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Abstract

In this note, some fundamental results including relationship between weighted distribution functions and mean advantage over inferiors functions are established. Ordering of reliability and/or distribution functions via mean advantage over inferiors functions and related functions for parent and weighted reliability functions are presented. Some applications and examples are given.

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1 Introduction

The usefulness and applications of weighted distributions to biased samples in various areas including medicine, ecology, reliability, and branching processes can be seen in Patil and Rao [11], Gupta and Kirmani [7], Gupta and Keating [6], Oluyede [10] and in references therein. When data is unknowingly sampled from a weighted distribution as opposed to the parent distribution, the survival function, hazard function, and mean residual life function (MRLF) may be under or overestimated depending on the weight function. If the weight function is monotone increasing and concave, then the weighted distribution of an increasing hazard rate (IHR) distribution is an IHR distribution. Similarly, the size-biased distribution of a decreasing mean residual (DMRL) distribution has decreasing mean residual life.

The MRLF has a mirror image referred to as mean advantage over inferiors function (Goldberger [5]). Let $(0, b)$ be the support of the probability density function f for the components or machines of interest. The mean advantage over inferiors function is the difference between the age x of a machine that has not broken down and the average age at breakdown of the machines that it has outlasted. The average age at breakdown of machines that broke down before age x is given by $\int_0^x tf(t)dt/F(x)$, where $f(t)/F(x)$ is the conditional probability that a machine broke down at age t, given that it did not survive to age $x > 0$. The mean advantage over inferiors of a machine that survives to exactly age x is defined as

$$\mu_F(x) = x - \int_0^x \frac{tf(t)dt}{F(x)}. \tag{1}$$

See Bergstrom and Bagnoli [1] for additional results.

In section 2, some basic results and utility notions are presented. Sections 3 and 4 contain inequalities and results on the ordering of parent and weighted reliability functions via the mean advantage over inferiors (MAOI) function and mean inactivity time (MIT). For additional results on MIT see Block, Savit, and Singh [2], Chandra and Roy [3], and Kayid and Ahmad [8], and references therein. Section 5 deals with MAOI function and weighted residual entropy. Some examples and applications are presented.

2 Some Utility Notions and Basic Results

In this section, some basic definitions and utility notions are presented. In a weighted distribution problem, a realization x of X enters into the investigators record with probability proportional to a weight function $W(x)$. The recorded x is not an observation of X, but rather an observation on a weighted random variable X_W. Let X be a nonnegative random variable with an absolutely continuous cumulative distribution function $F(x)$, survival function $\overline{F}(x) = 1-F(x)$ and probability density function (pdf) $f(x)$. Let $a = \inf\{x : F(x) > 0\}$ and $b = \sup\{x : F(x) < 1\}$. The reverse hazard rate function of F is $\tau_F(x) = f(x)/F(x)$ for $x > a$, and the hazard rate function of F is $\lambda_W(x) = f(x)/\overline{F}(x)$ for $x < b$. Let $W(x)$ be a positive weight function such that $0 < E_F(W(X)) < \infty$. The weighted survival or reliability function of X_W is given by

$$\overline{F}_W(x) = \frac{E_F[W(X)|X > x]}{E_F[W(X)]}\overline{F}(x), \qquad x \geq 0. \tag{2}$$

When $W(x) = x$, the resulting reliability function is called the length-biased reliability function and is given by

$$\overline{F}_W(t) = \frac{\overline{F}(t)(t + \delta_F(t))}{\mu}, \tag{3}$$

where $\delta_F(t) = E_F(X - t|X > t] = \frac{1}{\overline{F}(t)} \int_t^{+\infty} \overline{F}(x)dx$ is the mean residual life function(MRLF). We present some basic and important definitions.

