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FREQUENCY RESPONSE STUDIES FOR A WEDGE PROBE IN VISCOELASTIC FLUIDS\*

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ABSTRACT

The response of a hot-film wedge probe in viscoelastic fluids has been investigated by imposing on the probe a sinusoidal vibration of known amplitude and frequency. Root-mean-square (rms) velocities calculated from the displacement of the probe were compared to rms velocities obtained with a constant temperature anemometer. The tests were performed under turbulent flow conditions and also at flow rates where viscoelastic effects (i.e., decrease of heat transfer rates from the probe to the fluid and drag reduction) were observed.

The frequency range covered was narrow (< 100 cps). This limitation was imposed by the decision to superimpose the sinusoidal vibrations on the turbulence signal, in order to have dynamic conditions similar to those encountered in actual turbulence measurements. Measurements were performed in mineral oil and four solutions of polyisobutylene (Vistanex L-200) in mineral oil.

The experimental technique was established by measuring the response of the probe in mineral oil. These are the first data available in which the frequency response of a hot-film probe in a purely viscous liquid has been observed to be correct in the range studied. The ratios of velocities calculated using the two different methods were approximately 1.0. The results for viscoelastic fluids are similar with ratios ranging from 0.90 to 1.10. These results establish the validity of intensity measurements in viscoelastic fluids performed with hot-film wedge probes. They indicate that objections raised in the literature concerning the use of film probes in this type of fluid are not correct or, at least, not applicable to wedge probes.

INTRODUCTION

Metzner and Astarita<sup>1</sup> have raised objections to the use of velocity sensitive probes, such as hot-film and hot-wire probes, to measure turbulence intensities in viscoelastic fluids. Their objections are based on an elementary analysis of the effect of viscoelasticity on the heat transfer rate from heated surfaces in viscoelastic fluids. Their analysis predicts that at high velocities hot-film probes will lose all sensitivity to velocity due to a stagnation region developed in front of the probe. Based on this loss of sensitivity predicted for the probes, they questioned the use of hot-film probes in viscoelastic fluids. It is obvious that if there is a complete loss of sensitivity with a constant temperature anemometer the value of the root-mean-square voltage obtained will be zero (or almost zero, since there is always a certain amount of anemometer noise). The intensity of turbulence cannot be determined when loss of sensitivity of the probe occurs because

$$\frac{u'}{\bar{u}} = \frac{e'}{d\bar{E}/d\bar{u}}$$

and both  $e'$  and  $d\bar{E}/d\bar{u}$  are equal to zero. This type of behavior has been observed in certain polymer solutions for cylindrical<sup>5</sup> and wedge<sup>10</sup> film probes.

Moreover, one might also question whether intensities of turbulence in viscoelastic fluids calculated from D.C. voltage versus velocity calibration curves of unusual shape for wedge probes are correct, or if there are viscoelastic effects on the calibration curves which make intensities incorrect. It is also valid to question whether root-mean-square voltages obtained from the constant temperature anemometer are representative of velocity fluctuations in the fluid, or if there are viscoelastic effects on the voltage fluctuations. These are the two questions that need to be answered in order to establish the validity of the hot-film wedge probe as a measuring device in viscoelastic fluids. The object of this paper is to provide answers to these questions.

The experimental technique used consisted of applying a sinusoidal vibration to the probe in a steady flow and calculating the resulting relative velocity fluctuation by two independent methods: (1) by obtaining a voltage signal from the anemometer and (2) by measuring the displacement of the probe and assuming a perfect sinusoidal vibration. This type of measurement has not been reported in the literature for hot-film probes in liquids, although measurements have been performed with a wedge probe in air and with a hot-wire in water.<sup>2,3</sup>

Bellhouse and Schultz<sup>2</sup> measured the frequency response of a hot-film wedge probe in air by applying a sinusoidal motion to the probe. They observed a frequency response drop at a frequency close to 200 cps, and attributed this to a loss at higher frequencies of unsteady heat transfer from the probe to the air through the substratum on which the film was deposited. Their mathematical analysis predicts that this effect will be very small for liquids, due to the differences in the thermal impedance between liquids and gases. However, no measurements were obtained in liquids.

