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# Characteristics of Laser Anemometer Signals and Frequency Demodulators for Highly Seeded Flows

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CHARACTERISTICS OF LASER ANEMOMETER SIGNALS AND FREQUENCY DEMODULATORS FOR HIGHLY SEEDED FLOWS

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## ABSTRACT

An analysis of the general characteristics of laser anemometer signals for liquid flows is made. Three detection schemes are analyzed:

(1) Spectral analysis of the photodetector output,
(2) frequency trackers, and
(3) counters

Where known, the strengths and weakness of each method are analyzed.

## INTRODUCTION

There have been many analyses of the signal from laser anemometers. However, not much has been written on the behavior of real detectors of the signals. It is our intention here to outline the basic characteristics of the laser anemometer signal and to illustrate the effect of those characteristics on the behavior of various detectors. Mathematical and physical rigor will occasionally be sacrificed in the interest of clarity of the presentation. It is hoped, however, that enough details and references are included to enable the reader to perform a detailed analysis of any particular problem of interest. The discussion will be restricted to systems with high scattering particle densities such as are encountered in liquid flows.

## THE LASER ANEMOMETER SIGNAL

Most motion measurements by light scattering take direct advantage of the coherent properties of laser light. In this context, coherence can be defined as the ability of the light to form sharp diffraction patterns. Other light sources can be used to generate diffraction patterns, indeed diffraction patterns have been studied for decades, even centuries before the advent of the laser. However, the laser is an intense source of coherent light and "more coherent" than thermal light sources or even line sources such as filtered hollow cathode lamps. It is for this reason lasers are used in practical systems.

Let us reconsider Young's Double Diaphragm Experiment. (See Figure 1.) A coherent source of light impinges on a screen with two slits in it. A diffraction pattern consisting of alternating rows of bright lines and dark lines is formed on another screen placed beyond the slits. The dark lines are where the light from the two slits arrives out of phase and canel. The bright lines are where the light from the two slits arrive at the second screen in phase. It is a simple matter to compute the position of the bright lines (y) if one knows:[1]

(1) The light wavelength, ($\lambda$)
(2) the slit spacing (d), and
(3) the distance between the screens (R)

$$y = \frac{R\lambda}{d} \cdot m, \quad m = 1, 2, 3, \cdots$$

To get an idea of the fringe spacing, let

$$\lambda = 6.328 \times 10^{-5} \text{ cm}$$

$$d = 0.2 \text{ cm}$$

$$R = 100 \text{ cm}$$

$$y = \frac{6.328 \times 10^{-5} \times 100}{0.2} = 3.164 \times 10^{-4} \text{cm}$$

Suppose one removes the second screen and substitutes a moving point scatterer (such as a small dust particle) and lets it move in the plane of the screen. Let the light from the scatterer be detected by a light detector placed in a position out of direct illumination by the slits but able to see the scatterer. (See Figure 2.)

The detector sees light from both slits scattered from the particle. In the travel from the particle to the detector, there is no relative phase shift of the light from the two sources, so that the particle is perceived to be bright or dim depending on whether the light arriving at the particle from the two slits is in phase or not. Let the particle now move vertically at a steady velocity V. As long as the particle is visible, it will generate a sinusoidal signal of frequency $\frac{Vd}{R\lambda}$ , where all the symbols have the same meaning as before. A signal has been generated at the detector whose frequency is proportional to the velocity of the particle.

The above simple experiment describes the fundamental operation of laser anemometers and was presented in order to set the stage for the following presentation that will not use the notion of a "Doppler" shift, nor the terminology that accompanies that notion.

Consider now a more practical version of Young's Double Diaphragm Experiment. (See Figure 3.) These two configurations are the most popular for laser anemometers. In both configurations, the signal at the detector is determined by the interaction of two mutually coherent light beams from the same source. At the detector, a bright particle is seen if the particle is in a position where the beams are in phase and dark if the beams are out of phase.

Now, let's consider the E-field at a detector from a collection of particles whose positions at time t are given by $\vec{r}_n(t)$.

For a dual beam system (Figure 3a), the total E-field at the detector is the sum of the E-fields scattered from the two beams incident on the measurement volume.

$$\mathcal{E}_d(t) = \sum P_1(\vec{r}_n(t))\, e^{i(\vec{K}_1-\vec{K}_3)\cdot\vec{r}_n(t)} + P_2(\vec{r}_n(t))\, e^{i(\vec{K}_2-\vec{K}_3)\cdot\vec{r}_n(t)}$$

The functions $P_1$ and $P_2$ are amplitude weighting functions for the beams $E_1$ and $E_2$. The phases of the beams arriving at the particle n is given by the terms $e^{i\vec{K}_1\cdot\vec{r}_n}$ and $e^{i\vec{K}_2\cdot\vec{r}_n}$ . The additional phase change of the beam down to the detector is given by terms $e^{-i\vec{K}_3\cdot\vec{r}_n}$

The photodetector current is proportional to the intensity which is the square of the E-field, i.e., EE*. For the dual beam system:

$$i_T(t) =$$

$$\mathrm{Re}\Big\{\sum_n\sum_m P_1(\vec{r}_n(t))\,P_1(\vec{r}_m(t))\exp\left[i(\vec{K}_1-\vec{K}_3)\cdot(\vec{r}_n(t)-\vec{r}_m(t))\right]$$

$$+\sum_n\sum_m P_2(\vec{r}_n(t))\,P_2(\vec{r}_m(t))\exp\left[i(\vec{K}_2-\vec{K}_3)\cdot(\vec{r}_n(t)-\vec{r}_m(t))\right]$$

$$+\sum_n\sum_m P_1(\vec{r}_n(t))\,P_2(\vec{r}_m(t))\exp\left[i(\vec{K}_1-\vec{K}_3)\cdot\vec{r}_n(t)-i(\vec{K}_2-\vec{K}_3)\cdot\vec{r}_m(t)\right]\Big\}$$

where $\mathrm{Re}$ is real part.

The first two terms are not determined by interference between the two incident beams. They are determined by interference of the light from one beam scattered by two or more particles. When computing the autocorrelation or spectrum of the photodetector current, one finds that these terms are dependent on the relative motion of the particles and thus under normal circumstances (a small measurement volume), represents velocities much lower than the velocity of a particle. The spectrum for these terms is centered at 0 Hz and thus is not of interest to this discussion[2]. It is normally filtered out of the signal before detection. We will neglect it. The third term can be rewritten,

$$i(t) =$$

$$\mathrm{Re}\Big\{\sum_n P_1(\vec{r}_n(t))\,P_2(\vec{r}_n(t))\,e^{i(\vec{K}_1-\vec{K}_2)\cdot\vec{r}_n(t)}$$

$$+\sum_{n\neq m}\sum P_1(\vec{r}_n(t))\,P_2(\vec{r}_m(t))\exp\left[i(\vec{K}_1-\vec{K}_3)\cdot\vec{r}_n-i(\vec{K}_2-\vec{K}_3)\cdot\vec{r}_m\right]\Big\}$$

The first term is the so-called "incoherent" term and essentially the vector sum of the signals from each individual particle. The second term is the so-called "coherent" term and it represents the interference between the E-field from pairs of particles scattered from two input beams ($\vec{K}_1$ and $\vec{K}_2$). Normally, laser anemometers operate with rather low f number optics and under these circumstances, the coherence term is negligible compared to the incoherent term (3). So, we have left:

$$i(t) =$$

$$\mathrm{Re}\Big\{\sum_n P_1(\vec{r}_n(t))\,P_2(\vec{r}_n(t))\,e^{i(\vec{K}_1-\vec{K}_2)\cdot\vec{r}_n(t)}\Big\}$$

$$= Re\left\{\sum P(\vec{r}_n(t))\, e^{i\vec{K}\cdot\vec{r}_n(t)}\right\}, \text{ where } P = P_1 \cdot P_2 \text{ and } \vec{K} = \vec{K}_1 - \vec{K}_2 .$$

The vector $\vec{K}$ is referred to[2] as the scattering vector. It is a vector perpendicular to the planes of the interference pattern and its magnitude is given by:

$$|\vec{K}| = \frac{4\pi}{\lambda}\,\eta\, \sin\frac{\theta}{2}$$

, where $\eta$ is the refractive index of the medium where the beams intersect and $\theta$ is the angle between $\vec{K}_1$ and $\vec{K}_2$, and $\lambda$ is the vacuum wavelength of the laser.

