Post-buckling Shear Strength of a Cold-formed Steel Joist

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by

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1. INTRODUCTION

Plates with a large width to thickness ratio when subjected to direct compression or shear undergo elastic buckling at a critical value of the stress. Analytical studies have shown that thin plates do not collapse when the buckling stress is reached, but can carry additional load by means of redistribution of stress. This reserve capacity, known as post-buckling strength, has been verified in the case of plates under direct compression by means of experimental investigations. Empirical relationships have been derived for the computation of the ultimate strength and they form the basis for relevant code provisions. Presumably because of a lack of similar experimental evidence on shear capacity of plates without stiffeners, codes do not take into account the post-buckling strength in shear and the allowable shear stress in webs is therefore limited by the elastic buckling capacity. Because buckling in shear results from the principal compressive stress, post-buckling behaviour somewhat similar to that of plates under direct compression should be expected in plates in shear too. Analytical studies confirm that post-buckling strength does exist in plates subject to pure shear.

The paper deals with the post-buckling shear strength of cold-formed steel joists. The steel joists are part of a new composite floor system for which the authors were called in as consultants to determine the post-buckling shear capacity of the joists from both an experimental and analytical point of view. The experimental work comprised a total of six joists with different ratios of h/t (web height to web thickness). The paper presents details of the test specimens, the experimental setup and test results. The test evidence shows that significant post-buckling shear strength does in fact exist for the cold-formed joists. The analytical work derives post-buckling shear capacities on the basis of parallel results for plates under direct compression. Experimental results show a significant reserve of strength beyond elastic buckling and hence the paper concludes that post-buckling shear strengths in cold-formed steel joists can be taken into account in design.

2. EXPERIMENTAL PROGRAM

2.1 General

A composite floor system in use today in North America and elsewhere in the world consists of an open web steel joist with a special z-shaped top chord as shown in Fig. 1. The shape of the cold-formed top chord produces composite action between the concrete slab and the joist. While the open web steel joist has proven itself as economical in certain span and loading ranges, a cold-formed steel joist was developed for other span and loading combinations to extend the economic range of the composite floor system.

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Note that the cold-formed steel joist shown in Fig. 2 retains the special z-shaped top chord for composite action with the concrete slab and that it has the advantage of being fabricated from a single strip of steel sheeting. Prepunched holes in the web are utilized to support temporary formwork for casting of the slab and to facilitate passing through of electrical conduit and wiring. The presence of the thin web introduces the possibility of web buckling and in particular the potential for post-buckling shear strength which is the topic of this paper.

2.2 Test Specimens

Details of the six test specimens are given in Table 1 and Fig. 3. The three sections of 10, 12 and 14 in. nominal height were chosen for testing because they represented typical sizes to be used in practice. For each section two specimens were used in the investigation to obtain an indication of strength variability. The z-shaped top chord was reinforced with two longitudinal bars as depicted in Section A-A of Fig. 3 to prevent premature buckling of the folds. A 3/8 in. thick plate welded to the top chord replaced the normal concrete slab that would act compositely with the cold formed steel section in a typical floor construction. As shown in detail B of Fig. 3, care was taken in introducing the test loads into the webs of test specimens by means of stiffeners. This particular requirement arose because a two-point loading setup was utilized which produced severely concentrated loading rather than the more uniformly distributed loading normally encountered in practice.

2.3 Instrumentation

Instrumentation was provided at midspan to obtain deflections as well as strains at three different levels over the height of the joist section. Since the objective of the instrumentation was to investigate flexual behavior especially into the plastic region, the test deflections and strains are of no interest to the post-buckling capacity dealt with in this paper and hence will not be further discussed. No special instrumentation was provided to detect the post-buckling strength of the steel joists but rather judgement was used in interpreting buckled shapes and the overall loading behavior of the test joists. The web shear region of some joists was lightly polished and then painted to help in the study of buckled shapes.

2.4 Test Setup and Procedure

For the two-point loading setup shown in Fig. 3, joists were positioned in a standard beam bending machine. A summary of the span details for the six test joists is presented in Table 1. To ensure equal loading in each half of a joist, loads were introduced through rollers. Since in the composite cross section the steel top chord is supported laterally by the concrete slab, a similar restraint was provided for the test specimens by means of wooden brackets as indicated in Fig. 4. The wooden brackets, located at each of the load points,were carefully aligned prior to application of any load so that a small gap of about 1/16 in. remained between the top chord and each bracket. This arrangement ensured virtually free vertical movement of the joist under test loads while providing the required lateral restraint.