Delleur, Toebes, and Lin<sup>3</sup> performed experiments very similar to those reported in this paper with a hot-wire probe which was vibrated mechanically in a water jet. The authors indicated that experiments with a film probe were also performed, but no results were shown. They measured the response of the hot-wire in the frequency range 2-38 cps and indicated that there was a drop in the response as frequency increased at these low frequencies. Values of the ratio of the calculated velocity to that obtained from the probe displacement fell to 0.30. It is difficult to believe that this drop in response occurred, for one would not expect thermal inertia effects at these low frequencies based on the results of Bellhouse and Schultz and measured energy spectra<sup>4</sup>.

Thus, few results are available on the frequency response of hot-film probes in liquids, and none in viscoelastic fluids.

EQUIPMENT

The experimental equipment consisted of a 16-ohm, 30 watt loud speaker which was used to vibrate the wedge film probe inside a vertical 1-inch tube, a constant temperature anemometer, and a capacitive transducer system used to determine the displacement of the probe. The test fluids were pumped past the probe at a sufficiently high flow rate to prevent the probe from moving through its heated boundary layer.

The pumping unit used is shown in Figure 1. The unit consisted of an 8-inch diameter reservoir, a 200 gpm gear pump, a vertical 1-inch I.D. test section, and auxiliary equipment to monitor the flow and maintain a constant

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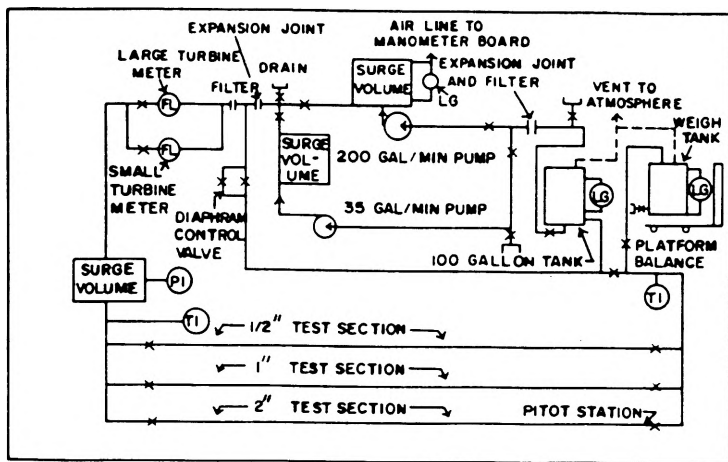


FIGURE 1. PUMPING UNIT

possibilities of end effects, which could affect the experimental results.

The wedge probe was connected through a 5 m coaxial cable to a DISA model 55A01 constant temperature anemometer. Overheat ratios in the range 1.05-1.10 were used in these experiments. The anemometer was equipped with a D.C. voltage suppressor which permitted D.C. voltages to be read accurately. This was required to establish accurate voltage versus velocity calibration curves.

The displacement of the probe was determined with a capacitive transducer. The transducer consisted of two sensing surfaces, one fixed and one movable. The capacitance between two surfaces is given by:

$$C = \frac{\epsilon A}{a}$$

where  $\epsilon$  is the dielectric constant of air,  $A$  is the area of the sensing surfaces, and  $a$  is the separation between the surfaces. The sensing surfaces were connected to a reactance converter that converted capacitance to voltage. A calibration curve of D.C. voltage versus displacement  $\Delta a$  was established, and from this curve values of root-mean-square (rms) velocities were calculated from the rms voltage measured by the reactance converter.

