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VELOCITY DISTRIBUTION VERSUS SEDIMENT IN THE MISSOURI RIVER



A Report Submitted to

The Department of the Army
Kansas City District, Corps of Engineers

By

University of Missouri - Rolla
Rolla, Missouri 65401

30 June, 1973

VELOCITY DISTRIBUTION VERSUS SEDIMENT IN THE MISSOURI RIVER



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The Department of the Army
Kansas City District, Corps of Engineers
for the Period of
14 August 1972 -- 30 June 1973

by

Glendon T. Stevens
Terence E. Harbaugh
Paul R. Munger
Civil Engineering Department
University of Missouri - Rolla

Contract No. DACW 41 - 73 - C - 0022

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GH
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1.0 Analysis of the large quantities of velocity and sediment data gathered on the Missouri River has not been undertaken in the past due to the heavy workloads of U.S. Army Corps of Engineers staff. The need to undertake this effort has long been recognized by both the Kansas City District and the Omaha Division. As a result of this need, the Kansas City District, Corps of Engineers, entered into a contract with the curators of the University of Missouri in September 1972. Dr. G. T. Stevens of the University's Rolla campus will be the principal investigator and will perform data analysis on the following measurement stations.

CORPS LOCATION (River Mile)	NAME	DATA SETS
732.3	Sioux City	140
641±	STC Data	423
615.9	Omaha	215
562.6	Nebraska City	225
448.2	St. Joseph	10
366.1	Kansas City	10
293.4 291.4 289.7	Waverly	125
97.9	Hermann	25

1.1 Due to the relatively short project period, September 1972 to July 1973, the analysis was restricted to a study of only the velocity distribution portion of the above station data. Typically, these data consist of point velocities taken at varying depths in the vertical and at numerous locations perpendicularly across the main channel. The number of vertical point velocities ranged from a minimum of five up to a maximum of seven. The number of verticals, i.e. stations in a cross section, ranged from one in the 1951 Sediment Transport Characteristics data to five in the remaining data. The station location generally being determined by a subsection width containing approximately 20 percent of the total channel flow. The usable data sets are shown for each Corps location in paragraph 1.0 and consists of a complete vertical velocity profile at a known station within the channel cross section. A number of other data sets were discarded due to erratic data points within the set. This was initially accomplished by manually scanning printed output and eliminating those data sets containing maximum velocities at the bottom. These data were not discarded from the data bank but were eliminated from the initial analysis. Thus, future analysis may be used to explain the apparent erratic behavior through a sound scientific basis. Additionally, the present data sets still contain some erratic points which should be eliminated. These erratic points are either due to recording errors or to

problems encountered during the measurement as a result of local disturbances such as sand bed configurations. Probably about seventy five percent of the total project time was devoted to obtaining a workable data bank.

1.2 The initial phase of this project involved a) a literature search and b) establishing a computerized data bank of available velocity measurement data. These tasks were undertaken simultaneously and continued throughout the project. The results of a portion of the literature search is provided in three volumes of material submitted under separate cover with this report. These volumes are xerox copies of the ASCE Progress Report, Task Committee on Preparation of Sedimentation Manual. (2, 6, 10, 22, 38, 39) Additional literature search revealed numerous publications and texts on the general subject of sedimentation, however, the ASCE Progress Report provides one of the best, most comprehensive discussions of the present approaches in sediment theory for alluvial channels. Additionally, a short bibliography of current literature relative to this study is included at the end of this report.

1.3 As noted in the contract, three basic velocity profile approaches were investigated to ascertain their suitability for use in predicting sediment transport on the Missouri River. The three sediment prediction methods were those developed by Ippen⁽⁴⁰⁾, Toffaleti⁽⁴²⁾, and by Alan and Kennedy⁽⁴¹⁾. Due to the nature of the data utilized by both Ippen and by

Alan and Kennedy, these two methods were not extensively utilized. Their results were from controlled flume studies using uniform size sediment and in their present formulation would not be adaptable to the field measurement data presently available on the Missouri River. It should be noted that if these methods are to be more thoroughly investigated, a more refined procedure of data collection will have to be undertaken at various locations throughout the Missouri River. This data collection would have to be developed from a study of the revised methods of Alan and Kennedy, and Ippen as applied to natural rivers. Of the three methods, the Toffaleti approach is the most applicable to the use of existing field measurements and as a result is the best approach toward developing a usable, accurate sediment prediction technique.

1.4 The practical applicability of Toffaleti's approach, however, is one of the major reasons for attempting to develop a generalized velocity profile relationship for the Missouri River. His work resulted from an analysis of several hundred point velocity and sediment measurements which produced a predictive equation in the Mississippi River for determining both typical velocity and sediment concentration profiles which when integrated over the channel depth will result in a total sediment load prediction. The necessary data for computing total daily sediment load from this procedure are:

- a) Mean section velocity, Q/A
- b) Section width, W
- c) Water temperature, °F
- d) Hydraulic mean depth, A/W
- e) D_{65} representative grain size
- f) Slope of the river
- g) Bed material composition

1.5 The apparent simplicity of this method is self-evident since all these data can be obtained from a discharge measurement, a temperature measurement, a slope measurement, and a sieve analysis of the bed material. The computer program provided by Toffaleti⁽²⁰⁾ has been altered and certain program changes made to convert it to the UMR IBM 360-50 system. This program is operational and was utilized on this project to evaluate the applicability of this procedure to the Missouri River.

