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Two-Dimensional Laser-Doppler Velocimetry in Turbulent Flows

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TWO-DIMENSIONAL LASER-DOPPLER VELOCIMETRY

IN TURBULENT FLOWS*

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ABSTRACT

Localized measurement of the velocity of flow in fluids may be accomplished by detecting the Doppler shift in frequency from coherent, monochromatic light scattered from micrometer sized contaminant particles within the flow. The versatility of applications of this technique, first used several years ago, has been expanded by recent research. The basic geometry of measurement, in which the scattered energy is recombined with light energy at the original frequency on the face of the photocathode of a photomultiplier tube, can be rearranged in many alternative configurations to meet the needs of the experimenter. By using a single laser, additional arrangements are possible for obtaining two- or three-dimensional measurements of velocity components. Several different velocimeters are described for making two-dimensional flow measurements, including both backscatter and forward scatter systems. Effects of flow system geometries on capabilities for measuring specific components are investigated. Effects of laser beam polarization are discussed and conclusions reached on methods of optimizing signal strength in each of two orthogonal measurement systems. Results of the application of a two-dimensional measurement system used for obtaining velocity profiles in turbulent flow in a smooth walled four-inch diameter pipe are presented. Operating at a Reynolds number of about 1.1×10^6, relative turbulent intensities were measured in the axial direction and normal to that direction. Standard techniques were utilized. A second readout system was employed to make a measurement of this parameter without the requirement of adding scattering centers to the flow.

INTRODUCTION

Measurement of more than one component of the velocity in turbulent fluid flow using the laser Doppler technique can be accomplished using a number of different approaches. The optical configuration chosen is dependent upon the interrelationship between the velocimeter and the flow system--involving the geometries, the velocity components desired, and the capability to detect the scattered energy. In this paper, a brief description of the operating principles of the device is given, followed by discussions of velocimeter systems employing single incident beam and dual incident beams. Application of a two-dimensional velocimeter is illustrated with results from measurements on a four-inch diameter pipe water flow system.

Results of measurement of relative turbulent intensities along the axis of flow and approximately normal to a diameter are given. A system is described which can be utilized with good results to measure turbulence parameters at low velocities or where addition of scattering particles is not practical. The latter use is restricted to systems where natural contaminants are present in sufficient quantity to produce several signals per second. The description of the velocimeter and results of its use provide a means of determining factors which must be specified in order to design a system for a particular application.

DESCRIPTION OF THE LASER DOPPLER TECHNIQUE

The initial use of the Doppler shift in monochromatic, coherent radiation scattered from particles suspended in a fluid flow to detect velocity was reported by Yeh and Cummins in 1964.[1] Subsequently, this work was extended and first applied in a practical instrument to measure localized flow in gases and liquids at these Laboratories.[2,3,4]

Measurements in turbulent flow have been reported by various workers. In order to summarize that work, it is necessary to describe the processes involved in the detection of velocity by the laser Doppler technique. A schematic representation of a one-dimensional system is shown in Figure 1.

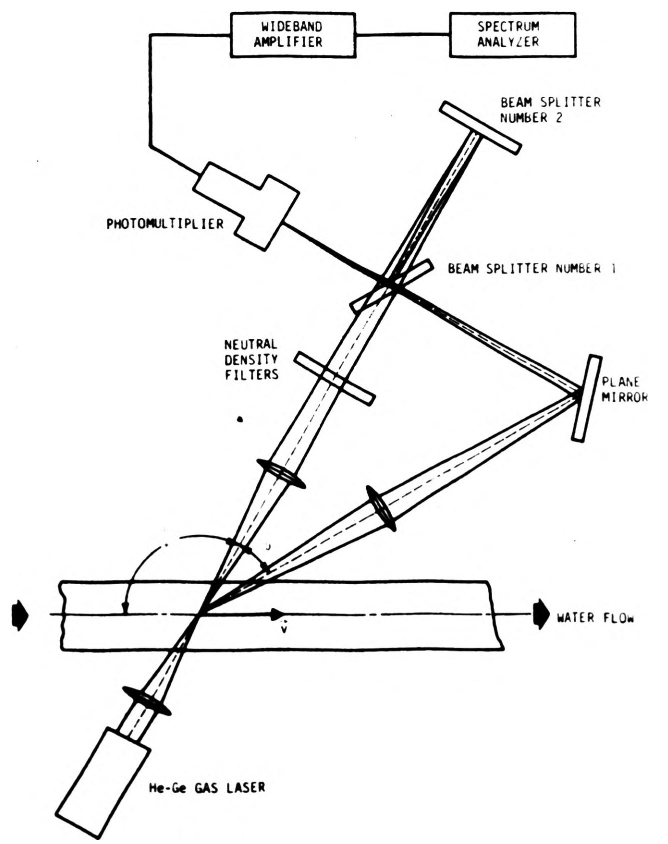

Figure 1. Geometry of One-Dimensional Laser Doppler Velocimeter

The Doppler equation, in terms of the parameters shown in Figure 2, is given by

$$f_D = \frac{1}{2\pi} (\bar{K}_s - \bar{K}_o) \cdot \bar{v} \tag{1}$$

where

$$\bar{K}_s = \frac{2\pi}{\lambda} \bar{\mu}_s \tag{2}$$

$$\bar{K}_o = \frac{2\pi}{\lambda} \bar{\mu}_o \tag{3}$$

In these expressions, the wavelength of the laser radiation in the scattering medium is taken to be approximately the same before and after scattering and