Loading consisted of increments of about one-tenth of the estimated joist capacity. At each load stage, instrumentation readings were taken and joist behavior was observed. The total testing time per joist amounted to about two hours.

3. TEST RESULTS AND DISCUSSION

A summary of the test results is presented in Table 2. Note that the yield strength, F_y, was determined from standard tensile coupon tests where coupons were machined from web and bottom chord material that was not affected by cold-forming. As will be explained more fully in Section 4 of the paper, the elastic buckling shear stress, F_v, in Table 2 is based on $142{,}650/(h/t)^2$ for the buckling of a slender web. Comparing the resultant elastic shear buckling capacity, V_{code}, with the test values, V_{test}, it is readily seen that all six test specimens achieved significant increases in strength beyond the elastic capacity. Theoretical considerations concerning this increase will be dealt with in Section 4. Note that V_{test} refers to the maximum shear that each specimen was able to carry.

The fact that post-buckling behavior was exhibited by the cold-formed steel joists is shown in Fig. 5. As depicted in Fig. 5a the web plate will be subjected to principal stresses equal to τ, where τ is the applied stress. At some critical value of these principal stresses the plate with a large h/t ratio will buckle in the direction of the compressive stress giving rise to the typical contours of a buckled shape shown in Fig. 5b. Fig. 5c presents a view of the buckled web of one of the test specimens which is in agreement with the theoretical shape of Fig. 5b. Although all six joists exhibited web buckling, test observations and measurements of deflected web shapes were not conclusive to what extent the full post-buckling capacity of each joist was reached. While each test was terminated when a maximum test load had been attained, this maximum load was influenced by two additional factors besides the post-buckling web capacity of a joist: Firstly, local web crippling below the stiffeners at points of load application was exhibited to some degree by all joists but particularly by the most slender web specimens 3 and 4. Secondly, the low lateral stiffness of the joists contributed to lateral rotations of the joists as can be seen in Fig. 5c by a view along the bottom chord. In spite of the presence of these secondary effects, a significant increase in buckling capacity beyond the elastic strength was achieved as indicated by the ratios of V_{test}/V_{code} in Table 2.

4. ANALYTICAL CONSIDERATIONS

When a beam web with a relatively small clear depth to thickness (h/t) ratio is subjected to shear stress, it is likely to fail by shear yielding beginning at the neutral axis at a stress equal to about $F_y/\sqrt{3}$ (0.577 F_y), where F_y is the yield stress. Webs with large h/t ratios fail by shear buckling. As shown in Fig. 5a, the web plate is subjected to principal stresses equal to τ the applied shear stress. At some critical value of τ the plate buckles in the direction of the compressive stress forming the typical contours shown in Fig. 5b. Analytical studies on the shear buckling of infinitely long plates (4,5) have shown that the elastic critical shear buckling stress is given by

$$\tau_{cr} = \frac{k\pi^2}{12(1-\mu^2)\ (h/t)^2} \qquad (1)$$

where k = a buckling coefficient
E = modulus of elasticity of steel
μ = Poisson's ratio

In the above equation, the value of k varies with the support conditions and the ratio of the width to the depth of the plate called the aspect ratio a/h. For a long plate and simple supports k is found to be 5.35.

When the above value of k, E = 29,500 ksi and μ = 0.3 are substituted in Eq. 1, the critical buckling stress works out to

$$\tau_{cr} = \frac{142,650}{(h/t)^2} \tag{2}$$

For webs having moderate h/t ratios, the computed theoretical value of τ_{cr} may exceed the proportional limit in shear. In such a situation, the critical shear buckling stress would be expected to be lower than that given by Eq. 2 because of a reduction in the modulus of elasticity. Studies made by Basler (2) have indicated that the following equation can be used to find the reduced shear buckling stress.

$$\tau_{cr} = \sqrt{\tau_{pr}\ \tau_{cre}} \tag{3}$$

where τ_{pr} = proportional limit in shear = 0.8 τ_y

τ_{cre} = elastic critical buckling stress, Eq. 1

For E = 29,500 ksi, Eq. 3 gives

$$\tau_{cr} = \frac{260\sqrt{F_y}}{(h/t)} \tag{4}$$

Based on the considerations cited above, the theoretical value of the critical shear stress for the entire range of h/t ratios is shown by curve 'a' in Fig. 6. It will be noted that an h/t ratio of $550/\sqrt{F_y}$ defines the boundary between the regions of inelastic and elastic shear buckling. The webs of all the specimens tested had h/t ratios larger than this value and were therefore expected to undergo elastic shear buckling.

