01 Sep 1975

Energy Spectrum and Turbulent Scales in a Plane Air Jet

V. W. Goldschmidt

M. F. Young

ENERGY SPECTRUM AND TURBULENT SCALES IN A PLANE AIR JET

V.W. Goldschmidt and M.F. Young*
Ray W. Herrick Laboratories
Purdue University
West Lafayette, Indiana 47906

ABSTRACT

The energy spectra and longitudinal length scales measured in a free plane jet are presented. The actual convective velocity was used in determining the scales. The results show a universal spectral distribution along the axis but no obvious similarity in the large scale motion off the axis. The results are limited to $x/D \leq 60$. The measured scales suggest a noticeable increase of the microscale along the lateral coordinate but an essentially constant value along the axis. However the macroscales, increasing linearly along the axis do not show any similarity off the axis.

INTRODUCTION

An earlier paper presented measurements of spectra and scales in a submerged circular water jet, Goldschmidt and Chuang (1972). This now presents similar measurements in a plane air jet. For brevity sake the reader is referred to the earlier publication, Goldschmidt and Chuang (1972), for definitions, nomenclature, and extensive references. There the warning was made that the concept of eddies and scales, useful as it is in some models of turbulent transport, has some physical weakness. This is particularly true in flows which are intermittently turbulent and non-turbulent.

The length scales of turbulence can be determined from measured energy spectra by knowing the velocity of propagation of the turbulent structure. This velocity, referred to as convective velocity, need not be equal to the local mean velocity. Taylor's frozen turbulence hypothesis, usually invoked when computing the scales, postulates this equality. In the results to be presented the scales will be computed based on both the local mean and convective velocities.

FLOW FIELD

The flow field considered is a subsonic, plane free jet. The flow emanates at an exit Reynolds number of $U_oD/\nu = 10{,}000$, through a rectangular slot 0.635 x 30.48 cm. In order for it to behave as an infinite plane jet the jet was confined by two horizontal walls. Measure of the mean velocity profiles showed them to satisfy the documented Reichardt and Gortler solutions, with a half-width given by:

$$\frac{b}{D} = 0.0875\ [\frac{x}{D} + 8.75] \qquad (1)$$

and an axial velocity decay rate

$$\left(\frac{U_m}{U_o}\right)^{-2} = 0.15\ [\frac{x}{D} + 1.25] \qquad (2)$$

agreeing well with documented values in the literature. Such are tabulated (Table 1) as follows, where

$$\frac{b}{D} = K_1\ [\frac{x}{D} - C_1]$$

$$\left(\frac{U_m}{U_o}\right)^{-2} = K_2\ [\frac{x}{D} - C_2]$$

ENERGY SPECTRA

The energy spectra were obtained from digital Fourier analysis of the hot-wire anemometer signal. The major instrumentation consisted of a Security Associates anemometer model 100, a HP 5465A analyzer, a standard TSI probe and a SKL variable low pass filter. The scales were obtained from the energy spectra (based on the wave number) from

$$\lambda_x = \sqrt{2}\ [\int_0^\infty F(k)k^2dk]^{-1/2} \qquad (3)$$

and

$$\Lambda_x = \frac{\pi}{2} \lim_{k \to 0} F(k) \qquad (4)$$

The energy spectra, based on wave number, were obtained from the frequency distribution by noting that

$$k = \frac{2\pi f}{U_c} \qquad (5)$$

*Presently at Texas A & M University, College Station, Texas.

Taylor's hypothesis in essence implies that the convective velocity, U_c, is equal to the local mean velocity, U.

The energy spectra were measured at x/D = 20, 30, 40 and 60 for different y/b values from 0 to 1.8. The spectra along the axis, when scaled by the distance from the origin, collapse onto one single curve as shown in Figure 1. However, in the intermittent region this is no longer the case. As an example the spectra at different lateral locations is shown in Figure 2. It is shown for the furthest x/D station where data resolution would be at its worst. As expected, the high wave number region (dependent on dissipation rate) does collapse onto one curve whereas the large scale motion, strongly dependent on intermittency, does not. This is in agreement with the results of reference 1, Goldschmidt and Chuang (1972).

TAYLOR LONGITUDINAL SCALES

The longitudinal macroscale can be estimated from the extrapolation of data taken at very low wave numbers (corresponding to frequencies as low as 2 Hz). The macroscale along the axis is found to satisfy the relationship,

$$\frac{\Lambda_{xo}}{D} = 0.0885\ [\frac{x}{D} - 8.5] \qquad (6)$$

in excellent agreement (except for a jump in virtual origin) with the measured half-width of equation (1). The corresponding lateral distribution of the macroscale (computed based on Taylor's frozen hypothesis) is shown in Figure 3, whereas that based on the actual convective velocities: Young and Ott (to be published), is shown in Figure 4 disproving the otherwise inferred reduction in macroscale towards the edge of the jet. (The convective velocities were found from the cross correlation between two probes one downstream from the other.)

