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MEASUREMENT OF TURBULENCE POWER SPECTRA  
IN AGITATED VESSELS OF DIFFERENT SIZE  
WITH A LASER-DOPPLER VELOCIMETER

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ABSTRACT

We have used the laser-Doppler velocimeter with a new noise reduction technique for measurements in water-filled, turbine-agitated vessels of several sizes, but of the same geometry. Mean velocities, turbulence intensities and turbulence power spectra were obtained in the impeller stream region.

With these results we hope, in the future, to put scaling rules for mixing vessels on a sounder basis.

INTRODUCTION

Agitated vessels are used in the chemical industry for a wide variety of processes, e.g. mixing, dispersion and suspension of solid particles. In most applications the characteristics of the (turbulent) fluid flow are crucial for the performance of the stirred tank. Measurements done in the past mainly concerned mean velocities and turbulence intensities<sup>1-8</sup>. However, turbulence intensities reported should be considered with some reserve since they might contain the periodically fluctuating component as well as the (random) turbulence. Data on the spectral distribution of the velocity fluctuations are scarce in any case<sup>6,8,9</sup>, because the lack of an appropriate measuring technique. For, the insertion of a hot-film/wire probe or a Pitot tube disturbs the circulating flow. Moreover, although measurements were done in vessels of different sizes, their geometry was different too and no unambiguous scaling rules can be derived from them.

Hitherto, a basic limitation to the usefulness of the laser-Doppler velocimeter as a turbulence measuring device was the so-called Doppler ambiguity. Its influence on the power spectrum of the turbulent velocity

fluctuations is shown in Figure 1, where it can be seen that for wave numbers higher than a certain value the turbulence power spectrum is completely obscured by the presence of the ambiguity noise. We therefore used a laser-Doppler velocimeter which was specially designed for measuring highly turbulent liquid flows and had the advantage of producing hardly any ambiguity noise in the turbulence power spectrum. With the improved equipment we measured mean and periodic velocities and turbulence power spectra and obtained from the investigated spectra the root mean square values (RMS) of the random fluctuations. We conducted the measurements at various points in the impeller stream region and repeated them in vessels of two other sizes but of exactly the same geometry.

EXPERIMENTAL ARRANGEMENT

The Laser-Doppler Velocimeter (LDV)

As the laser-Doppler technique has been discussed in detail elsewhere<sup>10,11</sup>, we confine ourselves to a short review of our set-up. Some attention will be paid to aspects which are important in connection with our application of the technique to flows with high turbulence intensities.

The optical configuration used in our experiments is shown in Figure 2. As a beam splitter we used a radial grating, manufactured to provide high intensities in the first-order beams. By rotating the grating around an axis perpendicular to it, a frequency preshift of the laser beams was obtained. It is essential to remove the directional ambiguity from the Doppler signal.

The scheme of the signal processing of the Doppler signal is given in Figure 3. The Doppler signal is fed to a frequency tracker of which the output is a voltage proportional to the instantaneous velocity. The mean velocity is determined from the time-averaged frequency

of the internal oscillator of the tracker. The power spectrum is calculated via autocorrelation of the tracker output and Fourier transformation of the autocorrelation function in the spectrum display.

Finally, it should be remarked that the scattering particle density was increased by addition of some milk to the water. This served a double purpose. The strong accelerations occurring in the flow under consideration might lead to too large frequency jumps after a drop-out period (in which no signal is received, for instance because no particle is present in the measuring volume). By increasing the particle density, we decreased the number of drop-outs as well as their duration, which minimises the frequency jumps. Moreover, a high particle density improves the frequency response of the system, so that higher turbulence frequencies can be observed.

#### Description of the Noise Reduction Technique

The basic idea is that a reduction in the noise level could be obtained if we were able to simultaneously produce two signals,  $s_1(t)$  and  $s_2(t)$ , both representing the velocity,  $v(t)$ , at the same point in space, but with different and uncorrelated noise signals. Thus:

$$\begin{aligned} s_1(t) &= v(t) + n_1(t) \\ \text{and} \\ s_2(t) &= v(t) + n_2(t) \end{aligned}$$

In such a case, cross-correlation of the two signals (instead of autocorrelation of one signal) would eliminate the ambiguity noise completely<sup>12</sup>.

