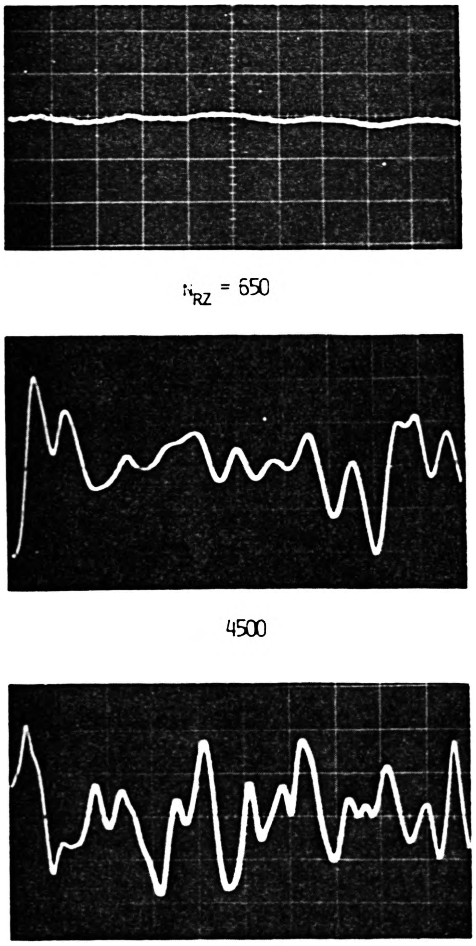
By registering the output of the probes with an oscilloscope and recording simultaneously the RMS value of the A.C. component with a true RMS meter, it was possible to establish the transition from the laminar to the turbulent flow regime. Typical oscilloscope traces at different flow conditions are shown in Figure 8 for a stationary pipe and in Figure 9 (using a different sweep rate than in Figure 8) for a rotating pipe at a fixed tangential Reynolds number and increasing axial Reynolds numbers. The results obtained were verified by visual diagnostic techniques utilizing dye streaks and hydrogen bubbles.

FLOW VISUALIZATION

Since flow visualization plays an important role in the present studies, it is appropriate to describe briefly the techniques that were used and how they were applied, and to report some of the results obtained.

Hydrogen Bubbles

The hydrogen bubble visualization technique was used for three purposes: (a) to calibrate the thermistor probes; (b) to establish the axial velocity profiles in the pipe; and, (c) to determine the axial Reynolds number at which transition from laminar to turbulent flow regimes occurs and to compare the results with those obtained using thermistors. The techniques used here are basically the same as those described by F. Schraub et al.[7] and W. Davis et al.[8]. The apparatus consisted of a D.C. power supply capable of supplying up to 0.1 amp at 500 volts; annealed platinum-rhodium (10%) 0.0015-inch diameter straight wire and 0.004-inch diameter kinked wire (0.1 inch pitch); and several 500 watt flood lamps used for lighting.



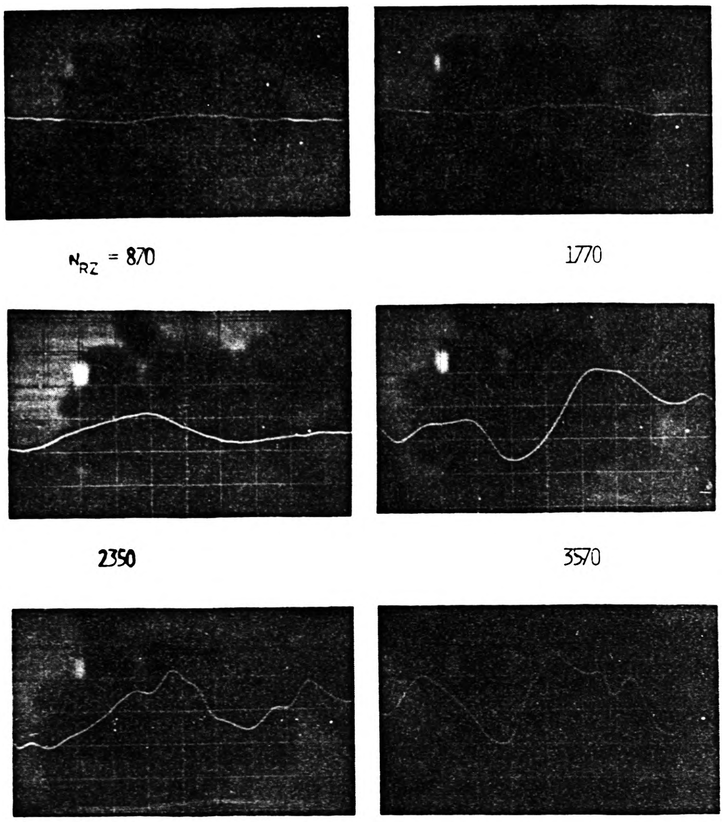
N_{RZ} = 650

4500

7000

FIG. 8. OSCILLOSCOPE TRACES AT INCREASING AXIAL REYNOLDS NUMBER WITH NO ROTATION ($N_{R\theta}$ = 0)