VELOCITY DISTRIBUTIONS

2.1 Typical velocity profiles of the Mississippi and Atchafalaya Rivers are shown in Figure 1 as obtained by Toffaleti⁽⁴²⁾. The resulting equation for the Mississippi River vertical velocity profile is

$$U_y = 1.15 \bar{U} \left(\frac{y}{d}\right)^{0.155}$$

in which

y = distance above bed

d = depth of a mean-depth section

\bar{U} = average velocity of flow in mean-depth section

U_y = point velocity at y distance from the bed

It should be noted that this equation is considerably simpler than the one proposed by Ippen⁽¹⁾.

$$\frac{u-U_{max}}{U^*} = \frac{1}{K} \ln\left(\frac{y}{y_0} - \psi \ln \frac{y}{y_0}\right)$$

u = point velocity

U_{max} = maximum vertical velocity

U^* = shear velocity, $\sqrt{gy_0 S_0}$

K = $\gamma / (a+y) U^*$

y = point depth

y_0 = section depth

ψ = trial and error depth ratio modifier

As a result of lack of necessary data, such S_0 and ψ , needed to fit Ippen's method it was not further investigated. However, several trial runs were performed using Toffaleti's

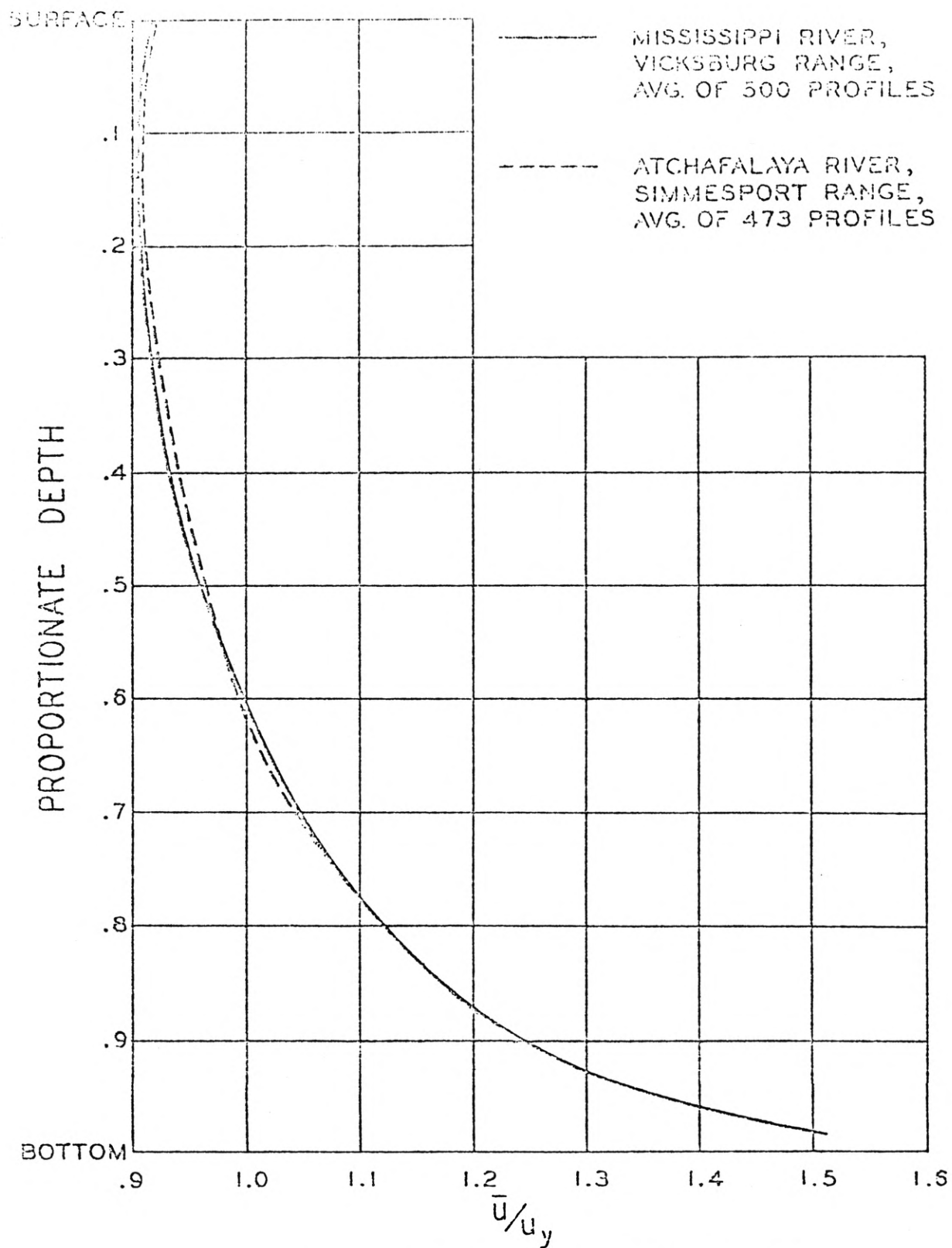


FIG. 1 VELOCITY PROFILES

technique on the Missouri River data.

2.2 Results of the Toffaleti approach as applied to the Missouri River produced values of 50 to 300 percent variation in sediment load in comparison with tabulated sediment loads provided by the U.S. Army Corps of Engineers. These calculations were made at both Kansas City and Hermann for random dates from 1965 to 1969.⁽⁴⁴⁾ Two apparent reasons for this discrepancy are a) a need to redefine the velocity distribution for the more shallow Missouri River and b) a need to redefine the typical sediment concentration curve for the Missouri River. However, the closeness of the estimates obtained by Toffaleti's approach indicates considerable merit in further pursuit of developing these two typical profiles.

2.3 In an attempt to determine a typical vertical velocity distribution for the Missouri River, the following three mathematical relationships were tested:

1. log - log
2. semi - log
3. rectangular

All of the relationships were also investigated with respect to various normalizing constants: a) average velocity V in the vertical obtained as the numerical average of the sum of the magnitudes of the vertical point velocities; b) mean velocity \bar{V} in the vertical obtained at the 0.37 depth on a semi-log plot; c) mean sectional velocity, $U=Q/A$; d) the numerical average of the point velocities at a selected depth

ration, UA; e) total depth D of water at the vertical section; and f) the mean hydraulic depth R of the cross section. A summary of these normalizing constants is shown below:

Corps Velocity Type	Corps Parameter					
	P ₁				P ₂	
Log-Log (L)	V	\bar{V}	U	UA	D	R
Semi-Log (SL)	V	\bar{V}	U	UA	D	R
Rectangular (RA)	V	\bar{V}	U	UA	D	R

2.4 The following general relationships now describe the typical velocity profiles investigated in this report.

$$\text{Log-Log} \quad U_y = P_1 (C_1) (Y/P_2)^{C_2}$$

$$\text{Semi-Log} \quad U_y = P_1 (C_2 \ln (\frac{Y}{P_2}) + C_1)$$

$$\text{Rectangular} \quad U_y = P_1 (C_2 (\frac{Y}{P_2}) + C_1)$$

where

U_y = point velocity

Y = depth measured from bottom

P_1 = velocity normalizing parameter

P_2 = depth normalizing parameter

C_1 = coefficient (sometimes called intercept)

C_2 = coefficient (sometimes called slope)

Typical results of these calculations are shown in "Typical Analysis Procedures for Missouri River Velocity Profiles," Vol. 2, June 1973 (provided in original copy only). These data are arranged to correspond to the summaries shown in the succeeding tabulations explained in the following section. Explanations of the data analysis formats and interpreting the computer printouts are to be provided at the July 1973 seminar in Kansas City, Missouri.

