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Analyses of Thin-Walled Sections under Localised Loading for General End Boundary Conditions – Part II: Buckling

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Analyses of thin-walled sections under localised loading for general end boundary conditions – Part 2: Buckling

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Abstract

Thin-walled sections under localised loading may lead to buckling of the sections. This paper briefly introduces the development of the Semi-Analytical Finite Strip Method (SAFSM) for buckling analyses of thin-walled sections under localised loading for general end boundary conditions. This method is benchmarked against the Finite Element Method (FEM).

For different support and loading conditions, different functions are required for flexural and membrane displacements. In Part 1- Pre-buckling described in a companion paper at this conference, the analysis provides the computation of the stresses for use in the buckling analyses in this paper. Numerical examples of buckling analyses of thin-walled sections under localised loading with different end boundary conditions are also given in the paper in comparison with the FEM.

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1. **Introduction**

Thin-walled plates and sections subjected to localised loading and experiencing plate buckling have been studied over a long period by numerous investigators who mainly focused on web plates of sections under concentrated load. Two comprehensive investigations in this research area were Khan and Walker (1972) for the buckling of plates under localised loading and Johansson and Lagerqvist (1995) for the resistance of plate edges under localised loading. In the application of the General Beam Theory (GBT), Natário, Silvestre, and Camotim (2012) further extended investigations for beams under concentrated loading. The results for plates, unlipped channel sections and I sections from the GBT have been benchmarked against previous research and the Shell Finite Element method (SFE).

The Finite Strip Method (FSM) developed by Cheung (1976) is an efficient method of analysis in comparison with the FEM. This method is used extensively in the Direct Strength Method (DSM) of design of cold-formed sections in the North American Specification for the Design of Cold-Formed Steel Structural Members AISI S100-2012 (2012) and the Australian/New Zealand Standard AS/NZS 4600:2005 (2005). It is therefore very important to extend the FSM of buckling analysis to localised loading. The SAFSM was applied in Chu, Ye, Kettle, and Li (2005) and Bui (2009) to the buckling analysis of thin-walled sections under more general loading conditions, where multiple series terms were used to capture the modulation of the buckles. The limitation of these investigations is that the transverse compression and shear are not included. Hancock and Pham (2013) applied the SAFSM to the buckling analysis of thin-walled sections subjected to shear forces. More recently, Hancock and Pham (2014) have extended the SAFSM to the analysis of thin - walled sections under localised loading for simply supported boundary condition using multiple series terms. In the longitudinal direction, a pre-buckling analysis was performed to compute stresses prior to the buckling analysis using these stresses. Solution convergence with increasing number of series terms was provided. However, in practice, cold-formed members are connected together by welds or bolts so that the end boundary conditions are expected to be different from simply supported. Thus, it is necessary to extend this method to the analysis of thin-walled sections under localised loading for general end boundary conditions.

In this Part 2 – Buckling, the paper briefly introduces the functions used to compute the stress distributions in the strips of the structural member for different end boundary conditions. In addition, the theory of the SAFSM for buckling analysis of thin walled sections under localised loading for general end boundary conditions is developed. Numerical examples have been performed by the SAFSM built into the THIN-WALL-2 program developed by the authors (Nguyen, Hancock, & Pham, 2015). The numerical solutions are compared with those from the analyses by the Finite Element Method (FEM) on ABAQUS (ABAQUS/Standard Version 6.13, 2013) to validate the accuracy.

2. Strip displacements

2.1. Flexural displacement

An isometric view of flexural displacements of a strip is shown in Fig.1 of the companion paper Part 1 - Pre-buckling.