For a reference beam system (Figure 3b), the E-field at the detector can be written

$$\mathcal{E}_d(t) = \sum_n P_1(\vec{r}_n(t))\, e^{i(\vec{K}_1-\vec{K}_2)\cdot\vec{r}_n(t)} + A$$

where A is the amplitude of the second beam ($\vec{K}_2$) at the detector since it is not scattered. The phase of this beam has been assumed to be zero at the detector.

$$i_H(t) = \left\{A^2 + \sum_m\sum_n P_1(\vec{r}_n(t))P_1(\vec{r}_m(t))\, e^{i(\vec{K}_1-\vec{K}_2)\cdot(\vec{r}_n(t)-\vec{r}_m(t))} + A\sum P_1(\vec{r}_n(t))\, e^{i(\vec{K}_1-\vec{K}_2)\cdot\vec{r}_n(t)}\right\}$$

The first term $A^2$ is a constant since we assume the one beam is not scattered (a reference beam). The second term, as in the previous section, represents a low frequency term due to interference of the light from pairs of particles scattered from one beam. Again, this term will be discarded. The reference beam system thus results in a similar expression to that of the dual beam system.

$$i_H(t) = \sum_n P_1(\vec{r}_n(t))\, e^{i\vec{K}\cdot\vec{r}_n(t)}$$

We now have, for both configurations of the laser anemometer, a similar expression for the instantaneous intensity at the photodetector as a function of the particle positions.

It is instructive to compute the spectrum or autocorrelation function of the photodetector current. By the Weiner-Khintchine theorem, the autocorrelation is the temporal Fourier transform of the spectrum and, of course, vice versa[4]. As we will see later, one can learn much about the expected behavior of various detectors from these functions.

The photodetector current can be rewritten:

$$i(t) = \sum_n P(\vec{r}_{no} + \Delta\vec{r}_n(t))\, e^{i\vec{K}\cdot\vec{r}_{no}}\, e^{i\vec{K}\cdot\Delta\vec{r}_n(t)}$$

where $\vec{r}_{no}$ is the initial position of the $n^{th}$ particle and $\Delta\vec{r}_n(t)$ is the change in position of the $n^{th}$ particle in time t. This format allows for the possibility that the particles may accelerate during the time they are observed.

We will assume that the flow field is such that a particle does not accelerate appreciably in the time it takes the particle to traverse one fringe spacing. Indeed, if this condition isn't fulfilled, one can no longer speak of "Doppler shifts" or velocity measurements[5].

With the above assumption, the autocorrelation $R(\tau)$ can be written[5]

$$R(\tau) = \overline{i(t)\,i^*(t+\tau)} = \rho\iint_{-\infty}^{\infty} P(\vec{r})P(\vec{r}+\vec{v}(\vec{r})\tau)\,V(\vec{v}(\vec{r}))\, e^{i\vec{K}\cdot\vec{v}(\vec{r})\tau}\, d\vec{r}\, d\vec{v}$$

where P( ) is the amplitude weighting function for the detected E-field, $V(\ )$ is the velocity probability density for a velocity $\vec{v}$, and $\rho$ is the density of particles.

The spectrum, $S(\omega)$, of the signal is the temporal Fourier transform of $R(\tau)$[4].

The mean, $\bar{\omega}$, and variance, $\overline{\Delta\omega^2}$, of the spectrum can be computed from the autocorrelation[4]

$$\bar{\omega} = -i\left.\frac{dR}{d\tau}\right|_{\tau=0} \Big/ R(0)$$

$$\overline{\Delta\omega^2} = -\left.\frac{d^2R}{d\tau^2}\right|_{\tau=0} \Big/ R(0) - \bar{\omega}^2$$

Since it is our purpose here to elucidate the general principles of laser anemometer detector, some assumptions will be made to simplify the mathematical expressions while retaining the essential behavior of the system. We assume:

(1) The $\vec{K}$ vector points in the direction of the mean flow which is lined up in the x-direction.

(2) The velocity fluctuations probability is independent of position.

(3) The contribution to the autocorrelation from particles moving perpendicular to $\vec{K}$ is negligible (see Equations 35 and 36 in reference 5).

(4) The gradient in velocity is perpendicular to the direction of the mean flow.

(5) The amplitude weighting function can be written:

$$P(\vec{r}) = P_x(x)\,P_y(y)\,P_z(z)$$

None of the assumptions above are necessary to do the analysis, however, they approximate accurately many physical situations. With the above assumptions, the velocity component in the x-direction can be written:

$v_x = \tilde{v}_x(y,z) + v_x'$, where , $\tilde{v}_x(y,z)$ is the mean velocity as a function of y and z, and $v_x'$ is the fluctuating component of the velocity. The time average of $v_x'$ is zero. Now,

$$R(\tau) = \rho \iiint_{-\infty}^{\infty} Q_x((\tilde{v}_x + v_x')\tau) P_y^2(y) P_z^2(z) V(v_x') e^{iK(\tilde{v}_x + v_x')\tau} \, dy\, dz\, dv_x',$$

where $$Q_x(v_x\tau) = \int_{-\infty}^{\infty} P_x(x) P_x(x + v_x\tau)\, dx$$

The spectrum $S(\omega)$ for positive $\omega$, can now be computed (See Figure 4).

$$S(\omega) = \rho \iiint \frac{1}{v_x} \hat{Q}_x\left(\frac{\omega - Kv_x}{v_x}\right) P_y^2(y) P_z^2(z) V(v_x')\, dy\, dz\, dv_x'$$

where $\hat{Q}_x$ is the spatial Fourier transform of $Q_x$.

The first two moments of the spectrum can be computed.

$$\bar{\omega} = \frac{\rho}{R(0)} \iiint_{-\infty}^{\infty} \left(Kv_x + i\,\dot{Q}_x(0)\right) P_y^2(y) P_z^2(z) V(v_x')\, dy\, dz\, dv_x'$$

Since $P_x$ is square integrable, it can be shown that $\dot{Q}_x(0) = 0$, thus

$$\bar{\omega} = \frac{\rho}{R(0)} \iiint_{-\infty}^{\infty} K(\tilde{v}_x(y,z) + v_x') P_y^2(y) P_z^2(z) V(v_x')\, dy\, dz\, dv_x'$$
$$= \frac{\rho}{R(0)} \iint K\tilde{v}_x(y,z) P_y^2(y) P_z^2(z)\, dy\, dz + \int K v_x' V(v_x')\, dv_x'$$

By definition,

$$\int v_x' V(v_x')\, dv_x' = \overline{v_x'} = 0, \quad \text{thus}$$
$$\bar{\omega} = \frac{\rho K}{R(0)} \iint \tilde{v}_x(y,z) P_y^2(y) P_z^2(z)\, dy\, dz \equiv K\bar{v}_x$$

The quantity $\bar{v}_x$ is the amplitude weighted mean velocity seen by the laser anemometer (2,11)

$$\overline{\Delta\omega^2} = \frac{\rho}{R(0)} \iiint \left(K^2(\tilde{v}_x^2 + v_x'^2 - \bar{v}_x^2) - \ddot{Q}_x(0)\right) P_y^2(y) P_z^2(z) V(v_x')\, dy\, dz\, dv_x'$$
$$\simeq \frac{\rho}{R(0)} \iiint \left(K^2(\tilde{v}_x^2 + v_x'^2 - \bar{v}_x^2) + \frac{\tilde{v}_x^2}{\sigma_x^2}\right) P_y^2(y) P_z^2(z) V(v_x')\, dy\, dz\, dv_x'$$

$\ddot{Q}_x(0)$ has been approximated by $-\frac{\tilde{v}_x^2}{\sigma_x^2}$ where $\sigma_x$ is the characteristic length of the measurement volume in the flow direction[5].

$$\overline{\Delta\omega^2} \approx K^2(\overline{v_x^2} - \bar{v}_x^2) + \frac{\overline{v_x^2}}{\sigma_x^2} + K^2\overline{v_x'^2},$$

where $\overline{v_x^2}$ is the amplitude weighted mean square velocity seen by the velocimeter.

## INSTRUMENT RESPONSE

### Spectrum Analysis (Autocorrelograph)

There exist instruments that will measure directly the autocorrelation or spectrum of a signal. At present, the most popular form of spectrum analyzer is a tunable filter with an amplitude detector attached. As the filter is tuned through a given frequency range, the output is a signal proportional to the square root of the power in the filter bandwidth. If this signal is squared and stored, an accurate representation of spectrum of the signal can be obtained[13]. The operation of this type of spectrum analyzer is akin to that of a standard AM radio. These devices are relatively slow in that the filter can only see a small part of the signal at a time.