The ratio $U_c/\overline{U}$ was noted to approximately follow the relationship

$$\frac{U_c}{\overline{U}} \doteq 1 + 0.7(\frac{y}{b})^2 \qquad (6)$$

The solid data points noted in Figure 4 were obtained from the longitudinal space correlation measured with two probes located at different separations (in the downstream direction) from each other. These scales when compared with the values computed from the energy spectra are well within the range of experimental error.

Similar results for the Taylor microscale are shown in Figures 5 and 6. The microscale, based on the approximate Taylor hypothesis appears essentially constant. However, based on the actual convective velocities it exhibits an interesting increase with lateral location. The agreement with values obtained from actual longitudinal conditions, noted by the shaded data points, is satisfactory. From the measured microscales the turbulent Reynolds number and corresponding Kolmogoroff Scales can be computed. To so do, the isotropic relationships,

$$Re_\lambda = \frac{\sqrt{\overline{u^2}}}{\sqrt{2}\nu}\ \lambda_x \qquad (7)$$

and

$$\eta = \frac{\lambda_x}{\sqrt{2}} \cdot \frac{1}{15^{\frac{1}{4}}} \cdot \frac{1}{Re_\lambda^{\frac{1}{2}}} \qquad (8)$$

may be used. These apply, at best, on the axis where isotropic conditions nearly exist. Comparison of dissipation and microscales with data inferred from Heskestad (1963), is shown in Table 2.

The measurements are in need of further refinement. It must be noted that the reported (non-conditional) measurements include both the turbulent and non-turbulent regions in the flow. The additional refinement would be through a measure of the conditional scales (in the turbulent region only).

CONCLUSIONS

The energy spectra along the axis of the jet was seen to have a universal shape when plotted in terms of a wave number made dimensionless by the axial distance from the origin. On the other hand, the energy spectra at different lateral positions showed no universality. This was attributed to the differing large scale structures as interpreted with Eulerian non-conditional averaging. At lateral locations off the axis a maximum of the energy spectra was seen to occur at a frequency of the order of 20 Hz. This was not so at the origin. One possible explanation would be the intermittent nature of the flow.

The longitudinal micro and macro length scales were computed from the measured spectra. This was done correctly by recalling the measured convective velocities. These values were compared with those computed erroeously assuming that the local mean velocity represented the convective velocity. The difference in these results was considerable for lateral stations larger than $y/b = 0.8$.

The lateral distribution of the macroscales did not convincingly suggest that similarity had been reached for $20 \leq x/D \leq 60$. In general the results did show a slight increase of the scale with y/b. The measured values were spot checked with macroscales computed from measured longitudinal correlation coefficients. This agreement also reinforced the confidence in the measured data. To the best of the authors' knowledge this is the first time that measurement of longitudinal macroscales in plane jets have been reported. Earlier measurements in circular jets: Laurence (1957), Wygnanski and Fiedler (1969), and Corrsin and Uberoi (1951), did present macroscales which are of the same order of magnitude as the present results.

The longitudinal microscales along the axis increased very slightly with axial location. On the other hand, the longitudinal microscales increased considerably with y/b (the values at $y/b = 1.5$ being more than double those at the axis). The data did show consistent trends at different x/D locations. Agreement with the scales obtained from the independent measurement of the longitudinal correlation coefficient was very satisfactory.

The results are useful in characterizing the flow. In addition, the scales and the form of the spectra may be of assistance in formulating models describing the flow and predicting its behavior. As an example, available predictors for the transfer of suspended particles require a knowledge of the macro-scales.

ACKNOWLEDGEMENTS

The results reported were partly supported by the NSF through grant GK 19317. Final stages of the work were done partly through ONR contract No. N000 14-67-A-0226. Their support is gratefully acknowledged.

REFERENCES

1. Goldschmidt, V.W., and Chuang, S.C., "Energy Spectrum and Turbulent Scales in a Circular Water Jet", Transactions of the ASME, J. Basic Engineering, Vol. 71, 1972, pp. 22-26.

2. Ott, E.S., III, "Convective Velocities in a Plane Jet", Masters Thesis, Purdue Univ, Sept. 1972. Also presented at CANCAM, 1973.

3. Laurence, J.C., "Intensity, Scale, and Spectra of Turbulence in Mixing Region of Free Subsonic Jet", NACA Report 1292, 1957.

4. Heskestad, G., "Two Turbulent Shear Flow Plane Jet II. A Radial Jet", The Johns Hopkins Univ., Dept. of Mechanics, Clearinghouse for Federal Scientific and Technical Information of the U.S. Dept. of Commerce, Document AD 605-695, 1963.