To apply this idea we placed photodiodes in beams 1 and 2 (see Figure 2), and the signals from these were fed into separate frequency trackers. In this set-up, which we call a twin-detector LDV, each laser beam acts as both a reference and a scattering beam. The ambiguity noises in the two photodiode signals were then uncorrelated, as can easily be understood from the following reasoning: Configurations of particles or irregularly shaped particles will give different intensities of scattered light in the directions of beams 1 and 2. Hence the signal from the photodiode in beam 1 might be above the drop-out level of the tracker connected to it, while the signal from the photodiode in beam 2 is below the drop-out level of its tracker, and vice versa. This hypothesis was confirmed by a measurement of the cross-correlation between the drop-out signals (a binary signal whether there is a Doppler signal or not) from the two frequency trackers. The cross-correlation appeared to be very small (see Figure 4), which can be regarded as proving that the two frequency trackers react to different particles or particle configurations.

Further confirmation was obtained by measuring the noise reduction as a function of the angle between the two laser beams. Since the configurations of particles in the measuring volume as seen by the different detectors are more different at larger angles, the noise reduction should increase with increasing scattering angle; this was indeed the case.

Completely uncorrelated phase changes can be expected when the two signals come from totally different particles. In the measurement of longitudinal velocity fluctuations, this can easily be achieved by displacing one beam somewhat (about one beam width) out of the plane of measurement. As the beam width in the focal plane is usually much smaller than the length of the volume of measurement, a reduction in spatial resolution need not be feared; in addition, alignment problems do not arise since the shift can be easily accomplished by inserting a microscope slide in one of the beams.

Figure 1 is an example concerned with turbulent pipe flow of water, where the noise is mainly caused by the finite residence time of the particles in the scattering volume. The three spectra of the longitudinal velocity component were all measured under identical circumstances; only the techniques used were different. This demonstrates the degree of noise reduction that can be obtained.

#### The Tank Geometry

As our first aim was to study scaling effects, no attempts were made to obtain an exact copy of an existing tank. The geometry of the Perspex tanks and the impellers, shown in Figure 5, was chosen mainly because of its ease of construction. It was covered with a Perspex disk to prevent air bubbles from being sucked in and to enable us to measure also the tangential velocity component.

All linear sizes were scaled up using the ratios between the tank diameters ( $D$ ), which were 0.12, 0.29 and 0.90 m, while in operation the power per unit mass, averaged over the vessel volume, was kept constant:  $\bar{\epsilon} = 5.7 \times 10^{-2} \text{ m}^2 \cdot \text{s}^{-3}$ . So the impeller frequency was inversely proportional to the  $-2/3$ -power of the scale factor, unless stated otherwise.

#### RESULTS AND DISCUSSION

In our first measurements we determined the mean velocities. Radial as well as axial cross-sections of the impeller stream were determined. Our results were in close agreement with those of Cooper and Wolf<sup>4</sup>, thus confirming the commonly used scaling rule in the case of

constant average power per unit mass, viz. mean velocity proportional to  $D^{1/3}$ . From the tangential and radial components the total mean velocity was calculated and used later to convert the observed frequency of turbulence into wave number.

By using the signal recovery mode of the correlator, the time-averaged amplitudes of the periodic radial and axial components were measured as a function of both radial and axial positions. The results for the axial dependence are given in Figure 6. As triggering occurred at a fixed position of the impeller with respect to the vessel, the relative phase, too, could be determined. Both results agree with the picture<sup>9</sup> of two trailing vortices, one above and one below the impeller centre plane, rotating in opposite directions. Measurement of the radial decrease in amplitude shows that at a radial position of twice the impeller radius the vortices have disappeared.