In order to establish the D.C. voltage versus displacement calibration curves, the sensing surface of the capacitance transducer that remained stationary during the frequency response measurements was moved with respect to the sensing surface attached to the probe holder, the position of which remained fixed. The stationary surface was mounted on a frame with two micrometers which were used to determine the displacement of the frame.

The speaker was driven with a signal generator that imparted a sinusoidal motion to the probe. The probe displacement was given by  $\beta \sin 2\pi nt$ , where  $n$  corresponds to the frequency at which the probe was vibrated. The instantaneous velocity of the probe was given by  $2\pi n\beta \cos 2\pi nt$ . Additional details of the experimental equipment are given elsewhere<sup>4</sup>.

fluid temperature. Expansion joints were used to isolate the reservoir and test section from the pump and eliminate vibrations. The flow was monitored by using a turbine flow meter in conjunction with a digital counter. A 50 micron filter was used to remove lint and dirt from the liquid. Constant liquid temperature was maintained by a heat exchanger placed inside the reservoir. Fluid temperature could be controlled to  $\pm 0.05^\circ\text{C}$ .

The reservoir and test section are shown in Figure 2. The test section consisted of a 1-inch I.D. seamless carbon steel tube which was placed inside the reservoir. The length to diameter ratio of the tube was 51. A 1-inch to 3-inch expander was welded to the top of the test section. The threads of the expander were removed and the inside surface smoothed. The expander

and test section were centered with respect to a flange welded to the top of the reservoir, and were held in position by three struts welded to the expander.

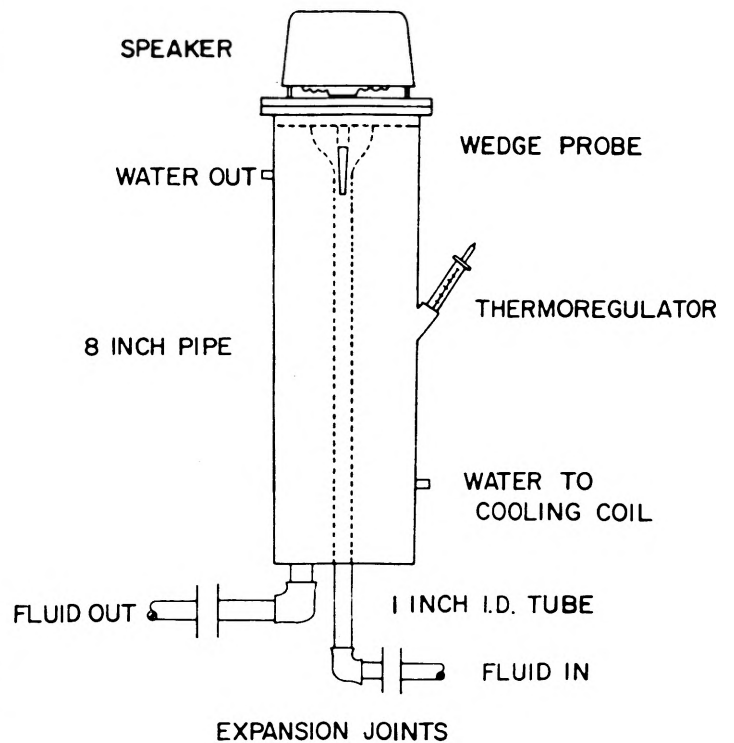


FIGURE 2. RESERVOIR AND TEST SECTION FOR THE VIBRATING PROBE

The speaker was bolted to a metal plate and the speaker and plate were bolted to the flange on top of the reservoir. The bottom of the metal plate consisted of a parabolic-shaped piece, especially designed to fit inside the expander. The metal plate was used to divert the fluid into the reservoir. A 0.337-inch hole in the bottom of the parabolic piece was used to introduce the probe into the test section. The sensing surface of the probe was centered inside the tube, 1- $\frac{1}{2}$  inches from the end of the tube. The length to diameter ratio of the entrance section was approximately 50, which is considered sufficient to develop the profile in the tube for purely viscous fluids. The probe was connected to a specially-designed probe holder which also provided the electrical connections for the displacement transducer and the anemometer system. The fluid was not subjected to a rapid deceleration as it left the 1-inch tube, since the expander had an inside diameter of 1- $\frac{1}{2}$  inches for a length of approximately 1.5 inches (measured from the end of the tube). This minimized the