The flexural deformations w of a strip can be described by the summation over μ series terms as:

$$w = \sum_{m=1}^{\mu} f_{1m}(y) X_{1m}(x) \tag{1}$$

where:

μ is the number of series terms of the harmonic longitudinal function,

$X_{1m}(x)$ is the curve for longitudinal variation, as described in Part 1 - Pre-buckling

$f_{1m}(y)$ is a polynomial for transverse variation. This function for the mth series term is given by:

$$f_{1m}(y) = \alpha_{1Fm} + \alpha_{2Fm}\left(\frac{y}{b}\right) + \alpha_{3Fm}\left(\frac{y}{b}\right)^2 + \alpha_{4Fm}\left(\frac{y}{b}\right)^3 \tag{2}$$

$\{\alpha_{Fm}\}$ are the vector polynomial coefficients for the mth series term which depend on the nodal line flexural deformations of the strip,

$$\{\alpha_{Fm}\} = \begin{bmatrix}\alpha_{1Fm} & \alpha_{2Fm} & \alpha_{3Fm} & \alpha_{4Fm}\end{bmatrix}^T$$

b and *L* are the strip width and length respectively.

The flexural deformations w can be written in matrix format:

$$w = \sum_{m=1}^{\mu} X_{1m}(x)\left[\Gamma_{FL}\right]\left[C_F\right]^{-1}\{\delta_{Fm}\} \tag{3}$$

where:

$\{\alpha_{Fm}\} = \left[C_F\right]^{-1}\{\delta_{Fm}\}$

$f_{1m}(y) = \left[\Gamma_{FL}\right]\{\alpha_{Fm}\}$

$\left[\Gamma_{FL}\right] = \begin{bmatrix}1 & (y/b) & (y/b)^2 & (y/b)^3\end{bmatrix}$

$\{\delta_{Fm}\}$ is the flexural displacement vector for nodal line displacements

$[C_F]$ is the evaluation matrix of the flexural displacement functions at the nodal lines

In the computation of the flexural potential energy described later, the derivatives of the flexural deformation are required. The derivatives used are as follows:

$$\frac{\partial w}{\partial x} = \sum_{m=1}^{\mu} X_{1m}^{'}(x)[\Gamma_{FL}]\{\alpha_{Fm}\} \tag{4}$$

$$\frac{\partial w}{\partial y} = \sum_{m=1}^{\mu} X_{1m}(x)\frac{1}{b}[\Gamma_{FT}]\{\alpha_{Fm}\} \tag{5}$$

where $\quad [\Gamma_{FT}] = \begin{bmatrix} 0 & 1 & 2(y/b) & 3(y/b)^2 \end{bmatrix}$

2.2. Membrane displacement

An isometric view of membrane displacements of a strip is shown in Fig.2 of the companion paper Part 1 - Pre-buckling.

The membrane deformations in the longitudinal and transverse directions of a strip can be described by the summation over μ series terms as:

$$v = \sum_{m=1}^{\mu} f_{vm}(y) X_{1m}(x) \tag{6}$$

$$u = \sum_{m=1}^{\mu} f_{um}(y) X_{2m}(x) \tag{7}$$

where:

$X_{1m}(x)$ and $X_{2m}(x)$ is the longitudinal variation curve for the membrane transverse v and longitudinal u deformations respectively, as described in Part 1 - Pre-buckling

$f_{vm}(y)$ and $f_{um}(y)$ are the transverse variations. These functions for the mth series term are given by:

$$f_{vm}(y) = \alpha_{1Mm} + \alpha_{2Mm}\left(\frac{y}{b}\right) \tag{8}$$

$$f_{um}(y) = \alpha_{3Mm} + \alpha_{4Mm}\left(\frac{y}{b}\right) \tag{9}$$

$\{\alpha_{Mm}\}$ is the vector of polynomial coefficients for the mth series term which depend on the nodal line membrane deformations of the strips

$$\{\alpha_{Mm}\} = \begin{bmatrix} \alpha_{1Mm} & \alpha_{2Mm} & \alpha_{3Mm} & \alpha_{4Mm} \end{bmatrix}^T$$

The membrane deformations of the strip can be written in matrix format:

$$v = \sum_{m=1}^{\mu} X_{1m}(x)[\Gamma_{Mv}][C_M]^{-1}\{\delta_{Mm}\} \tag{10}$$

$$u = \sum_{m=1}^{\mu} X_{2m}(x)[\Gamma_{Mu}][C_M]^{-1}\{\delta_{Mm}\} \tag{11}$$

where:

$\{\alpha_{Mm}\} = [C_M]^{-1}\{\delta_{Mm}\}$

$f_{vm}(y) = [\Gamma_{Mv}]\{\alpha_{Mm}\}$ and $f_{um}(y) = [\Gamma_{Mu}]\{\alpha_{Mm}\}$

$[\Gamma_{Mv}] = [1 \quad (y/b) \quad 0 \quad 0]$ and $[\Gamma_{Mu}] = [0 \quad 0 \quad 1 \quad (y/b)]$

$\{\delta_{Mm}\}$: is the membrane displacement vector

In the computation of the membrane potential energy described later, the derivatives of the membrane deformations are required. The derivatives used are as follows:

$$\frac{\partial v}{\partial x} = \sum_{m=1}^{\mu} X'_{1m}(x)[\Gamma_{Mv}]\{\alpha_{Mm}\} \tag{12}$$

$$\frac{\partial u}{\partial x} = \sum_{m=1}^{\mu} X'_{2m}(x)[\Gamma_{Mu}]\{\alpha_{Mm}\} \tag{13}$$

3. Membrane stresses

3.1. Membrane stresses calculation

The membrane stresses of a strip are given by:

$$\{\sigma_{Mm}\} = [D_M]\{\in_{Mm}\} \tag{14}$$

where $\{\in_{Mm}\}$ is the membrane strain vector:

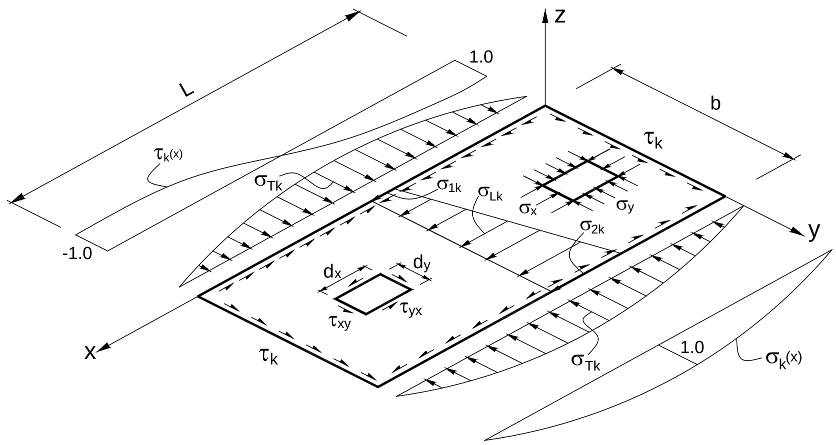
$$\{\in_{Mm}\} = [B_{Mm}]\{\alpha_{Mm}\} \tag{15}$$

Hence:

$$\{\sigma_{Mm}\} = [D_M][B_{Mm}]\{\alpha_{Mm}\} \tag{16}$$

3.2. Stress distribution in a strip

A strip subjected to loading will have complex stresses as shown in the Fig.1 where the stresses due to the k=1 series term are drawn. The stresses are obtained from the pre-buckling analysis step described in Part 1- Pre-buckling.



x,y,z are local axes aligned with strip

Figure 1: Stress distribution of a strip with both ends simply supported (k=1)

The longitudinal stress for buckling analysis which is obtained from Equation (16) varies in both the longitudinal and transverse directions and is given by:

$$\sigma_x(x)=\sum_{k=1}^{\mu}\sigma_{1k}(x)\sigma_{L1k}+\sum_{k=1}^{\mu}\sigma_{2k}(x)\left[\sigma_{L2k}+\sigma_{L3k}\frac{y}{b}\right] \tag{17}$$

where:

k is the series term of the stress functions

$\sigma_x(x)$ is the longitudinal stress

$\sigma_{L1k},\sigma_{L2k},\sigma_{L3k}$ are the amplitude components of the longitudinal stress for series term k

$$\sigma_{L1k}=\alpha_{2Mk}\frac{E_{12}}{b};\sigma_{L2k}=\alpha_{3Mk}E_2 \text{ and } \sigma_{L3k}=\alpha_{4Mk}E_2$$