ANALYSIS

3.1 Analysis of existing data rapidly demonstrated the lack of fit of the rectangular velocity distribution. Due to the limitation of computer funds late in the study period, this distribution was not pursued further. The log-log profile, however, was extensively tested since this type of distribution is much more useful in the application of the Toffaleti method to the Missouri River. A tabulation of the range in values of C_1 , C_2 , P_1 , and P_2 is shown for the log-log distribution in Tables 1, 2, 3, and 4. Examination of these tables indicates considerable variation in the coefficients, standard errors, and correlation coefficients. The typical log-log distribution has a much higher correlation coefficient by station in comparison to a fitting for the entire cross section at particular locations, i.e. Omaha, Nebraska City, Sioux City, etc. Arranged in downstream order the general log-log equations for all data at each location is:

Sioux City	$U_y = V(1.77) \left(\frac{Y}{D}\right)^{0.30}$
Omaha	$U_y = V(1.66) \left(\frac{Y}{D}\right)^{0.282}$
Nebraska City	$U_y = V(1.63) \left(\frac{Y}{D}\right)^{0.280}$
Waverly	$U_y = V(1.26) \left(\frac{Y}{D}\right)^{0.180}$

Composite plots showing these data points are given in Vol. 3 (provided in original copy only) of the appended material. Other plots for various locations and dates are also provided

TABLE I

LOG-LOG DATA SUMMARY FOR SIOUX CITY (701)
 $P_1=V$ $P_2=D$

C ₁	200			Station				260				310		
	C ₂	STDE	Cor. Coef.	C ₁	C ₂	STDE	Cor. Coef.	C ₁	C ₂	STDE	Cor. Coef.			
1.322	.194	.02	.99	1.102	.079	.02	.95	1.249	.152	.03	.97			
1.471	.274	.07	.97	1.148	.093	.03	.92	1.963	.522	.23	.95			
1.329	.198	.04	.97	1.223	.137	.04	.96	1.424	.249	.06	.97			
1.275	.166	.01	.99	1.221	.138	.11	.91	1.291	.176	.01	.99			
1.305	.184	.03	.98	1.179	.112	.03	.95	1.188	.117	.01	.99			
1.610	.291	.09	.95	1.295	.177	.03	.97	1.651	.368	.11	.96			
1.228	.172	.10	.95	1.370	.223	.19	.93	1.352	.212	.03	.98			
1.327	.202	.14	.91	1.218	.137	.04	.96	1.361	.209	.02	.99			
1.301	.180	.01	.99	1.339	.201	.01	.99	1.233	.143	.01	.98			
1.344	.210	.13	.94	1.396	.233	.02	.99	1.505	.295	.10	.97			
1.183	.114	.01	.97	1.239	.147	.03	.97	1.331	.196	.04	.97			
1.351	.178	.02	.99	1.387	.228	.03	.99	1.280	.168	.01	.99			
1.210	.157	0.00	.99	1.097	.075	.04	.89	1.162	.103	.04	.94			
1.256	.155	.05	.98	1.224	.140	.13	.93	1.331	.197	.03	.99			
1.298	.178	.01	.99	1.132	.085	.07	.85	1.333	.199	.01	.99			
1.309	.185	.01	1.00	1.310	.188	.04	.97	1.219	.136	.02	.97			

TABLE I (cont.)

460				660				Ave			
C ₁	C ₂	STDE	Cor. Coef.	C ₁	C ₂	STDE	Cor. Coef.	C ₁	C ₂	STDE	Cor. Coef.
1.118	.089	.02	.95	1.324	.234	0.0	1.0	1.220	.152	.17	.80
2.486	.715	.12	.97	1.236	.173	.01	.98	1.516	.300	.78	.68
1.255	.184	.01	.98	1.225	.195	.01	.98	1.260	.156	.60	.53
2.042	.538	.01	1.00	1.152	.138	.02	.92	1.269	.165	1.19	.47
1.219	.163	.01	.98	--	--	--	--	1.169	.104	.36	.53
1.257	.188	.03	.96	1.142	.109	.07	.76	1.443	.277	.15	.87
1.449	.310	.04	.97	1.228	.172	.02	.96	1.305	.200	.77	.61
1.330	.198	.03	.98	1.445	.361	.01	.99	1.270	.163	.61	.55
1.381	.267	.09	.95	1.237	.209	.13	.68	1.277	.174	.32	.67
1.600	.339	.03	.99	1.337	.246	.09	.87	1.423	.254	.37	.79
1.212	.131	.03	.95	1.391	.281	.03	.97	1.241	.143	.39	.61
1.228	.170	.04	.96	1.204	.176	.01	.95	1.173	.099	.77	.50
1.277	.156	.02	.98	1.153	.121	.21	.78	1.115	.071	.64	.44
1.556	.318	.01	.98	1.274	.198	.05	.94	1.282	.172	2.69	.38
1.473	.278	.04	.98	1.197	.147	.02	.96	1.266	.163	.33	.69
1.219	.161	.01	.99	1.173	.133	.03	.92	1.237	.148	.19	.74

TABLE II

LOG-LOG DATA SUMMARY FOR OMAHA (801)