(SWEEP = 0.5 SEC/DIV; SCALE = 1 MV/DIV)



N_{RZ} = 870

1770

2350

3570

5570

7050

FIG. 9. OSCILLOSCOPE TRACES AT INCREASING AXIAL REYNOLDS NUMBER FOR A MODERATE ROTATION ($N_{R\theta}$ = 1735)

(SWEEP = 0.2 SEC/DIV; SCALE = 1 MV/DIV)

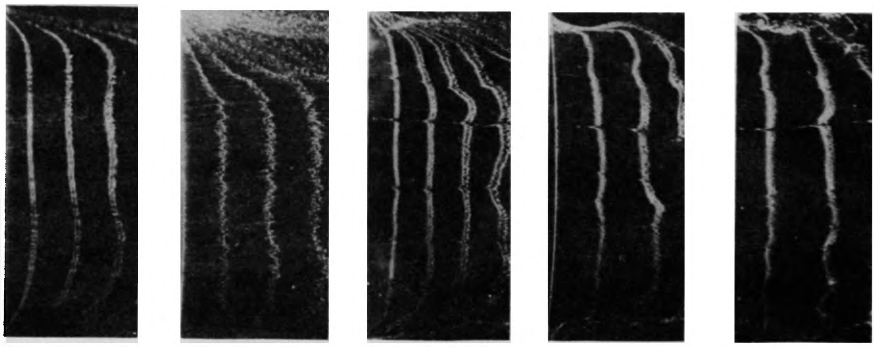
The thermistor calibration and determination of the velocity profiles were carried out with the straight wire which received a series of timed pulses of current of known characteristics. In the resulting photographs, shown in Figure 10, local axial velocities can be established from the distance between subsequent profiles and the pulse rate.

In exploring the transition from laminar to turbulent flow regimes, the streak lines produced by the kinked wire were employed. A series of photographs taken in the course of these experiments is shown in Figure 11.

Dye Streaks

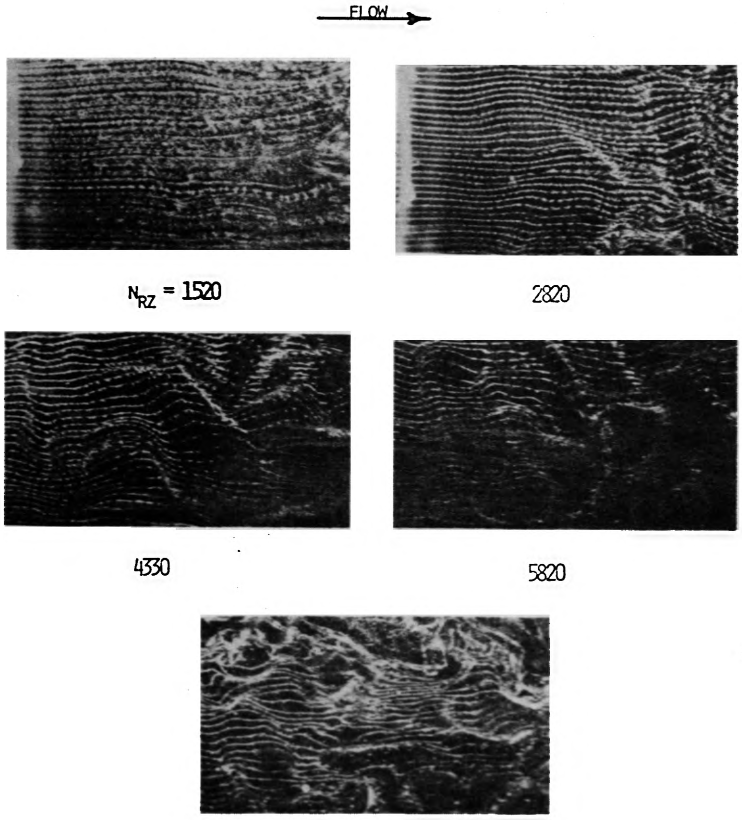
The observation of the flow field with the aid of dye streaks, a technique that probably originated with the unprecedented investigation of O. Reynolds[9] in 1883. was of great benefit in the present study.

The dye used consisted of malachite green crystals dissolved in water. Whenever necessary, alcohol was added in minute quantities to the solution to produce corrections in density. The dye was introduced into the rotating pipe parallel to its axis through four equally spaced probes positioned along a radial line, and passing through the upstream porous plug. The innermost one is at the axis and the outermost is located approximately 1/16 inch from the pipe wall. The probes are made of 0.026-inch inside diameter and 0.042 inch outside diameter stainless steel tubing. The dye is injected a small distance downstream of the porous plug. Since the plug and hence the probes rotate with the pipe, it appears that the dye is injected with no tangential velocity relative to the flow.



