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ABSTRACT

This is an exploratory study dealing with the effect of gas bubbles on the turbulence intensity of the liquid in a two-phase flow.

Distributions of time-mean and fluctuating velocities were measured in a two-dimensional channel. These were measured for single-phase flow and on the two sides of a bubble layer, which simulated the two-phase flow.

In single-phase flow the distribution of the time-mean velocities agrees well with other data, while the distribution of the intensity of turbulence is lower than expected.

The measurements of the time-mean velocity and turbulence intensity across the bubble layer revealed that they are discontinuous at that point. This supports the hypothesis that the presence of bubbles in liquid flows tends to lower the intensity of turbulence.

INTRODUCTION

Out of the voluminous literature on two-phase flow there seems to be only a small fraction focused on the basic turbulent characteristics of the flow. The difficulties involved in an investigation of this sort are both theoretical and experimental. Some studies of two-phase flow are summarized here and a method for determining the effect of bubbles on the turbulence structure of the main flow is presented.

The theoretical studies were started by Tchen¹, who derived an equation for a solid particle suspended in an unsteady velocity field, by extending the solution of Basset-Boussinesq-Oseen to the Lagrangian equation of particle motion. For the i -component of flow, in the Stokes-law regime, he obtained:

$$\frac{4}{3} a^3 \rho_d \dot{u}_{di} = 6\pi\mu a (u_i - u_{di}) - \frac{4\pi a^3}{3} \frac{\partial p}{\partial x_i} + \frac{2\pi a^3 \rho}{3} (\dot{u}_i - \dot{u}_{di}) + 6a^2 \sqrt{\pi\mu} \int_0^t \frac{\dot{u}_i(\tau) - \dot{u}_{di}(\tau)}{\sqrt{t-\tau}} d\tau + F_i \quad (1)$$

where a is the radius of the particle, μ is the viscosity of the continuous fluid, p is the pressure, F is the external field force acting on the particle, u and u_d are the velocities of the fluid and particle, ρ and ρ_d are the mass densities of the fluid and particle, respectively. The dot denotes the material derivative following the motion of the particle, i.e.

$$\dot{} = \frac{D}{Dt} = \frac{\partial}{\partial t} + u_{di} \frac{\partial}{\partial x_i} \quad (2)$$

Corrsin and Lumley² suggested that the pressure gradient term in Equation 1 may be expressed as:

$$\frac{\partial p}{\partial x_i} = \rho \frac{Du_i}{Dt} - \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (3)$$

Hinze³ further suggested to consider particles of small size and to limit the discussion to particles in uniform fields. In this case we have:

$$\frac{u_d}{a^2 (\partial^2 u / \partial x_j^2)} \gg 1 \quad (4)$$

With Equations (3) and (4), Equation (1) simplifies considerably and is more amenable to solution. Hinze solved the resulting equation for some limiting cases, and represented his results as the ratio of the Lagrangian energy spectrum of the particle to that of the flow field. For a solid particle in a turbulent airflow under atmospheric condition, this ratio is shown to decrease from unity to about 0.2 with increasing frequency. This result implies that the turbulent fluctuations of the particles have lower amplitude than those of the main stream. One may, therefore, intuitively conclude that the presence of discrete particles in fluid flow tends to lower the intensity of turbulence of the main stream.

Similar results were obtained by Chao⁴, who solved Tchen's equation by using Fourier transformations. His solution applies for simplified systems where the suspensions are dilute, there are no wall effects and where Stokes' drag law is applicable. With these assumptions Chao obtained a ratio between the energy spectra and the turbulent intensities of solid particles and those of the undisturbed flow field. His results also indicate that since the particles fluctuate less than the undisturbed field, they tend to lower the intensity of turbulence of the continuous phase of a two-phase flow.

The theoretical results of Hinze³ and of Chao⁴ were extended by Hetsroni and Sokolov⁵. They calculated the ratio between the energy spectrum of a two-phase (air - liquid droplets) jet to that of a single-phase air jet, and showed that this ratio is smaller than unity. Hetsroni and Sokolov also measured the turbulent properties of a two-phase axially-symmetrical jet and found that the turbulence intensities were lowered due to the presence of the discontinuous phase. They showed that the velocity profile of the jet becomes less blunt as the flow rate of the discontinuous phase increases.

Other experimental studies with two-phase (air-liquid) flows dealt with the gross effect of the liquid droplet on the characteristics of the main flow.

Gill, et al.⁶ investigated air upflow in a vertical tube. They found that when liquid droplets were added to the main flow, the gas velocity profiles became less blunt. The velocity profiles still obeyed the logarithmic law, but for higher rates of liquid flow the constant in this law diminished from its classical value, i.e. the velocity profile could be described by von Karman's velocity-deficiency-law

$$\frac{u-u}{u} = \frac{1}{k} \ln y/r_0 \quad (5)$$

with $k = 0.2$ to 0.25 compared with $k = 0.4$ for the single-phase flow.

Experimental observations carried out by Dr. Hassid and his group in Italy⁷ further substantiated these findings. They measured the linear velocity of the gas in a gas-liquid mixture, and found that the profile can be described by Equation (5), with k varying from 0.27 to 0.31.