Definition 2.1 *Let X and Y be two random variables with distribution functions F and G respectively. We say $F <_{st} G$, stochastically ordered, if $\overline{F}(x) \leq \overline{G}(x)$, for $x \geq 0$ or equivalently, for any increasing function $\Phi(x)$,*

$$E(\Phi(X)) \leq E(\Phi(Y)). \tag{4}$$

Definition 2.2 *A distribution function F is an increasing hazard rate (IHR) distribution if $\frac{\overline{F}(x+t)}{\overline{F}(t)}$ is decreasing in $0 < t < \infty$ for each $x \geq 0$. Similarly, a distribution function F is a decreasing hazard rate (DHR) distribution if $\frac{\overline{F}(x+t)}{\overline{F}(t)}$ is increasing in $0 < t < \infty$ for each $x \geq 0$.*

Let $X_{(t)} = \{t - X|X < t\}$, $t \in \{x : F(x) > 0\}$ denote the inactivity time (IT) or reversed residual life (See Block, Savit, and Singh [1], Chandra and Roy [2], Kayid and Ahmad [8] for details).

Definition 2.3 *Let F and G be absolutely continuous distribution functions with probability density functions f and g respectively. We say F is smaller than G in mean inactivity time order ($F \leq_{MIT} G$) if*

$$E_F[t - X|X < t] \geq E_G[t - Y|Y < t], \qquad \forall t > 0. \tag{5}$$

We say F is smaller than G in increasing concave order ($F \leq_{ICV} G$) if

$$\int_0^x F(y)dy \geq \int_0^x G(y)dy, \qquad \forall x > 0. \tag{6}$$

Definition 2.4 *A distribution function F has smaller mean advantage over inferiors function than a distribution function G, denoted by $F \leq_{MAOI} G$ if for any x and t, $\mu_F(x) \leq \mu_G(x)$, that is*

$$\int_0^x \frac{tf(t)dt}{F(x)} \geq \int_0^x \frac{tg(t)dt}{G(x)}. \tag{7}$$

Remark: Note that $\mu_{F}(x) = x - \int_0^x \frac{tf(t)dt}{F(x)} = \frac{H(x)}{H'(x)}$, where $H(x) = \int_0^x F(t)dt$. This follows from the fact that $\int_0^x tf(t)dt = xF(x) - \int_0^x F(t)dt$, so that

$$\mu_{F}(x) = x - \frac{xF(x) - \int_0^x F(t)dt}{F(x)} = \frac{H(x)}{H'(x)}. \tag{8}$$

Note that, if $\mu_{F}(x) = \mu_{G}(x)$, then F and G are said to be equivalent. This leads to the following result on the ordering of mean advantage of inferiors functions $\mu_{F}(x)$ and $\mu_{G}(x)$ for the distribution functions F and G respectively.

Proposition 2.5 *Let $\mu_{F}(x)$ and $\mu_{G}(x)$ denote the mean advantage over inferiors functions for the distribution functions $F(x)$ and $G(x)$ respectively. Then $\mu_{F}(x) \le \mu_{G}(x)$ if and only if $\frac{\int_0^x F(t)dt}{F(x)} \le \frac{\int_0^x G(t)dt}{G(x)}$, where $F(x) = \int_0^x f(t)dt$ is the distribution function with pdf f.*

Example 2.6 *Consider the power function distributions with survival functions $\overline{F}(x) = 1 - x^{\alpha_1}$, and $\overline{G}(x) = 1 - x^{\alpha_2}$, for $0 < x \le 1$ and $\alpha_i > 0$, for $i = 1, 2$. Note that $\mu_{F}(x) = \frac{x}{1+\alpha_1}$, and $\mu_{G}(x) = \frac{x}{1+\alpha_2}$, so that $\mu_{F}(x) \le \mu_{G}(x)$ if and only if $\alpha_1 \ge \alpha_2$.*

Lemma 2.7 *The function $\mu_F(x)$ is monotone increasing if and only if $H(x)$ is log-concave.*

Proof: See Goldberger [5], and Bergstrom and Bagnoli [1].

Definition 2.8 *Let F and G be two absolutely continuous distribution functions having reverse rate functions $\tau_F(x)$ and $\tau_G(x)$ and hazard rate functions $\lambda_F(x)$ and $\lambda_G(x)$ respectively. We say F is smaller than G in reverse hazard rate order, denoted by $F \le_{rh} G$, if $\tau_F(x) \le \tau_G(x)$ for all x for which $\tau_F(x)$ and $\tau_G(x)$ are defined. Also, F is smaller than G, in hazard rate order, denoted by $F \le_{hr} G$ if $\lambda_F(x) \ge \lambda_G(x)$ for any x for which $\lambda_F(x)$ and $\lambda_G(x)$ are defined.*