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is represented by λ; $\bar{\mu}_s$ and $\bar{\mu}_o$ are unit vectors in the direction of scatter and in the direction of the incident beam respectively, and $\bar{v}$ is the velocity of the flow in the plane of measurement. These vectors are shown in Figure 2. Using the angles defined in this figure, the scalar equation for the Doppler shift is

$$f_D = \frac{2\bar{v}}{\lambda} \sin\frac{\theta}{2} \sin\left(\psi + \frac{\theta}{2}\right) \tag{4}$$

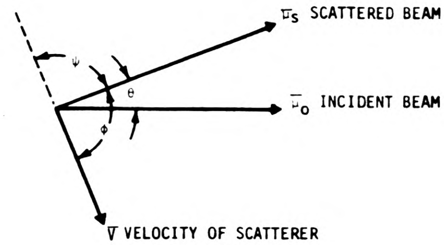

Figure 2. Relationship Between Original and Scattered Wave Vector

The volume within the scattering medium from which a measurement is made is determined by the interrelationship between the focusing optics and the laser beam diameter and dispersion. If diffraction limited optics are employed, the smallest diameter region that may be focused upon is given by

$$D = \frac{1.22\lambda f}{D'} \tag{5}$$

where D' is the diameter of the receiving aperture and f is the focal length of the receiving lens. The length of the measurement volume is several times this dimension. The incident beam is focused to dimensions approximating the receiving system capabilities. While this picture is considerably simplified, the measurement volume may be thought of as being produced by the intersection of two cylinders. The measurement volume used here was 75 by 1,000 micrometers.

Various methods have been used to make measurements in turbulent flow. Goldstein and Hagen[5] reported on use of a spectrum analyzer as a readout method. Welch and Tomme[6] used a time correlation method to read out velocity profiles at a point. Lewis, et al[7] used a phase-locked loop to record instantaneous velocities. Huffaker, Fuller, and Lawrence[8] reported on the use of another type frequency tracker using the output of a discriminator to provide the control signal for the feedback loop. All of the methods previously used depend on a more or less continuous signal being present in order to respond correctly. The readout technique described in this paper does not depend upon a near continuous signal being present.

TWO-DIMENSIONAL GEOMETRIES

The choice of geometry for the optical system to be employed in making a two-dimensional measurement is dependent upon the measurement directions required and upon a number of limitations imposed by the velocimeter system. These limitations must be determined on the basis of particular configurations.

The scattered radiation from a single beam incident upon a desired volume may be detected at two different positions in the forward scatter direction to obtain a two-dimensional measurement (see Figure 3). A geometry available for obtaining the velocity component nearly parallel to the incident beam is shown in Figure 4. The first of these set-ups is described in the section of this paper entitled Measurements of Turbulent Flow in a Four-Inch Pipe. The latter set-up requires that the radiation in the backscatter direction be sufficient to detect. For 0.5 micrometer diameter polystyrene latex spheres, the ratio of intensities in the near forward scatter direction compared to intensities in the far backward scatter direction is nearly 50 to 1 according to Mie scattering theory.[9] If the Doppler spectrum is spread out due to turbulent flow, considerably greater power must be used in the incident beam to detect the signal. Experiments in the four-inch diameter pipe for which data are reported here indicated that a helium-neon laser with 25 milliwatt output was not sufficient to produce a useable signal at the Reynolds numbers of flow used.