Since k is a measure of the turbulence of the main flow, one may conclude that the results of Hinze and Chao were qualitatively verified. However, no serious experimental attempt was made to find an exact quantitative correlation between the flow rate and properties of the discontinuous phase and the turbulence of the main stream.

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Hinze³ also studied the case when the discontinuous phase is much less dense than the main stream e.g. small gas bubbles in turbulent liquid. He concluded that for this case the intensity and the amplitude of the fluctuating-bubble motion are greater than those of the carrying fluid. Chao also arrived at similar conclusions, but did not elaborate on the subject. From this, one may conclude that the effect of small gas bubbles is to increase the intensity of turbulence of the main stream. This conclusion does not seem to us physically reasonable, and we will discuss it further in a later section.

There is, however, no data known to us on the basic characteristics of two-phase (liquid-bubbles) flows. This paucity of data is mainly due to experimental difficulties, as will be outlined below.

The measurements of references 6 and 7 were conducted with isokinetic probes, which are essentially similar to total-head-impact tubes, with the pressure inside the probe being equal to the pressure outside it. This is achieved by applying suction on the probe and measuring the pressure required to reach such an equilibrium.

Isokinetic probes are well suited for measurements of time-average characteristics such as linear velocity and concentration of droplets.

However, these probes are restricted in use for several reasons:

- a) they are primarily useful for measurements of time-average properties.

Measurements of intensity of turbulence, for example, are impossible with these probes;

- b) the isokinetic probes measure space-average values because of their relatively large size. Wherever velocity gradients prevail, space averaging is undesirable.

Hot-wire and hot-film techniques overcome some of the difficulties of isokinetic sampling. The small size of the sensing element makes local measurements feasible, in addition to having short response time for measurements of fast turbulent fluctuations. Goldschmidt^{8,9} successfully used a constant-temperature hot-wire anemometer for measurement of aerosol concentration and flow characteristics of a two-phase plane jet. Hetsroni and Sokolov¹⁰ measured turbulent properties of a two-phase (air-droplets) axially symmetrical jet using a somewhat simpler technique. These studies were all conducted in systems where gas was the continuous phase, which makes hot-wire anemometry applicable. Hsu, et al.¹¹ used hot-film techniques simultaneously with a cinematographic method for measuring void fraction in a two-phase water-air system. They found good correspondence between the signal obtained from the probe and the visual observations. Their study was, however, more qualitative and no conclusive results were obtained. Delhaye¹² used hot-film techniques, together with more advanced electronics, to find the local void fraction. He then compared the average of his results with the mean void fraction obtained by γ -ray attenuation method, and found good agreement. The main difficulties involved in quantitative measurements of the turbulent velocities, in the liquid phase of a two-phase system, arise from the fact that bubbles tend to collect on the hot-film probe, even with very low heating ratios. These bubbles change the heat transfer characteristics of the probe and make calibration quite impossible.

The aim of this exploratory study was to devise an experimental set-up in order to overcome these experimental difficulties, and determine the effect of gas bubbles on the turbulence characteristics of the liquid flow.

EXPERIMENTAL

The experimental system was designed to facilitate measurements of

time-average and fluctuating velocities of a two-phase (liquid-bubbles) flow, while minimizing the usual experimental difficulties. This was achieved by producing a thin layer of bubbles in a two-dimensional liquid flow. The effect of the bubbles on the flow field was determined by measuring the turbulence intensity of the liquid on the two sides of the bubble layer, without performing any measurements in the layer itself. Since the hot-film probe was never really exposed to two-phase flow, its calibration was not affected. Furthermore, since the flow rate of the bubbles was negligible compared with the liquid flow rate, the bubble-layer exerted no other effects on the flow - such as changing the pressure drop, etc.

Test Loop

The test loop, shown in Figure 1, consisted of the test section, tubing, storage tank, a centrifugal pump and water treatment units.

The test section, Figure 2, was a 40±0.4 mm high, 600±0.5 mm wide and 2400±1 mm long parallelepiped, constructed of plexiglass and anodized aluminum. The length of the test section was such that a fully developed velocity profile was attained at a location less than halfway downstream. The aspect ratio of the test section was 1:15, which was considered sufficient for securing two-dimensional flow.

The hydrogen bubble-layer was produced in the test section by electrolysis on a thin stainless steel wire. The wire was located across the test section at a distance y_1 from the top plate. This wire was connected to the cathode of a DC power supply, while the anode was a stainless steel strip located at an arbitrary location downstream of the wire. The wire was 0.1 mm in diameter and the voltage needed to produce a continuous layer of bubbles was about 100 volts. The diameter of the bubbles was estimated to be 0.05 mm¹³ and their life-time about 3 sec. The rising velocity of the bubbles was calculated and found to be 0.1 cm/sec, implying that the bubble-layer was essentially parallel to the walls of the test section, at axial flow rates of the order of 1.0 m/sec. This was also visually verified.

The wire itself (with no bubble) did not affect the velocity profile at the location of measurements, which was a few hundred wire-diameters downstream.