 $P_1=V$
 $P_2=D$

C_1	Station								315			
	180 C_2	STDE	Cor. Coef.	C_1	250 C_2	STDE	Cor. Coef.	C_1	C_2	STDE	Cor. Coef.	
				1.105	.067	.13	.83	1.172	.107	.09	.94	
1.166	.310	.17	.82	1.092	.072	.07	.90	1.146	.110	.04	.95	
1.209	.130	.05	.93	1.032	.025	.00	.93	1.060	.047	.02	.91	
1.451	.317	.12	.93	1.192	.143	.07	.93	1.138	.105	.06	.93	
1.263	.172	.20	.88	1.164	.102	.02	.98	1.090	.059	.08	.84	
1.110	.070	.04	.90	1.154	.097	.20	.87	1.218	.132	.08	.96	
1.161	.103	.15	.92	1.200	.124	.08	.97	1.275	.168	.05	.99	
1.339	.214	.29	.86	1.121	.094	.04	.93	1.139	.106	.06	.95	
1.278	.169	.02	.98	1.095	.073	.02	.96	1.127	.096	.03	.96	
1.360	.215	.10	.94	1.472	.275	.18	.95	1.047	.037	.05	.44	
1.148	.093	.03	.94	1.198	.121	.01	.99	1.127	.080	.01	.97	
1.177	.112	.04	.95	1.095	.062	.11	.83	1.236	.143	.08	.96	
1.277	.167	.06	.97	1.115	.088	.00	1.00	1.262	.159	.02	.99	
1.368	.217	.02	.99	1.064	.050	.03	.88	1.226	.166	.26	.87	
1.560	.314	.08	.98	1.182	.135	.12	.91	1.191	.143	.08	.93	
1.231	.142	.03	.98	1.187	.117	.15	.91	1.181	.115	.08	.95	

TABLE II (cont.)

C ₁	Station				C ₁	Ave					
	C ₂ ³⁶⁰	STDE	Cor. Coef.	C ₂ ⁴⁶⁰		STDE	Cor. Coef.	C ₂	STDE	Cor. Coef.	
1.303	.184	.10	.97	1.224	.168	.03	.95				
1.256	.155	.12	.95	1.156	.121	.28	.82	1.195	.149	1.88	.44
1.582	.33	.18	.97	1.163	.123	.08	.92	1.231	.179	1.40	.55
1.822	.441	.09	.99	1.609	.413	.72	.89	1.456	.321	1.17	.69
1.187	.116	.04	.93	1.112	.085	.00	.93	1.141	.080	2.24	.22
1.328	.199	.10	.97	1.139	.105	.01	.98	1.176	.102	2.30	.33
1.493	.283	.05	.99	1.345	.253	.15	.88	1.249	.145	2.91	.43
1.182	.113	.03	.98	1.182	.138	.09	.96	1.229	.183	1.24	.57
1.291	.177	.05	.98	1.160	.127	.02	.98	1.225	.174	1.26	.55
1.397	.236	.03	.99	1.107	.083	.05	.81	1.295	.194	.78	.62
1.122	.078	.03	.95	1.172	.108	.01	.98	1.154	.096	.52	.55
1.212	.132	.07	.97	1.131	.099	.02	.94	1.162	.098	1.15	.44
1.598	.340	.46	.94	1.312	.225	.07	.96	1.304	.197	.51	.79
1.312	.188	.01	.99	1.213	.159	.02	.99	1.268	.200	1.14	.62
1.415	.241	.06	.99	1.376	.236	.16	.96	1.364	.255	.96	.72
1.491	.286	.16	.97	1.125	.097	.02	.96	1.226	.138	1.42	.52

TABLE III

LOG-LOG DATA SUMMARY FOR NEBRASKA CITY (951)

 $P_1=V$ $P_2=D$

Station											
165				230				290			
C_1	C_2	STDE	Cor. Coef.	C_1	C_2	STDE	Cor. Coef.	C_1	C_2	STDE	Cor. Coef.
1.154	.097	.02	0.97	1.131	.083	0.00	1.00	1.414	.246	.12	0.98
1.328	.194	.01	1.00	1.170	.107	0.03	0.98	1.128	.081	.01	0.99
1.096	.062	.12	0.82	1.175	.109	0.07	0.96	1.078	.050	.02	0.96
1.424	.247	.10	0.98	1.249	.152	0.05	0.98	1.117	.077	.36	0.80
1.167	.105	.01	0.99	1.670	.411	2.48	0.80	1.214	.132	.07	0.95
1.299	.181	.02	0.99	1.314	.190	0.10	0.97	1.385	.227	.02	1.0
1.398	.236	.35	0.92	1.667	.368	0.41	0.95	2.295	.550	.42	0.95
1.258	.157	.04	0.98	1.109	.070	0.01	0.98	1.141	.090	.02	0.98
1.240	.149	.07	0.97	1.162	.102	0.09	0.93	1.099	.063	.00	0.99
1.329	.198	.04	0.99	1.384	.226	0.02	0.99	1.177	.111	.07	0.94
1.074	.048	.06	0.83	1.054	.042	0.01	0.96	1.167	.120	.06	0.95
1.062	.041	.04	0.86	1.156	.098	0.00	1.00	1.205	.127	.04	0.97
1.272	.163	.06	0.96	1.142	.090	0.03	0.97	1.174	.108	.01	0.99
1.153	.097	.08	0.88	1.051	.033	0.01	0.93	1.225	.143	.05	0.97
1.322	.192	.04	0.98	1.227	.139	0.06	0.96	1.200	.125	.07	0.96

TABLE III (cont.)

340				500				Ave			
C ₁	C ₂	STDE	Cor. Coef.	C ₁	C ₂	STDE	Cor. Coef.	C ₁	C ₂	STDE	Cor. Coef.
1.150	.096	.14	0.88	1.162	.123	0.00	1.00	1.179	.104	2.23	.35
1.192	.118	.02	0.98	1.150	.114	0.00	0.98	1.172	.100	1.73	.36
1.320	.193	.13	0.96	1.209	.180	0.00	1.00	1.127	.063	4.80	.11
1.071	.047	.29	0.53	1.060	.049	0.06	0.27	1.172	.094	3.42	.26
1.112	.072	0.0	0.99	1.181	.137	0.00	0.99	1.233	.137	2.34	.40
1.220	.136	.05	0.97	1.402	.289	0.02	0.98	1.284	.163	2.05	.49
1.379	.226	0.0	1.00	1.174	.154	0.03	0.95	1.682	.401	1.34	.73
1.184	.115	.02	0.98	1.097	.074	0.00	0.99	1.145	.085	1.16	.39
1.241	.147	.08	0.96	1.126	.097	0.04	0.84	1.159	.094	1.14	.42
1.118	.076	.03	0.95	1.399	.299	0.04	0.92	1.225	.125	2.71	.32
1.193	.145	.02	0.99	1.205	.152	0.01	0.99	1.105	.066	1.43	.32
1.168	.107	.15	0.87	1.150	.116	0.01	0.95	1.123	.065	2.84	.19
1.258	.156	.07	0.96	1.333	.246	0.01	0.98	1.185	.101	3.09	.25
1.269	.165	.09	0.96	1.142	.108	0.01	0.96	1.157	.098	0.78	.48
1.704	.394	.31	0.93	1.289	.210	0.03	0.95	1.295	.173	1.63	.49