Another type of device used to obtain the autocorrelation or spectrum of a signal is parallel processors that use all of the signal from a given time period to construct the spectrum. A computer Fast Fourier Transform routine or commercial correlation computer are typical devices of this kind. These machines can measure the spectrum to a given degree of accuracy two or three magnitudes faster than can a tunable filter type. Any instrument corrections to spectrum obtained in this fashion are well known and so will be ignored in this analysis.

As was shown above, the mean frequency of the spectrum obtained by direct spectral analysis of the photodetector output is the amplitude weighted mean of velocity in the sample region. This measured velocity is not necessarily the velocity at the center of the measurement volume. However, the correction is caused chiefly by the curvature of the velocity[11] and is only on the order $0.25 \frac{d^2 v_x}{dz^2} \sigma_z^2$ where $\sigma_z$ is the characteristic dimension of the measurement volume in the direction of the gradient. Measurement volumes are usually too small to sample much of the curvature in most flow profiles, so that the correction in the measured mean due to gradients is usually less than 1% of the measured velocity. Many examples have appeared in the literature of velocity measurements using a spectrum analyzer in the presence of gradients. Figure 5 shows the results of mean velocity measurements near the wall of a 2" pipe in turbulent flow using a spectrum analyzer. The solid line is the "Law of the wall" plot, $v^+ = y^+$ obtained from the volumetric flow rate. As one can see from the data, quite accurate measurements of mean velocity can be made.

The width of the spectrum has three components:

(1) $\overline{v_x^2}/\sigma_x^2$ This term has been called the "finite transit time ambiguity". This is unfortunate terminology, since there is no real ambiguity caused by this term. All that is required is that a smooth enough spectrum be obtained to get a good statistical measure of the mean and variance of the spectrum.

(2) $K^2(\overline{v_x^2} - \overline{v}_x^2)$ This term is the broadening due to a mean velocity gradient in the flow seen by the laser anemometer. This term is on the order $a^2\sigma_z^2/4$ where a is the gradient of the mean velocity.

(3) $K^2\overline{v_x'^2}$ This term is the broadening due to fluctuations in velocity. It is proportional to the local variance of the fluctuating velocity.

The turbulence intensity can be obtained from a measurement of the second central moment of the spectrum.

$$\frac{\overline{\Delta\omega^2}^{1/2}}{\overline{\omega}} = \left[\frac{\overline{v_x'^2}}{\overline{v}_x^2} + \frac{\overline{v_x^2}}{\overline{v}_x^2}\left[1 + \frac{1}{K^2\sigma_x^2}\right] - 1\right]^{1/2}$$

The term $K\sigma_x$ is a measure of the number of fringes in the measurement volume. The more fringes, the smaller the finite transit time correction.

A gradient broadening term can be computed from a plot of the mean velocity(6). Figure 6 shows measurements of turbulence intensity as function of position in a 2" pipe. The measurements have about ± 10% error in them. More accurate measurements could have been made by averaging longer.

## Signal to Noise Ratio

The primary noise component of a laser anemometer signal is the shot noise due to the essential particulate nature of the photons arriving at the detector(3). The spectrum of the noise is "white" (i.e., a flat spectrum) and the power/unit bandwidth of the noise is proportional to the photodetector current(3).

The power/unit bandwidth of the spectrum is proportional to the square of photodetector current. For a given averaging time, the area under a signal spectrum is constant(2) independent of velocity; however, the width of the spectrum $(\overline{\Delta\omega^2})^{1/2}$ increases as the velocity increases. Therefore, the power/unit bandwidth of the signal decreases as the velocity increases.

Fortunately, the shot noise spectrum appears as a flat baseline under the signal spectrum, and can usually be subtracted. Thus, signal to noise ratios less than one, on a peak signal power/unit bandwidth to noise power/unit bandwidth basis, can be tolerated.

## TRACKERS

To a crude approximation, a laser anemometer signal has a frequency proportional to the velocity in the measurement volume. There are many types of detectors that attempt to measure the instantaneous frequency of the anemometer signal. The most popular type of frequency detector is the tracker. The tracker is a phase lock or frequency lock loop, where a local oscillator is continually controlled to track the frequency of the incoming laser anemometer signal. These devices can be viewed as tracking filters as the bandwidth seen by the device is approximately the speed of the control sytem for the local oscillator(14).

Before discussing frequency detectors in detail, it is necessary to examine the frequency content of the signal. By the previous analysis, the signal clearly contains more than one frequency.

If the signal is written in the form:

$$i(t) = A(t)\cos(\theta(t))$$

an ideal frequency detector gives an output

$$\theta(t) = \frac{d\theta}{dt} \equiv \dot{\theta}.$$

Initially, we will assume that the flow is laminar and steady with no mean gradient.

The phase of the laser signal can be written:

$$\theta = \vec{K}\cdot\vec{v}t + \arctan\left[\frac{\sum_n P(\vec{r}_{no}+\vec{v}t)\sin\vec{K}\cdot\vec{r}_{no}}{\sum_n P(\vec{r}_{no}+\vec{v}t)\cos\vec{K}\cdot\vec{r}_{no}}\right]$$

where $\vec{r}_{no}$ is the initial position of particle n.

The phase of the signal contains two terms:

(1) A linear phase term $\vec{K}\vec{v}t$

(2) A fluctuating term that is a function of the shape of the measuring volume and the initial positions of the particles

One can visualize the behavior of this second term by means of a vector picture (see Fig. 7). The resultant vector $\vec{R}$ is made by adding vectors with components $(P(\vec{r}_{no} + \vec{v}t)\sin\vec{K}\cdot\vec{r}_{no},\ P(\vec{r}_{no} + \vec{v}t)\cos\vec{K}\cdot\vec{r}_{no})$. As time evolves, some particles enter or leave the measurement volume causing the vector $\vec{R}$ to change length and direction. Indeed, when the resultant vector is very small due to cancellation, one can see how the angle of the resultant vector could very quickly change from $\pi/2$ to $-\pi/2$. Since the output of an ideal detector is the time derivative of the total phase, this type of occurrence can cause large fluctuations in $\dot{\theta}$.

$$\dot{\theta} = \vec{v} \cdot \left[ \vec{K} + \frac{\sum_n \sum_m \frac{\partial P}{\partial r}(\vec{r}_{no}+\vec{v}t) P(\vec{r}_{mo}+\vec{v}t) \sin \vec{K}\cdot(\vec{r}_{no} - \vec{r}_{mo})}{\sum_n \sum_m P(\vec{r}_{no}+\vec{v}t) P(\vec{r}_{mo}+\vec{v}t) \cos \vec{K}\cdot(\vec{r}_{no} - \vec{r}_{mo})} \right]$$

Figure 8 shows an oscillograph of: 1) a carefully simulated laser anemometer signal for steady laminar flow, 2) the output of a $\dot{\theta}$ detector, detecting that signal. Note the large fluctuations in the output, even though the flow is steady. These fluctuations have been termined "ambiguity noise".

If the device is locked to the signal (i.e., the phase error is less than $\pi/2$ radians), the output is indeed $\dot{\theta}$. It can be shown (8) that

$$\overline{\dot{\theta}} = \overline{\omega}.$$

Thus, a tracker can give as accurate a measure of the mean velocity as does a spectrum analyzer.

In principle, therefore, the "ambiguity noise" is no hindrance to measurements of velocities; however this term can cause problems for real devices - such as phase lock or frequency lock loops. Real devices cannot handle arbitrarily large or arbitrarily fast fluctuations. The fluctuations have a spectrum whose bandwidth is on the order of the finite transit time ambiguity (7). If the response speed of the detector is slower than this, it will not be "locked" much of the time and its output will not correspond to the desired output, $\dot{\theta}$. In the measurement of cross flows this width can be a substantial fraction of the signal frequency, and the detector's bandwidth should be adjusted accordingly.

In velocity gradients, due to the intrinsic nonlinearity of the tracker, beat frequencies will be generated in the tracker corresponding to the difference in frequency generated by particles in different positions in the measurement volume*. The width of the signal due to the gradient is

$$\frac{K}{\pi} \left( \overline{v_x^2} - \overline{v_x}^2 \right)^{1/2} .$$

The maximum beat frequency the tracker must follow is approximately this width. These beat frequencies can be a significant fraction of the signal frequency, expecially in measurements near walls. In order to give a correct output, a tracker must have a response frequency higher than beat frequencies encountered. In many measurements of mean velocity in the presence of gradients, the tracker speed has apparently been too low and erroneous measurements have resulted. Apparently, the tracker loses lock rather often and only samples the signal. Since the scattering particles are randomly positioned in space, more fast moving particles go through the measurement volume/unit time than the slow particles. The tracker thus seems to weigh the fast measurements more than the slow measurements. Erroneously high velocities are obtained. Since the tracker is clearly not locked and, thus, it is not a $\dot{\theta}$ detector, no rigorous theory exists for correction of this bias. The experimentalist should be sure that his (or her) tracker is fast enough to accurately detect the signal.