Some representative turbulence power spectra observed are given in Figures 7, 8 and 9. These spectra were measured at equivalent positions in the impeller stream regions of the differently sized vessels. The shape of the spectra agrees roughly with the expected form. There is a, somewhat anisotropic, part at low wave numbers and a high-wave-number part showing local isotropy, the dividing wave number being inversely proportional to the scale of the vessel (see Figures 7 and 8). The high-wave-number part is a straight line over several decades of the spectrum, and is determined by  $\epsilon$  only (see Figures 7 and 9). However, although the Reynolds number of turbulence in the biggest vessel is of the order of  $10^2$ , which is rather high<sup>13</sup>, the slope is not  $-5/3$ , but rather  $-5/2$ . This value was not only observed in all spectra of the turbine impeller, but we also measured it in the jet of a propeller. It agrees with a few spectra known from the literature<sup>6,9</sup> and recent laser-Doppler measurements in the wake of a cylinder<sup>14</sup>, showing a slope of  $-5/2$  to  $-2$  at high wave numbers. Up to now no explanation has been found for this observation.

About the low-wave-number parts for the different scales, it should be remarked that the ratios between their spectral levels are larger than  $D^{5/3}$ , as expected from a  $-5/3$ -slope of the high-wave-number part. A low-wave-number level proportional to  $D^{5/3}$  would give rise to an increase in the RMS value of the velocity fluctuations by  $D^{1/3}$ , so proportional to the mean velocity. However, our measurements show that these RMS values, calculated from the integrated power spectra, appear to increase by  $D^{1/2}$  rather than by  $D^{1/3}$ . This implies that the low-wave-number level is proportional to  $D^2$ , which

is in better agreement with our results than a proportionality to  $D^{5/3}$ . So we are confronted with the paradoxical situation that the relative turbulence intensity in the impeller stream region increases by  $D^{1/6}$ , at least for vessels with  $D$  smaller than 1 m. Our results show a tendency to a smaller increase in the bigger vessels. On the other hand, integration of the curves of Figure 8 shows too small an RMS value for the low-velocity curve. So we might think that we had operated at too low Reynolds number. But all our experiments were performed at Reynolds numbers well above  $10^4$ , a value which is commonly considered high enough for scaling rules to be applicable.

#### CONCLUSIONS

As far as the laser-Doppler velocimetry is concerned, we conclude that

- a large reduction in ambiguity noise in the turbulence power spectrum can be achieved by cross-correlation of two signals obtained at the same measuring points. This can be achieved because the noise in the signals is not correlated.
- when the velocity fluctuations in the direction of the mean flow are measured, an additional reduction in the noise level can be obtained if at their focal points the beams are one beam diameter apart. Since the length of the measuring control volume is generally larger than its width, this technique does not imply a reduction in spatial resolution.

For the turbulence in the impeller stream region of mixing vessels of equal geometry and operating at equal  $\bar{\epsilon}$ , the following scaling rules can be derived from our measurements:

- The mean velocity scales as  $D^{1/3}$ .
- The RMS value of the turbulent fluctuations increases more steeply than the mean velocity; the turbulence intensity is roughly proportional to  $D^{1/6}$ .
- The low-wave-number level of the power spectrum is proportional to  $D^2$  rather than to  $D^{5/3}$ .
- The high-wave-number part falls off by roughly the wave-number to the power  $-5/2$ , its value being determined by the local dissipation.

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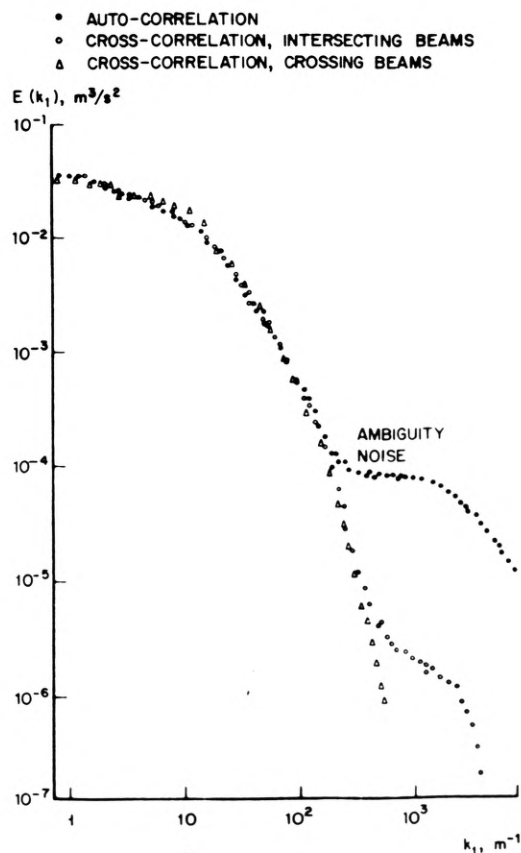