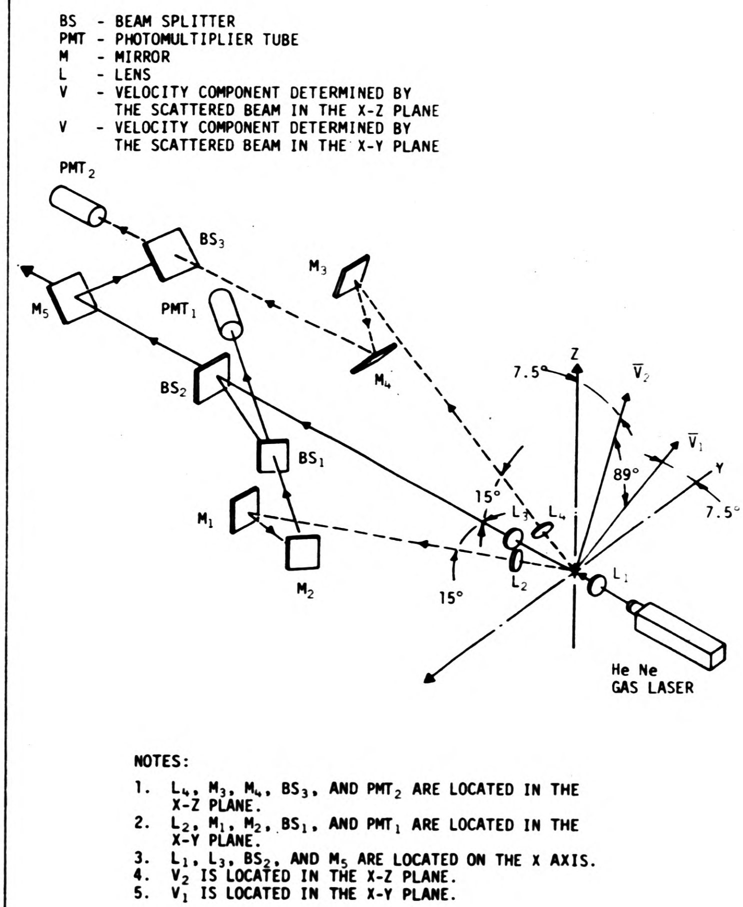

Figure 3. Two-Dimensional Single Incident Beam Velocimeter

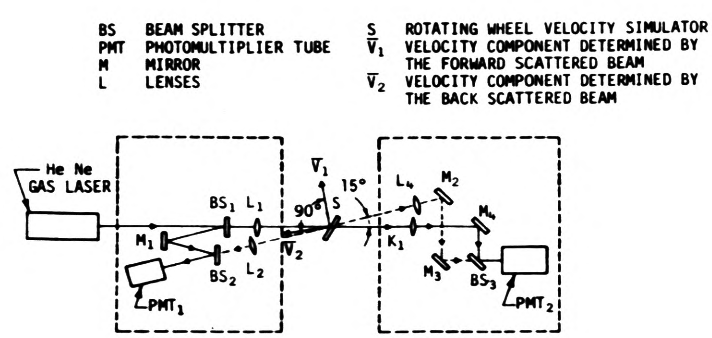

Figure 4. Forward and Backscatter Velocimeter System

Another system may be employed to obtain orthogonal components relatively easily. This system employs two incident beams to obtain two directions of near forward scatter radiation. Figure 5 illustrates such a system. A velocimeter constructed with this geometry offers symmetry of measured velocity components with respect to the flow axis geometry, which might be a considerable asset insofar as "tracking" is concerned as will be seen. The components thus measured, however, are not those of most interest for turbulent flow in most channels. In addition, Beam 1 can heterodyne with scattered energy from Beam 2 and vice versa and the scattered beams can heterodyne with each other. An extensive analysis of this system is found elsewhere.[10]

When more than one component of velocity in a system is measured, effects of the polarization of the incident and scattered beams become more difficult

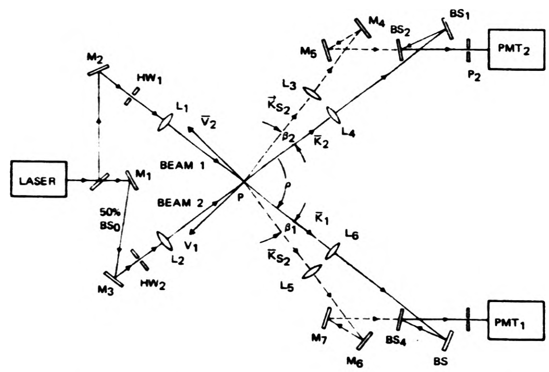

Figure 5. Two-Dimensional Twin Incident Beam Velocimeter

to allow for in setting up the geometry of the velocimeter. An experiment was performed to find ways of minimizing polarization effects. The plane of polarization of the output of a Model 124 Spectra-Physics laser was rotated by using an accessory polarization rotator. A thin mylar wheel was used as a flow simulator to produce the data shown in Figure 6. The plane of polarization was varied from vertical (0^o) to horizontal (90^o). Curves 1 and 2 were obtained by observing the intensity of the heterodyne signal on a Hewlett-Packard Model 8551 A spectrum analyzer using the backscatter and forward scatter systems of Figure 4. These systems were mounted in the horizontal plane. In the backscatter system, a variation of 2.5 decibels resulted from the rotation of the polarization plane. In contrast, the forward scatter system produced a signal level that varied over 9 decibels.

These losses are incurred at reflecting surfaces within the system. A dielectric surface (such as a glass beam splitter) will change the plane of polarization of the reflected beam from the original plane except under specific conditions; if the plane of polarization is perpendicular to the plane containing the incident and reflective beam or if the incident beam strikes the reflecting surface near the normal, then the plane of polarization is least affected. Reflection from metallic surfaces, in general, reduces the intensity in a given plane of polarization by elliptically polarizing the reflected beam. Arranging the system to provide near normal incidence on metallic reflectors also reduces this loss. Curves 3 and 4 were obtained with two experimental systems which were designed with no angle of incidence greater than 25^o. The maximum signal loss experienced was 3 1/2 decibels. A two-dimensional system must use these principles if the losses are to be kept to a minimum. The desirability of this is readily apparent in reducing cost of the laser to be used and in keeping the bulk of the system as low as practical.