5. Miller, D.R., and Comings, E.W., "Static Pressure Distribution in the Free Turbulent Jet", J. Fluid Mechanics, Vol. 3, No. 1, Oct. 1957, pp. 1-16.

6. Van Der Hegge Zijnen, B.G., "Measurements of the Velocity Distribution in a Plane Jet of Air", Applied Science Res., Section A, Vol. 7, 1958, pp. 293-313.

7. Foss, J.F., "A Study of Incompressible Bounded Turbulent Jets", Ph.D. Thesis, Purdue University, Jan. 1965, pp. 70-72.

8. Householder, M.K., and Goldschmidt, V.W., "Turbulent Diffusion of Small Particles in a Two-Dimensional Free Jet", Report Submitted to the Nat. Sci. Foundation, Tech. Rpt. FMTR-68-3, Purdue Res. Foundation 5307, 1968, pp. 132-143.

9. Flora, J.J., Jr., and Goldschmidt, V.W., Virtual Origins of a Free Plane Turbulent Jet", AIAA Journal, Vol. 7, No. 12, 1969, pp. 2344-2346.

10. Kaiser, K.F., "An Experimental Investigation of the Interaction of an Acoustic Field and a Plane Turbulent Jet", Masters Thesis, Purdue University, Jan.1971, p.92.

11. Jenkins, P.E., and Goldschmidt, V.W., "Mean Temperature and Velocity in a Plane Turbulent Jet", ASME Transactions, J.F.E., Dec. 1973, pp. 581-584.

12. Young, M.F., Ott, E.S., and Goldschmidt, V.W., "Turbulent Convective Velocities (Broadband and Wave Number Dependent) in a Plane Air Jet", to be published.

13. Wygnanski, I., and Fiedler, H., "Some Measurements in the Self-Preserving Jet", J. Fluid Mechanics, Vol. 38, Part 3, 1969, pp. 327-361.

14. Corrsin, S., and Uberoi, M.S., "Spectra and Diffusion in a Round Turbulent Jet", NACA Tech. Report 1040, 1951.

TABLE 1. CHARACTERIZATION COMPARISON OF VARIOUS FREE TURBULENT JETS.

Investigator	K_1	K_2	C_1	C_2	Re
Miller & Comings (5)	0.0983	0.227	-1.572	-1.572	$1.78(10^4)$
Van der Hegge Zijnen (6)	0.100	0.205	0	-1.70	$1.33(10^4)$
Foss (7)	0.085	0.2565	-2.0	6.50	$5.5(10^4)$
Heskestad (4)*	0.110	0.364	5.3	5.3	$2.5(10^4)$
Householder (8)	0.0908	0.1927	-1.46	6.98	$4\text{-}8(10^4)$
Flora (9)	0.109 to 0.130	0.158 to 0.227	-15.0	2.0	$2\text{-}3(10^4)$
Kaiser (10)	0.101	0.208	-2.6	0	$1\text{-}4(10^4)$
Ott (2)	0.0968	0.228	-3.0	7.0	10^4
Jenkins (11)	0.085	0.160	-6.1	4.0	$1.45(10^4)$
Present	0.0875	0.150	-8.75	-1.25	10^4

*Heskestad's K_2 is larger than that of other investigators because of mixing at the exit.

TABLE 2. COMPARISON OF TURBULENT SCALES

Author	x/D	$\lambda_x(ft)10^2$	λ_x/D	Re_λ	$\eta/D(10^3)$	Re_η
Heskestad (4)	80	1.62	0.3885	385.7	7.1	7.047
	70	1.56	0.3744	406.8	6.73	7.3
	60	1.50	0.3600	411.1	6.37	7.26
	50	1.455	0.3492	440.0	5.98	7.53
	40	1.45	0.3485	475.0	5.74	7.82
	35	1.45	0.3384	493.0	5.476	7.978
Present	60	1.01	0.487	358.0	9.248	3.047
	40	0.981	0.468	390.0	8.515	3.097
	30	0.952	0.4575	415.0	7.79	3.175
	20	0.931	0.446	445.0	7.6	3.413