$\sigma_{1k}(x),\sigma_{2k}(x)$ are the functions for the variation of the longitudinal stress

$$\sigma_{1k}(x)=X_{1k}(x) \text{ and } \sigma_{2k}(x)=X_{2k}^{'}(x)$$

The transverse stress for buckling analysis which is obtained from Equation (16) is the average transverse stress in a strip and is given by:

$$\sigma_y(x)=\sum_{k=1}^{\mu}\sigma_{1k}(x)\sigma_{T1k}+\sum_{k=1}^{\mu}\sigma_{2k}(x)\sigma_{T2k} \tag{18}$$

where:

$\sigma_y(x)$ is the transverse stress

$\sigma_{T1k},\sigma_{T2k}$ are the amplitude components of the transverse stress for series term k

$$\sigma_{T1k} = \alpha_{2Mk}\frac{E_1}{b} \text{ and } \sigma_{T2k} = \left[\alpha_{3Mk}E_{12} + \frac{1}{2}\alpha_{4Mk}E_{12}\right]$$

$\sigma_{1k}(x), \sigma_{2k}(x)$ are the functions for the variation of the transverse stress

$$\sigma_{1k}(x) = X_{1k}(x) \text{ and } \sigma_{2k}(x) = X'_{2k}(x)$$

The shear stress for buckling analysis which is obtained from Equation (16) is the average stress in a strip and is given by:

$$\tau_{xy}(x) = \sum_{k=1}^{\mu}\tau_{1k}(x)\tau_{1k} + \sum_{k=1}^{\mu}\tau_{2k}(x)\tau_{2k} \tag{19}$$

where:

$\tau_{xy}(x)$ is the shear stress

τ_{1k}, τ_{2k} are the amplitude components of the shear stress for series term k

$$\tau_{1k} = \left[\alpha_{1Mk}G + \frac{1}{2}\alpha_{2Mk}G\right] \text{ and } \tau_{2k} = \alpha_{4Mk}\frac{G}{b}$$

$\tau_{1k}(x), \tau_{2k}(x)$ are the functions for the variation of the shear stress

$$\tau_{1k}(x) = X'_{1k}(x) \text{ and } \tau_{2k}(x) = X_{2k}(x)$$

For different boundary conditions, different functions are required for flexural and membrane displacements, as described in Part 1 - Pre-buckling

4. Strain energy and potential energy

In order to compute the stiffness matrix of a strip according to conventional finite strip theory (Cheung, 1976), it is necessary to define the strain energy in a strip under deformation and the potential energy of the membrane stresses.

4.1. Strain energy of a strip

The flexural strain energy U_F and the membrane strain energy U_M are given in Part 1 - Pre-buckling.

4.2. Potential energy of the membrane stresses

The flexural potential energy of the membrane stresses is given by:

$$V_F = -\frac{1}{2}\int_0^L\int_0^b \begin{bmatrix} \sigma_x(x)\left(\frac{\partial w}{\partial x}\right)^2 + \sigma_y(x)\left(\frac{\partial w}{\partial y}\right)^2 + \\ +\tau_{xy}(x)\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right) + \tau_{xy}(x)\left(\frac{\partial w}{\partial y}\right)\left(\frac{\partial w}{\partial x}\right) \end{bmatrix} tdydx \tag{20}$$

Substitution of equations (17), (18), (19) into equation (20) and using Equations (4), (5) results in:

$$V_F = V_{FL} + V_{FT} + V_{FS1} + V_{FS2} \tag{21}$$

where:

$$V_{FL} = -\frac{1}{2}\int_0^L\int_0^b \sum_{m=1}^{\mu}\sum_{n=1}^{\mu} \{\alpha_{Fm}\}^T [\Gamma_{FL}]^T X'_{1m}(x)\left(\sum_{k=1}^{\mu}\sigma_{1k}(x)\sigma_{L1k} + \sum_{k=1}^{\mu}\sigma_{2k}(x)\left[\sigma_{L2k} + \sigma_{L3k}\frac{y}{b}\right]\right) X'_{1n}(x)[\Gamma_{FL}]\{\alpha_{Fn}\} tdydx \tag{22}$$