The tubing and storage tank were made of PVC and all the valves and fittings were made of inert materials such as neoprene. This was done in order to maintain a highly clean system, with water of high and constant resistivity. Furthermore, a bypass flow was continuously circulated through an ion exchange column and a micronite filter.

A jet pump at the top of the storage tank was used to extract gases from the water. It was observed during operation that the water flow did not contain any gas bubbles, except for the bubble layer in the test section.

The pump was a well balanced centrifugal horizontal pump made of stainless steel. Its capacity was 500 m³/hr with 15 m. head. The pump was installed on a separate base and it was isolated from the rest of the loop by means of flexible rubber connections, in order to eliminate vibrations and noise.

Instrumentation

The total flow rate through the test section was measured by means of two stainless steel orifice plates, located in the two pipes connecting the test section to the storage tank. The orifices were carefully calibrated using a weighing tank. Their calibration accuracy was better than 1%.

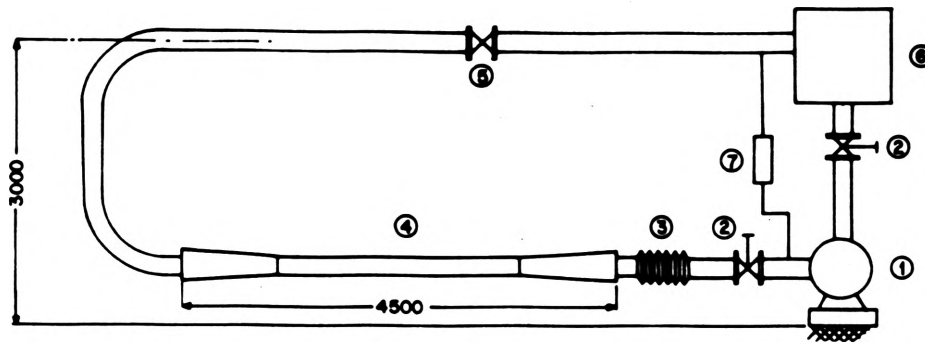


Figure 1 The test loop.
1) the pump 2) valves 3) flexible connections 4) the test section 5) orifice plates 6) storage tank 7) ion exchange column and filter units.

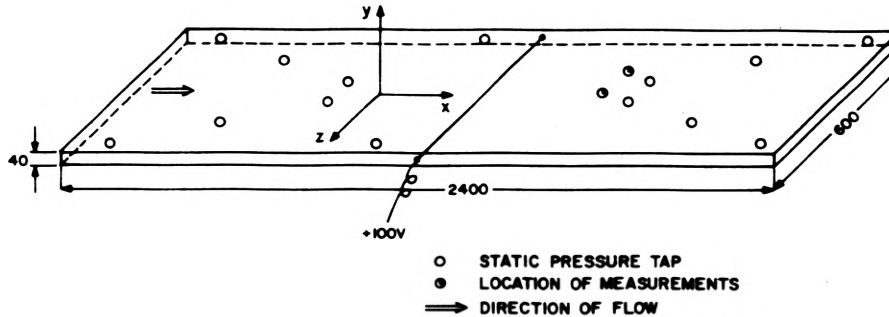


Figure 2 The test section and coordinate system. The dimensions are in millimeters.

The static pressures at the test section were sensed by means of seven pairs of pressure taps located on the upper plate of the test section. These pairs were located symmetrically around the centerline of the test section (at $z=0$) and were staggered in such a manner that no pressure tap affected the next ones downstream. Also, the upper plate of the test section was machined smooth after drilling the holes and installing the pressure taps and other penetrations in the upper plate, in order to eliminate burrs and other disturbances. The static pressures were measured by means of an accurate manometer (with nominal accuracy of ± 0.05 mm) which contained a standard fluid.

The velocity distribution in the test section was determined by means of a Pitot tube and a hot-film probe. Both probes penetrated the test section through the top plate and were attached to micrometers in order to ascertain their height in the test section with an accuracy of ± 0.02 mm.

The Pitot tube was a standard one, made of stainless steel, with 2 mm O.D. and 1 mm I.D. The accuracy of the 90° bend was found to be within $\pm 1/4^\circ$. The Pitot was connected to a U-tube manometer, made by R. Fuess, with a nominal accuracy of ± 0.05 mm.

The hot-film probe was a type 1236 W quartz coated wedge probe, made by Thermosystems, Inc. The probe was connected to a type 55D01 constant-temperature anemometer made by DISA. The output from the anemometer was measured by means of a Hewlett-Packard DC voltmeter and a Bruel and Kjaer true RMS voltmeter.

The hot-film probe was calibrated in place using the Pitot tube as a standard. The calibration curve was found to be a straight line on log-log paper, in the range of velocities studied. The calibration was checked several times during each run.

Preliminary Tests

At the start of the experimental program the dimensions of the test section were verified. Furthermore, the two-dimensional properties of the flow were confirmed by traversing a probe along the z -axis of the test section, i.e. parallel to the upper plate and perpendicular to the flow direction. The flow was found to have a uniform velocity within $\pm 1.5\%$. Later, the velocity distribution in the y -direction was measured at a few locations along the stream. The profiles at the location of measurements and further downstream were substantially constant and it was concluded that the velocity profile was fully developed at the location of measurements.