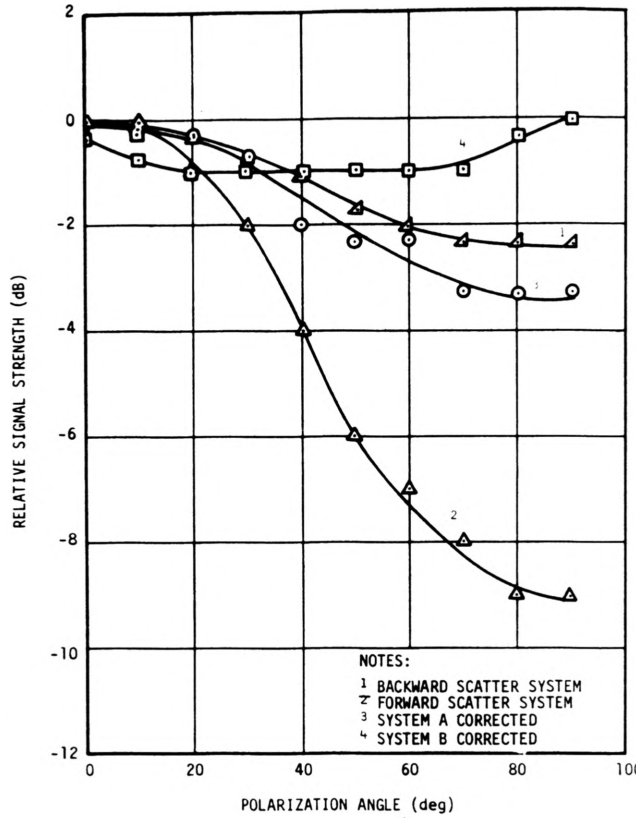

Figure 6. Effects of Polarization Rotation on Received Signal

MEASUREMENTS IN TURBULENT FLOW IN A FOUR-INCH PIPE

A system similar to that shown in Figure 3 was set up on a 4-inch outside diameter glass pipe flow system. Two modes of operation were attempted. In order to better observe the component of velocity in the z-direction (defined in Figure 3), a direction normal to the axis of the pipe, modifications were made in the processing circuitry for signals centered about zero velocity. First, the reference beam was generated by passing the laser beam through an acoustic-optical cell to shift the frequency of a portion of the beam by 30 megahertz. Secondly, the reference beam was passed around the flow system in order to minimize amplitude variations. The heterodyne signal produced by this system is centered about 30 megahertz rather than zero frequency. This procedure makes it possible to view low velocity spectra on a spectrum analyzer without being troubled by the zero frequency signal present to some degree in all spectrum analyzers and, more important, to get rid of the low frequency noise generated in the laser. Mode interaction in the laser can create spurious signals of narrow bandwidth at low frequencies. Once a signal centered about 30 megahertz is obtained, it may be mixed with a constant frequency radio frequency signal to locate the frequency corresponding to zero velocity at any position in the spectrum desired. A block diagram of this apparatus is shown in Figure 7.

The flow system used is a closed loop, four-inch diameter unit. The loop is driven by a 60-horsepower motor driving a pump rated at 1100 gal/min at a total head rise of 150 feet of liquid at 1700 rpm. Bulk flow rates through the tube are measured with a propeller-type volumetric flowmeter located in the 4-inch diameter line on the suction side of the pump. The velocimeter measurements were made in a glass-walled section of the system. The system is vibration isolated from the surroundings by rubber pads on the supports and a flexible coupling to the pump. A vacuum system was used to remove

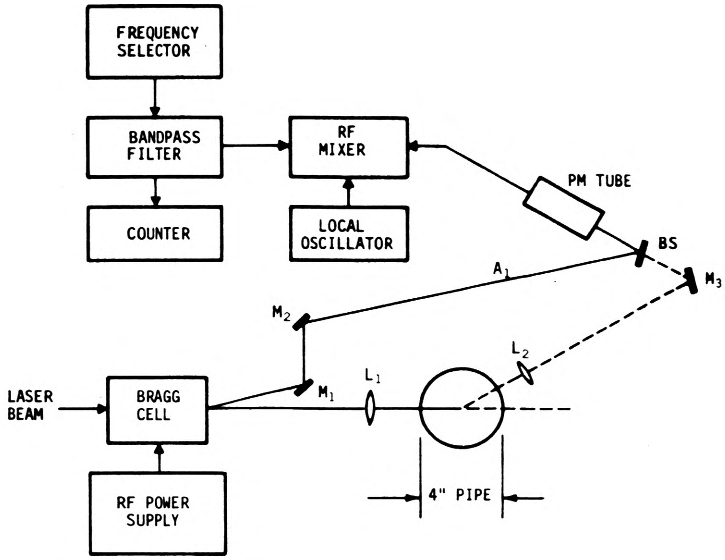

Figure 7. Double Conversion Heterodyne System

dissolved air from the water in the loop so as to prevent bubble formation. The ultimate Reynolds number of the system is 1.8×10^6. The flow system heats up during runs because of friction. Measurements utilized here were made over a water temperature range of 95 to 105°F corresponding to a Reynolds number range of $1.12 \pm 0.06 \times 10^6$ at the velocity used. The velocimeter receiving aperture was set to produce a readable signal at all measurement positions within the tube. An exception to these operating conditions was in measurements with the double conversion heterodyne system, where the length of time necessary to make the measurement extended the operating temperatures to between 90 and 114°F.