If the frequency fluctuations are primarily due to turbulent velocity fluctuations, the situation is somewhat less severe. The turbulence scale is usually larger than the measurement volume, so large gradients are usually not observed. Further frequency changes due to turbulence velocity changes are far slower than the fluctuations due to ambiguity noise. As long as the tracker is fast enough to track the "ambiguity noise", the mean frequency should be $\overline{\omega}$ as computed above.

Since the "ambiguity noise" fluctuations occur in the output of trackers even with no velocity fluctuations, it is impossible to "instantaneously" measure velocity with a tracker. Further, it can be shown that the theoretical power in the "ambiguity noise" is infinite (7). However, any real system has a limited output bandwidth. Consequently, a large, but finite, RMS power can be measured in the complete absence of turbulence.

Figure 9 illustrates the ability of various types of detectors to track turbulence fluctuations. There are four tracks on the oscillograph:

(1) The turbulence velocity (turbulence intensity = 4.4%)
(2) The output of a wide band second order phase-lock loop (a $\dot{\theta}$ detector)
(3) The output of a counter (zero crossing detector)
(4) The output of a frequency lock loop (a $\dot{\theta}$ detector)

Note that all the detectors follow the general trends in the turbulence but they all are noisy. The noise is the "ambiguity noise". Under these conditions, the output spectrum of any of the devices appears to consist of the low frequency turbulence spectrum, plus a flat wide band spectrum. (See Figure 10.) At the low

*One can convince onself of this by simply computing $\dot{\theta}$ for the case of two particles moving at a different velocity going through the sample volume.

frequencies, the power/unit bandwidth of the turbulence is much higher than the power/unit bandwidth of the noise and the noise spectrum can be approximated by a flat spectrum. A measure of the quality of the recovery of the turbulence signal is the ratio of the height of turbulence spectrum to the height of the noise. If one assumes that the turbulence spectrum can be approximated by a Lorentzian of width $\Lambda$ (Hz), one finds the approximate ratio is (7)

$$\mathrm{R_{ATIO}} \approx \frac{5.4\,K^2}{\Lambda \cdot 2\pi \cdot \left(\overline{\Delta\omega^2} - K^2\overline{v_x'^2}\right)^{1/2}}$$

The presence of a velocity gradient can cause fluctuations in the output of a tracker or a counter. To my knowledge, the width of the spectrum of these fluctuations has never been measured in detail, although the power density has been shown to be proportional to the strength of the gradient (7,13). This effect has been included in the above expression.

If there is no mean velocity gradient,

$$\mathrm{R_{ATIO}} \approx 5.4 \cdot \frac{\overline{v_x'^2}}{\bar{v}_x} \cdot K\sigma_x \cdot \frac{f_0}{\Lambda},$$ where $f_0$ is the signal frequency.

Note that the ratio is higher for large measurement volumes. Let's compute the ratio for a realistic case of interest - turbulent pipe flow, $K\sigma_x = 40$, $\frac{(\overline{v_x'^2})^{1/2}}{\bar{v}_x} = 0.04$ and $\frac{f_0}{\Lambda} = 10{,}000$

$$\mathrm{R_{ATIO}} \approx 3{,}455$$

Despite the problems of the "ambiguity noise", quite good resolution can be obtained of the turbulence fluctuations spectra.

A word should be said about the fidelity of the measured spectra to the velocity spectrum of the particles. Lars Lading and I have measured the cross correlation of the output of a $\dot{\theta}$ detector with the "turbulence signal" on a laser anemometer simulator (14). At average particle densities from 300 particles/measurement volume to 3 particles/measurement volume and with measurement volumes 10 fringes long to 100 fringes long, we found the output to be accurately represented by the true turbulence spectrum plus the ambiguity noise. The simulations were run under conditions where the turbulence scales are large compared to the fringe spacing. George & Lumley (7) have shown that a distortion of the apparent turbulence spectrum occurs when the measurement volume size can contain appreciable portions of the turbulence scale. Essentially the spectrum looks low pass filtered, and an attenuation of the high wave number portion of the spectrum is predicted. To my knowledge, this last result has not been experimentally tested; however, the prediction is a strong signal to the experimentalist to be careful in this flow regime.

Since the output of tracker consists of a wide band signal plus a narrow band signal, the probability density of the output is a function of the output bandwidth. A narrow bandwidth will primarily sample the statistics of the low frequency signal and a wide bandwidth will primarily sample the statistics of the wide bandwidth signal. If one wishes to measure the RMS turbulence intensity, one can use two output filters with different bandwidths, both wide enough to contain at least 90% of the turbulence spectral power. The RMS output is then measured for both filters and then plotted on a graph of RMS vs. filter bandwidth. If the line determined by the two points is extrapolated to 0 Hz, the intercept is the RMS of the turbulence (9). To be more accurate, one should use 3 or more filter bandwidths.

Signal to Noise

As was discussed previously, in order to work correctly, the filter bandwidth of the tracker must be larger than the signal bandwidth. The minimum allowed filter bandwidth must increase as the signal frequency increases since the signal's bandwidth also increases. This means that the minimum amount of noise seen by the tracker necessarily increases as the frequency increases. Since the total signal power is constant, the detected signal to noise ratio decreases with increasing frequency (velocity).

Some preliminary results of ours from simulator experiments show that shot noise affects the output of a tracker in the same way as ambiguity noise. A broad, flat spectrum is generated in the output. The effect of shot noise is very small as long as the total signal power is greater than the noise power seen by the tracker.

COUNTERS

In liquid flows, where there are usually many particles, counters are zero crossing detectors. In steady flow, the number of zero crossings per second also fluctuates due to particles entering and leaving the measurement volume. The mean number of zero crossings, unfortunately, does contain a bias. The mean, $\overline{zcc}$, can be written (8,10,16)

$$\overline{zcc} = \frac{1}{\pi}\left(-\frac{1}{R(0)}\frac{d^2R}{d\tau^2}\Big|_{\tau=0}\right)^{1/2} = \frac{1}{\pi}\left(\overline{\Delta\omega^2} + (K\bar{v}_x^2)\right)^{1/2}$$

$$\overline{zcc} = \frac{1}{\pi} K\bar{V}_x\left(1 + \frac{\overline{\Delta\omega^2}}{K^2\bar{V}_x^2}\right)^{1/2}$$

In general, the mean output will be biased toward higher frequencies. I refer the reader to references 8, 10 and 15 for a detailed exposition of the behavior of the number of zero crossings detected per second as a function of signal bandwidth and threshold level. A bias should appear in the output of counters used in high scattering particle densities such as liquid flows. However, to my knowledge, this relation has never been experimentally tested for laser anemometers.

Our simulator experiments indicate that for large measurement volumes $(K\sigma_x > 30)$ the counter output has an identical spectrum to that of the phase lock loop. Under these circumstances, RMS turbulence intensities should be measured in the same manner as for trackers as was outlined in the previous section.

Signal to Noise

Counters are usually designed with a fixed filter in front of the zero crossing detector. The filter has a bandwidth that covers the entire range of signals expected. For instance, in a highly turbulent wake flow a velocity ratio of 10 to 1 may be encountered, and thus the filter would have at least a 10 to 1 ratio between the low pass cutoff and the high pass cutoff frequencies. A tracker could detect the same signal with a bandwidth only on the order of 3% of the bandwidth of the counter's filter. The bandwidth of the tracker only determines how fast it can move over a range of 10 to 1 in frequency. Consequently, the zero crossing counter will see a higher noise power than the tracker and a given shot noise level will result in a much higher noise level in the output of a counter (8).

SUMMARY

Spectral analysis of the photodetector output can yield very accurate ($<$ 1%) mean velocities and turbulence intensities. There are no intrinsic biases in these measurements. Accuracy can be retained at very low signal to noise ratios. The method is usually slow.