The pressure gradient along the test section was then measured and compared with the literature. The pressure readings at any two taps of one pair were identical (another indication of the two-dimensionality of the flow), and the pressure gradient along the test section was linear and checked standard correlations.

Finally, the time-average velocity distribution across the test section were measured for various Reynolds numbers by means of the Pitot tube. The velocity profiles are depicted in Figure 3 for Reynolds numbers range 81,000 to 250,000. These profiles can be represented by von Karman's logarithmic law (in this form they are independent of the Reynolds number), i.e.

$$u^+ = A + B \log y^+$$

where $u^+ = \frac{u}{u^*} = u \sqrt{\frac{\rho}{\tau_0}}$ - the friction velocity

$y^+ = \frac{yu^*}{\nu}$ - the friction distance parameter

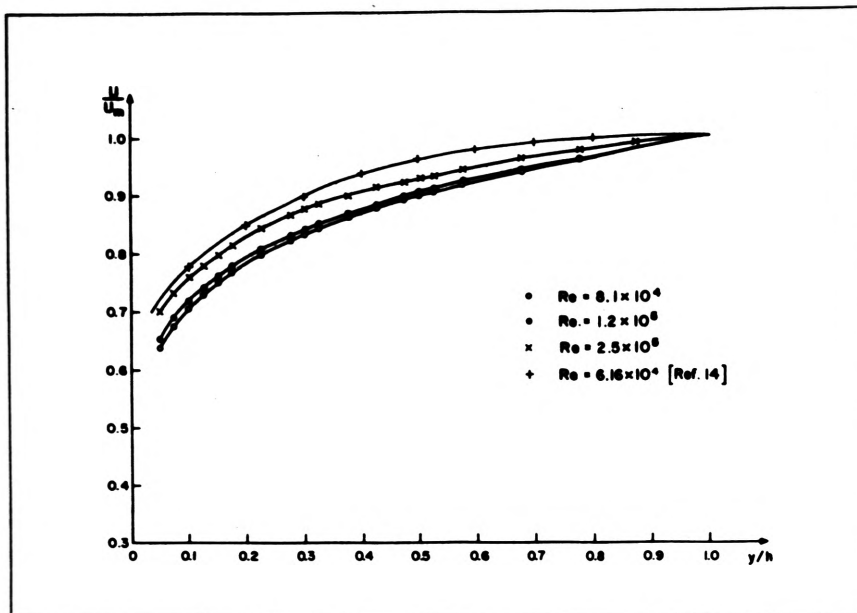


Figure 3 Distributions of time-average velocities, for various Reynolds numbers.

with constants $A=5.5$, $B=6.51$ for $y^+ > 40$. These constants may be compared with Laufer's¹⁴ values $A = 5.5$ and $B = 6.9$. Notice, however, that Laufer's data were measured in air with Reynolds numbers ranging from 12,300 to 61,000. Laufer's data for $Re=61,000$ is also included in the figure for comparison purposes. It can be noted that his data is somewhat higher than our data, which is also implied by his coefficient B which is higher than ours.

Experimental Procedure

Prior to each test the loop was operated until reaching constant temperature (usually around 52°C). The flow rate was then measured by means of the orifice plates and a well-type manometer.

Each test was begun by calibrating the hot-film probe in place, with the Pitot tube serving as a standard.

The static pressure along the test section was measured and recorded. The pressures indicated by the two taps of any pair were always within 0.4% of each other.

The distributions of the time-mean velocities \bar{u} and fluctuating velocities u' were then measured by means of the hot-film probe. The time-mean velocity \bar{u} was sensed by the probe and measured with the DC voltmeter. This distribution was then integrated and compared with the total flow rate as measured with the orifice plates. The agreement was better than $\pm 1.4\%$. The accuracy of measuring the time-mean velocities was estimated at $\pm 1\%$.

A total of four flow rates was used for the experimental program i.e. 52, 69, 82 and 126 m³/hr. These flow rates correspond to an average velocity in the test section of 0.6, 0.8, 0.95 and 1.46 m/sec and to Reynolds numbers of 8.1×10^4 , 11.0×10^4 , 13.0×10^4 and 20.0×10^4 , respectively.

For each flow rate there were three tests made: the first test was with no bubbles generated on the wire, i.e. with single-phase flow; the second test was performed with a bubble layer at a distance $y_1=5.5$ mm (i.e. $y_1/h = 0.275$) from the upper wall and the third test with $y_1 = 10$ mm (i.e. $y_1/h = 0.5$).

RESULTS AND DISCUSSION

The test results are depicted in Figures 4 through 11.

In figure 4 the distribution of time-average velocity for the single- and two-phase flows are compared. In the upper half of the figure the bubble-layer is located at $y_1/h=0.275$, while the lower half of the figure is for the case when $y_1/h=0.500$. The Reynolds number was 81,000. It can be observed from this figure, and the corresponding figures for higher Reynolds numbers, that there is a discontinuity in the curve, at the location of the bubble-layer. At this discontinuity no measurements were performed (in order to avoid bubbles collecting on the probe), but one may observe that the velocity gradient between the wall and the bubble-layer is less steep than for the single-phase flow. At the bubble-layer itself the velocity gradient is steeper than for the single-phase case. This result, however, is not conclusive and should be further investigated.