FIGURE 1. NOISE REDUCTION IN A TURBULENCE POWER SPECTRUM OF PIPE FLOW OF WATER

Some data: Pipe diameter = 5.0 cm,  $Re \approx 3 \times 10^4$ , optical constants as in Fig. 2. Longitudinal component of velocity measured at centre line of pipe. The wave number  $k_1$  is defined as  $\frac{\text{frequency}}{\text{mean velocity}}$ .

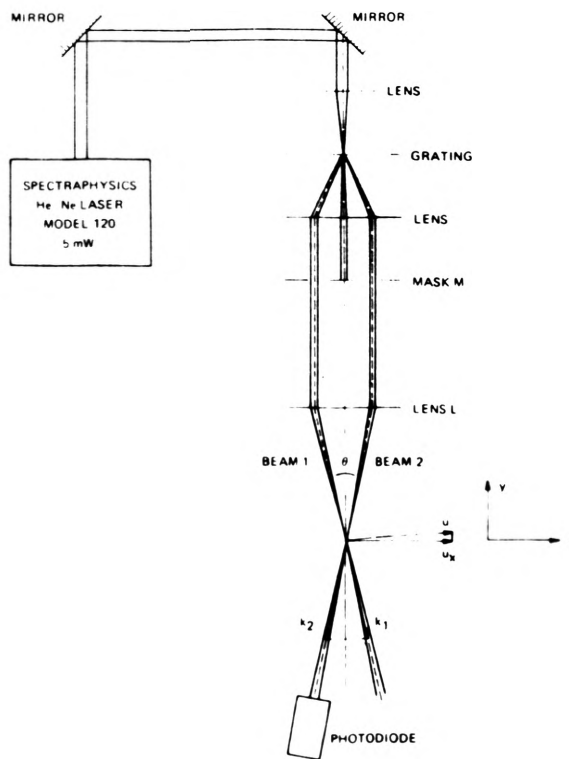


FIGURE 2. OPTICAL SYSTEM OF LASER-DOPPLER VELOCIMETER

For a focal length of lens  $L = 120$  mm,  $\theta = 14^\circ 38'$ ,  $\lambda = 632.8$  nm and  $u_x = 1$  m/s the Doppler shift is 402.5 kHz. Dimensions of measuring volume (in air): in  $y$  direction: 1.45 mm; in  $x$  direction: 0.19 mm.

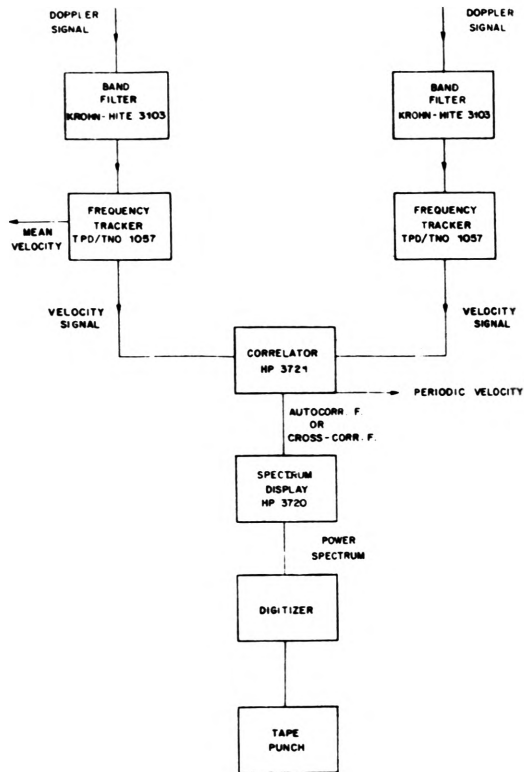


FIGURE 3. FLOW SCHEME OF THE DOPPLER SIGNAL PROCESSING

The second branch of band filter and tracker is only used when the spectrum is measured with the cross-correlation technique.