A properly designed tracker (one whose bandwidth is wider than the bandwidth of the detected signal) can make accurate, non-biased mean velocities and, so long as the turbulent scales are bigger than the measurement volume, accurate turbulence spectra can be obtained. Moderate (4:1) signal to noise ratios can be tolerated with little degradation of performance.

Counters can make accurate mean velocity measurements; however, a bias can be presented in the data. Turbulence spectra can also be measured using them if the measurement volume is large $(K\sigma_x > 30)$. Counters are only good when the signal to noise ratio is very high.

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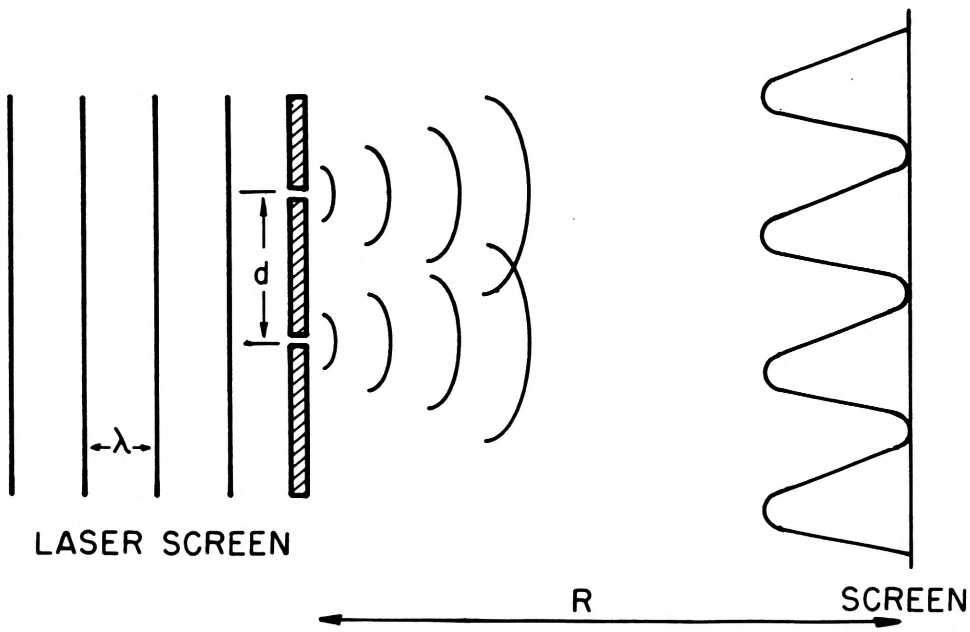