$N_{RZ} = 1520$ 2820 4330 5820 7100

FIG. 10. MEAN AXIAL VELOCITY PROFILES FOR INCREASING AXIAL REYNOLDS NUMBERS WITH NO ROTATION ($N_{R\theta} = 0$), OBTAINED BY USING HYDROGEN BUBBLES.

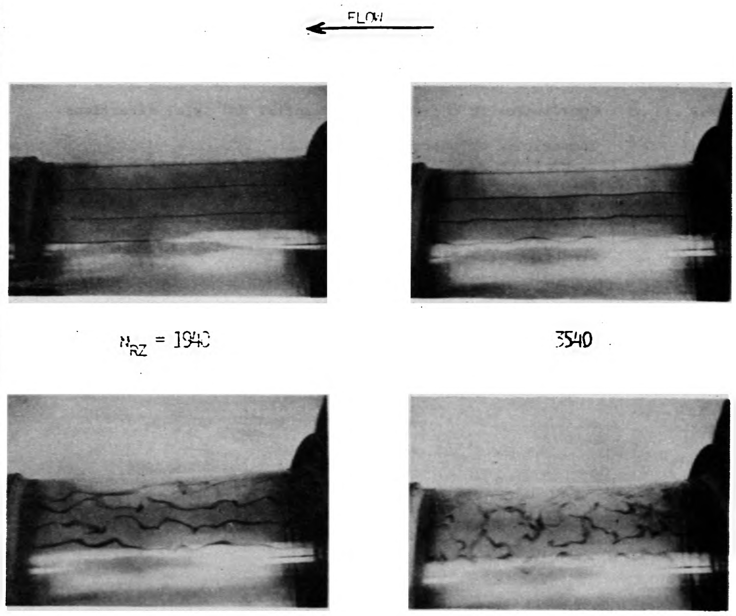


$N_{RZ} = 1520$ 2820

4330 5820

7100

FIG. 11 HYDROGEN BUBBLE STREAKS FOR INCREASING AXIAL REYNOLDS NUMBERS WITH NO ROTATION ($N_{R\theta} = 0$)



$N_{RZ} = 1940$ 3540

4500 5000

FIG. DYE STREAKS FOR INCREASING AXIAL REYNOLDS NUMBERS WITH NO ROTATION ($N_{R\theta} = 0$)

The dye streaks were used in connection with the measurements of the mean flow. When the fluid is in solid body rotation the streamlines having originated at points along a radial line, appear like a ribbon along the axis of the pipe. Otherwise the streak lines assume the appearance of concentric helical spirals of different pitch. Thus the dye streaks indicate the time required, after a change in operating conditions, for solid body rotation to be achieved.

The dye streak observations revealed that when a fixed tangential Reynolds number was used and the axial Reynolds number was increased, the time required to reach solid body rotation was substantially longer than in the case when the axial Reynolds number was fixed and the tangential Reynolds number increased. Such observations also corroborated the finding that in the present set-up (using the 2% dense porous plugs) solid body rotation could not be achieved when the swirl ratio is larger than four.

The dye streaks were also useful in verifying the onset of turbulence inferred from thermistor and hydrogen bubble data. This can be seen by comparing the thermistor data of Figure 8 with the streak lines of hydrogen bubbles and dye in Figures 11 and 12.

CONCLUSIONS

The present investigation demonstrates that thermistors are suitable to determine transition from laminar to turbulent flow regimes despite their limited frequency response. The main advantages of systems of this type are:

A. The high resistance of thermistors (compared to hot-wires and hot-film sensing elements) yields signals of high intensity permitting the use of inexpensive and convenient electronics (without the need for amplification) and of simple brushes and slip rings.

B. Due to the large temperature coefficients of electrical resistivity of thermistors, small velocity changes produce large variations in the signals, permitting easy filtering of the noise introduced by the brushes and slip rings.

C. Thermistors are rugged, inexpensive and commercially available.

D. The relatively small size of thermistors leads to compact probe design that minimizes the flow disturbances.

SYMBOLS

D	Rotating pipe inside diameter
E	Potential difference across thermistor
E_o	Potential difference across thermistor at zero flow velocity
L	Length of rotating pipe
N_{Rz}	Axial Reynolds number, defined as $\overline{W}D/\nu$
$N_{R\theta}$	Tangential Reynolds number, defined as $\omega D^2/2\nu$
ΔP_r	Radial pressure difference due to rotation
ΔP_z	Axial pressure drop across the porous plug
R	Dimensionless radius
R_e	Electrical resistance of thermistor
R_o	Electrical resistance of thermistor at room temperature
R_∞	Reference electrical resistance of thermistor
r,θ,z	Coordinates in the radial, tangential and axial directions
T	Temperature of thermistor
T_o	Reference temperature of thermistor
U,V,W	Velocity components in the r,θ and z directions
$\overline{W}$	Average axial velocity
Γ	Swirl ratio, defined as $N_{R\theta}/N_{Rz}$
ν	Kinematic viscosity
ω	Angular velocity

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