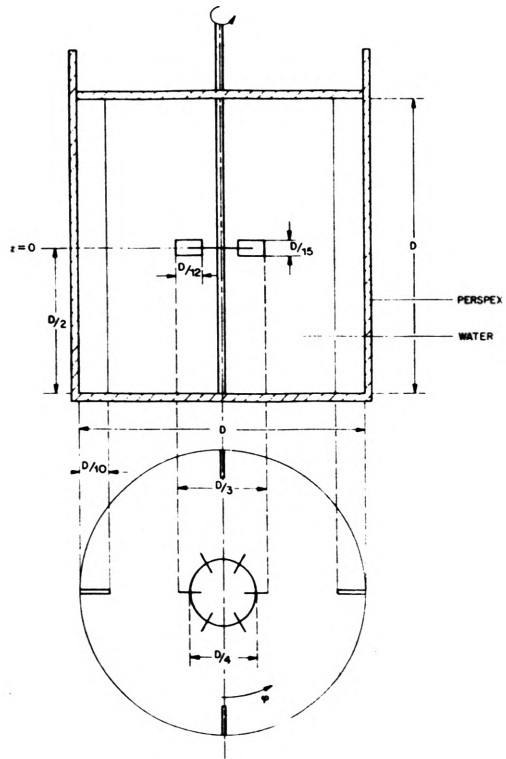


FIGURE 5. SCHEMATIC DRAWING OF THE MIXING VESSEL GEOMETRY

$D = 0.12, 0.29$  and  $0.90$  m.