Two factors associated with optical system parameters have been cited as causing line broadening of the Doppler signal. Examination of Equation 4 for the Doppler frequency shows that if scattered radiation is received over a finite angular aperture, the Doppler spectrum of the radiation is broadened accordingly. Differentiation of Equation 4 with respect to θ and ψ yields

$$df_D = \frac{2\bar{v}}{\lambda}\left\{\left[\frac{1}{2}\sin\frac{\theta}{2}\cos\left(\psi + \frac{\theta}{2}\right) + \frac{1}{2}\cos\frac{\theta}{2}\sin\left(\psi + \frac{\theta}{2}\right)\right] d\theta + \sin\frac{\theta}{2}\cos\left(\psi + \frac{\theta}{2}\right) d\psi\right\} . \quad (6)$$

For a system such as that shown in Figure 6, $\psi = 90°$ and the fractional shift is

$$\frac{df_D}{f_D} = \cot\theta\, d\theta - \frac{1}{2}\tan\frac{\theta}{2}\, d\psi . \quad (7)$$

The signal produced by a scatterer passing through the measurement volume is in the form of a pulse. The time of occurrence of the pulse is

$$t = \frac{D}{v_n} \quad (8)$$

where D is a dimension of the measurement volume such as defined in Equation 5 and v_n is the velocity component across that dimension. The spectrum of such a pulse has frequency spread of

$$\Delta f_t \cong \frac{1}{t} \quad (9)$$

It has been pointed out that for diffraction limited optics, the frequency spread caused by the spread in scattering angles observed should be zero for rays received from the bundle at the focus of the optical system.[12] The magnitude of possible velocity spread in the system used here may be estimated by taking a value of about $\theta = 15°$ approximate mean vertical component measurement angle, $d\theta = \frac{3}{140}$ (3 mm aperture at a distance of 140 mm), $d\psi = 0$ (for purposes of this order of magnitude estimate) to get from Equation 7

$$\frac{df_D}{f_D} \cong 0.08 \quad . \quad (10)$$

This value may not be observed in practice. In addition to the reason for a narrower spectrum given above, the heterodyning efficiency at the photomultiplier is dependent on how nearly colinear the scatterer and reference beams are, and the beam intensities drop off at the edges of the scattering volume.

The frequency spread in the measured vertical component is also affected by introduction of a component of the axial velocity into the measurement by the finite angle of acceptance in the system. At the center of the flow, under the same conditions cited above to calculate maximum frequency spread due to the effect of finite aperture on the measurement of the desired component, the maximum frequency component introduced into the measurement due to this effect may be calculated from Equation 4. If the system is carefully aligned to be symmetrical about a component normal to the axial flow, an angle of

$$\theta = \frac{3}{280} \text{ deg} \quad (11)$$

is used in the equation. This yields

$$df_D = \pm 195 \text{ kilohertz} \quad (12)$$

where the mean Doppler frequency produced by the velocity component along the system axis is 3.60 megahertz.

Using a mylar wheel at the measurement point, the frequency spread due to all factors mentioned here amounts to

$$\frac{df_D}{f_D} \cong 0.04 . \quad (13)$$

Wave trains from the wheel appear to extend over a longer period of time than would be expected from a particle traversing the measurement volume. For this reason, the frequency spread due to pulse broadening would be expected to be less than with particulate material. Another measurement was made with the mylar wheel placed so that the velocity was approximately normal to the plane containing the scattered and reference beams. An effective scattering angle of

$$\theta = 0.62 \text{ deg} \quad (14)$$

was computed on the basis of an observed Doppler mean frequency of 150 kilohertz. The frequency spread was 70 kilohertz. This provides an upper limit for the pulse broadening for the wheel, since the angular aperture should produce the frequency speed noted. Computation of this frequency spread yields

$$f_D = \pm 70 \text{ kilohertz} \quad (15)$$

which is double the measured value. However, at such a small effective scattering angle small errors in wheel position produce large errors in this measurement. In computations of the frequency spread due to the angular aperture, the value computed from the above measurements was used:

$$d\theta = 0.762 \text{ deg.} \quad (16)$$

This value is 60 percent of the value computed from the aperture size.