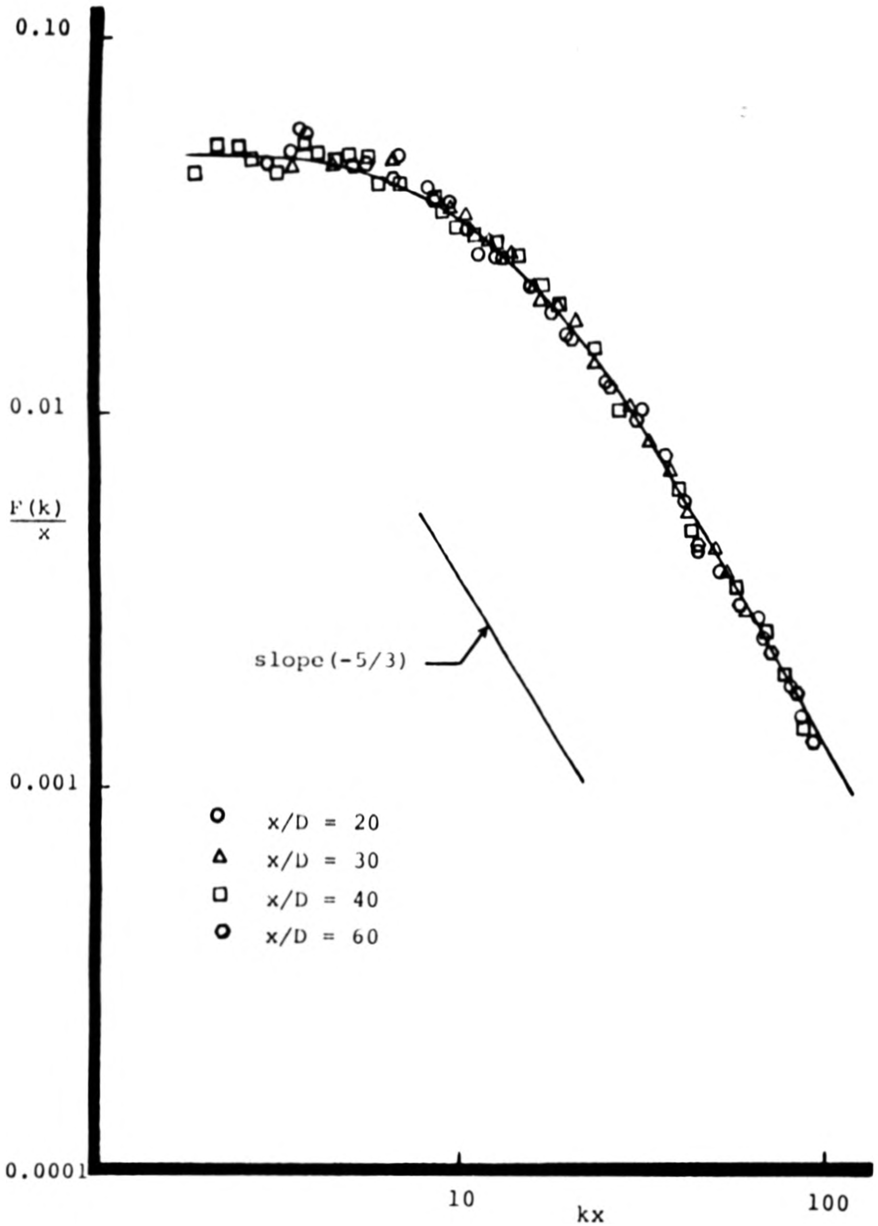

Figure 1: Dimensionless Energy Spectra Along the Jet Axis.