Example 2.9 *Consider the inverse Weibull distribution used to model degradation of mechanical components such as pistons, crankshafts of diesel engines, as well as breakdown of insulating fluid to mention just a few areas. The probability density function f is given by*

$$f(x) = \frac{c_1}{\alpha_1^{c_1} x^{c_1+1}} e^{(-\frac{1}{\alpha_1 x})^{c_1}}, \qquad x > 0 \text{ and } c_1 > 0,\ \alpha > 0. \tag{9}$$

Note that $\tau_{F}(x) = \frac{f(x)}{F(x)} = c_1 \alpha_1^{-c_1} x^{-c_1-1}$ and $\tau_{G}(x) = \frac{g(x)}{G(x)} = c_2 \alpha_2^{-c_2} x^{-c_2-1}$, so that for fixed $c_1 = c_2 > 0$, $\tau_{F}(x) \le \tau_{G}(x)$, if $\alpha_1 \ge \alpha_2$. Also, note that $xf(x) = c_1 F(x)(-lnF(x))$, so that

$$\tau_{F}(x) = \frac{c_1(-ln(F(x)))}{x}. \tag{10}$$

Consequently, $\tau_{F}(x) \le \tau_{G}(x)$, if and only if $F(x) \ge G(x)$, and $\alpha_1 \ge \alpha_2$.

Theorem 2.10 *Let X_W be a weighted random variable defined on an interval $(0, b) \subset (0, \infty)$, with nondecreasing weight function $W(x) \geq 0$. The reverse hazard function of the random variable $\{X_W - t | X_W \leq t\}$ is decreasing (increasing) if and only if*

$$\{X_W - t | X_W \leq t\} \geq_{rh} (\leq_{rh}) \{X_W - s | X_W \leq s\} \qquad 0 < t \leq s < b, \tag{11}$$

and

$$X_W + t \leq_{rh} (\geq_{rh}) X_W + s \qquad 0 < t \leq s < b. \tag{12}$$

Proof: The cumulative distribution function and reverse hazards function of $\{X_W - t | X_W \leq t\}$ are given by

$$F_{W_t}(x) = \frac{F_W(x+t)}{F_W(t)} \qquad \text{and } \tau_{F_{W_t}}(x) = \tau_{F_W}(x+t),\, 0 < x+t < b, \tag{13}$$

respectively. Therefore, $\tau_{F_{W_t}}(x)$ decreasing (increasing) if and only if

$$\tau_{F_{W_t}}(x) \geq (\leq) \tau_{F_{W_s}}(x), \qquad \forall x + t > 0 \text{ whenever } 0 < t \leq s < b. \tag{14}$$

The reverse hazard function of $X_W + t$ is given by $\tau_{F_W}(x - t)$, $0 < x - t < b$. Therefore

$$X_W + t \leq_{rh} (\geq_{rh}) X_W + s, \qquad 0 < t \leq s < b, \tag{15}$$

which is equivalent to

$$\tau_{F_W}(x - t) \geq (\leq) \tau_{F_W}(x - s), \qquad \forall x - t > 0 \text{ whenever } 0 < t \leq s < b. \tag{16}$$

Consequently, τ_{F_W} is decreasing (increasing) on $(0, b)$. This completes the proof of the Theorem.

Proposition 2.11 *Let F be an absolutely continuous distribution function with pdf f and the expected inactivity time (EIT) be given by*

$$m_F(x) = \frac{1}{F(x)} \int_0^x F(t) dt, \tag{17}$$

then $\mu_F(x)$ and $m_F(x)$ are equivalent.

Proof: The result follows from the fact that $\int_0^x t f(t) dt = x F(x) - \int_0^x F(t) dt$.

Remark: Note that if F is a decreasing reverse hazard rate (DRHR) distribution then $\mu_F(x)$ is increasing in $x > 0$. Also, for a distribution function F

and mean advantage over inferiors function $\{\mu_F(x) : x \in (a,b)\}$, the distribution function is uniquely determined as

$$F(x) = exp\Big[-\int_x^b \frac{1-\mu_F^{'}(y)}{\mu_F(y)}dy\Big], \tag{18}$$

where $\mu_F^{'}(x) = \frac{d\mu_F(x)}{dx}$, so that the pdf $f(x)$ is given by

$$f(x) = exp\Big[-\int_x^b \frac{1-\mu_F^{'}(y)}{\mu_F(y)}dy\Big]\Big[\frac{1-\mu_F^{'}(y)}{\mu_F(y)}\Big]. \tag{19}$$

3 Mean Advantage Over Inferiors Order

In this section, results on mean advantage over inferiors ordering for the parent and weighted distributions including residual and equilibrium distribution functions are presented.