#### EXPERIMENT DESIGN

Performance of the frequency response studies under conditions similar to those encountered under actual turbulence measurements in viscoelastic fluids was the objective. The fundamental variable was the velocity of the fluid moving past the probe since the rate of deformation of the viscoelastic fluid in front of the probe depends on velocity. It is known that large rates of change of the rate of deformation such that the characteristic time for the change of rate of deformation approaches the relaxation time of the viscoelastic fluid, will induce viscoelastic effects.

The measurements were performed under turbulent flow conditions, since the experiments were performed in the same size tube (1-inch I.D.) and at the

same velocities (7-15 ft/sec) used in other turbulence measurements.<sup>4</sup> Only if a much smaller tube or a much more viscous fluid had been used could the experiments have been performed under laminar conditions. Laminar flow would have simplified the data analysis since the sinusoidal signal due to the motion of the probe would not have been superimposed on the turbulent signal sensed by the anemometer. However, it was not possible to use a smaller tube since the diameter of the probe was 0.275 inches, and a smaller tube would have created experimental difficulties. The viscosity of the fluid would have had to be increased by factor of 10 (≈ 50 cp) in order to maintain laminar flow in the 1-inch tube at the desired velocity.

Since the sinusoidal signal due to the probe motion was superimposed on the turbulent signal, it was necessary to separate the two contributions to the total signal obtained from the anemometer. One alternative was to use a band-pass filter with an extremely narrow band width. However, this was not available and the sinusoidal component was determined by measuring the root-mean-square voltage due to the turbulent signal and that due to the sinusoidal signal. From the definition of root-mean-square quantities:

$$\overline{(e^2_{\text{turb}} + e^2_{\text{sin}})}^{\frac{1}{2}} = \overline{(e^2_{\text{turb}} + 2e_{\text{turb}}e_{\text{sin}} + e^2_{\text{sin}})}^{\frac{1}{2}}$$

Thus,

$$\overline{e^2_{\text{turb}} + e^2_{\text{sin}}} = \overline{e^2_{\text{turb}}} + \overline{e^2_{\text{sin}}}$$

if the correlation between the turbulent signal and the sinusoidal signal is neglected. For a perfectly random turbulent signal, the correlation  $\overline{e_{\text{turb}} e_{\text{sin}}}$  should be equal to zero.

If there is a certain periodicity to the flow, and if it is assumed that the periodicity has a frequency  $n^*$ , a contribution to  $\overline{e_{\text{turb}} e_{\text{sin}}}$  would be obtained only if the probe were vibrated at this frequency. If the total turbulent signal is of the same order of magnitude as the sinusoidal signal, the contribution from one particular frequency to the total turbulent signal will be much smaller than the sinusoidal signal, and thus the correlation  $\overline{e_{\text{turb}} e_{\text{sin}}}$  will be much smaller than  $\overline{e^2_{\text{sin}}}$ . (Furthermore, the correlation can be a positive, zero, or negative value depending on the phase angle between the periodic fluctuation in the turbulent signal and the sinusoidal signal). Thus, if there is periodicity in the flow, the error introduced should be small and random. Based on these considerations, the correlation term was ignored in calculating  $e'_{\text{sin}}$  from:

$$e'^2_{\text{sin}} = e'^2_{\text{turb}} + e'^2_{\text{sin}} - e'^2_{\text{turb}}$$

There were experimental limitations on the frequency range and the maximum velocity that could be investigated. The limit on the velocity was due to vibrations in the experimental system caused by the fluid striking the metal plate on top of the reservoir. Vibrations were greatly damped by the mass of the fluid in the reservoir at velocities below approximately 15 ft/sec. The limit on the frequency was due to the superposition of the sinusoidal signal on the turbulent signal. Since the amplitude of the sinusoidal signal was proportional to  $(1/n^2)$  for a given power input to the speaker, at high frequencies the amplitude of the probe motion was greatly decreased, and the sinusoidal signal could not be distinguished from the turbulent signal. It was found experimentally that the limit on the frequency was approximately 100 cps for the power of the speaker used.