$$V_{FT} = -\frac{1}{2}\int_0^L\int_0^b \sum_{m=1}^{\mu}\sum_{n=1}^{\mu} \frac{1}{b^2}\{\alpha_{Fm}\}^T [\Gamma_{FT}]^T X_{1m}(x)\left[\sum_{k=1}^{\mu}\sigma_{1k}(x)\sigma_{T1k} + \sum_{k=1}^{\mu}\sigma_{2k}(x)\sigma_{T2k}\right] X_{1n}(x)[\Gamma_{FT}]\{\alpha_{Fn}\} tdydx \tag{23}$$

$$V_{FS1} = -\frac{1}{2}\int_0^L\int_0^b \sum_{m=1}^{\mu}\sum_{n}^{\mu} \{\alpha_{Fm}\}^T [\Gamma_{FL}]^T X'_{1m}(x)\left[\sum_{k=1}^{\mu}\tau_{1k}(x)\tau_{1k} + \sum_{k=1}^{\mu}\tau_{2k}(x)\tau_{2k}\right] \frac{1}{b} X_{1n}(x)[\Gamma_{FT}]\{\alpha_{Fn}\} tdydx \tag{24}$$

$$V_{FS2} = -\frac{1}{2}\int_0^L\int_0^b \sum_{m=1}^{\mu}\sum_{n}^{\mu} \{\alpha_{Fm}\}^T [\Gamma_{FT}]^T \frac{1}{b} X_{1m}(x)\left[\sum_{k=1}^{\mu}\tau_{1k}(x)\tau_{1k} + \sum_{k=1}^{\mu}\tau_{2k}(x)\tau_{2k}\right] X'_{1n}(x)[\Gamma_{FL}]\{\alpha_{Fn}\} tdydx \tag{25}$$

Note that in Equations (21) to (25), summation is taken over the k=1 to μ series term for stress as well as the m, n=1 to μ modal terms.

The membrane potential energy of the membrane stresses is given by:

$$V_M = -\frac{1}{2}\int_0^L\int_0^b \left(\sigma_x(x)\left(\frac{\partial v}{\partial x}\right)^2 + \sigma_x(x)\left(\frac{\partial u}{\partial x}\right)^2\right) tdydx \tag{26}$$

As stated in Plank and Wittrick (1974), it is believed that there are no membrane instabilities associated with transverse stress and shear stress so that there are no term in above equation associated with these.

Substitution of equation (17) into equation (26) and using equations (12), (13) results in:

$$V_M = V_{Mv} + V_{Mu} \tag{27}$$

where:

$$V_{Mv} = -\frac{1}{2}\int_0^L\int_0^b \sum_{m=1}^{\mu}\sum_{n=1}^{\mu} \{\alpha_{Mm}\}^T [\Gamma_{Mv}]^T X'_{1m}(x)\left(\sum_{k=1}^{\mu}\sigma_{1k}(x)\sigma_{L1k} + \sum_{k=1}^{\mu}\sigma_{2k}(x)\left[\sigma_{L2k} + \sigma_{L3k}\frac{y}{b}\right]\right) X'_{1n}(x)[\Gamma_{Mv}]\{\alpha_{Mn}\} tdydx \tag{28}$$

$$V_{Mu} = -\frac{1}{2}\int_0^L\int_0^b \sum_{m=1}^{\mu}\sum_{n=1}^{\mu} \{\alpha_{Mm}\}^T [\Gamma_{Mu}]^T X'_{2m}(x)\left(\sum_{k=1}^{\mu}\sigma_{1k}(x)\sigma_{L1k} + \sum_{k=1}^{\mu}\sigma_{2k}(x)\left[\sigma_{L2k} + \sigma_{L3k}\frac{y}{b}\right]\right) X'_{2n}(x)[\Gamma_{Mu}]\{\alpha_{Mn}\} t dy dx \qquad (29)$$

5. Stability matrix

5.1. Flexural and membrane stiffness matrices

The flexural and membrane stiffness matrices are given in Part 1 – Pre-buckling.