Theorem 3.1 *If $F \geq_{MAOI} G$, then*

$$F_W(x) \leq \frac{\mu_G F(x)}{\mu_F} \qquad and \qquad G_W(x) \geq \frac{\mu_F G(x)}{\mu_G} \tag{20}$$

respectively, where $F_W(x)$ and $G_W(x)$ are the weighted distribution functions corresponding to the the distribution functions F and G with weight function $W(x) = x$.

Proof: Note that $F \geq_{MAOI} G$, is equivalent to

$$\begin{aligned}
&\Longleftrightarrow \quad x - \int_0^x \frac{tf(t)dt}{F(x)} \geq x - \int_0^x \frac{tg(t)dt}{G(x)} \\
&\Longleftrightarrow \quad \int_0^x \frac{tf(t)dt}{F(x)} \leq \int_0^x \frac{tg(t)dt}{G(x)} \\
&\Longleftrightarrow \quad \frac{\mu_F}{F(x)}\int_0^x \frac{tf(t)dt}{\mu_F} \leq \frac{\mu_G}{G(x)}\int_0^x \frac{tg(t)dt}{\mu_G} \\
&\Longleftrightarrow \quad \frac{\mu_F F_W(x)}{F(x)} \leq \frac{\mu_G G_W(x)}{G(x)}. \qquad (21)
\end{aligned}$$

Now using the fact that $G_W(x) \leq G(x)$ and $F_W(x) \leq F(x)$ for all $x \geq 0$, we have

$$F_W(x) \leq \frac{\mu_G F(x)}{\mu_F} \qquad \text{and} \qquad G_W(x) \geq \frac{\mu_F G(x)}{\mu_G} \tag{22}$$

respectively.

Theorem 3.2 *If $F \geq_{MAOI} G$, and $\mu_G \leq \mu_F$, then $\frac{F_W(x)}{F(x)} \leq \frac{G_W(x)}{G(x)} \leq 1$.*

Proof: The result follows immediately on the application of the condition that $\mu_G \leq \mu_F$, to Theorem 3.1.

Definition 3.3 *An absolutely continuous distribution function F*

(i) is said to be increasing mean advantage over inferiors (IMAOI) if $\mu_F(x)$ is increasing in $x \in (a,b)$,

(ii) is said to be decreasing mean advantage over inferiors (DMAOI) if $\mu_F(x)$ is decreasing in $x \in (a,b)$,

(iii) is said to be constant mean advantage over inferiors (CMAOI) if $\mu_F(x)$ is constant in $x \in (a,b)$.

Clearly, $\mu_F(x)$ is increasing in x if and only if $\tau_F(x)\mu_F(x) < 1$ for $x \in (a,b)$, so that F is IMAOI if and only if $\tau_F(x)\mu_F(x) < 1$ for $x \in (a,b)$, F is DMAOI if and only if $\tau_F(x)\mu_F(x) > 1$ for $x \in (a,b)$, and F is CMAOI if and only if $\tau_F(x)\mu_F(x) = 1$ for $x \in (a,b)$.

Theorem 3.4 *Let F_W and G_W be two absolutely continuous distribution functions with densities f_W and g_W respectively. $F_W \leq_{MAOI} G_W$ if and only if $F_W \leq_{MIT} G_W$.*

Proof: Note that $\mu_{F_W}(x) = \frac{\int_0^x F_W(t)dt}{F_W(x)}$, so that $F_W \leq_{MAOI} G_W$ if and only if

$$\frac{\int_0^x F_W(t)dt}{\int_0^x G_W(t)dt} \qquad \text{is decreasing in } x \geq 0. \tag{23}$$

However, $F \leq_{MIT} G$ if and only if

$$\frac{\int_0^x F(t)dt}{\int_0^x G(t)dt} \qquad \text{is decreasing in } x \geq 0. \tag{24}$$

Consequently, for weighted distributions F_W and G_W with increasing weight function $W(x) > 0$, it follows that $F_W \leq_{MAOI} G_W$ if and only if $F_W \leq_{MIT} G_W$.