Thus, the frequency response of a wedge hot-film probe was investigated under dynamic conditions similar to those encountered under actual turbulence measurements. Frequencies below 100 cps were investigated with the fluid velocity in the range 7-15 ft/sec. The viscoelastic solutions used

(PIB L-200 in mineral oil) were drag reducing in the 1-inch tube at the velocities investigated.

## RESULTS AND DISCUSSION

In order to determine the frequency response of the probe, the root-mean-square velocity of the sinusoidal fluctuations calculated from the anemometer data were compared to the root-mean-square velocity calculated from the known displacement of the probe. The rms velocity calculated from the probe displacement is given by:

$$v'_{\text{pd}} = 0.707 \beta (2\pi n)$$

where  $\beta$  is half the peak-to-peak displacement of the probe in feet. The rms velocity from anemometer data is given by:

$$v'_{\text{anem}} = \frac{e'_{\text{sin}}}{\frac{d\overline{E}}{dU}}$$

where  $d\overline{E}/dU$  is the slope of the D.C. voltage versus velocity calibration curve.

The ratio R, defined by:

$$R = \frac{v'_{\text{anem}}}{v'_{\text{pd}}}$$

was used as an indication of the response of the probe. The value of R should equal 1.0 for a perfect response.

Intensities of turbulence, defined by  $\frac{u'}{U}$ , were obtained at the same time the probe response studies were performed for mineral oil and the polymer solutions. It should be noted that  $v'$  is the rms velocity associated with the sinusoidal motion of the probe, while  $u'$  is the rms velocity of turbulent fluctuations. Turbulence intensities were calculated from:

$$\frac{u'}{U} = \frac{e'_{\text{turb}}}{\frac{d\overline{E}}{dU}}$$

The reason for performing the frequency response studies was to determine whether intensity values obtained with hot-film probes in viscoelastic fluids were correct, and if not, what corrections should be applied. Intensities of turbulence for mineral oil and 0.028, 0.05, 0.20, and 0.40% PIB in mineral oil obtained in this system are given in Figure 3. The turbulence intensities at the center of the tube for mineral oil were in the range 0.043-0.045, which agree satisfactorily with previous intensity

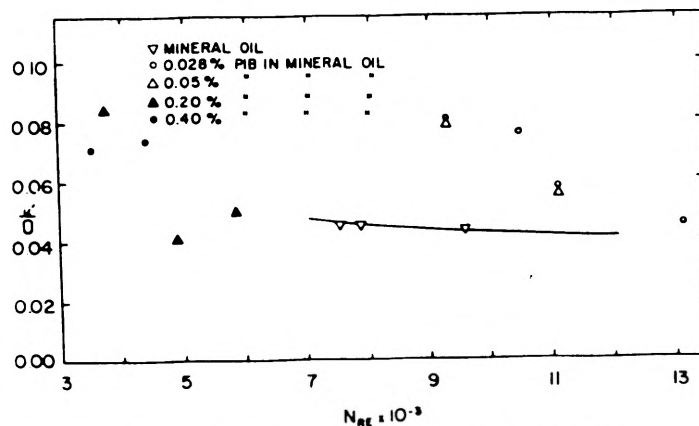


FIGURE 3. TURBULENCE INTENSITY VERSUS REYNOLDS NUMBER

measurements in the 1-inch tube in this Reynolds number range<sup>4</sup>. The data for the polymer solutions show intensities higher than for the solvent at low flow rates. Thus, the results of the response studies should establish whether these intensity values are correct or are due to viscoelastic effects on the probe.