The pulse broadening of the spectrum may be estimated from Equations 8 and 9. Near the axis of the tube, the mean flow velocity is large enough with respect to the turbulent velocity that the time for a particle to go across the beam may be estimated from the mean velocity alone. This holds for measurements normal to the axis of the system as well, since velocities in these directions are small compared to mean axial velocity. With a 75-micrometer wide beam and a peak mean velocity of 8.65 meters per second, the frequency spread is

$$\Delta f_t = 115 \text{ kilohertz} \quad (17)$$

This spread is dependent on the position within the flow, of course.

The four-inch diameter flow system was used to study the interrelationship between system geometries and the factors affecting the Doppler frequency output. The horizontal component of velocity measured was parallel to the tube axis while the velocity component measured normal to this axis was slightly off the vertical. With a fixed receiver geometry, the resultant measurement direction at three positions within the tube is shown in Figure 8. As can be seen, the scattering angle changes as the measurement volume is moved in from the edge of the tube. The scattering angle must be determined at each point and used to calculate the conversion factor from frequency to velocity. In this system, the velocimeter geometry remains fixed during the scan. The vertical component may be directly measured, but a system of that type would require realignment at each measurement position. Since the scattered energy in the horizontal component system is contained within a principal plane, the scattering angle remains fixed during scan. This component was measured at a slight angle to the axial direction.

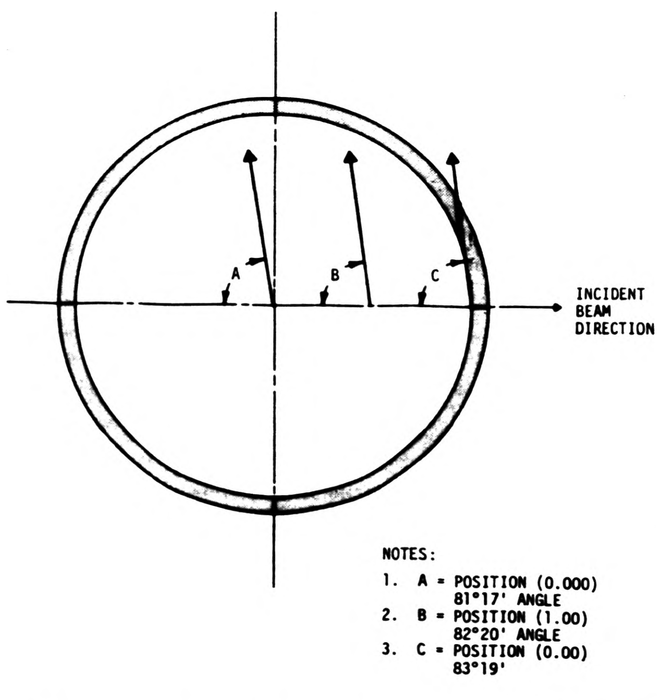

Figure 8. Variation of Measured Near-Vertical Component With Radial Position

Measurements in a tube with these components must take account of the different "tracking" rates of the measurement volumes. Figure 9 shows the relationship between the distance the velocimeter is traversed and the distance from the edge of the tube the measurement volume is moved. The two measurement volumes do not move at the same rate as the system is traversed, and, in fact, the near vertical component measurement volume is traversed non-linearly with respect to the system motion. The two volumes, if set to coincide at the edge of the tube, will be separated by 0.7 inch at the center. By making the volumes coincident a quarter of the way across the tube, the maximum error is about 0.3 inch, which is still too large for any correlation-type measurement. Simultaneous measurements of these two components at the same measurement position requires a complex traversing scheme, with the two component systems traversing at different rates.

The mean velocity profile was measured from a spectrum analyzer output. The results of this measurement are shown in Figure 10. The mean velocity at the center of the tube was 8.65 meters per second as measured by the velocimeter as compared to a velocity of 8.58 meters per second computed from the volumetric measurement of the flowmeter.

All data using a spectrum analyzer output was obtained by adding 0.500 micrometer polystyrene latex spheres to the water in the system. Measurements near the wall with the apparatus are not expected to have much significance because of the length of the measurement volume along the reference beam--about 1 millimeter to the half power points. At points far from the wall, the significance of the line broadening factors can be estimated. At the center of the tube, by using mean velocity of 8.65 meters per second, the horizontal component yields