TABLE IV

LOG-LOG DATA SUMMARY FOR WAVERLY, MO. Range 7
 $P_1=V$ $P_2=D$ Mile 293.4

Station											
C_1	90 C_2	STDE	Cor. Coef.	C_1	215 C_2	STDE	Cor. Coef.	C_1	290 C_2	STDE	Cor. Coef.
1.442	.258	.04	.98	1.533	.299	.03	.98	1.364	.215	.01	.99
1.243	.149	.01	.99	1.460	.268	.07	.97	1.388	.230	.02	.99
1.445	.225	.11	.93	1.175	.111	.02	.95	1.120	.077	.10	.79
1.318	.166	.07	.95	1.353	.214	.15	.92	1.241	.167	.16	.93
	75				225				440		
1.249	.154	.02	.97	1.152	.096	.01	.98	1.240	.152	.22	.88
1.415	.250	.01	.99	1.280	.205	.04	.98	1.450	.260	.07	.97
1.194	.128	.11	.78	1.208	.134	.05	.94	1.191	.133	.95	.68
1.213	.134	.06	.92	1.233	.143	.03	.98	1.583	.336	.09	.98
1.260	.161	.02	.95	1.390	.231	.11	.92	1.652	.357	.19	.97
1.324	.192	.03	.97	1.418	.242	.02	.99	1.096	.074	.02	.94
1.279	.293	.01	.99	1.125	.132	.00	1.00	1.282	.289	.07	.95
	1				2				3		
1.114	.072	.08	.78	1.106	.081	.06	.89	1.088	.067	.02	.92
1.252	.152	.07	.88	1.153	.084	.14	.87	1.535	.385	.60	.89
1.389	.232	.10	.95	1.385	.227	.14	.97	1.415	.214	.30	.91
1.313	.187	.02	.98	1.139	.088	.01	.98	1.242	.149	.00	1.00
1.570	.329	.03	.98	1.167	.125	.02	.98	1.162	.124	.07	.96

TABLE IV (cont.)

Station												
C ₁	480			758				C ₁	Ave			Cor. Coef.
	C ₂	STDE	Cor. Coef.	C ₁	C ₂	STDE	Cor. Coef.		C ₁	C ₂	STDE	
1.211	.159	.04	.94	1.175	.131	.00	1.00	1.349	.218	.16	.85	
1.183	.135	.01	.98	1.274	.199	.02	.95	1.305	.193	.11	.87	
1.290	.177	.14	.93	1.014	.012	.18	.34	1.286	.192	.41	.66	
1.645	.354	.13	.97	1.209	.128	.03	.92	1.360	.230	.79	.69	
Station												
C ₁	550			690				C ₁	Ave			Cor. Coef.
	C ₂	STDE	Cor. Coef.	C ₁	C ₂	STDE	Cor. Coef.		C ₁	C ₂	STDE	
1.100	.077	.01	.96	1.145	.109	.02	.95	1.186	.133	.74	.52	
1.415	.291	.02	.99	1.160	.121	.02	.95	1.363	.252	.33	.81	
1.162	.122	.13	.74	1.368	.267	.02	.97	1.208	.137	1.07	.42	
1.208	.188	.01	.99	1.225	.165	.03	.96	1.301	.201	1.30	.58	
1.111	.085	.06	.90	1.346	.252	.02	.99	1.370	.247	1.52	.53	
1.184	.139	.05	.94	1.136	.103	.01	.95	1.262	.195	.27	.75	
1.109	.123	.02	.95	1.042	.049	.02	.80	1.074	.075	.34	.65	
Station												
C ₁	4			5				C ₁	Ave			Cor. Coef.
	C ₂	STDE	Cor. Coef.	C ₁	C ₂	STDE	Cor. Coef.		C ₁	C ₂	STDE	
-----	-----	---	---	1.080	.054	.02	.39	1.112	.099	5.70	.12	
1.173	.129	.23	.71	1.432	.258	.05	.96	1.181	.101	1.24	.47	
1.022	.022	.20	.32	1.055	.047	.03	.22	1.286	.158	2.65	.30	
1.163	.103	.09	.94	1.076	.049	.01	.79	1.186	.116	1.69	.41	
1.147	.112	.03	.98	1.054	.035	.03	.61	1.281	.238	3.24	.42	

in Vol. 3. The values of C_1 and C_2 appears to decrease in magnitude with downstream location thus indicating a change in flow characteristics. However, the large standard error associated with these grouped data equation fits will not allow a reliable use of these equations without a sensitivity analysis of their influence on a typical sediment prediction method such as the one proposed by Toffaleti. Grouped data, analyzed in this manner, do not distinguish between velocity relationships developed in the shallow sections of the cross section and those developed in the deeper navigational channel. Neither does this analysis allow for local disturbances, stage variations, changes in temperature, or etc. All of these factors will influence any attempt to predict vertical velocity profiles on a daily basis. Thus, over a sustained period of time the average profile, as given by the above equations, should be within one standard error two thirds of the time.

3.2 Changing the normalizing constant does little to improve the variability. Graphical demonstration of the effect of altering the normalizing constant from V to U as described in paragraph 2.3, are shown in "Typical Analysis Procedures for Missouri River Velocity Profiles", Vol. 2, June 1973. These relationships are shown for St. Joseph, Kansas City, and Hermann. A summary of the values of C_1 and C_2 are shown for both constants in Table 5. As can be seen, the product C_1V and C_1U remains constant thus proving that the two relation-

TABLE V
 LOG-LOG DATA SUMMARY -
 COMPARISON OF NORMALIZING CONSTANT FOR
ST. JOSEPH

1280						1180						1080					
$P_1=V$ C_1	$P_2=D$ C_2	$P_2=D$ C_1V	$P_1=U$ C_1	$P_2=D$ C_2	$P_2=D$ C_1U	$P_1=V$ C_1	$P_2=D$ C_2	$P_2=D$ C_1V	$P_1=U$ C_1	$P_2=D$ C_2	$P_2=D$ C_1U	$P_1=V$ C_1	$P_2=D$ C_2	$P_2=D$ C_1V	$P_1=U$ C_1	$P_2=D$ C_2	$P_2=D$ C_1U
1.85	.386	6.77	2.03	.386	6.77	1.17	.110	5.94	1.79	.110	5.95	1.21	.127	5.70	1.72	.127	5.71
1.19	.123	6.05	1.55	.123	6.05	1.10	.063	5.98	1.53	.063	5.98	1.34	.209	6.01	1.54	.209	6.01