Figure 4 shows the experimental results for mineral oil, presented as R versus frequency. These measurements were performed to establish the reliability of the experimental techniques used. Values of R at the highest frequencies investigated were mostly in the range 1.05-1.10. Intensity of turbulence measurements using constant temperature anemometry are usually good to  $\pm 5\%$ <sup>9</sup>. Thus, it is reasonable to expect a 5% error in  $v'_{anem}$ . Considering the assumptions made in the calculation of  $v'_{pd}$ , a total error of only 10% can be considered to be quite good.

At the lowest frequencies investigated, particularly at 5 cps, extremely large values of R were obtained. The values of R were 1.70 and 1.46 at 5 cps, and 1.26, 1.22 and 1.10 at 10 cps. These high values of R are due to experimental error caused by a tendency of the probe movement to become unstable at low frequencies, the tip of the probe was observed to move in a lateral motion when the probe was vibrated in air at low frequencies. Results below 20 cps are, therefore, invalid.

The results obtained in mineral oil provide confidence in the experimental techniques. The drop in frequency response observed by Delleur et al.<sup>3</sup> in their measurements with a hot-wire probe in water at very low frequencies is not observed in these data, and indicates that experimental error must have been introduced in their measurements. In accordance with Bellhouse's<sup>2</sup> predictions, the drop in frequency response at low frequencies observed for a vibrating hot-film wedge probe in air was not observed in these liquid measurements.

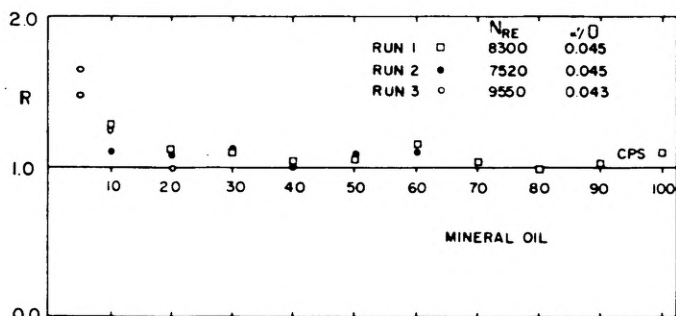


FIGURE 4. VIBRATING PROBE DATA

James<sup>5</sup> and Friehe and Schwarz<sup>6</sup> have obtained D.C. voltage versus velocity calibration curves for polymer solutions different from those obtained for purely viscous fluids. These results were obtained with cylinder hot-film probes. James observed a complete loss of sensitivity to velocity. Friehe observed a similar effect, but a regain in sensitivity was observed to occur at high velocities. These results appear to be characteristic of cylinder probes and are due to the development of a stagnation region in front of the cylinder. The regain in sensitivity observed by Friehe and Schwarz was probably due to the "washing away" of this stagnation region at very high velocities and not to "vortex shedding" as suggested by them. Friehe's energy spectra did not show regular vortex shedding, which would appear as peaks at characteristic frequencies. The stagnation region appears to develop from a balance between normal stresses and the impact pressure of the fluid against the cylinder. The normal stress developed in front of the cylinder may increase relatively slowly with velocity and since the impact pressure depends on the square of the velocity, the stagnation region in front of the cylinder is not formed at high velocities.

Figure 5 shows a typical calibration curve for a viscoelastic fluid obtained with a wedge probe. The fluid for this calibration curve was 0.028% PIB in mineral oil. This calibration curve does not conform to the equation  $E^2 =$

$A_1 + B_1 \bar{U}^{-0.50}$ , which calibration curves for purely viscous fluids usually follow. This type of calibration curve is also not predicted by Metzner's simplified analysis. Dimensional analysis considerations indicate that the calibration curves obtained for the wedge probe result from normal stress effects on the thickness of the boundary layer developed over the sensing surface of the probe<sup>4</sup>.