It is known that thin plates subjected to compression do not collapse when the buckling stress is reached. Additional load can be carried by the element after buckling by means of redistribution of stress. The post-buckling strength of a thin plate can be estimated by using the concept of effective width proposed by von Karman in 1932 (6). As suggested by him, the ultimate strength of the plate can be computed by assuming that the plate is stressed to a stress F_y on an effective width given by

$$b_e = \sqrt{\frac{\sigma cr}{F_y}} \tag{5}$$

where τ_{cr} is the elastic buckling stress and b is the overall width of the plate. The utlimate stress σ_u is then given by

$$\sigma_u = \sigma_{cr}\ F_y \tag{6}$$

Based on extensive experimental investigations on light-gage cold-formed steel sections, Winter (7) indicated that Eq. 6 is equally applicable to the element in which the stress is below the yield point, but that in each case it needs to be modified to take into account factors such as the initial imperfections, residual stresses and eccentricity of loads. The following expression used by the AISI specifications (1,8) for the effective width of plate elements subjected to direct compression is based on Winter's work.

$$\frac{b_e}{b} = \sqrt{\frac{\sigma_{cr}}{f_{max}}} \left(1-0.218 \sqrt{\frac{\sigma_{cr}}{f_{max}}}\right) \tag{7}$$

where f_{max} is the maximum stress in the plate.

Although the codes have consciously utilized the post-buckling strength of thin plate elements in direct compression and of webs under flexural compression, the post-buckling strength in shear has not been used. Previous theoretical studies (3) have shown that thin plates indeed have considerable post-buckling shear strength, but there is not enough experimental work available on the shear strength of long webs without stiffeners. It would however be reasonable to assume that an estimate of such strength could be obtained from Eq. 7 derived for plates under direct compression. Replacing σ_{cr} by τ_{cr} (Eq. 2) and f_{max} by τ_y in Eq. 7, the ultimate shear stress τ_y is given by

$$\tau_u = \frac{h_e}{h} \times \tau_y$$

and

$$\frac{h_e}{h} = \frac{378.0}{\sqrt{\tau_y}} \frac{t}{h} \left(1- \frac{82.4}{\sqrt{\tau_y}} \frac{t}{h}\right) \tag{8}$$

where τ_y, the yield stress in shear, is equal to 0.577 F_y.

For a steel with yield stress of F_y = 54 ksi, τ_y will be 31 ksi. When this value is substituted in Eq. 8, the following expression is obtained for the ultimate strength

$$\tau_u = \frac{2110}{(h/t)} \left[1- \frac{14.76}{(h/t)}\right] \tag{9}$$

Equation 9, which has been plotted in Fig. 6 as curve 'b', shows that the post-buckling strength increases with the web slenderness.

5. COMPARISON BETWEEN THEORY AND TEST EVIDENCE

It will be noted from Table 2 that the yield stress of the specimens tested ranges from a value of 53 to 55 ksi. The curves in Fig. 6 have therefore been plotted for an average value of F_y equal to 54 ksi. For this value of F_y, the boundary between the regions of inelastic and elastic buckling is defined by an h/t ratio of 75.5. The specimens tested in this study all have h/t ratios considerably larger than the above value and should therefore be expected to possess significant post-buckling strength.

Before a comparison is made between the theoretical estimates of strength and the test results, two factors that may affect the buckling strength need to be considered. The theoretical values of strength are based on a value of $k = 5.35$ which is applicable to very long plates whose aspect ratios can be considered to be infinite. For finite plates with simply supported edges, the buckling coefficient k is given by

$$k = 5.35 + \frac{4.0}{(a/h)^2}$$

The aspect ratios of the test specimens range from 4.7 to 5.0. For this value of the aspect ratio k is approximately equal to 5.5. The actual value may be somewhat less because the edges at the ends of the span are free rather than simply supported. Even if a value of 5.5 is used for k, the theoretical values of the critical buckling stress will increase by less than 3% and those of the ultimate strength by less than 1.5%.

The presence of holes in the web of the specimens tested may be expected to reduce both the critical buckling loads and the ultimate failure loads. An exact theoretical consideration is not possible. The reduction in area due to the holes is in the range of 4 to 6% of the gross area of web; the resulting effect on the shear strength is therefore likely to be small.