In Figure 5 the distribution of the fluctuating velocities of the single-phase flow is compared to that in the presence of the bubble-layer. In this figure, as well as corresponding figures for higher flow rates, u' indicates the root-mean-square of the fluctuating velocities in the longitudinal direction i.e. $u' = (\overline{u'^2})^{1/2}$. The fluctuating velocities are given as the ratio of u' to the time-mean velocity at the center of the channel U_m .

It should be noted in Figure 5 that although the data is self consistent and was repeatable, it is lower by a factor of roughly three than the data of Laufer¹⁴. This discrepancy may be due to the poor frequency response of the hot-film probe or some systematic difficulty in the instrumentation - but no complete explanation can be offered at present, and the subject is being further investigated. The main point brought out by the data of Figure 5 is, however, quite independent of the absolute magnitude of the velocity fluctuations.

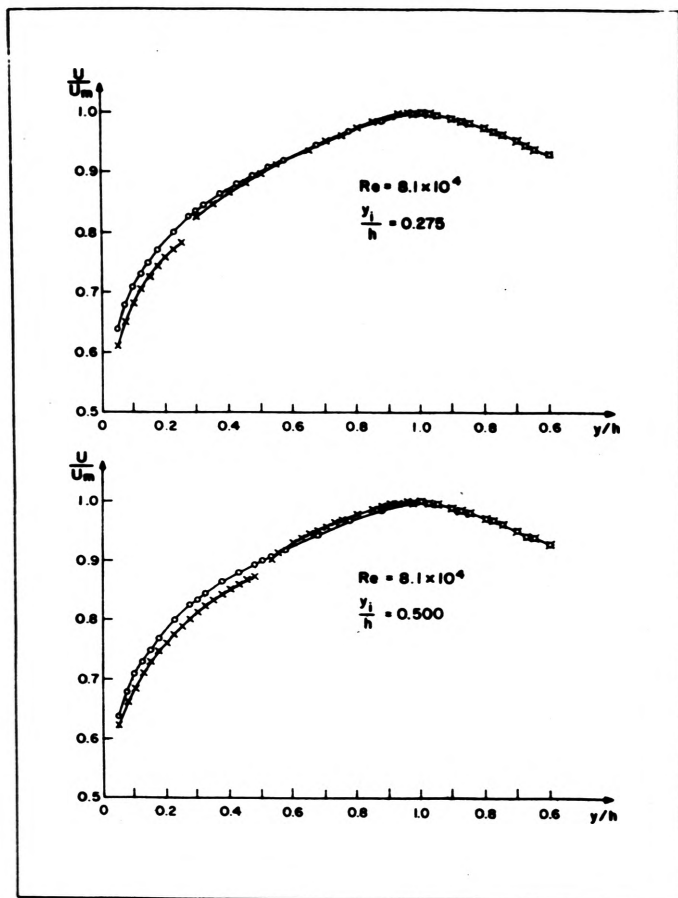


Figure 4 Distribution of time-average velocities, O - with no bubbles, x - with the bubble-layer.

The point of interest in Figure 5 is that the intensity of fluctuations u' is lower in the presence of the bubble-layer than in a single-phase flow. Furthermore, it can be seen that the gradient of the u' distribution is higher at the location of the bubble-layer than at a corresponding location when there are no bubbles. This effect is more pronounced at the upper half figure, namely when the bubble-layer is closer to the upper wall. Since these effects are only apparent in the part of the channel where the bubble-layer is, one can conclude that the fluctuations of the longitudinal velocity are reduced due to the presence of bubbles. One may hypothesize that in general the intensity of turbulence of liquid flow is reduced due to the presence of bubbles. This hypothesis appears to be in contrast with the theoretical conclusions discussed previously. However, the theoretical analysis was based on the tacit assumption that the bubbles are small and spherical, and the only effects which were considered were those due to the difference in density. In physical systems bubbles are neither very small nor do they maintain their spherical shape. Rather, bubbles change their shapes in a complex manner, as a result of external forces exerted on them by the flow field¹⁵.

It is therefore tentatively suggested that the results of the experiments can be interpreted in the following manner: the bubbles, being amenable to change in shape due to field forces, reduce the intensity of turbulent fluctuations of the main liquid flow. Since this effect is the dominant one, rather than the difference in density, the analyses of Hinze and Chao do not apply to the system under consideration. Accompanying the reduced intensity of turbulence, the distribution of time-mean velocities is also changed.