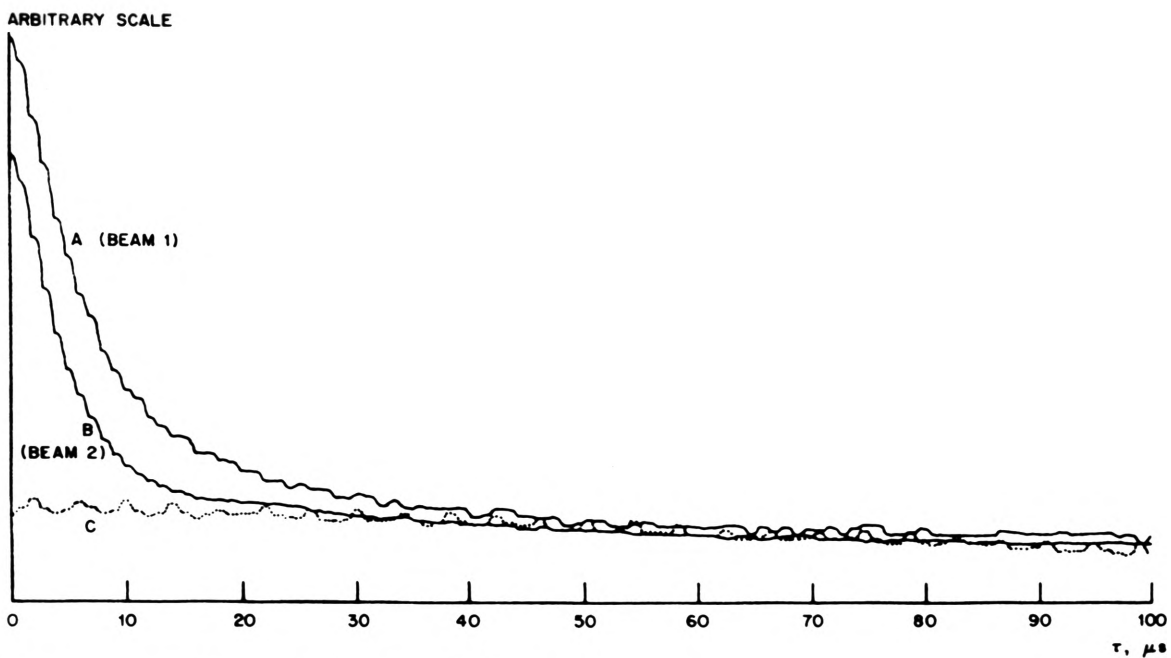


FIGURE 4. CORRELATION OF THE DROP-OUT SIGNALS

A and B are autocorrelation functions, C is the cross-correlation function

FIGURE 6. AMPLITUDE DISTRIBUTION OF THE PERIODIC  $r$  AND  $z$  COMPONENTS IN TWO AXIAL CROSS-SECTIONS THROUGH THE IMPELLER STREAM OF THE SMALLEST VESSEL, WITH  $D = 0.12$  m

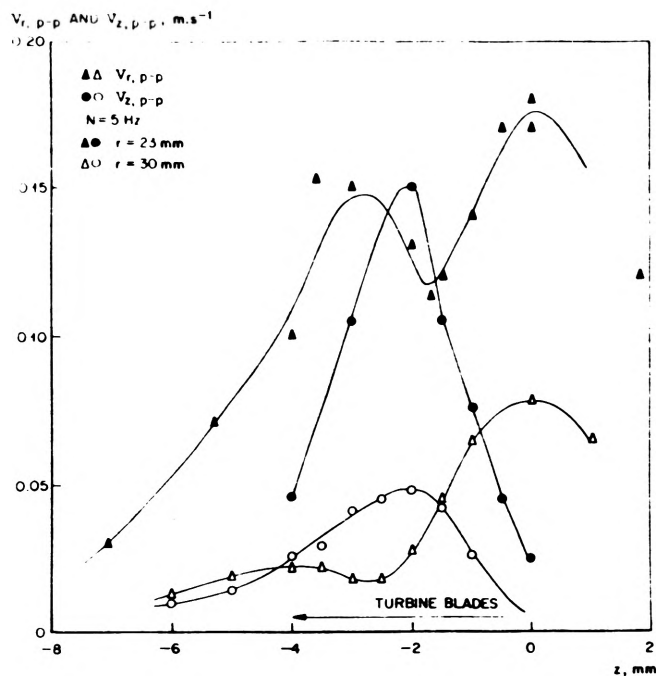


FIGURE 7. POWER SPECTRA OF THE RADIAL COMPONENT  
Position: impeller centre plane at  $D/4$  from the axis.  
• :  $D = 0.90$  m  
\* :  $D = 0.20$  m  
o :  $D = 0.12$  m

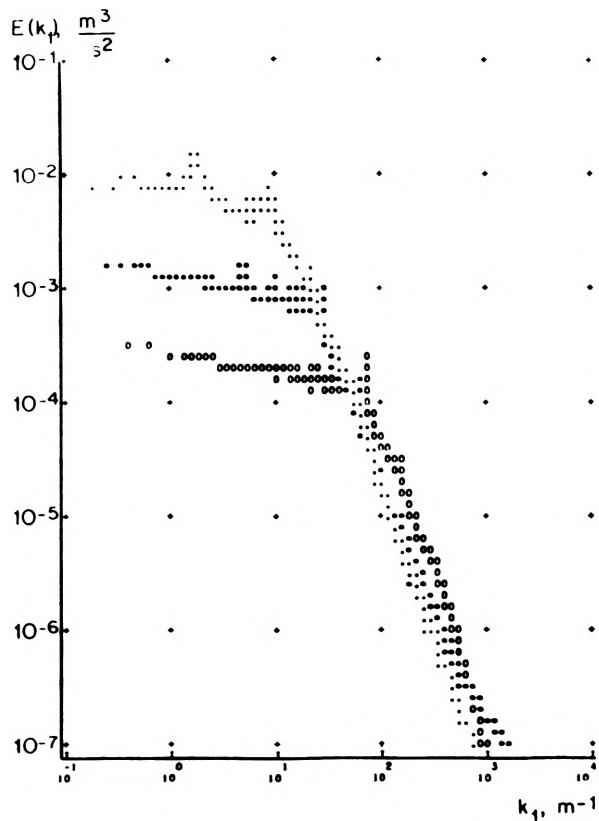


FIGURE 8. POWER SPECTRA OF THE RADIAL COMPONENT AT TWO IMPELLER VELOCITIES FOR  $D = 0.90$  m  
 Position: impeller centre plane at  $D/3$  from the axis.  
 • : Impeller frequency  $N = 1.30$  Hz, the frequency used for the spectra of Figures 7 and 9.  
 \* :  $N = 0.18$  Hz.