The two factors mentioned above have a small effect on the shear strength of the webs. Also, the two effects tend to compensate for each other. For these reasons they have been ignored in the following discussion. The theoretical values of the shear strengths are compared in Fig. 6 with the results obtained from the tests. It is evident that the web plates possess significant reserve of strength beyond the critical value for elastic buckling. The experimental values however do not reach the theoretical estimates based on test evidence for plates under compression. As discussed earlier, this is at least partly due to premature failure caused by web crippling and lateral buckling of the beams.

6. CONCLUSIONS

Based on the experimental and analytical work presented in the paper, the following conclusions are reached.

1. Theoretical considerations show that post-buckling shear strength of cold-formed steel joists is to be expected in much the same way as in thin plates subjected to compression.

2. The experimental work involving six joists with three different slender web sections indicates that significant strength beyond elastic buckling can be achieved. In this particular experiments the increase in strength varied between 33 and 68 percent.

3. Based on the theoretical and experimental results, post-buckling shear strength of slender web sections can be taken into account in design. The exact magnitude of the post-buckling capacity cannot be determined reliably from the few tests reported in this paper but would have to be based on a more extensive experimental investigation.

ACKNOWLEDGEMENTS

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APPENDIX I - REFERENCES

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APPENDIX II - NOTATION

a = unstiffened width of plate

A_v = area of web

a/h = aspect ratio

b = width of plate

b_e = effective width of plate

E = Young's modulus

f_{max} = maximum stress in plate

F_v = elastic buckling shear stress

F_y = yield stress

h = height of web

h_e = effective height of web

h/t = web slenderness ratio

k = buckling coefficient

TABLE 1 SUMMARY OF TEST SPECIMEN DETAILS

Section	Specimen no.	Nominal height (in.)	Web height, h (in.)	Web thickness, t (in.)	$\frac{h}{t}$	Span, L (in.)	Shear span, c (in.)
1	1	10	8.87	0.073	121.5	109	42.5
	2	10	8.87	0.073	121.5	109	42.5
2	3	12	10.87	0.071	153.1	140	55.0
	4	12	10.87	0.071	153.1	140	55.0
3	5	14	12.87	0.094	136.9	146	61.0
	6	14	12.87	0.094	136.9	146	61.0

Note: 1 in. = 25.4 mm

TABLE 2 SUMMARY OF TEST RESULTS

Section	Specimen no.	h (in.)	t (in.)	h/t	A_v (in^2)	F_y (ksi)	V_{test} (k)	τ_u (ksi)	F_v* (ksi)	V_{code} (k)	V_{test}/V_{code}
1	1	8.87	0.073	121.5	0.648	55	8.75	13.5	9.66	6.26	1.40
	2	8.87	0.073	121.5	0.648	55	8.35	12.9	9.66	6.26	1.33
2	3	10.87	0.071	153.1	0.772	53	7.90	10.2	6.09	4.70	1.68
	4	10.87	0.071	153.1	0.772	53	7.30	9.5	6.09	4.70	1.55
3	5	12.87	0.094	136.9	1.210	55	12.20	10.1	7.61	9.21	1.33
	6	12.87	0.094	136.9	1.210	55	12.40	10.3	7.61	9.21	1.35

* $F_v = \tau_{cr} = \frac{142{,}650}{(h/t)^2}$, $V_{code} = F_v^* A_v$

Note: 1 in. = 25.4 mm; 1 kip = 4.45 kN; 1 ksi = 6.89 MPa.

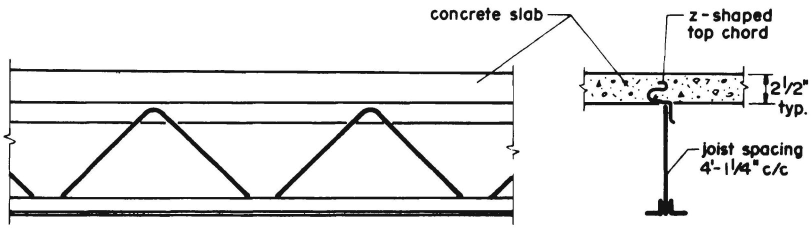


Fig. 1 Composite floor with open web steel joist (1 in. = 25.4mm)