Figure 1

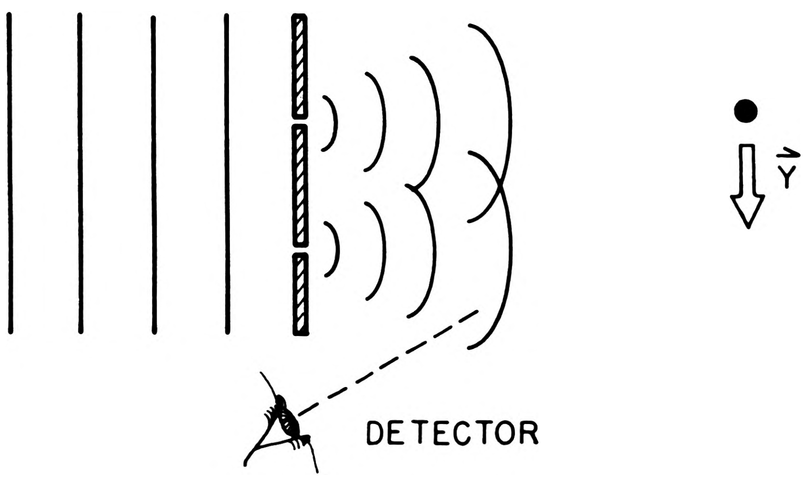


Figure 2

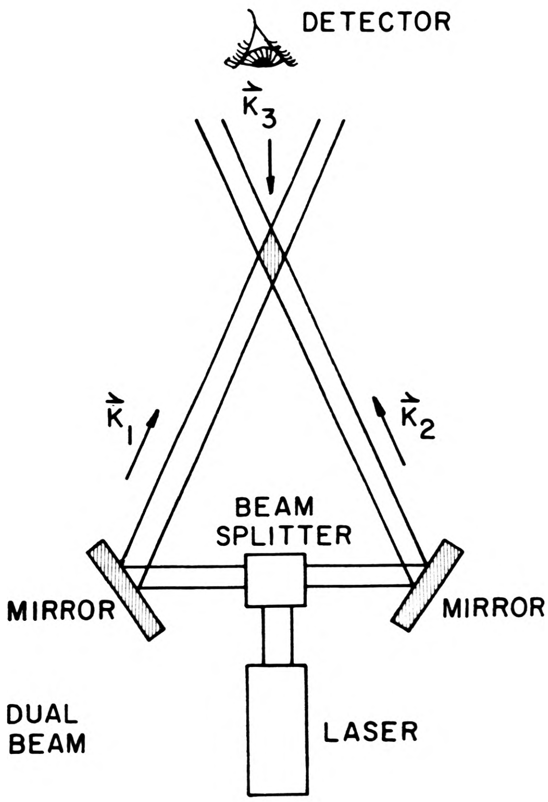


Figure 3a

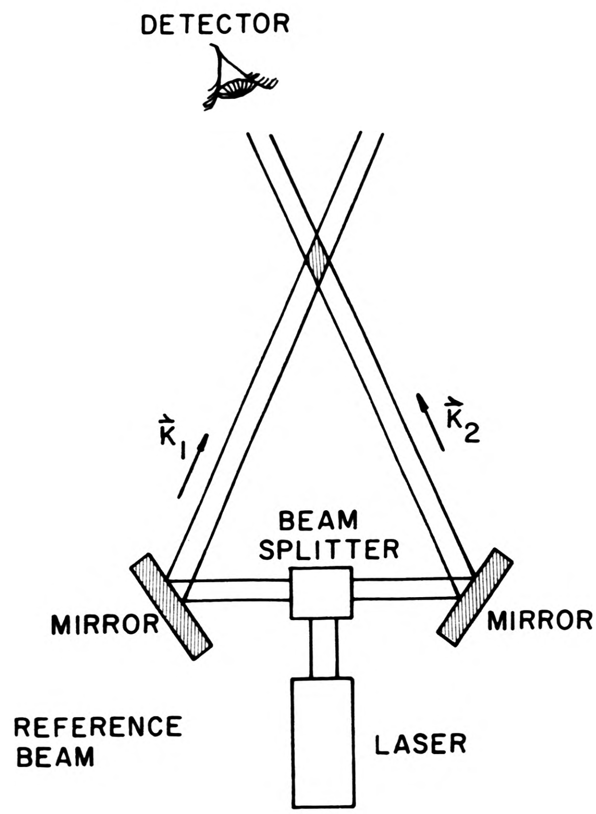


Figure 3b

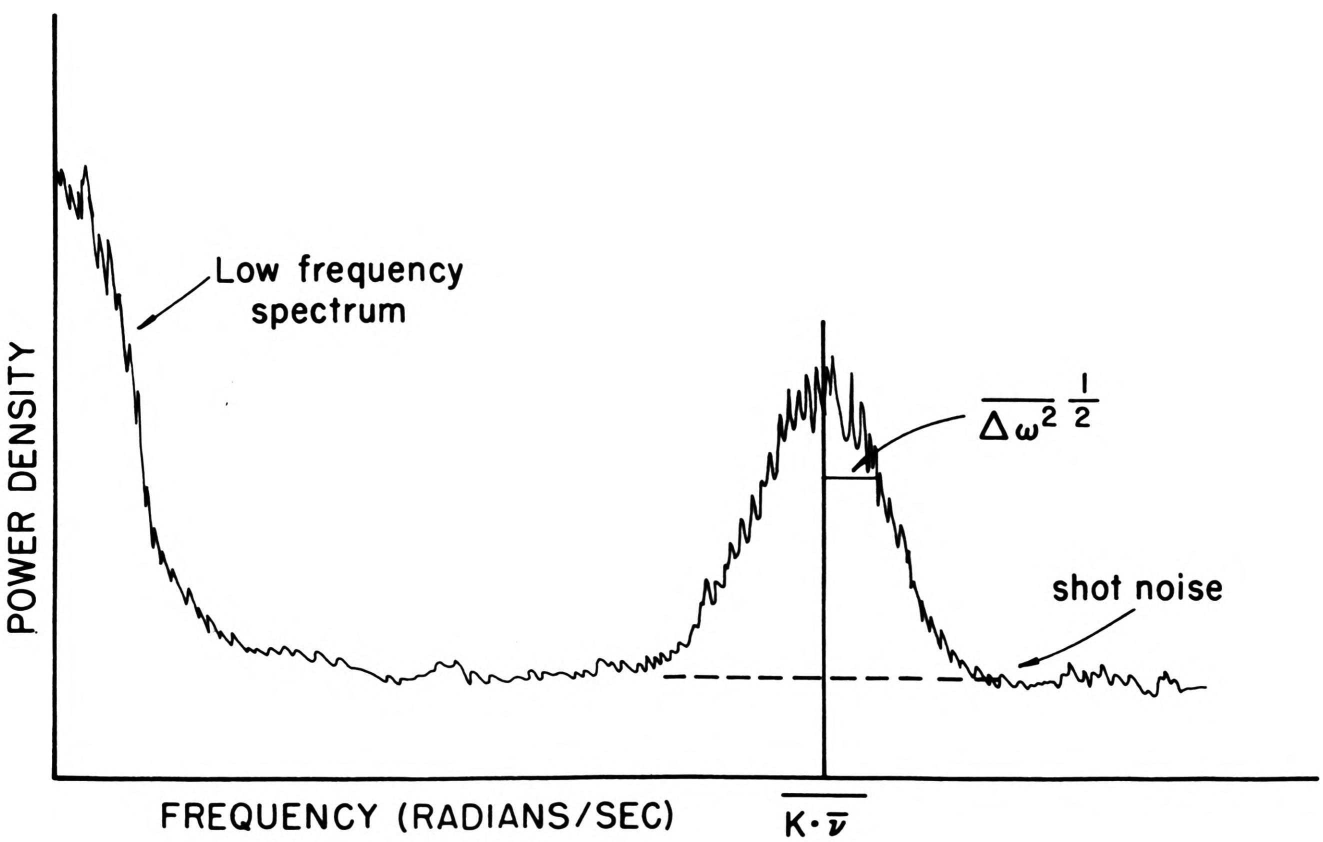