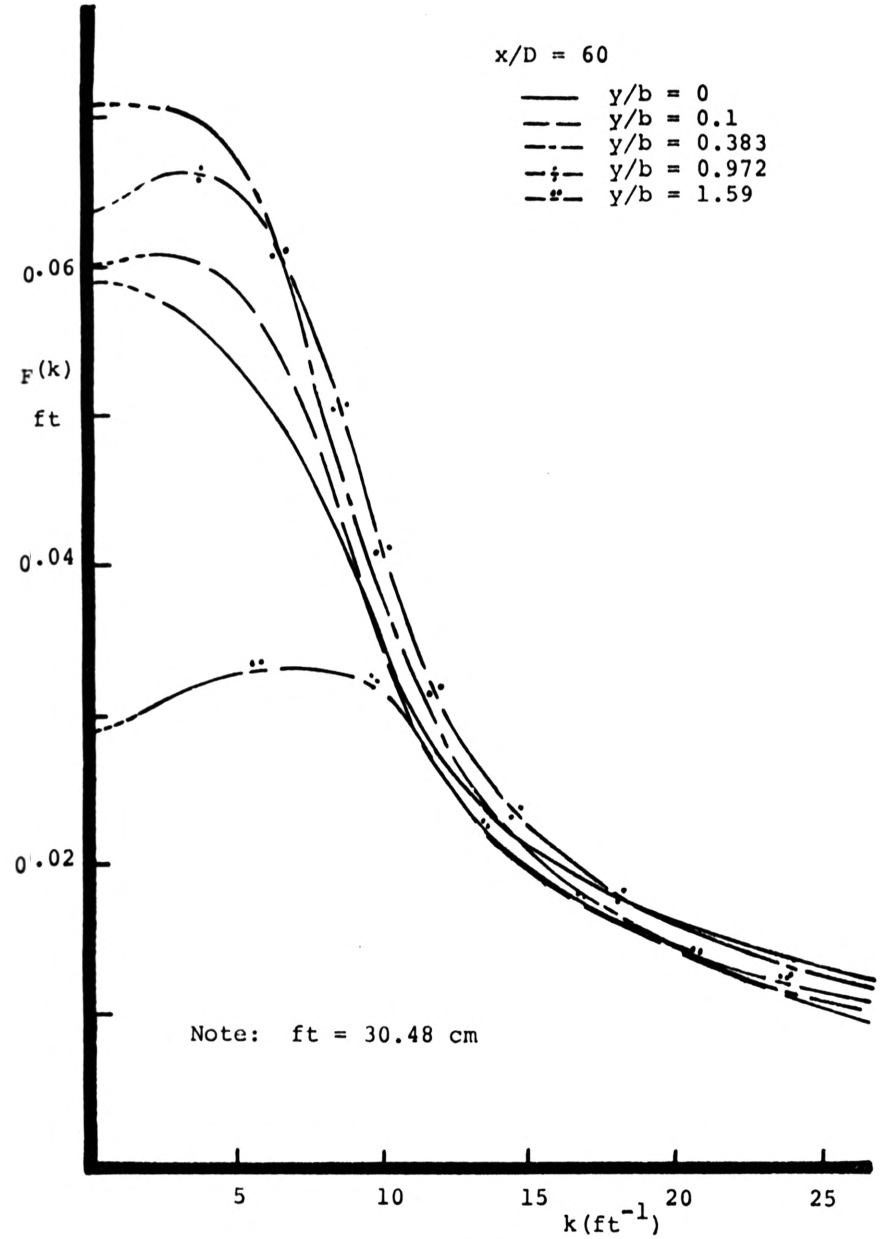

Figure 2: Normalized Energy Spectra at x/D = 60.

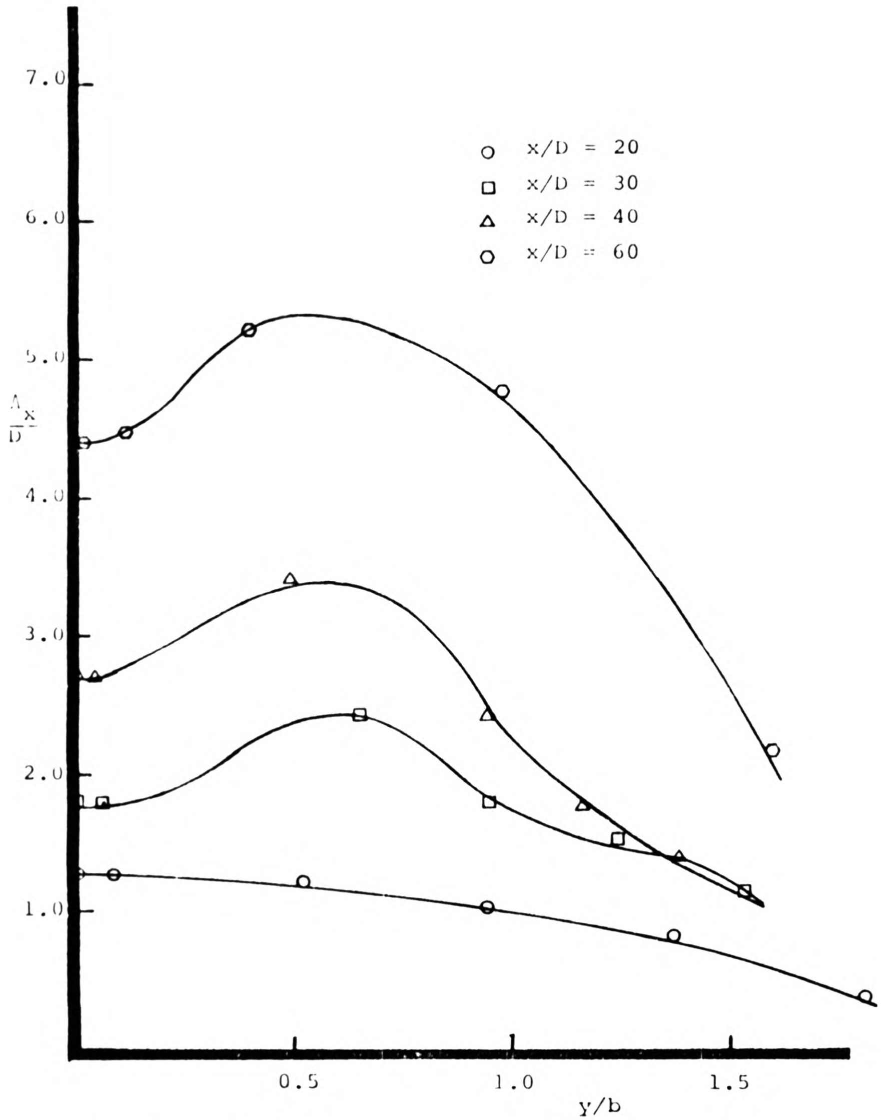

Figure 3: Distribution of Longitudinal Macroscale based on $U_c = U$.