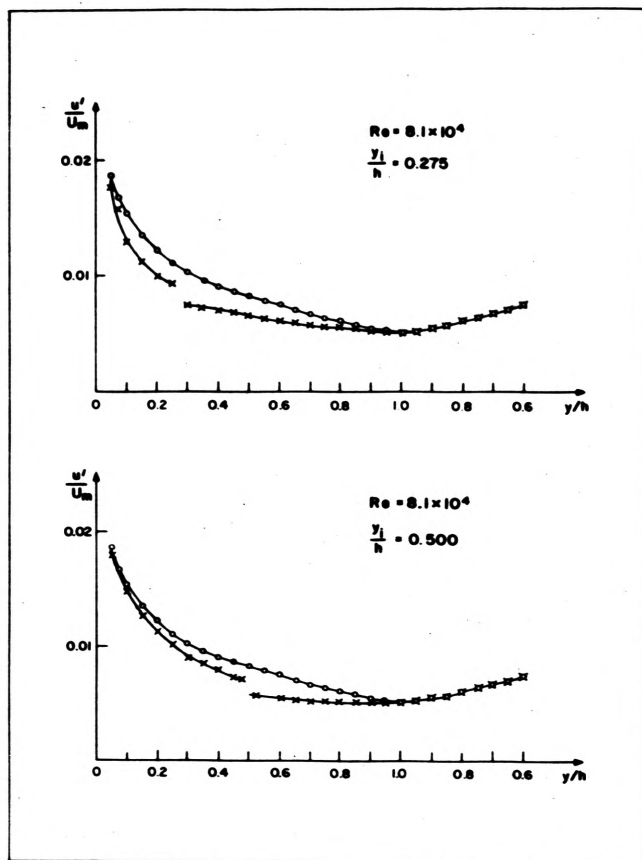


Figure 5 Distribution of fluctuating velocities, O - with no bubbles, x - with the bubble-layer.

In figures 6 through 11 similar data are presented, for increasing Reynolds number. While the basic effects which were observed for $Re=81,000$ and which were discussed previously, are still observable, yet their size diminishes as the flow rate increases. This fact is quite understandable if one realizes that the rate of production of bubbles is constant. Therefore, as the flow rate increases the bubble-layer becomes thinner and less effective. For the highest flow rate, with $Re=200,000$, the effect of the bubble layer on the flow is hardly noticeable - as in Figure 11.

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SYMBOLS

a	radius of solid particle
k	a constant in velocity deficiency law
p	pressure
r_0	dimension of conduit
u	fluid velocity

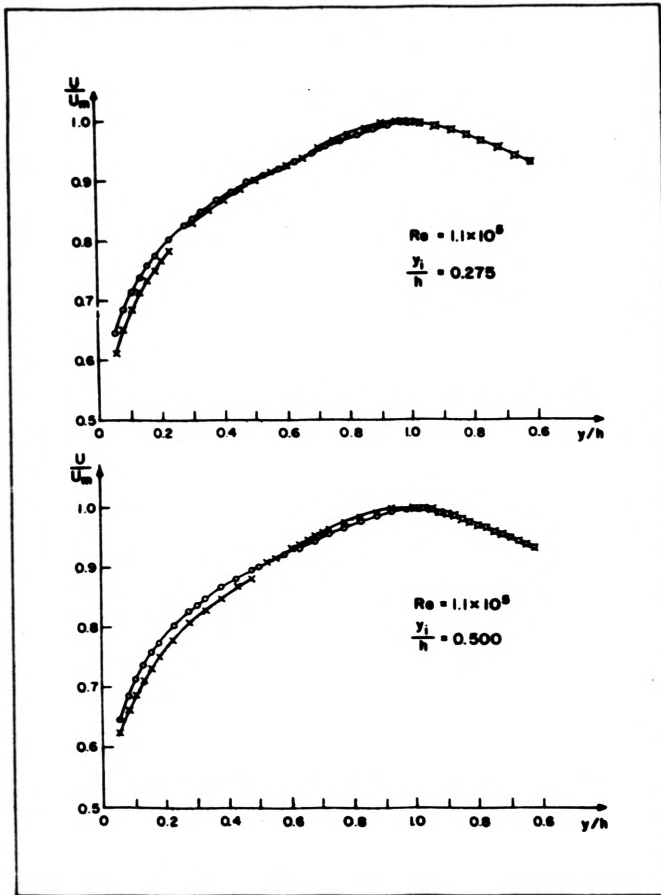


Figure 6 Distribution of time-average velocities, 0 - with no bubbles, x - with the bubble-layer.