KANSAS CITY

115						265						385					
$P_1=V$ C_1	$P_2=D$ C_2	$P_2=D$ C_1V	$P_1=U$ C_1	$P_2=D$ C_2	$P_2=D$ C_1U	$P_1=V$ C_1	$P_2=D$ C_2	$P_2=D$ C_1V	$P_1=U$ C_1	$P_2=D$ C_2	$P_2=D$ C_1U	$P_1=V$ C_1	$P_2=D$ C_2	$P_2=D$ C_1V	$P_1=U$ C_1	$P_2=D$ C_2	$P_2=D$ C_1U
1.89	.118	5.26	1.28	.118	5.26	1.85	.451	7.59	1.84	.451	7.59	1.28	.153	5.09	1.23	.153	5.08
1.43	.256	7.08	1.44	.256	7.08	1.39	.237	6.59	1.34	.237	6.59	-----	-----	-----	-----	-----	-----

HERMAN

225						450						650					
$P_1=V$ C_1	$P_2=D$ C_2	$P_2=D$ C_1V	$P_1=U$ C_1	$P_2=D$ C_2	$P_2=D$ C_1U	$P_1=V$ C_1	$P_2=D$ C_2	$P_2=D$ C_1V	$P_1=U$ C_1	$P_2=D$ C_2	$P_2=D$ C_1U	$P_1=V$ C_1	$P_2=D$ C_2	$P_2=D$ C_1V	$P_1=U$ C_1	$P_2=D$ C_2	$P_2=D$ C_1U
1.12	.101	2.72	.65	.101	2.71	1.18	.112	4.98	1.19	.112	4.98	1.67	.374	6.68	1.60	.374	6.68
1.10	.078	3.55	.86	.078	3.55	1.45	.270	5.52	1.34	.270	5.52	1.19	.146	5.77	1.40	.146	5.78
1.17	.104	3.47	.82	.104	3.47	1.42	.249	6.03	1.42	.249	6.04	1.37	.222	6.22	1.46	.222	6.22

TABLE V (cont.)

ST. JOSEPH

980						864					
$P_1=V$	$P_2=D$		$P_1=U$	$P_2=D$		$P_1=V$	$P_2=D$		$P_1=U$	$P_2=D$	
C_1	C_2	C_1V	C_1	C_2	C_1U	C_1	C_2	C_1V	C_1	C_2	C_1U
1.16	.087	5.05	1.52	.087	5.05	1.60	.288	5.07	1.52	.288	5.07
1.37	.187	6.97	1.79	.187	6.98	1.16	.094	4.66	1.19	.094	4.65

KANSAS CITY

515						651					
$P_1=V$	$P_2=D$		$P_1=U$	$P_2=D$		$P_1=V$	$P_2=D$		$P_1=U$	$P_2=D$	
C_1	C_2	C_1V	C_1	C_2	C_1U	C_1	C_2	C_1V	C_1	C_2	C_1U
1.30	.180	7.13	1.73	.180	7.14	1.46	.260	3.66	0.89	.260	3.66
1.34	.199	8.63	1.75	.199	8.62	1.10	.070	5.89	1.94	.070	5.88

HERMAN

825						1025					
$P_1=V$	$P_2=D$		$P_1=U$	$P_2=D$		$P_1=V$	$P_2=D$		$P_1=U$	$P_2=D$	
C_1	C_2	C_1V	C_1	C_2	C_1U	C_1	C_2	C_1V	C_1	C_2	C_1U
1.25	.153	5.04	1.21	.153	5.04	1.19	.118	5.73	1.37	.118	5.73
1.30	.171	4.01	0.97	.171	4.02	1.15	.094	6.67	1.62	.094	6.68
1.27	.146	5.63	1.33	.146	5.63	1.12	.074	5.13	1.21	.074	5.13

ships are predicting the same value. A change in the depth normalizing constant will likewise shift the data.

3.3 Examination of the comparison of fit of the log-log and semi-log relationships was performed for the Waverly location. These data, for range 7, are given in "Typical Analysis Procedures for Missouri River Velocity Profiles", Vol. 3, June 1973. A summary of these relationships is shown in Tables 6 and 7. Results of this analysis would leave little doubt that the choice of the velocity relationships can either be log-log or semi-log with little loss if any in the accuracy. The results proved by the analysis conducted thus far point out that the velocity profile can be described by either a log-log or semi-log relationship.

3.4 As can be seen in Table 7, grouping the data by station rather than by location gives significantly improved fits for both log-log and semi-log approaches. This behavior is due to a variety of influencing factors which tend to distort the velocity profile, i.e. the depth at the station, the bed configuration; the temperature; the time history of the stage relationship; the changing shear on the bed due to depth, velocity near the bed, and sediment concentration. The reduced correlation coefficient obtained by grouping velocity data for the entire cross-section is readily apparent throughout the length of the Missouri River studied in this project. It would be necessary to enlarge this study to include those and other combinations of parameters in order

TABLE VI

SEMI-LOG SUMMARY FOR WAVERLY, MO. Range 7
 $P_1=U$ $P_2=R$ Mile 293.4

Station											
C_1	75	STDE	Cor. Coef.	225				440			
	C_2			C_1	C_2	STDE	Cor. Coef.	C_1	C_2	STDE	Cor. Coef.
0.918	.139	.031	.966	1.108	.197	.027	.985	1.110	.084	.019	.929
0.911	.126	.012	.982	1.373	.113	.007	.986	1.306	.193	.264	.854
0.986	.204	.022	.985	1.484	.252	.055	.970	1.136	.223	.105	.949
0.766	.096	.115	.766	1.284	.149	.063	.929	1.306	.201	.989	.668
1.122	.272	.013	.993	1.161	.133	.000	.999	1.073	.226	.113	.924
	90				215				290		
0.934	.127	.014	.980	1.000	.210	.100	.953	0.966	.177	.026	.982
1.001	.210	.085	.956	1.065	.230	.030	.987	0.985	.167	.004	.996
1.028	.172	.124	.927	1.310	.115	.026	.952	1.246	.092	.104	.793
0.931	.124	.010	.980	1.008	.219	.100	.950	1.002	.183	.030	.980
0.728	.113	.090	.931	0.929	.151	.207	.895	1.236	.163	.108	.955
	410				560				760		
1.092	.169	.017	.983	1.148	.200	.048	.966	1.001	.144	.040	.949
0.960	0.94	.036	.897	1.022	.156	.069	.907	1.470	.239	.037	.983

TABLE VI (cont.)