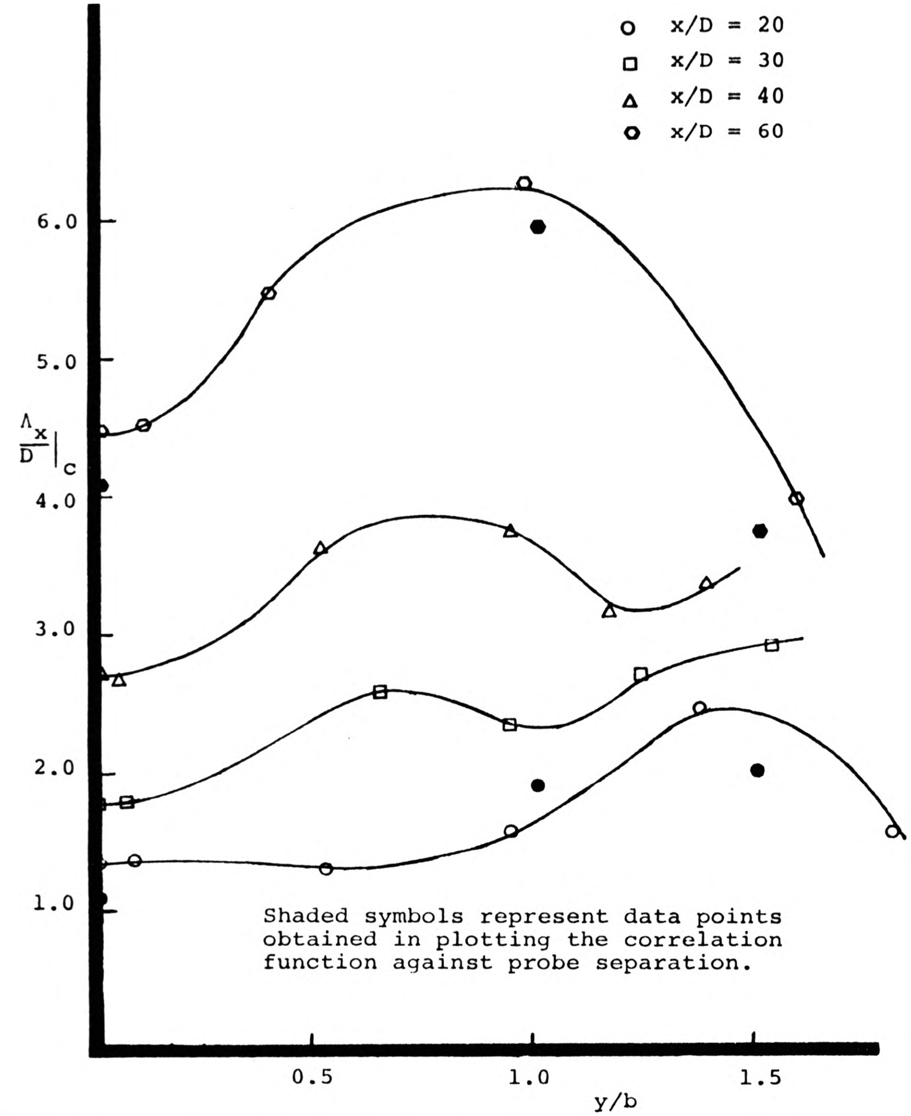

Figure 4: Distribution of the Longitudinal Macroscale, based on Measured U_c.