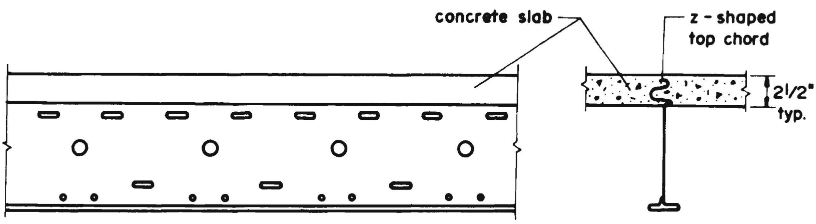


Fig. 2 New cold-formed steel joist for composite floor (1 in. = 25.4mm)

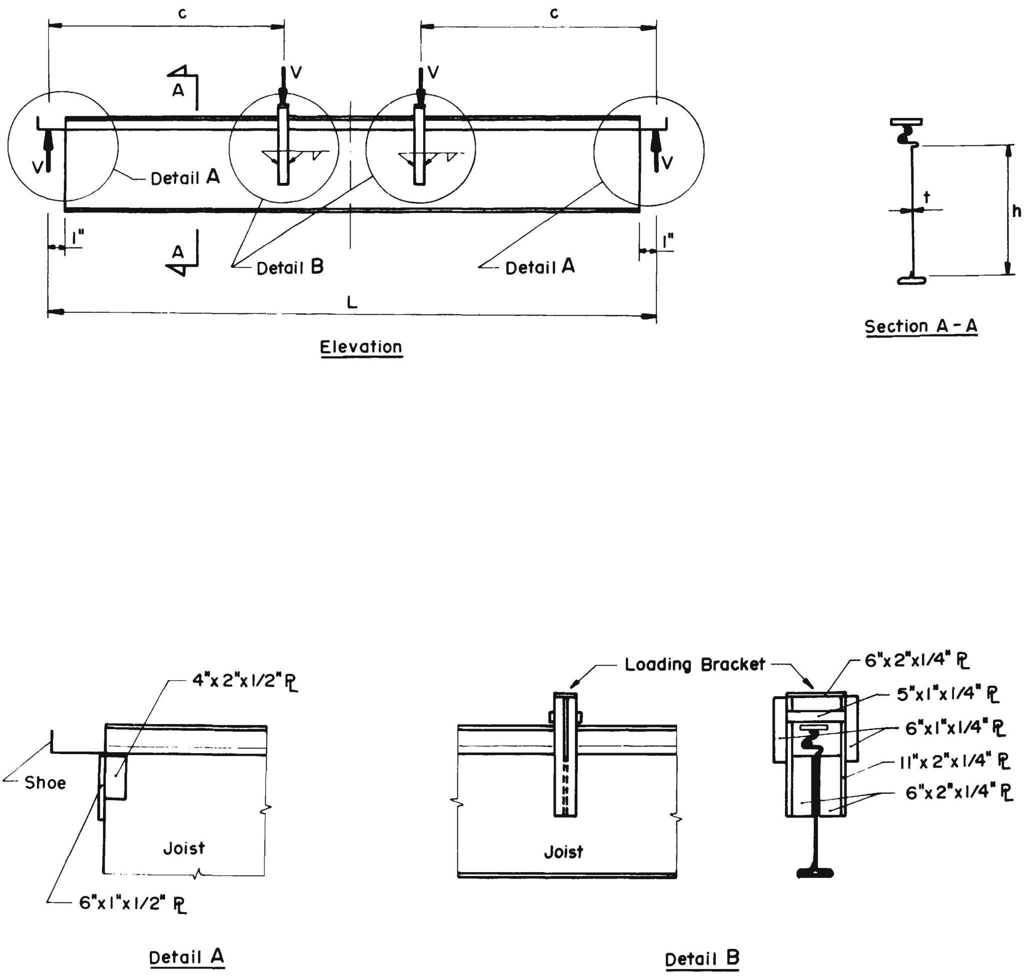


Fig. 3 Details of cold-formed steel joist test specimens (1 in. = 25.4mm)

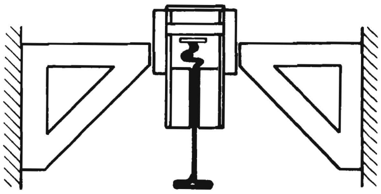
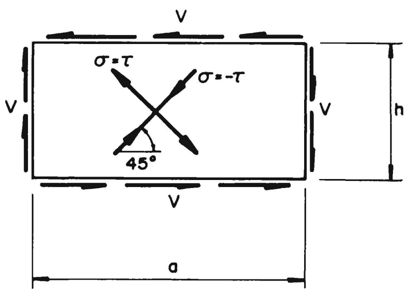
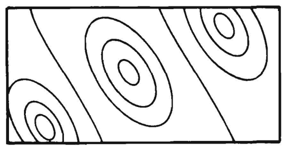