Station											
C ₁	550			C ₁	690			C ₁	Ave		
	C ₂	STDE	Cor. Coef.		C ₂	STDE	Cor. Coef.		C ₂	STDE	Cor. Coef.
1.313	.152	.041	.950	1.143	.091	.014	.954	1.095	.123	0.401	.582
1.532	.095	.010	.963	1.178	.103	.019	.945	1.205	.088	0.907	.313
1.522	.298	.005	.998	1.131	.123	.027	.943	1.199	.184	0.538	.659
1.317	.106	.122	.755	0.897	.158	.014	.979	1.100	.130	1.093	.400
1.248	.128	.025	.937	1.420	.064	.017	.808	1.183	.116	0.474	.435
	480				758				Ave		
1.005	.108	.006	.986	0.898	.122	.018	.965	0.973	.153	0.080	.904
1.057	.147	.040	.944	0.815	.088	.001	.998	1.008	.181	0.093	.913
1.347	.201	.172	.911	1.053	.034	.176	.342	1.202	.127	0.475	.592
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
0.908	.214	.208	.944	0.724	.068	.026	.929	0.892	.140	0.926	.615
	885				1025				Ave		
1.294	.117	.012	.969	1.265	.133	.001	.998	1.153	.156	0.279	.700
1.767	.211	.003	.998	1.035	.087	.037	.855	1.256	.162	1.090	.450

TABLE VII

DATA FOR COMPARISON OF LOG-LOG AND SEMI-LOG
 Waverly, Mo. Range 7 Mile 293.4
 $P_1=U$ $P_2=R$

Station																	
75						225						440					
Log-Log			Semi-Log			Log-Log			Semi-Log			Log-Log			Semi-Log		
Cor. Coef.	STDE	SQD	Cor. Coef.	STDE	SQD	Cor. Coef.	STDE	SQD	Cor. Coef.	STDE	SQD	Cor. Coef.	STDE	SQD	Cor. Coef.	STDE	SQD
.997	.001	.004	.999	.000	.001	.997	.001	.004	.999	.000	.001	.948	.078	.243	.924	.113	.338
.779	.110	.440	.766	.115	.460	.944	.050	.203	.929	.063	.251	.689	.938	3.84	.668	.989	3.96
.993	.011	.044	.985	.022	.088	.981	.035	.108	.970	.055	.166	.965	.073	.298	.949	.105	.419
.973	.017	.070	.982	.012	.046	.985	.008	.030	.986	.007	.027	.882	.216	.898	.854	.264	1.06
.969	.029	.116	.966	.031	.125	.987	.024	.094	.985	.027	.108	.936	.017	.052	.929	.019	.057
90						215						290					
.950	.067	.342	.931	.090	.452	.921	.158	.654	.895	.207	.829	.933	.159	.494	.955	.108	.323
.990	.010	----	.980	.010	----	.970	.070	----	.950	.100	----	.990	.020	----	.980	.030	----
.934	.113	.567	.927	.124	.620	.954	.025	.099	.952	.026	.105	.793	.103	.414	.793	.104	.414
.978	.043	.185	.956	.085	.339	.986	.033	.132	.987	.030	.121	.994	.007	.028	.996	.004	.017
.985	.011	.043	.980	.014	.058	.967	.071	.289	.953	.100	.399	.988	.017	.068	.982	.026	.104
410						560						760					
.901	.035	.140	.897	.036	.145	.920	.059	.180	.907	.069	.208	.979	.044	.177	.983	.037	.147
.975	.025	.102	.983	.017	.066	.959	.057	.233	.966	.048	.192	.966	.027	.110	.949	.040	.160

TABLE VII (cont.)

550						690						Ave					
Log-Log			Semi-Log			Log-Log			Semi-Log			Cor. Coef.	STDE	SQD	Cor. Coef.	STDE	SQD
Cor. Coef.	STDE	SQD	Cor. Coef.	STDE	SQD	Cor. Coef.	STDE	SQD	Cor. Coef.	STDE	SQD						
.948	.021	.064	.937	.025	.076	.802	.018	.054	.808	.017	.052	.431	.476	11.1	.435	.474	10.9
.742	.128	.385	.755	.122	.367	.969	.021	.063	.979	.014	.041	.406	1.09	28.8	.400	1.09	28.4
.993	.017	.053	.998	.005	.014	.955	.021	.065	.943	.027	.081	.658	.539	13.6	.659	.538	13.4
.964	.010	.030	.963	.010	.031	.947	.018	.055	.945	.019	.058	.313	.908	23.9	.313	.907	23.6
.940	.049	.149	.950	.041	.123	.952	.014	.043	.954	.014	.041	.573	.408	10.3	.582	.401	10.0
480						758						Ave					
.965	.134	.554	.944	.208	.832	.924	.028	.113	.929	.026	.106	.618	.921	26.1	.615	.926	25.9
.929	.138	.564	.911	.172	.687	.348	.175	.703	.342	.176	.703	.597	.471	13.8	.592	.475	13.8
.939	.042	.128	.944	.040	.119	.995	.001	.003	.998	.001	.002	.922	.083	2.17	.913	.093	2.42
.980	.008	.025	.986	.006	.017	.955	.023	.071	.965	.018	.054	.908	.076	1.98	.904	.080	2.07
885						1025						Ave					
.997	.003	.009	.998	.003	.008	.869	.034	.102	.855	.037	.111	.451	1.09	27.8	.450	1.10	27.4
.973	.010	.030	.969	.012	.035	.997	.001	.004	.998	.001	.003	.693	.285	7.47	.700	.279	7.26

to fully explore the typical velocity profile for the Missouri River.