4 Ordering Weighted Distributions

In this section we obtain useful inequalities for residual reliability functions. Let $\{X_i\}_{i=1}^{\infty}$ be a sequence of operating times from a repairable system that start functioning at time $t = 0$. The sequence of times $\{X_i\}_{i=1}^{\infty}$ form a renewal-type stochastic point process. Following Kijima [9], if a system has virtual age

$T_{m-1} = t$ immediately after the $(m-1)^{th}$ repair, then the length of the m^{th} cycle X_m has the distribution

$$F_t(x) = P(X_m \leq x | T_{m-1} = t) = \{F(x+t) - F(t)\}/\overline{F}(t), \tag{25}$$

$x \geq 0$, where $\overline{F}(x) = 1 - F(x)$ is the reliability function of a new system. When $t = \sum_{i=1}^{j} X_i$, $j = 1, 2, ..., m-1$, minimal repair is performed, keeping the virtual age intact and when $t = 0$ we have perfect repair. The virtual age of system is equal to its operating time for the case of minimal repair. The corresponding reliability function is given by

$$\overline{F}_t(x) = \overline{F}(x+t)/\overline{F}(t), \qquad x \geq 0. \tag{26}$$

The next result shows that mean advantage over inferiors order is stronger than increasing concave (ICV) order. It is well known that a differentiable probability density function f is log concave if $f^{'}(x)/f(x)$ is decreasing in x, that is, if for every constant k, $\frac{f^{'}(x)}{f(x)} - k$ has at most one sign change, + to − if one occurs. Please see Shaked and Shanthikumar [12], for additional results on log-convexity (log-concavity) and ICV order.

Theorem 4.1 *Let F_W and G_W be two absolutely continuous weighted distribution functions with increasing weight function $W(x) > 0$ and densities f_W and g_W respectively. If $F_W \leq_{MAOI} G_W$, then $F_W \leq_{ICV} G_W$.*

Proof: Note that since the weight function $W(x) > 0$ is increasing, it follows that

$$log\left[\frac{\int_0^x F_W(t)dt}{\int_0^x G_W(t)dt}\right] \qquad \text{is decreasing in } x \text{ for } G_W(x) > 0, \tag{27}$$

so that

$$\frac{\int_0^x F_W(t)dt}{F_W(x)} \geq \frac{\int_0^x G_W(t)dt}{G_W(x)} \qquad x > 0. \tag{28}$$

Now, since

$$\frac{\int_0^x F_W(t)dt}{\int_0^\infty F_W(t)dt} \geq \frac{\int_0^x G_W(t)dt}{\int_0^\infty G_W(t)dt} \qquad x > 0, \tag{29}$$

and $F_W \leq_{MAOI} G_W$ implies

$$\frac{\int_0^x F_W(t)dt}{\int_0^x G_W(t)dt} \geq \lim_{x\to\infty} \frac{F_W(x)}{G_W(x)}, \tag{30}$$

the result follows immediately. Recall that the residual life at age t, is a weighted distribution, with survival function given by

$$\overline{F}_t(x) = \overline{F}(x+t)/\overline{F}(t), \tag{31}$$

for $x \geq 0$. The weight function is $W(x) = f(x+t)/f(x)$, where $f(u) = dF(u)/du$. It is clear that if F is IHR (DMRL) distribution, then F_t is IHR (DMRL) distribution.

Theorem 4.2 *Let $F_t(x)$ and $G_t(x)$ be residual life distribution functions with weight functions $f(x+t)/f(x) > 0$ and $g(x+t)/g(x) > 0$ that are log-convex for $x \geq 0$. If $F_t \leq_{MAOI} G_t$, then $F_t \leq_{ICV} G_t$.*

Proof: Let $f(x+t)/f(x)$ be log-convex, then $f(x+t)/f(x)$ increasing in $x \geq 0$. Similarly, $g(x+t)/g(x)$ log-convex implies $g(x+t)/g(x)$ increasing in $x \geq 0$. Applying the previous Theorem, the result follows immediately.