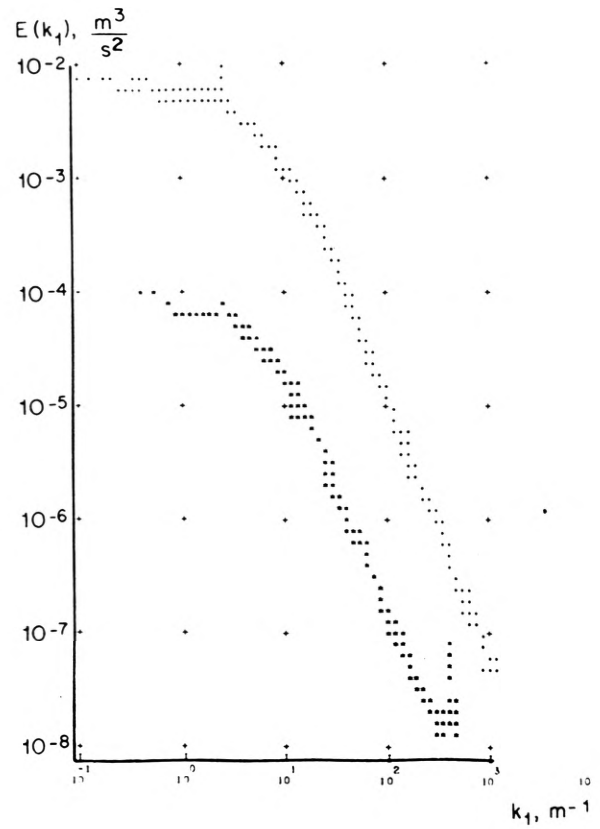
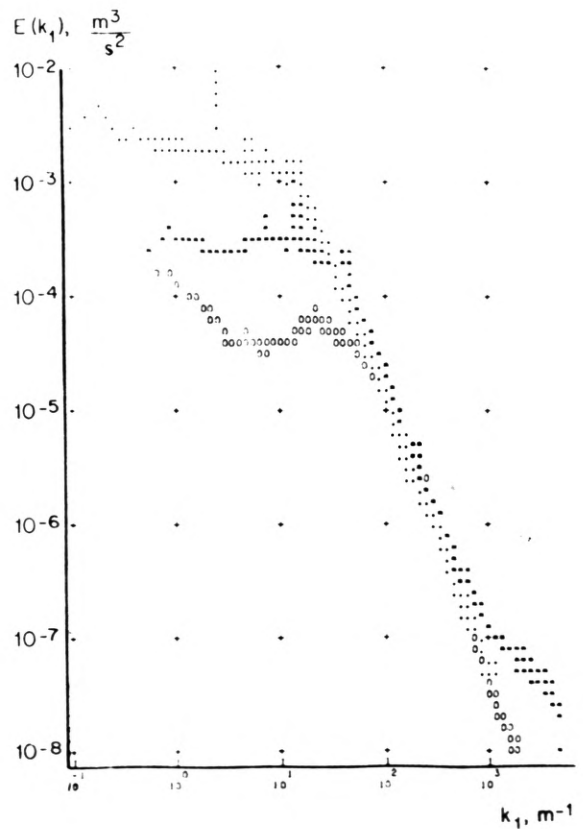


FIGURE 9. POWER SPECTRA OF THE AXIAL COMPONENT  
 Position: impeller centre plane at  $D/3$  from the axis.  
 For further details, see Figure 5.





## DISCUSSION

Phil McConnell, Westinghouse, Washington: First of all have you correlated the power spectra with "the degree of mixing or mixing effectiveness"?

van der Molen: No, I didn't correlate the mixing quantities with my velocity measurements. We only measured the turbulence characteristics.