Figure 4

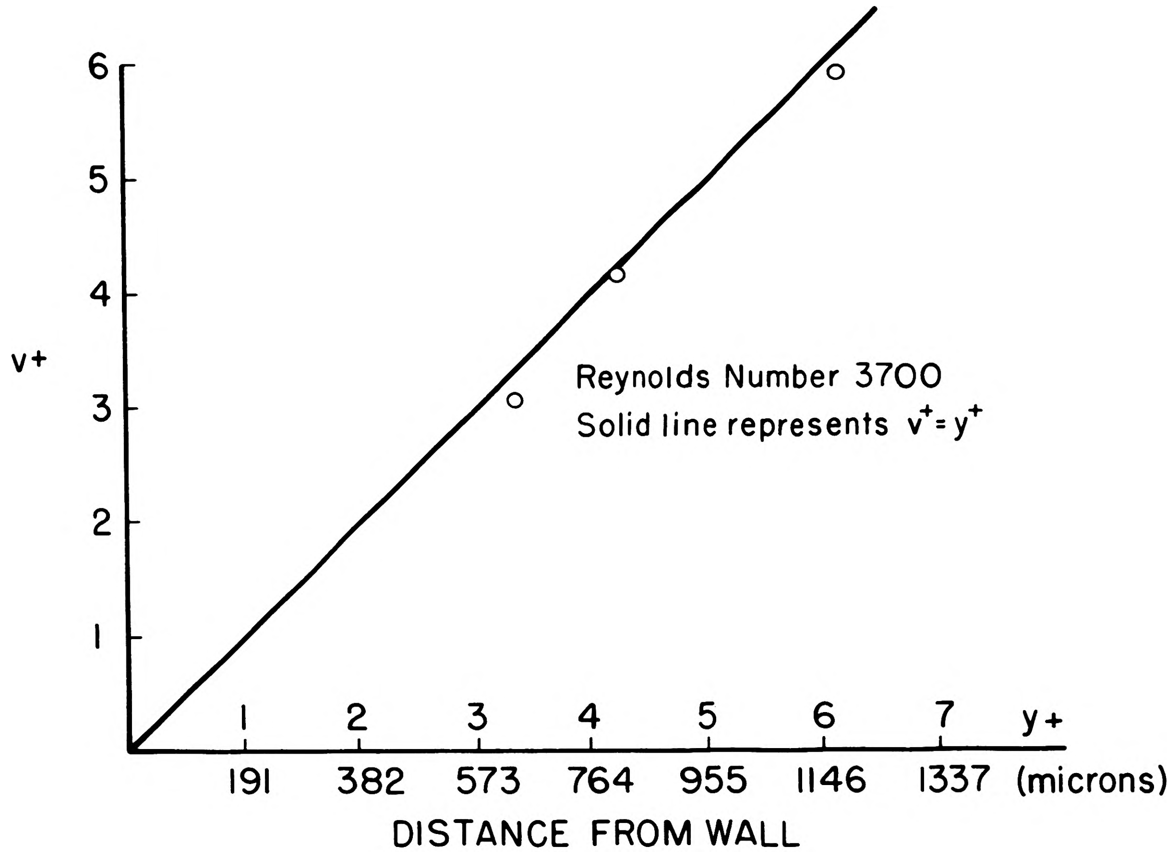


Figure 5

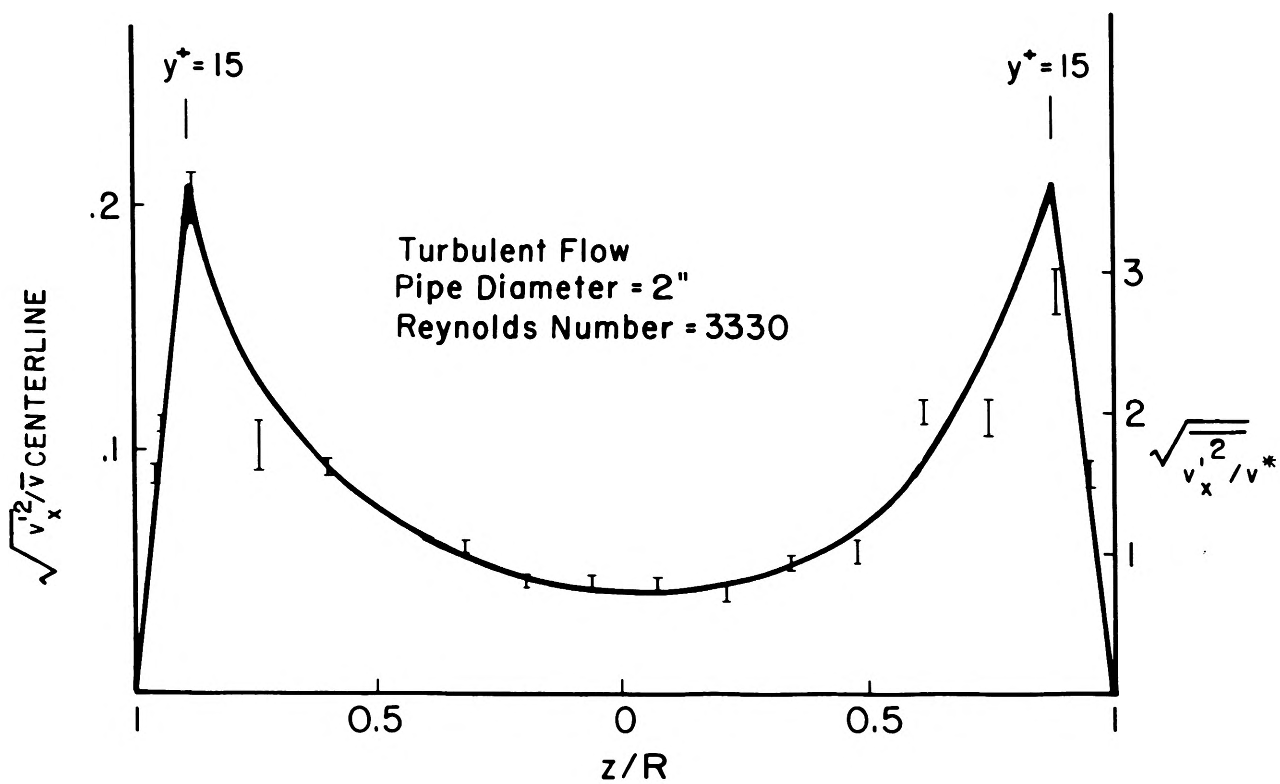


Figure 6

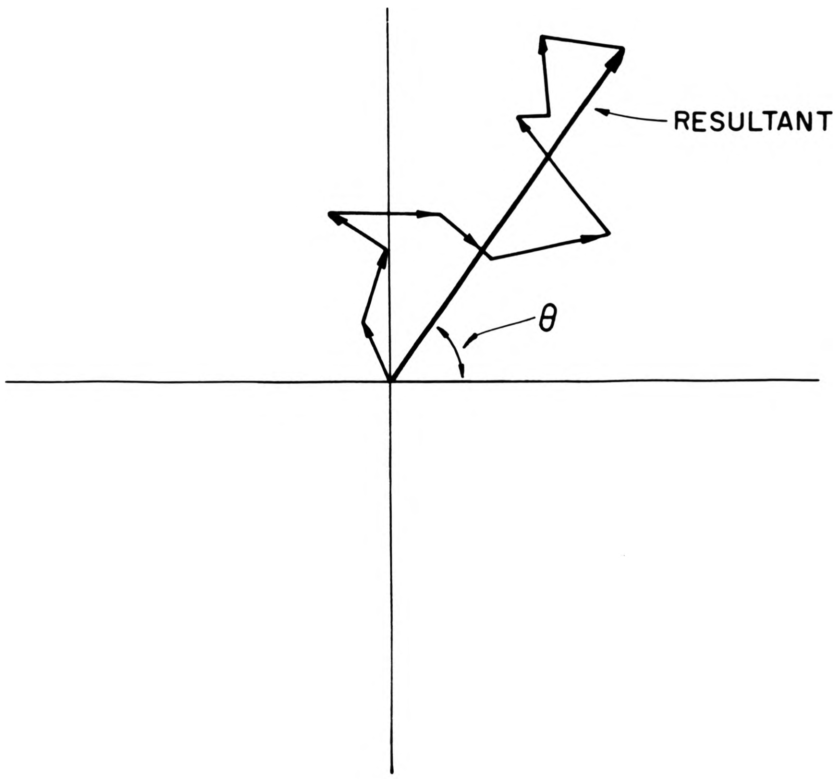


Figure 7

a.

b.