3.5 One of the most beneficial aspects of this study has been the assembly of most of the existing velocity data on the Missouri River. These data are reproduced and provided on cards with this report. A listing of all data is provided in "Typical Analysis Procedures for Missouri River Velocity Profiles", Vol. 3, June 1973, with an explanation of the format for the various stations. These data should be updated as additional measurements are made on the Missouri River.

RECOMMENDATIONS

4.1 The future of the Missouri River Basin will depend to a large extent upon the ability of the engineer and the alluvial river to interact in a manner to keep nature in balance. Much of this future work in maintaining navigation depths, providing flood control, controlling scour and sedimentation, etc., will depend upon design, operation and maintenance studies conducted by means of mathematical models. Some of these models will by nature be very broad in their treatment of the actual hydraulics of the river, while others will be quite detailed insofar as the necessary information relative to velocities, resistance, flood plain influence, constriction reaction, and the effect of river training works. It is these latter areas wherein this project has laid some initial groundwork for future studies.

4.2 An overall treatment of the proposed long-range research program to be undertaken by the Corps of Engineers on the Missouri River Basin should be established. The vast number of projects which could be undertaken can only be cost effective if they are considered as a part of an overall plan. This present project is a beginning which can be utilized to provide the necessary basic data for various types of operation, maintenance and design.

4.3 Normalizing techniques which have been used in this brief skirmish with the Missouri River velocity data have not

provided the type of typical velocity profile which one finds on rivers such as the Mississippi. However, this is to be expected since the Mississippi has depths of over one hundred feet whereas the Missouri rarely attains depths over thirty-five feet. This behavior of the existing normalizing techniques demonstrates the need to re-analyze these data and utilize normalizing techniques which are dependent upon temperature, changing shear within the section, the dynamics of the flow, etc. Thus far, only velocity and depth have been used to normalize the velocity profile. It is felt that additional parameters will more fully describe the mechanics of alluvial channel flow.

4.4 In order to provide a continuity of applied research as a long-range plan is being developed, it is suggested that the following studies be conducted during the next two years.

a) Examination of the many types of normalizing constants and the semi-log and log-log velocity plots reveal a scatter of results which is indicative of the need to further examine the cause for the variability. Analysis of the data by location, section, date, and lumped together reveals that in fact either a semi-log or log-log plot will suffice to describe a normalized velocity curve for the Missouri River. Changes in the coefficients C_1 , C_2 , P_1 , and P_2 are broad enough in their range to indicate that a phenomenon is occur-

ring unexplained by these normalizing techniques and should be further explored using normalizing parameters which reflect the dynamics of the flow.

b) The phenomenon of the effects of temperatures should be further examined. Results by Toffaleti for the Mississippi revealed, for example, that the coefficient of the log-log relationship followed an equation

$$C_1 = 1.00 + C_2$$

Analysis of the Missouri River data for Station 200 at Sioux City, however, revealed a relationship of the form

$$C_1 = \frac{C_2 + 0.489}{0.516}$$

Additionally, the exponent C_2 , varied according to the relationship expressed by

$$C_2 = 0.161 + 0.000367(T)$$

where

$$T = \text{water temperature in } ^\circ\text{F}$$

This temperature relationship compares to Toffaleti's Mississippi equation

$$C_2 = 0.1198 + 0.00048(T)$$

Comparison of the Toffaleti Mississippi equations versus Missouri River data and equations developed in this report are shown below in calculating a velocity distribution for Station 200 at Sioux City.

Temperature	Toffaleti Approach	Missouri River Equations	
		observed	calculated
66°F	$U_y = 1.144Y \left(\frac{Y}{D}\right)^{0.144}$	$U_y = 1.301V \left(\frac{Y}{D}\right)^{0.180}$	$U_y = 1.306V \left(\frac{Y}{D}\right)^{0.185}$
43°F	$U_y = 1.140Y \left(\frac{Y}{D}\right)^{0.140}$	$U_y = 1.228V \left(\frac{Y}{D}\right)^{0.172}$	$U_y = 1.290V \left(\frac{Y}{D}\right)^{.177}$

The temperature dependence of the C_2 exponent is thus a major contributor to the variability encountered in the attempt to obtain a typical velocity profile and should be studied further to define its influence throughout the Missouri River.

c) The trend shown in paragraph 3.1 shows a behavior of decreasing values of C_1 and C_2 with downstream location. As additional data and normalizing techniques are developed this trend should be examined to help define the locational dependence of the velocity profile shape.

d) For a particular measuring station on the Missouri River the analysis of all the data encompasses a wide range of temperature, stage, discharge, sediment concentration, season and location within the channel cross sections. This lumping of all data without regard to these changing factors will tend to produce the wide data scatter apparent in the velocity plots. Due to the short time span of the present project these variations could not be investigated; however, an excellent data base now exists which should be expanded to allow a more refined approach toward defining a "typical velocity profile" for the Missouri River. The use of Geo-

logical Survey Forms 9207 will provide data and information on depth, width, discharge, etc., and will allow a fuller description of the available data. A revised format for computer retrieval of these data should be developed and existing Missouri River data should be incorporated into the data base. This revised format should be developed to include both velocity and sediment data and to include as much information as can be gathered for the lower portion of the Missouri River, i.e. St. Joseph to the mouth. (See paragraph 1.0.)

e) Typical velocity and sediment concentration curves should be developed for adaptation into a revised Toffaleti method for use on the Missouri River. This will require some major program modifications but can be accomplished much more rapidly than developing completely new techniques since Toffaleti's method has already been debugged and is running on our system.

f) In order to provide the maximum information transfer between the contractor and the Kansas City District personnel, it is suggested that during the middle portion of the contract a series of nine one-day meetings at monthly intervals would be held in the Kansas City Corps Office for purposes of explaining the processes of river mechanics and the approaches used in conducting the proposed project. These briefing sessions can be two-way meetings wherein the input of the Corps personnel could be provided continually throughout the project.

4.5 A long range research program leading toward the development of design, operation and maintenance techniques should be initiated as soon as possible. The present and future use of mathematical models in these areas is becoming increasingly apparent. The basic studies outlined in this proposal will allow development of the knowledge necessary to properly define the mathematical structure of many of these future models. Thus the proposed work should allow a more rapid development of these models and thus reduce the over-all effort needed to develop techniques to improve design, operation and maintenance.

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