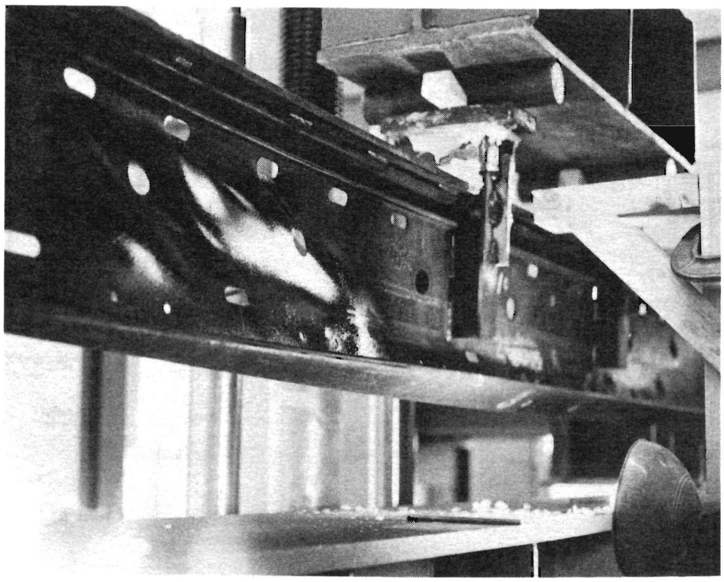
Fig. 4 Lateral support system



(a) Principal stresses



(b) Contours of buckled shape



(c) View of buckled web of a test joist

Fig. 5 Shear buckling of test plates

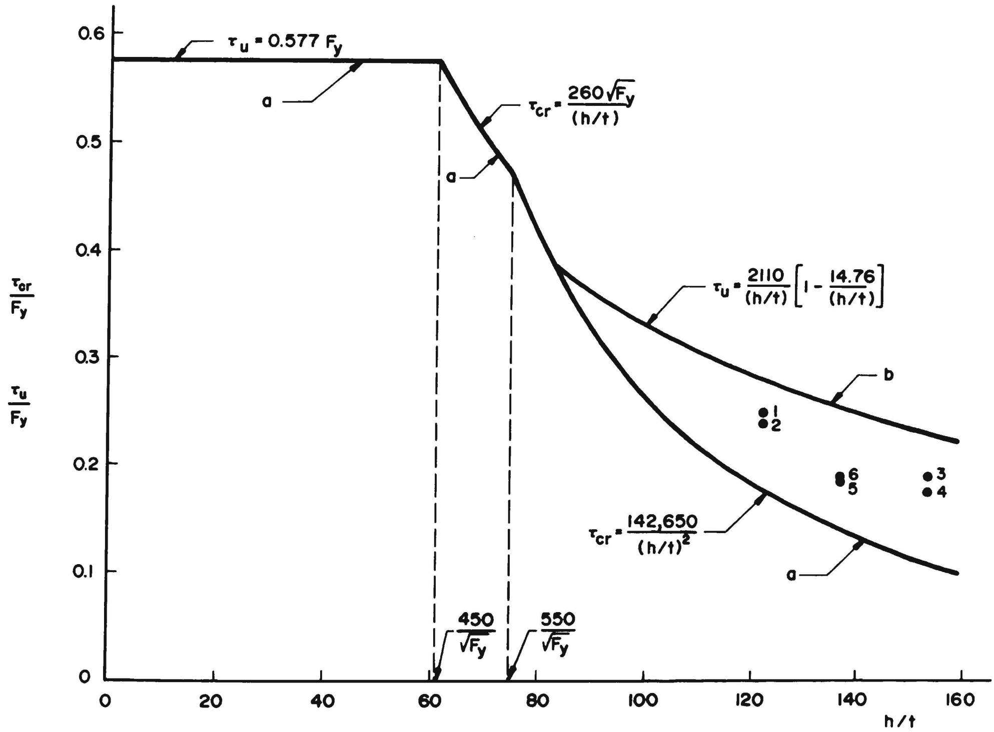


Fig. 6 The ultimate shear strength of web plates (F_y = 54 ksi)