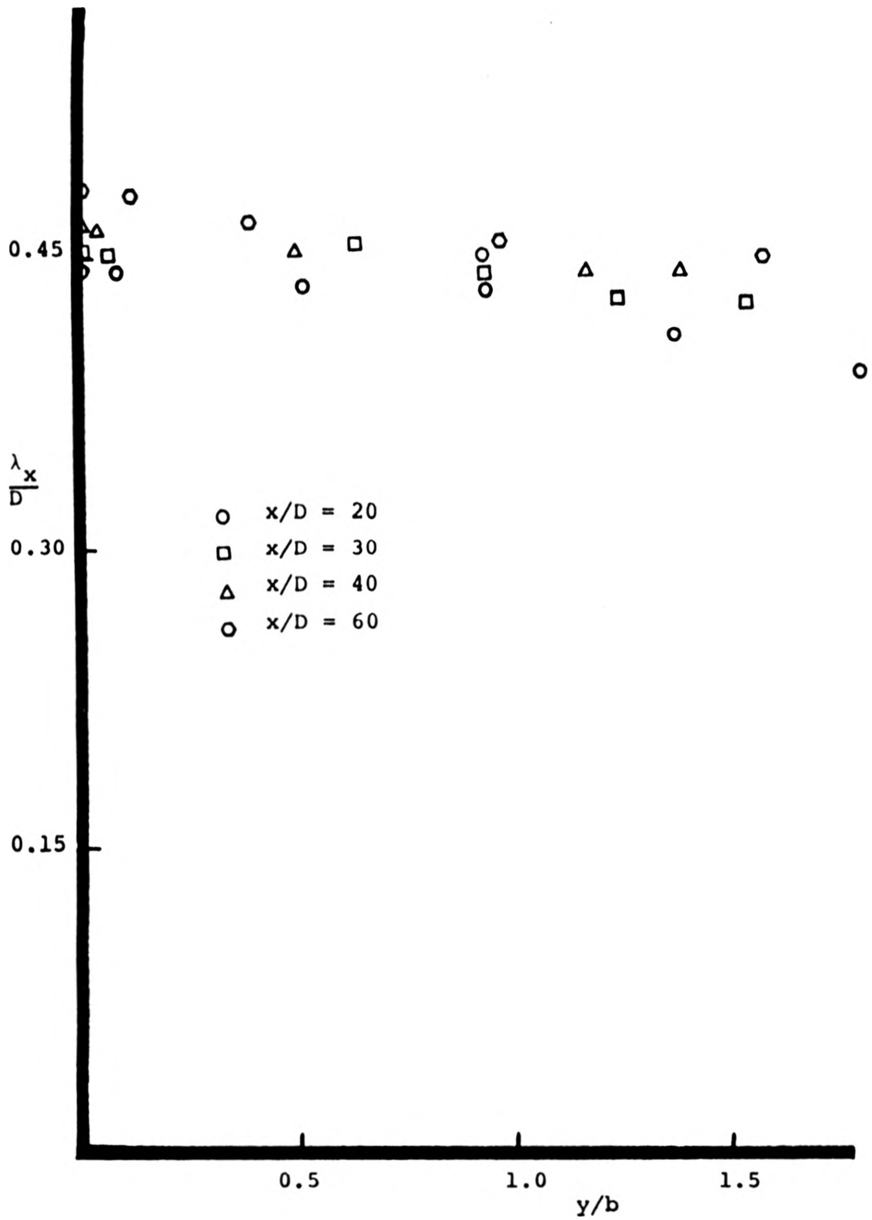

Figure 5: Distribution of the Longitudinal Microscale, based on $U_c = U$.