$$\frac{\Delta v_h}{v_{xmax}} = 0.043 \quad (18)$$

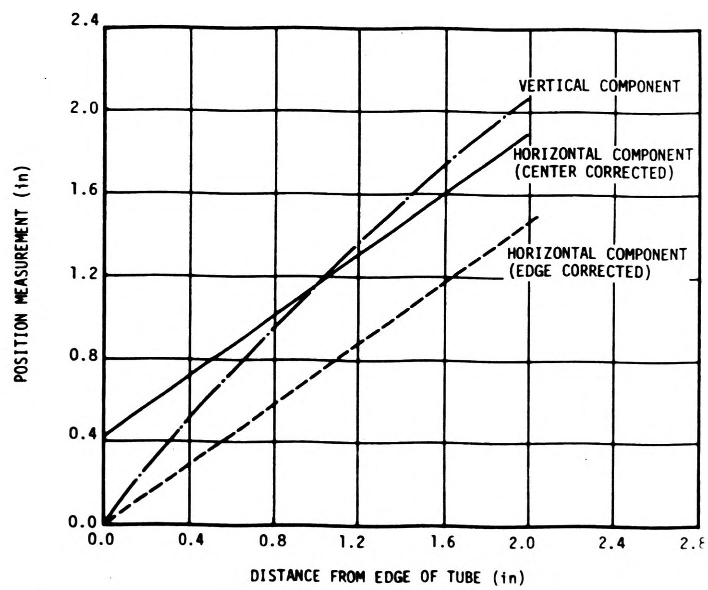

Figure 9. Measurement Volume Position "Tracking" of Velocimeter Motion

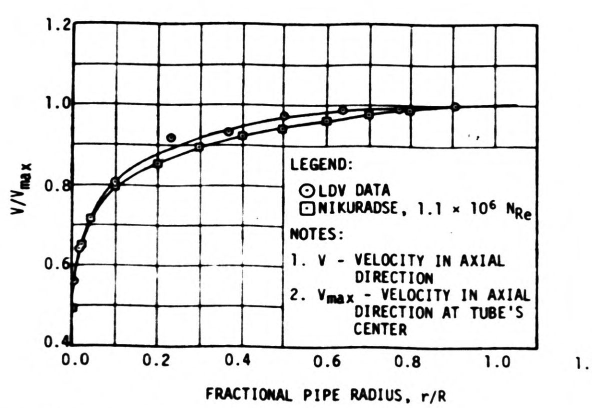

Figure 10. Mean Velocity Profile in 4-Inch Tube

and the near vertical component yields

$$\frac{\Delta v_v}{v_{xmax}} = 0.030 \qquad (19)$$

The values are computed from measurement of one-half the frequency spread between half amplitude points. Corrective factors that can be applied have been discussed above. These include the frequency spread due to pulse width from Equation 17

$$\Delta f_t = 115 \text{ kilohertz} \qquad (20)$$

and the frequency spread due to the angular aperture from Equation 13

$$df_{D_H} = 140 \text{ kilohertz} \qquad (21)$$

for the horizontal component and from the angular aperture computed from Equations 13 and 7

$$df_{D_v} = 50 \text{ kilohertz} \qquad (22)$$

for the vertical component.

These corrective factors are applied on the basis of assuming each frequency spread is an independent Gaussian distribution. The corrected spread will be

$$\left(\frac{(\Delta v)}{(v)_{xmax}}\right)_{corr} = \frac{1}{1.18}\left[\left(\frac{\Delta v}{v_{xmax}}\right)^2 - \left(\frac{1}{2}\,\frac{\Delta v_t}{v_{xmax}}\right)^2 - \left(\frac{1}{2}\,\frac{dv_D}{v_{xmax}}\right)^2\right]^{\frac{1}{2}} \qquad (23)$$

where the one-half factors in the pulse and angular aperture terms enter because the desired spread is the root mean square deviation from the mean. The frequency spreads must be converted to equivalent velocity spreads to use this equation, Δv_t corresponding to the pulse broadening term, dv corresponding to the angular aperture term and Δv to the observed spread. By using values from Equations 18 and 19, converted to velocities, the corrected values become

$$\left(\frac{\Delta v_h}{(v)_{xmax}}\right)_{corr} = 0.0335 \qquad (24)$$

$$\left(\frac{\Delta v_v}{(v)_{xmax}}\right)_{corr} = 0.028 \ . \qquad (25)$$

The angular aperture correction factor significantly affects the horizontal component, but not the vertical. The pulse correction factor significantly affects both. The percent correction produced by these terms is at the most 23 percent. As one moves toward the edge of the tube, the angular aperture correction factor becomes less important for the horizontal component and remains about the same for the vertical component. The pulse correction factor becomes less important as the edge of the tube is approached. The corrected curves of relative turbulent intensities are plotted in Figure 11. The frequency spreads Δf_v and Δf_H were obtained by measuring between $\pm 3\sigma$ points on the Gaussian shaped spectrum analyzer traces.

The two components of relative turbulent intensity measured here were compared to the results of Laufer presented in Hinze.[14] Laufer's data, taken with hot-wire anemometers, were obtained at a slightly smaller Reynold's number than here. The agreement between the data is good except that the vertical component measured appeared more constant with radial distance in the current measurement. Also, near the wall, the laser velocimeter measured horizontal component increased at a greater distance from the wall than did the previous result. As explained earlier, the measurement volume was largest along this axis and with this particular set-up one could not expect to get the resolution required to measure turbulent intensities at the wall.