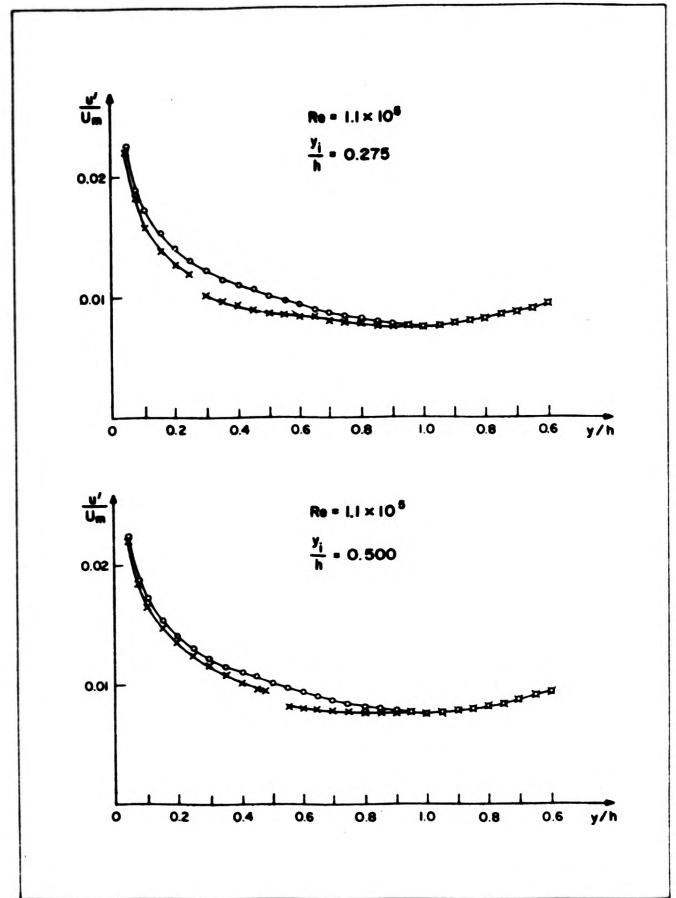


Figure 7 Distribution of fluctuating velocities, 0 - with no bubbles, x - with the bubble-layer

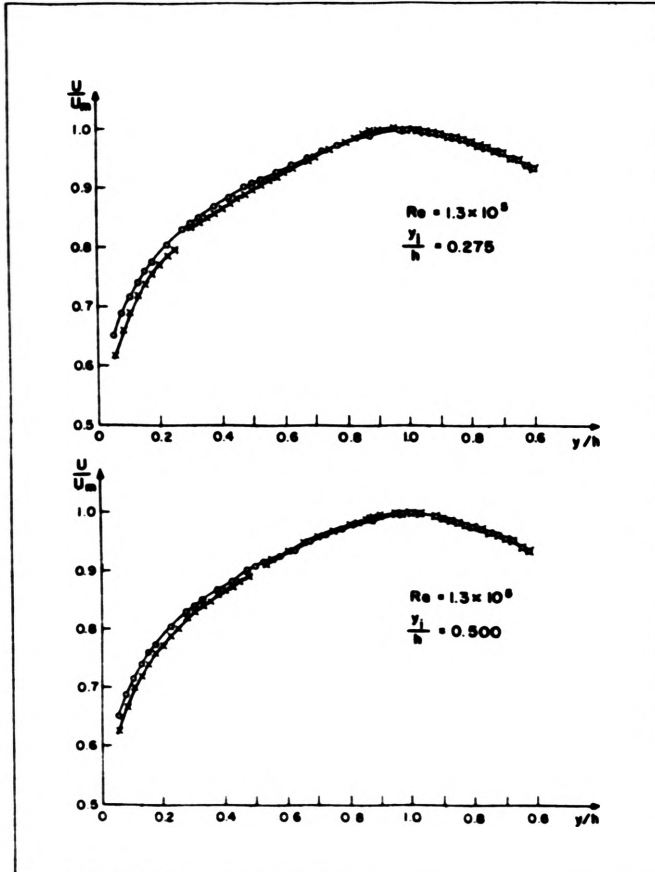


Figure 8 Distribution of time-average velocities, 0 - with no bubbles, x - with the bubble-layer.

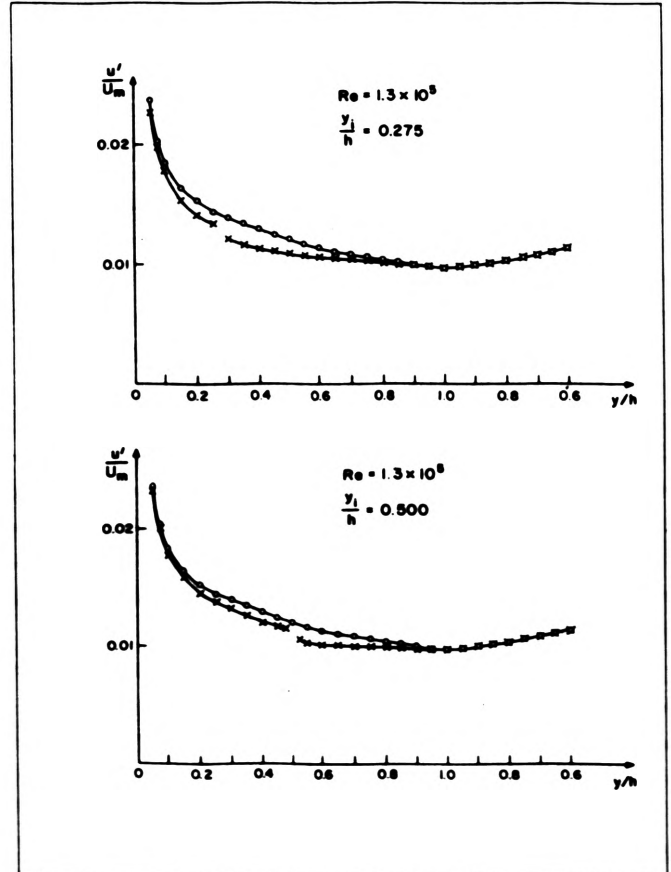


Figure 9 Distribution of fluctuating velocities, 0 - with no bubbles, x - with the bubble-layer.