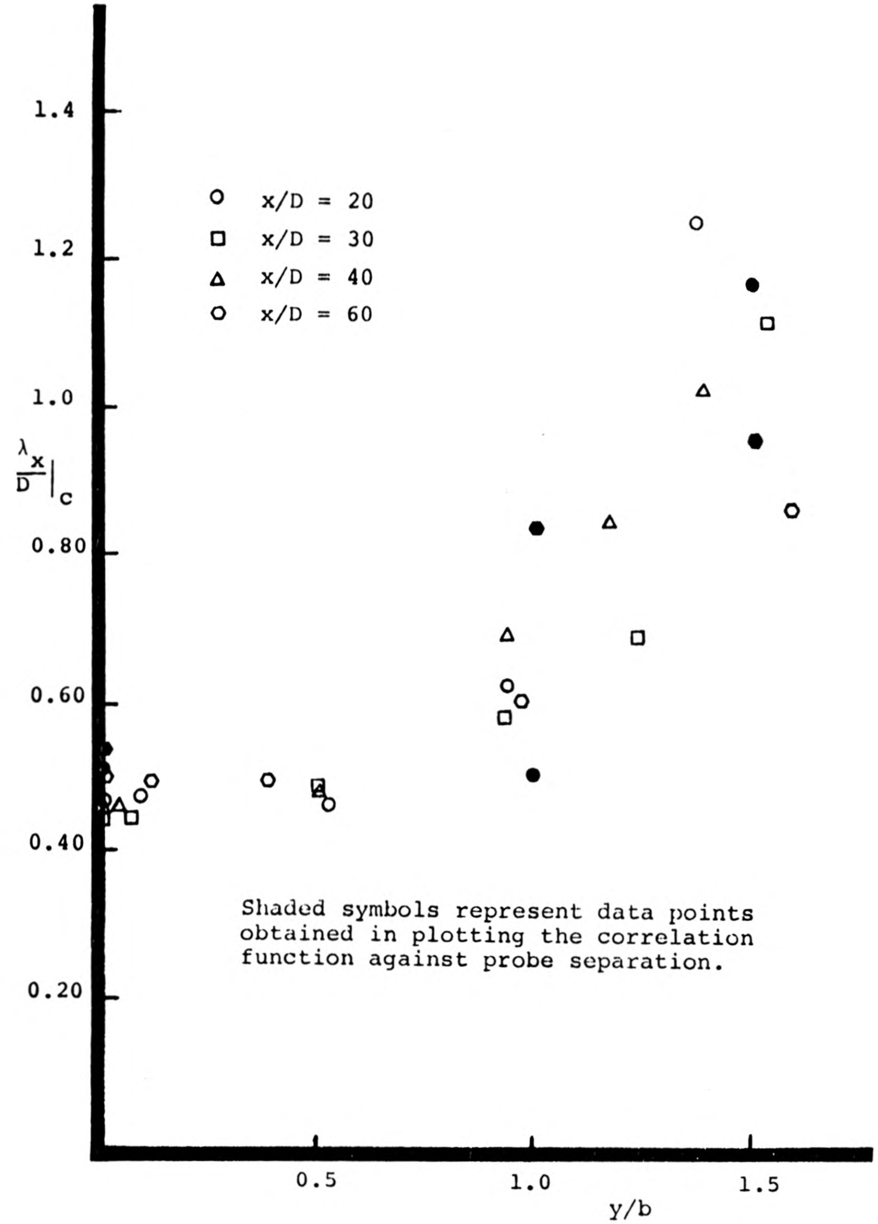

Figure 6: Distribution of the Longitudinal Microscale, based on Measured U_c.

DISCUSSION

W. Willmarth, University of Michigan: You have found that the small scales move faster than the local mean velocity. Does that mean that they are riding on top of the larger scales? Why do small scales move faster than the local mean velocity?

Goldschmidt: The question is related to measurements of convective velocity (at different frequencies) presented as reference data. The measurements showed that for $y/b < 1.0$, the convective velocities of small wave number structures was less than the local mean velocity, whereas for large wave number structures the ratio of convective velocity to local mean velocity is larger than one. This suggests that the small structures move faster than the larger structures. Whether or not this means that the little scales ride on the big scales I don't know - although I wish I could say so. I'm afraid we won't be able to explain what this all means until we repeat this same kind of measurement in a conditional sense.

J. A. Miller, Max-Planck Institute: How sharp are the sought maxima in the $\theta - \Delta\tau$ plane?

Goldschmidt: The question relates to curves not included in the paper showing how the convective velocities were determined. This included a measure of the time delay for a maximum in the space-time correlation curve.

As an example, at $x/D = 40$, $y/b = 0.25$ and $\theta = 1.3°$, the time delay for maximum correlation at a separation between the wires of 1.27 cm was at about 3.8 m seconds, whereas its value decreased by at least 10% for a delay of 3.3 and 4.5 m seconds (on either side of the maximum). The companion plot to determine θ_m was generally not as sharp, leading to a possible 20% uncertainty in the measure of θ_m. These details will be presented in Reference 12.

B. G. Jones, University of Illinois: Spencer's data for conditioned sampling of turbulent/non-turbulent structure convection velocities show that U_c is $\Delta\overline{U}_{cL}$ across at least half of the mixing layer.

We too have found that specific frequency structure has specific convection velocities. For long wave length structure $U_c < \overline{U}$ and for small wave length structure $U_c > \overline{U}$.

H. M. Nagib, Illinois Inst. of Tech.: In your Figure 2, the spectra has a peak in the low frequency range. Are these peaks related to the intermittency of the flow for $y/b \geq 0.1$? If so, what effect would that have on Figures 3 to 6 and the comparisons between them?

Goldschmidt: The peak is not noticed at the axis where the intermittency is 1. It occurs as we move outwards. Thus, it can be attributed to the intermittent nature of the flow, as you suggest. (The data were not conditionally sampled).

There is some uncertainty in the values of the macroscales. These were obtained by the extrapolation of curves such as those in Figure 2. The values obtained, however, are not in serious disagreement with those from actual space correlation (as noted by the shaded symbols in Figures 4 and 6).

G. Comte-Bellot, Ecole-Centrale de Lyon: Have you taken time correlation measurements for two probes which are only separated in the transverse direction? In that case, I think that the maxima can occur at time delays which are either positive or negative according to the shape of the two point space correlation contours.

Goldschmidt: No, we have not. Generally we carried the measurements up to angles of 30 to 60° from the longitudinal direction. I agree that two maxima might occur.

A. Hussain, University of Houston: We have carried out an experiment on controlled excitation of a plane turbulent air jet and determined the phase velocities of the vorticity waves introduced at different frequencies. Though the vorticity wave is different from large or small eddies, we have found that the phase velocity is typically 60% of the centerline velocity. It increases slightly with a frequency or Strouhal number but never went as high as the local mean velocity.

Goldschmidt: I would expect the phase velocity, as described, to be related more to the mean velocity across the jet (hence about 60% of the centerline velocity) than to the convection velocity. I believe your results may be comparable to Stiffler's work (Ph.D. thesis Penn. State, circa 1972).