5.2. Flexural stability matrix

The total flexural potential energy of the membrane stresses can be written as:

$$V_F = -\frac{1}{2}\{\delta_{Fm}\}^T [g_{Fmn}]\{\delta_{Fn}\} \qquad (30)$$

where $[g_{Fmn}]$ is the flexural stability matrix corresponding to the mth and nth series terms and $\{\delta_{Fn}\}$ is the flexural displacement vector of a strip corresponding to the nth series term. The matrix $[g_{Fmn}]$ is given in the Research Report 959 (Nguyen, Hancock, & Pham, 2016). The coefficients C_{Lw1mnk} , C_{Lw2mnk} , C_{T1mnk} , C_{T2mnk} , C_{S11mnk} , C_{S12mnk} , C_{S21mnk} , C_{S22mnk} in the report have been evaluated exactly for the displacement functions satisfying different boundary conditions as described in Part 1 - Pre-buckling.

5.3. The membrane stability matrix

The total membrane potential energy of the membrane stresses can be written as:

$$V_M = -\frac{1}{2}\{\delta_{Mm}\}^T [g_{Mmn}]\{\delta_{Mn}\} \qquad (31)$$

where $[g_{Mmn}]$ is the membrane stability matrix corresponding to the mth and nth series terms and $\{\delta_{Mn}\}$ is the membrane displacement vector of a strip corresponding to the nth series term. The matrix $[g_{Mmn}]$ is given in the Research Report 959 (Nguyen et al., 2016). The coefficients C_{Lv1mnk} , C_{Lv2mnk} , C_{Lu1mnk} , C_{Lu2mnk} in the report have been evaluated exactly for the displacement functions satisfying different boundary conditions as described in Part 1 - Pre-buckling.

5.4. The stability matrix of whole section

The stability matrix of a strip is assembled from both the flexural stability matrix and the membrane stability matrix in local coordinates. These matrices are transformed to global coordinates by a multiplication with transformation matrices. The stability matrix of the whole section for each series term is assembled from the stability matrices of individual strip. Finally, the complete stability matrix of the whole section is assembled from the stability matrices taken over the series terms, thus the size of this matrix is 4 times the node number and times the number of series terms.

6. Buckling analysis

The total potential energy is the sum of the elastic strain energy stored in a strip and the potential energy of the membrane stresses, thus:

$$\phi = U + V \tag{32}$$

The principle of minimum total potential energy requires that:

$$\left\{ \frac{\partial \phi}{\partial \{\delta_b\}} \right\} = \{0\} \tag{33}$$

Thus, we have:

$$\left([K] - \lambda[G]\right)\{\delta_b\} = \{0\} \tag{34}$$

where

[K] and [*G*] are the system stiffness and stability matrix respectively

λ is the load factor against buckling

$\{\delta_b\}$ are the vector of nodal line displacements which are the buckling mode

r is the size of the stiffness matrix [*K*] and the stability matrix [*G*],

$$r = 4 \times \mu \times n$$

μ is the number of series terms

n is the number of nodes in the section

Equation (34) is called a Linear Eigenvalue Problem. The r values of λ for which the determinant of ([K]- λ[G]) is zero are called the Eigenvalues. The r eigenvalues are the load factors for buckling in the r different modes. Obviously the section will buckle at the lowest calculated value of λ. The eigenvalue λ is obtained from this equation by using Eigenvalue routines in Matlab. The values of $\{\delta_b\}$ corresponding to the values of λ are called the Eigenvectors. They are the buckling modes of the section which are obtained from Eq (34). Each eigenvector $\{\delta_b\}$ corresponds to a practical eigenvalue λ in the above equation. The eigenvectors are computed by solving Eq (34) in Matlab. In the calculation, the buckling mode is the eigenvector corresponding to the minimum eigenvalue.

7. Numerical example

A buckling analysis has been performed for a lipped channel section with rounded corners and lips under localised loading using the THIN-WALL-2 program. The geometry of the beam and the loading are shown in Fig.2. The beam is analysed with different boundary conditions for the web and the flanges of the end sections. In addition, lateral restraints are applied along the beam at Nodal Lines 11 and 35 to avoid twisting caused by eccentric loading. The results from the buckling analysis of the beam under localised loading include buckling modes and load factor. The buckling modes are obtained from Nodal Line 23 for all sections.