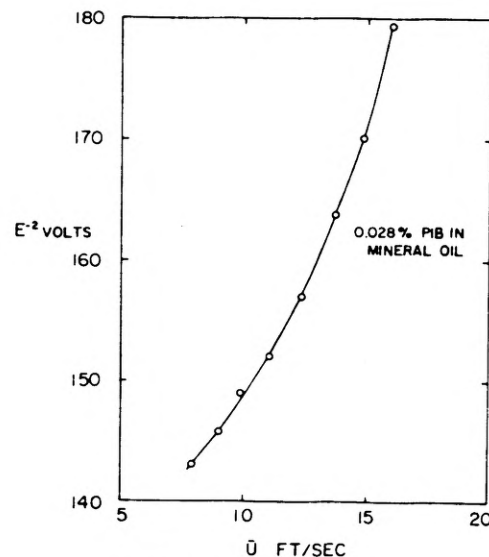


FIGURE 5. CALIBRATION CURVE

It is known from experimental measurements<sup>7,8</sup> that velocity profiles in viscoelastic fluids under drag reducing conditions differ from those for purely viscous fluids. These differences occur mainly because of a thickening of the boundary layer under drag reducing conditions. The maximum value of the ratio  $\bar{U}_{actual}/\bar{U}_{pv}$  at the center of the tube obtained by Florez<sup>7</sup> in his velocity profile measurements with highly drag reducing solutions was approximately 1.10. The effect of viscoelasticity on the velocity profiles was not considered in the present investigation since they are not well established. However, the error introduced is probably less than 10%, assuming that values of  $\bar{U} \frac{dE}{dU}$  can be obtained accurately from local curve slopes. If  $\bar{U}_{actual}/\bar{U}_{pv}$  remains constant over a short velocity range, no error is introduced in calculating  $\bar{U} \frac{dE}{dU}$ , even if there is an uncertainty in the velocity.

Results for the frequency response studies in the PIB in mineral oil solutions are given in Figures 6 and 7. The results for 0.05% and 0.20% PIB in mineral oil are representative of the results obtained for all solutions and only these will be discussed. Measurements were also performed for 0.028% and 0.40% PIB in mineral oil.

A total of 5 runs are shown in these figures. The results for all runs were similar. Values of R at the highest frequencies investigated were close to 1.0, and no trends existed for the deviations observed, indicating that these were due to experimental error. The low frequencies show the same trend observed at low frequencies for mineral oil. In the mineral oil results, deviations became significant at 10 cps. In the polymer solution data, large values of R were observed at 20 cps. The only difference observed between the mineral oil and the polymer solutions results is the small shift in frequency at which the large values of R start to occur. It is possible that the tendency of the probe to be unstable at low frequencies is enhanced by the viscoelasticity of the polymer solutions.