5 Mean Advantage Over Inferiors Function and Weighted Residual Entropy

In this section, we establish results on the link between MAOI function and weighted residual entropy. The following definitions on weighted entropies of residual lifetimes and past lifetimes are due to Di Crescendo and Longobardi [4].

Definition 5.1 *Let $(0,\infty]$ be the support of the pdf $f(x)$. The weighted residual entropy at time t of a random lifetime is the differential weighted entropy of $[X|X>t]$ and is given by*

$$H^w(t) = -\int_t^{+\infty} x\frac{f(x)}{\overline{F}(t)} log \frac{f(x)}{\overline{F}(t)} dx, \tag{32}$$

and the weighted past entropy at time t of a random lifetime X is the differential weighted entropy of $[X|X<t]$ is given by

$$\overline{H}^w(t) = -\int_0^{t} x\frac{f(x)}{\overline{F}(t)} log \frac{f(x)}{\overline{F}(t)} dx. \tag{33}$$

We establish a bound and inequality that relates the length-biased reliability function $F_W(x)$ to the weighted differential entropy.

Theorem 5.2 *Let $\overline{F}_W(t) = \frac{\overline{F}(t)(t+\delta_F(t))}{\mu}$ be the length-biased reliability function, where $\delta_F(t) = E_F(X-t|X>t] = \frac{1}{\overline{F}(t)}\int_t^{+\infty}\overline{F}(x)dx$. If the hazard function $\lambda_F(t)$ is decreasing in t, $0 < t \leq \infty$, then*

$$\overline{F}_W(t) \geq \frac{-H^w(t)\overline{F}(t)}{\mu log \lambda_F(t)}. \tag{34}$$

Proof: Note that since $\lambda_F(t)$ is decreasing in t, then by [4], we have

$$H^w(t) \geq -\frac{log\lambda_F(t)}{\overline{F}(t)} \int_t^{+\infty} xf(x)dx. \tag{35}$$

Now,

$$\mu \int_t^{+\infty} \frac{xf(x)dx}{\mu} \geq \frac{-H^w(t)\overline{F}(t)}{log\lambda_F(t)}, \tag{36}$$

that is

$$\overline{F}_W(t) \geq \frac{-H^w(t)\overline{F}(t)}{\mu log\lambda_F(t)}. \tag{37}$$

Example 5.3 *One of the most useful model in reliability studies is the exponential failure model with pdf given by*

$$f(t;\theta) = \theta^{-1}exp(-t/\theta), \qquad t > 0 \text{ and } \theta > 0. \tag{38}$$

The hazard function, mean residual life function, and length-biased reliability function are given by $\lambda_F(t) = \theta^{-1}$, $\delta_F(t) = \theta$, *and* $\overline{F}_W(t) = \frac{(t+\theta)exp(-t/\theta)}{\theta}$ *respectively. The weighted residual entropy is given by*

$$H^w(t) = t + 2\theta - (t+\theta)log(\theta)^{-1}, \qquad t \geq 0. \tag{39}$$

Consequently, the result of Theorem 5.2 *holds.*

Example 5.4 *The Pareto distribution has applications in a wide variety of settings including clusters of Bose-Einstein condensate near absolute zero, file size distribution of internet traffic that uses the TCP protocol, values of oil reserves in oil fields, standardized price returns on individual stocks to mention a few area. Consider the survival or reliability function given by*

$$\overline{F}(x;c,\alpha) = \begin{cases} \left(\frac{c}{x}\right)^{\alpha} & \text{if } x > c, \\ 1 & \text{otherwise.} \end{cases}$$

Clearly, the hazard function, length-biased version of the distribution function F *and its hazard function are given by* $\lambda_F(x) = \alpha/x$,

$$\overline{F}_W(x;c,\alpha) = \begin{cases} \left(\frac{c}{x}\right)^{\alpha-1} & \text{if } x > c, \\ 1 & \text{otherwise.} \end{cases}$$

and $\lambda_{F_W}(x) = \frac{\alpha-1}{x}$ *respectively. The failure rate function* $\lambda_F(x)$ *is decreasing. Consequently, an application of Theorem* 5.2 *leads to*

$$\overline{F}_W(t) \geq \frac{-H^w(t)\overline{F}(t)}{\mu log\lambda_F(t)}. \tag{40}$$

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