A buckling analysis of the beam has been performed using the ABAQUS software with an equivalent loading and boundary condition. It was meshed into 5mm x 5mm, except at the section's corners. The corners were modelled with 1mm x 5mm mesh to accurately represent the influence of corner radius. The buckling mode values are obtained from Nodal Line 23 for all sections.

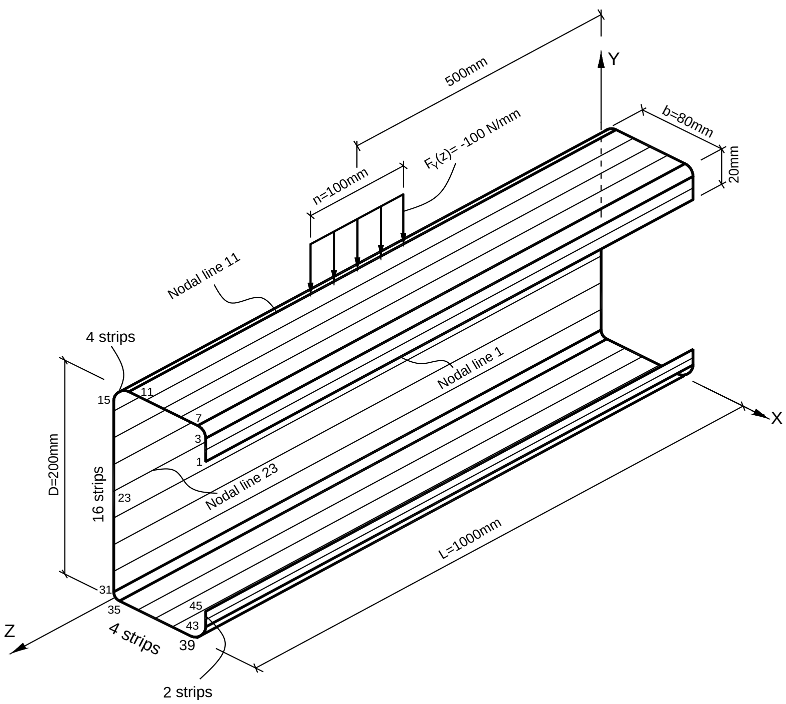


Figure 2: Lipped channel section under localised loading

A buckling analysis of the section has been performed for different boundary conditions by both the SAFSM and the FEM. The detail comparison of the buckling load factor λ for the different boundary conditions is shown in Table 1. It is clear that the SAFSM provides accurate estimates of buckling load factor in comparison with the FEM.

Table 1: Buckling load factor (λ) comparison

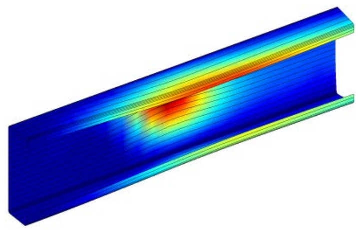
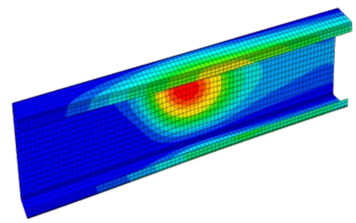
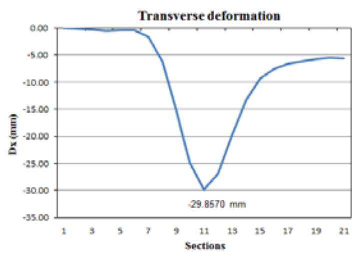
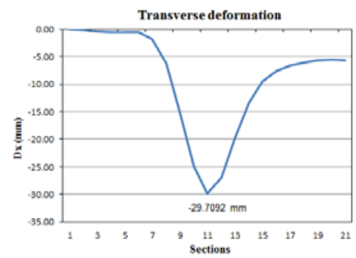
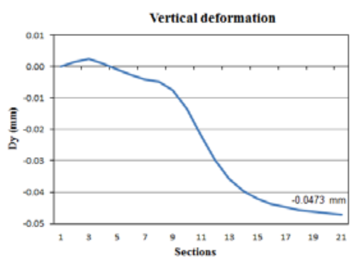
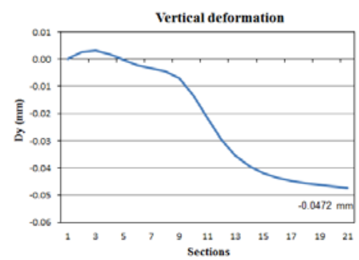
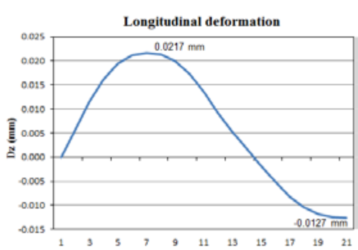
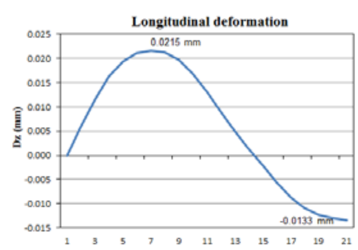
Boundary conditions	**SAFSM (THIN-WALL-2) (15 series terms)**	**FEM (Abaqus)**	**Different (%)**
SS	2.88402	2.87770	0.2196%
SC	3.23008	3.19310	1.1582%
SF	3.30027	3.30360	0.1008%
CC	3.45942	3.43930	0.5850%
CF	3.18921	3.18080	0.2643%
FF	2.88612	2.88610	0.0007%