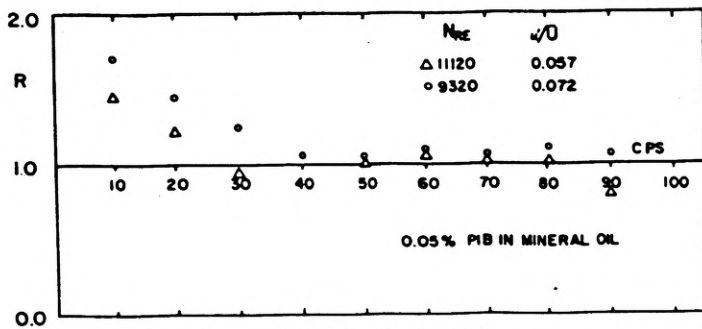


FIGURE 6. VIBRATING PROBE DATA

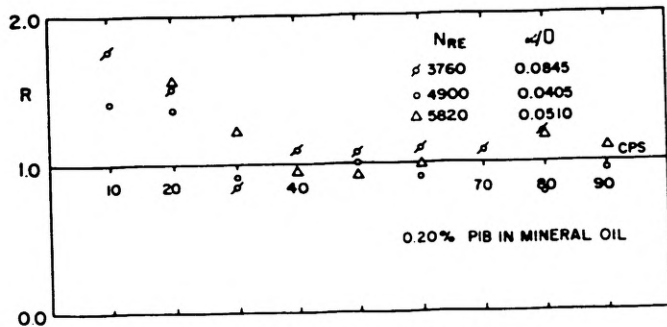


FIGURE 7. VIBRATING PROBE DATA

These high values of  $R$  were considered in detail. The values of  $R$  were independent of the level of turbulence intensity measured. Values of  $u'/\bar{U}$  given in Figures 6 and 7 and shown in Figure 3 indicate that intensities at  $N_{Re} = 11,120$  (for the 0.05% solution) and at  $N_{Re} = 4,900$  and  $5,820$  (for the 0.20% solution) were nearly normal, while intensities at  $N_{Re} = 9,320$  (for the 0.05% solution) and at  $N_{Re} = 3,760$  (for the 0.20% solution) were much higher than normal. However, the large values of  $R$  occur for both high intensities and normal intensities. Thus, it is not possible to associate large values of  $R$  with high intensities of turbulence. Filtering the turbulence signals at 20 cps indicated that approximately 10% of the signal was under 20 cps. This indicates that the high values of  $R$  obtained at low frequencies cannot account for the high intensities of turbulence observed in the PIB solutions which were approximately twice the normal values.

One might question whether low frequency vibrations might transfer energy to higher frequencies with the result that high intensities of turbulence might result from low frequency vibrations. Energy spectra were measured for the turbulent signal and for the turbulent signal with the sinusoidal vibrations superimposed. It was observed that the low frequency (as well as the high frequency) sinusoidal vibrations had no effect on the energy spectra, except in the bands covering the frequency at which the probe was vibrated. This indicated that there was no energy transfer mechanism taking energy from one frequency and transferring it to other frequencies.

#### CONCLUSIONS

It is concluded that the response of hot-film wedge probes in viscoelastic fluids is correct and that intensities of turbulence can be measured, even when the calibration curves for D.C. voltage versus velocity differ from those obtained for purely viscous fluids, provided the value of  $d\bar{E}/d\bar{U}$  can be determined accurately. The objections raised by Metzner and Astarita<sup>1</sup> have been shown not to be valid for wedge hot-film probes, and high intensities of turbulence observed for drag reducing viscoelastic fluids are not due to anomalous probe response.

#### SYMBOLS

A	area of sensing surface in capacitive transducer
$A_1$	constant in the equation $\bar{E}^2 = A_1 + B_1 \bar{U}^{0.50}$
a	separation between sensing surfaces of capacitive transducer
$B_1$	constant in the equation $\bar{E}^2 = A_1 + B_1 \bar{U}^{0.50}$
$\bar{E}$	D.C. voltage from the anemometer
e	instantaneous value of the fluctuating voltage
$e'$	root-mean-square value of the fluctuating voltage
n	frequency, cps
$n^*$	frequency of flow periodicity, cps
R	ratio of $v'_{anem}/v'_{pd}$
t	time, seconds
$u'$	root-mean-square value of turbulent velocity fluctuation, ft/sec
$\bar{U}$	average point velocity, ft/sec
$v'$	root-mean-square value of sinusoidal velocity fluctuation, ft/sec
$\beta$	one half of the peak-to-peak displacement of the probe
$\epsilon$	dielectric constant of air

#### SUBSCRIPTS

anem	obtained from the anemometer
pd	obtained from probe displacement
pv	related to a purely viscous fluid
sin	related to sinusoidal fluctuations
turb	related to turbulent fluctuations

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