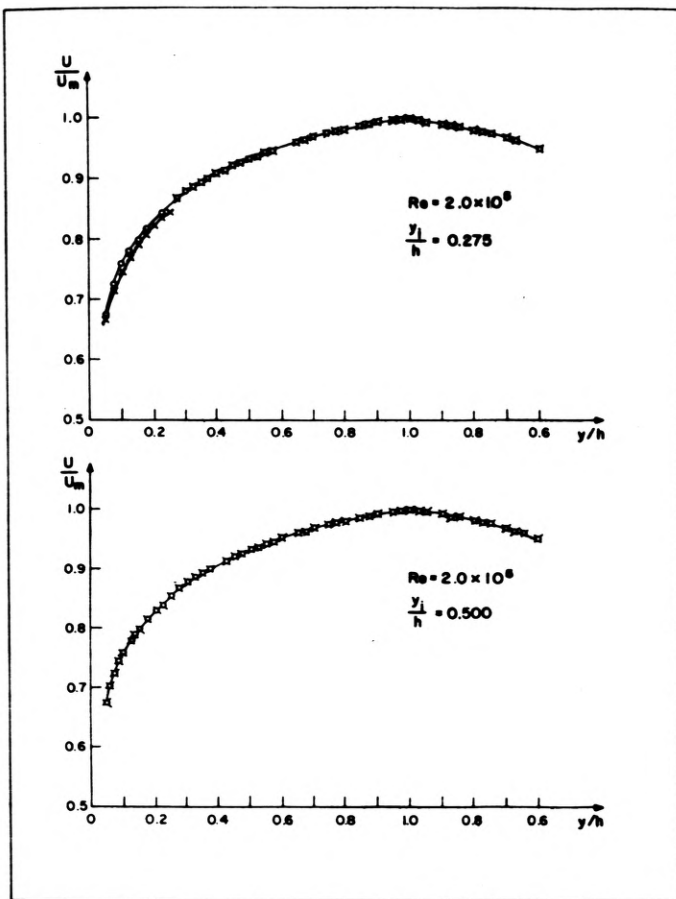


Figure 10 Distribution of time-average velocities, 0 - with no bubbles, x - with the bubble-layer.

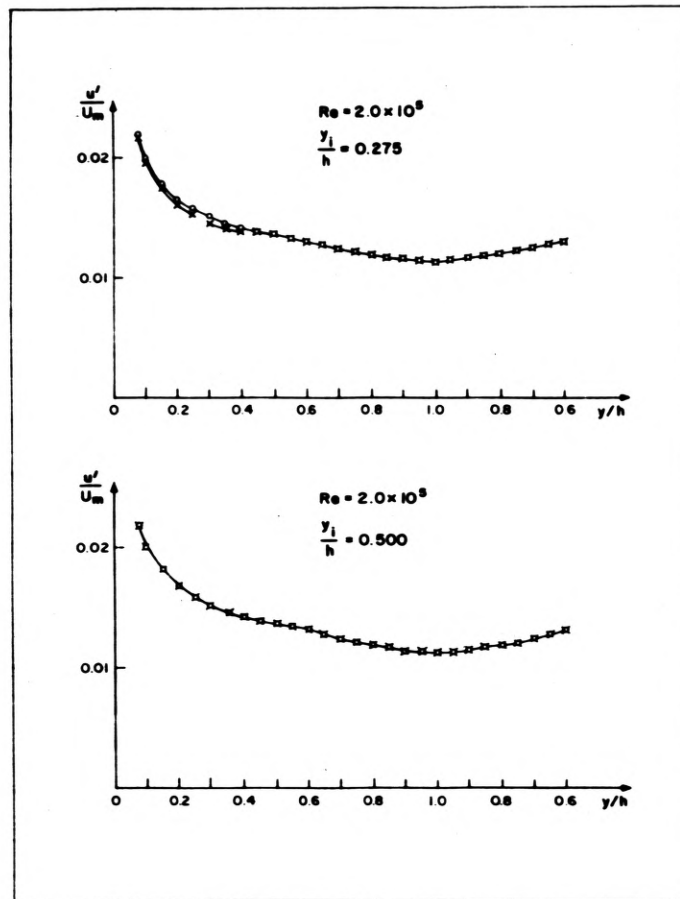


Figure 11 Distribution of fluctuating velocities, 0 - with no bubbles, x - with the bubble-layer.

- u_i fluid velocity in i -direction
- u_{di} solid particle velocity in i -direction
- u_m fluid velocity along centerline of conduit
- u^+ the friction velocity, u/ν^*
- u^* $\sqrt{\tau_0/\rho}$
- x_i coordinate
- y distance from wall
- y^+ friction distance from wall, yu^*/ν
- μ fluid dynamic viscosity
- ν fluid kinematic viscosity
- ρ mass density of fluid
- ρ_d mass density of solid particle
- τ_0 shear stress at the wall

A dot above a symbol indicates the material derivative - Eq. (2).

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