McConnell: Could your measuring technique be applied to a continuous flow mixing reactor in which flow is coming in and exiting from a vessel so there is a finite residence time of the fluid particles? This is a very important problem for me. I am working with liquid sodium mixing in a nuclear reactor inlet plenum, and we would like to be able to predict some of the mixing properties a priori by knowing just the geometry and the flow rates. Right now I know of no viable way of attacking this problem other than experimental.

van der Molen: I agree with you. After this work we should go into the theory of mixing or suspending small particles or making emulsions in some mixing vessel. I told you already, we started this work because we had to check some new scaling rules for the pelletizing of particles with a second immiscible liquid in it. Colleagues of mine found scaling rules different from what they had expected and they asked us to use our laser equipment to look at the fluid flow and the turbulence structure.

G. Comte-Bellot, Ecole-Centrale de Lyon, Ecully, France: What is the prevalent Reynolds number of your turbulent field?

van der Molen: We did make our spectra dimensionless with the Kolmogoroff scale. We did not present them as it could also be done with the outer scale of the vessel and  $u'$ .

Comte-Bellot: I think this turbulent Reynolds number is too small to get a large -5/3 slope region.

van der Molen: It is remarkable that we found such straight lines over several decades in this spectrum. When you find such a straight spectrum you would expect it to be a -5/3 slope.

Robert S. Brodkey, Ohio State Univ.: There is work going on at the Technical Univ. at Zurich on a similar problem using laser Doppler in a mixing tank. I think the gentleman working on it is Reed. You may not be aware of it because it has not been published yet.

van der Molen: I did not.

Brodkey: What was your impeller speed?

van der Molen: The impeller speed was 5 revolutions per second in the smallest vessel and in the biggest it was 1.3.

Brodkey: I agree with Dr. Comte-Bellot that the Reynolds number seems low because in our measurements in roughly the same size vessels, we have several hundred based on the Taylor microscale using  $u'$ . One would expect, and we did observe, a -5/3 region in our spectra, but the Reynolds numbers of the turbulence were maybe a factor of 3 larger, 300 to 400.

van der Molen: I calculated  $Re_\lambda = \frac{u'\lambda}{\nu}$  from  $\epsilon = 15 \nu u'^2/\lambda^2$  using the average value of  $\frac{\epsilon}{\nu}$ . For the biggest vessel we then find  $Re_\lambda$  of several hundreds.

Gary K. Patterson, Univ. of Missouri: With regard to the energy spectra measured in the tank, they could be very important in determining the rate of turbulence energy dissipation because the dissipation rate seems to be the most important variable determining the small scale mixing rates. Corrsin has in his work shown that this is true. So if the energy spectra could be used to calculate reliable rates of dissipation, they would be very important in our efforts to model mixing rates.

van der Molen: I want to mention two points. We hope to measure the turbulence dissipation rates from these spectra when we have found this -5/3 slope and we can be sure that we have the inertial sub-range with the dissipation to the 2/3 power in it. Then you can use the spectra to determine local dissipation.

But from the other side I would say to those people working in this mixing problem, you are going the reverse way. In the past they could only measure the average dissipation and from that they determined mixing quality. But now we are able to measure the true local velocities and these are the important quantities to use in modelling the mixing process. So

van der Molen (cont.): you don't have to use the dissipation, you should make a theory to relate your fluid flow velocities to your mixing qualities.

Patterson: I really feel that the velocity measurements are important in mixing, but in a particular regard. It's helpful to look at what's going on in any kind of mixing vessel or jet or anything else as being divided into two processes. In one you've got a blending effect or a gross large-scale mixing of components, or whatever happens to be in the system, and this can be defined based on the large scales and the velocities associated with them in the turbulence. On the other hand, you have the small scale mixing processes which bring things together down to molecular scales and it is at the small scales that the rate of turbulence energy dissipation becomes important in determining the final rate of the mixing. If you have processes which involve chemical reactions or similar interactions between mixed components, this is probably the most important consideration in the mixing process.