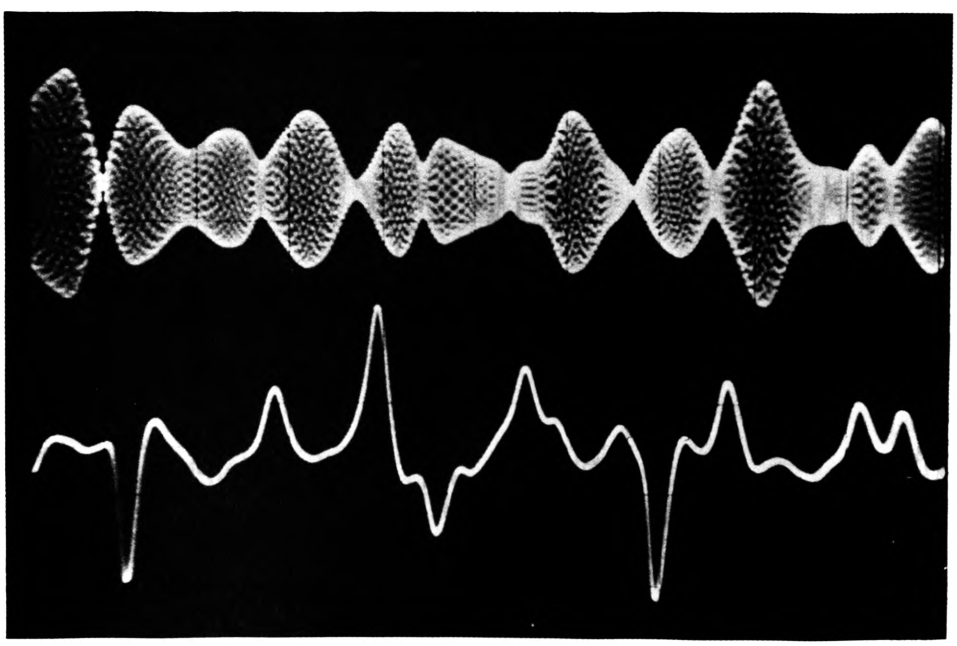

Figure 8

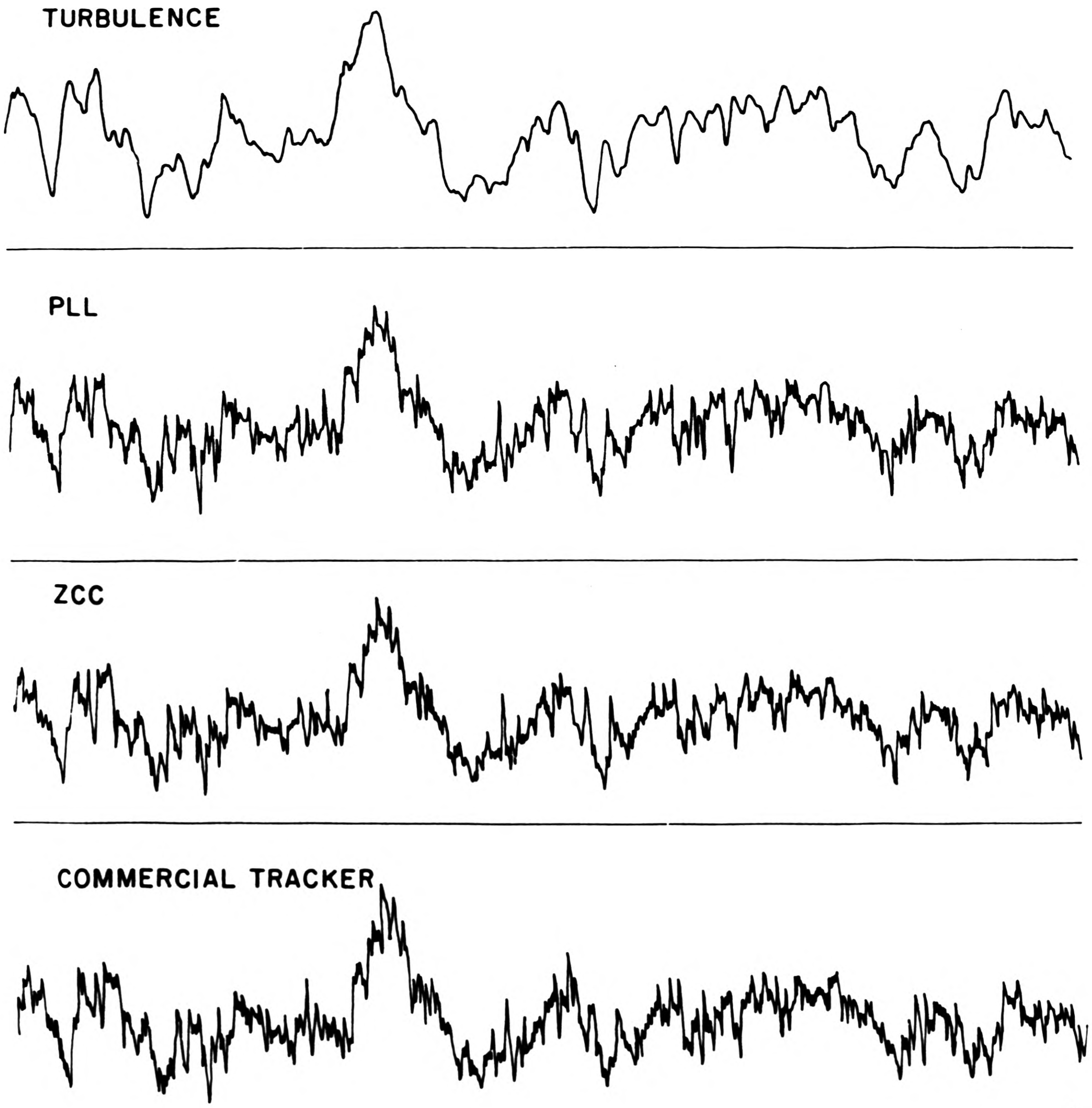


Figure 9

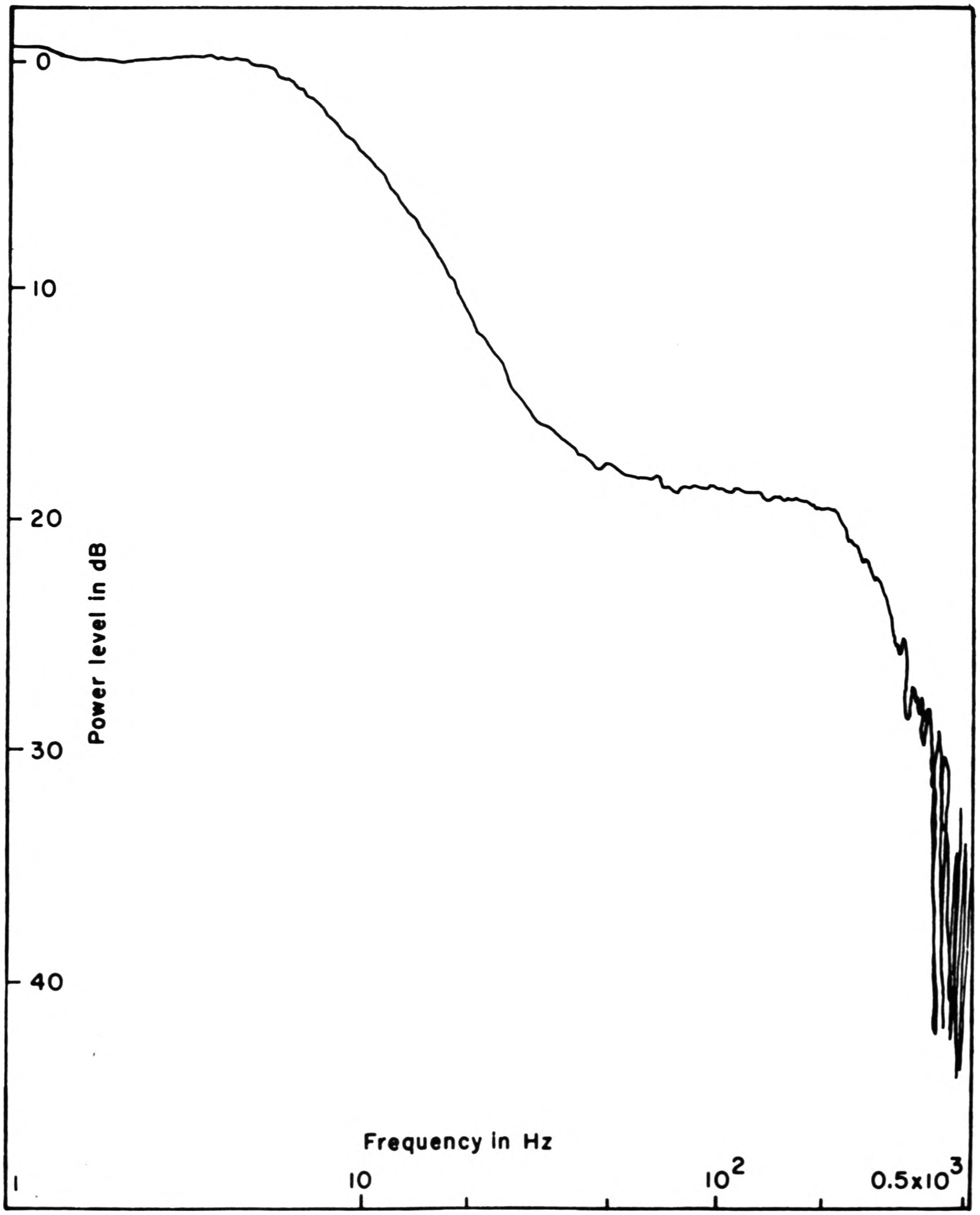


Figure 10

DISCUSSION

Bill Tiederman, Oklahoma State University: First of all, congratulations, you did a wonderful job of telling us what spectrum analyzers do, and in emphasizing that we need to look very carefully at the processing of the signal. I would like to make one comment and get your reaction to it about zero crossing counters. I don't believe that a zero crossing counter is any good unless it includes some method for verifying the signal. If you have only one verification then you don't reject very much noise. But if you have a sliding phase comparison or some other mechanism by which you have sequentially say 10 validations that you are measuring the proper frequency or that the frequency is repeating itself, I believe you can reject the noise. The essential point is that for rejecting noise it is the number of comparisons that matter.

Edwards: Verification circuits do help to mitigate some of the problems with noise encountered with zero crossing counters. People's experience certainly has been that the more verification (more cycles sampled), the better the performance of the counter.

Lars Lading, Danish Atomic Energy Commission: I would like to make one comment about this business about tracks and validation circuits. If you introduce validation circuits in counters it seems to me that you try to approach what phase lock loops do, but I have seen no counter which can do it as well as the phase lock loop.

Edwards: In liquid flows with high seeding density, our experience has been that trackers are far more reliable than counters. This apparently can be attributed to the ability of the tracker to filter noise to a higher degree than does the counter.

Bill Willmarth, University of Michigan: How long will it be, if ever, before we can get a laser anemometer to make measurements of individual velocity traces in turbulent flows and look at individual events without a lot of noise in the signal? How small can the sensing volume be made for research in boundary layers?

Edwards: With proper low pass filtering in a tracker output, signals that are almost indistinguishable from hot-wire signals can be attained even with sample volumes on the order of 30 microns. Particles have to be matched to sample volume. Recall, the power in the "ambiguity noise" can be very large, however, it is spread over a much larger frequency range than the turbulence. Therefore, in the low frequency range, the signal is primarily due to the fluid notion.

There are several research groups working on laser anemometer design and commercial equipment is available.

D. McLaughlin, Oklahoma State University: I have a comment with regard to using individual realization (counter) type LDA's in water flows. We use this system with a high degree of success simply by managing the seed and verifying the signal which is counted. Our data has agreed very well with the traditional hot-wire and hot-film data (i.e., Laufer, Eckelman and Reichart, etc.).

Comparison of the Reischman-Tiederman results using a counting technique in a two-dimensional channel with data of Rudd in square ducts using tracking techniques shows disagreements of about 50% in turbulence intensity. We don't know if this is because of the use of the square duct or if it is a problem with the tracker or the drop out.

What would be extremely useful is a direct comparison in a standard flow (highly turbulent) between the tracking system and the individual realization system. We need to know how well trackers perform and how dropout influences the important results. If the errors are small we can forget about them and put the instrument to use.

Edwards: In the complete absence of noise, we have demonstrated that trackers and counters give essentially the same output. However, in the presence of noise (for the counter) or if the signal's bandwidth is too wide for the tracker, neither one is working correctly so it's not surprising they give different answers. It is not a conclusive comparison to operate one or both of the devices where they aren't working correctly.

R. Adrian, University of Illinois: I would like to address myself to the question on comparisons because one has to be extremely careful in finding new criteria

Adrian (cont.): and in describing what you mean when you say something compares well or not. There are a great number of effects that are operative in one type of frequency demodulating system or another. I have done some work with Whitelaw at Imperial College, where we examined the effect of signal to noise ratio, as determined by how signal to noise ratio affected the measurement of mean velocity, not simply fluctuating velocities, but the simple mean velocity measurement. We found that using either simulated signals or real measurements it was possible with counting systems to have errors in mean velocity measurement up to 2-300% if you have low enough signal to noise ratio. And typically the signal to noise ratios required to give adequate accuracy was something like 10 to 1, higher than you were indicating. That is a very high signal to noise ratio. This is the power ratio. With the tracker we could go down to a signal to noise ratio of 1 to 1 and get the same accuracy. Now that's simply the signal to noise ratio effect. I think, on the other hand, that the tracker does have a problem in that it can lie to you sometimes. What we did in those experiments was to make sure that the tracker was always working correctly. Any general comparison must be made on the basis of whether or not the tracker is locking on to the signals and is locking on to the right signals. And there, I think, the counter does offer some advantage and it's difficult to quantify that sort of comparison.

Edwards: As a practical matter, trackers continue to work correctly at lower signal to noise ratios than would a counter. We (Lading and I) have noticed no mean error in the tracker's velocity measurements due to noise. At low signal to noise ratios, and if the filter in front of the tracker has a peak, the tracker will tend to bias the measured velocity toward the peak of the prefilter.

J. Hornkohl, Spectrum Development Labs: Counter processors work best when only a single particle is in the probe volume at any given time. Signal trackers work best when many particles are simultaneously passing through the probe volume. There is a range of particle concentration over which both counter and tracker processors will operate. This fuzzy region is, at the present time, ill-defined and further work is required.

Edwards: In the range from 10 particles/volume and up, the tracker seems to be less trouble to run than a counter.