The comparison between the results from the SAFSM and the FEM are shown in Table 2 for the Clamped - Free (CF) case which uses the Bradford and Azhari (1995) displacement functions with 15 series terms. The results for other boundary conditions can be seen in the Research Report 959 (Nguyen et al., 2016).

8. Convergence study

A study has been performed for the lipped channel section in 7 with different boundary conditions and different number of series terms to find the required number of series terms for a converged buckling analysis. The relationships between the load factor (λ) and the number of series terms are shown in Fig.3 for different boundary conditions. There is convergence of the buckling load factor (λ) from 0.0007% to 1.158% when the number of series terms reaches 15 in comparison with ABAQUS as shown in Table 1. It means that a smaller number of series terms is required for buckling analysis in comparison with the number of series terms for pre-buckling analysis as described in Part 1 - Pre-buckling.

Table 2: Buckling modes comparison for CF case (Nodal Line 23)

	SAFSM (THIN-WALL-2) (15 series terms)	**FEM (Abaqus)**
Mode		
D_x (mm)		
D_y (mm)		
D_z (mm)		

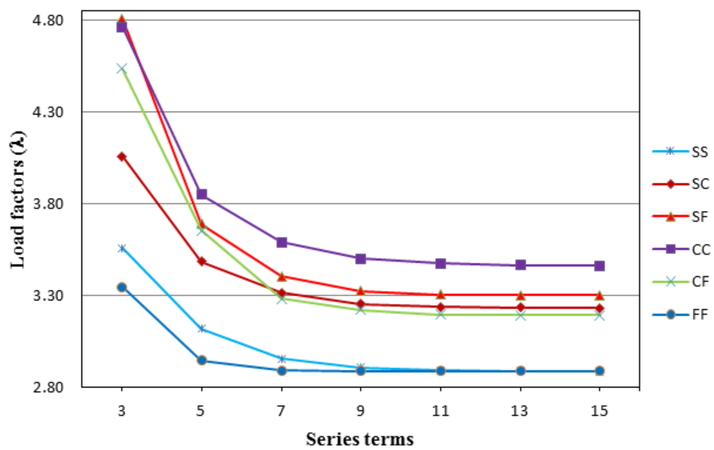


Figure 3: Convergence of load factor (λ)

9. Conclusion

The Semi-Analytical Finite Strip Method of buckling analysis of thin-walled section under localised loading has been developed for general end boundary conditions. This method has proven to be accurate and efficient in comparison with the Finite Element Method.

Different displacement functions are required for flexural and membrane displacements for different support and loading conditions. The buckling analysis requires a smaller number of series terms than the pre-buckling analysis to obtain the converged buckling load factor and buckling modes in comparison with the FEM.

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