The second mode of operation described above was used with the vertical

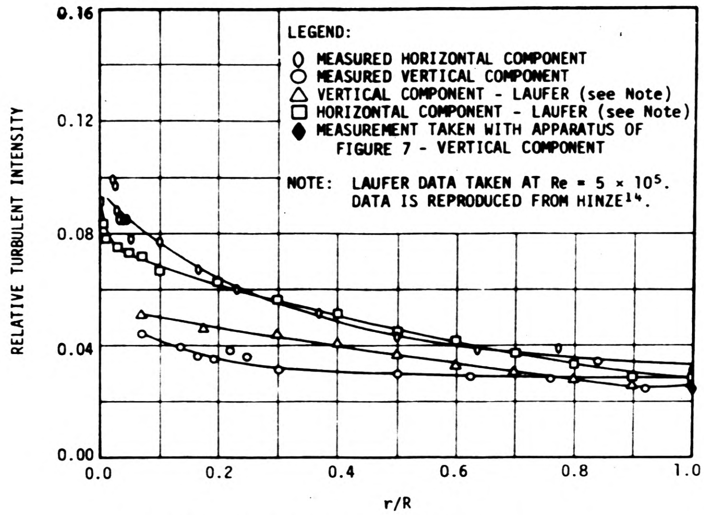

Figure 11. Relative Turbulent Intensities

component at one point at the center of the tube. The relative turbulent intensity measured in this manner was

$$\left(\frac{\Delta v_v}{v_{xmax}}\right)_{corr} = 0.025 \ . \qquad (26)$$

A plot of the counting rate versus frequency is shown in Figure 12. A Gaussian distribution has been plotted with the same root mean square deviation using the same mean. The noise level indicated on the plot was obtained by blocking the scattered beam and observing the count. The significance of the measurement is that the count was made with no artificial contaminants added to the water. A spectrum analyzer output of this signal could not be used to determine the frequency spread because of the small number of signals being produced.

An ambiguity is present in applying the pulse width correction factor when artificial contaminants are added to the system. In such a case, more than one particle is present within the measurement volume at any one time and the resultant pulse train appears to often exceed the number of cycles expected from the dimensions of the volume. The system with no artificial contaminants present is such that individual particles produce the heterodyne signals received.

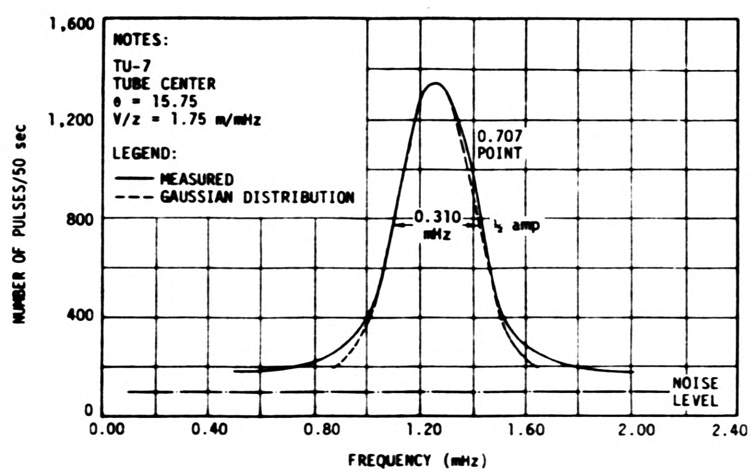

Figure 12. Counting Rate as a Function of Frequency

The technique as presently used is cumbersome, witnessed by the fact that with this flow system only one data point could be obtained with a reasonable expenditure of time. The total length of time utilized in collecting data used in the plotted spectrum shape was approximately one and a half hours. In addition, two shut-downs were necessary to let the water cool to stay within the temperature operating range specified earlier. During this time, contaminant level had to remain constant and no air bubbles could be allowed to form. Additional experiments are planned to decrease the length of time required for a measurement and to explore any additional limitations.

CONCLUSIONS

Two-dimensional measurements of turbulence parameters in fluid flow using a laser Doppler velocimeter require detailed analysis of velocimeter-flow system interactions to produce meaningful results. Frequency broadening of the signal is produced by both pulse width and angular aperture effects. While both factors can be controlled, other considerations enter into the capability to vary them. A smaller aperture size produces a reduced signal level. In this case, the signal level was adjusted to the minimum consistent with reliable data taking. A smaller measurement volume can be produced, but this requires increased aperture sizes and increases frequency spread due to pulse width.

Measurements may be made in large systems where it is impractical to add scattering particles so long as conditions within the fluid remain constant over the measurement time. Low frequency laser noise and low frequency characteristics of electronic readout systems can be circumvented in low velocity turbulent flow measurements by use of a dual conversion heterodyne system. Results comparable to more conventional methods are obtainable with proper attention to the operating parameters.

SYMBOLS

f_D	Doppler frequency
$\overline{K}_s$	scattering frequency
$\overline{K}_o$	incident frequency
$\overline{\mu}_s$, $\overline{\mu}_o$	unit vectors
λ	wavelength in scattering medium
$\overline{v}$	fluid flow velocity
D	smallest diameter of beam
f	focal length of receiving lens
D'	receiving aperture
θ, ψ	angles defined in figure 2
t	time of scattering pulse
v_n	velocity in plane normal to scattered beam
Δf_t	frequency spread of pulse
v_n	horizontal velocity component
v_v	vertical velocity component
$v_{x\ max}$	centerline velocity

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