Intermediate Stiffeners for Thin-walled Members

George Winter

T. P. Desmond

Teoman Peköz

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Desmond, T. P.,[1] Peköz, T.[2] and Winter, G.[3]

INTRODUCTION

This paper is the second of a two-paper sequence describing the results of an investigation of the structural behavior of longitudinally stiffened compression elements of thin-walled members. The first paper (Ref. 5) dealt with edge stiffened elements, while the present paper deals with intermediately stiffened elements. In both cases, the stiffener is longitudinal, i.e., parallel to the direction of the applied in-plane compressive stresses. A detailed description of the complete investigation may be found in the research report listed as Ref. 4.

A typical component element of a thin-walled member is the stiffened element shown in Fig. 1a. A stiffened plate element by definition is adequately supported on each longitudinal edge. Local plate buckling is a usual design consideration for thin-walled members. One way to increase the local buckling load and to enhance structural efficiency is by providing a secondary or intermediate stiffener between the main longitudinal stiffeners or webs. As shown in Fig. 1b, the intermediate stiffener breaks up the wave-like pattern of the stiffened element of Fig. 1a such that the two plate strips, one on either side of the intermediate stiffener, behave as two stiffened elements.

The research described in this paper explored the structural behavior of intermediately stiffened elements analytically and experimentally. Based primarily on the experimental results, stiffener requirements were developed.

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1) Stanford Research Institute, Menlo Park, CA, formerly Research Assistant Cornell University, Ithaca, N.Y.

2) Associate Professor of Structural Engineering, Cornell University, Ithaca, N.Y.

3) Professor Emeritus, Cornell University, Ithaca, N.Y.

Methods were also formulated for predicting ultimate strengths of elements that are supported by such stiffeners.

BUCKLING BEHAVIOR

Two distinct buckling modes characterize the buckling behavior of intermediately stiffened elements. One is the *stiffener buckling mode,* where the instability of the assembly is initiated by buckling of the stiffener in a direction perpendicular to the plane of the flange it is intended to support. Stiffener buckling will necessarily induce local buckling of the component plate elements. The second mode is the *local plate buckling mode.* Both buckling modes are shown schematically in Fig. 2.

An independent critical buckling analysis characterized the behavior of each buckling mode. For the stiffener buckling mode the buckling equation is given by Barbré (Ref. 2), and for the local plate buckling mode the equation is given by Bryan (Ref. 3). The detailed derivations of these equations are adequately documented in the cited references and are not given here.

Solutions to these buckling equations can be given as a relationship between the buckling coefficient k_b and the aspect ratio ϕ_b. The subscript b denotes that the buckling coefficient and aspect ratio are expressed as functions of b, the total width of the assembly. Alternatively, the buckling coefficient could also be expressed as a function of the flat width w, where w is the width between the intermediate stiffener and the supported longitudinal edge, so that

$$k_b = (b/w)^2\, k_w \tag{1}$$

For an intermediate stiffener of relatively small width and located in the center of the plate, w is approximately one half of the total width b, and

$$k_b = 4 \cdot k_w \tag{2}$$

For the local plate buckling mode, the Bryan equation gives a minimum k_w value of 4. To be consistent with the notation for the stiffener buckling mode, from Equation 2 the minimum local plate buckling stress coefficient will be taken as k_b equals 16.

The relationship between k_b and ϕ_b is shown in Fig. 3 for both stiffener and local plate buckling modes. These curves are very similar to those for the edge stiffened element shown in Fig. 3 of Ref. 5.

The plate buckling stress coefficients obtained from the buckling equations are plotted in Fig. 4 against a non-dimensional stiffener parameter A_s/A_s^*. A_s is the actual cross-sectional area of the stiffener, and A_s^* is the stiffener area which would lead to simultaneous stiffener and local plate buckling.

EXPERIMENTAL INVESTIGATION

The influence of the longitudinal stiffener rigidity on the performance of the intermediately stiffened plate assembly was studied by a series of tests. In this study, the depth of the longitudinal stiffener was systematically varied for each of several flat plate dimensions.

The investigation involved four beam test series with w/t ratios of 47, 70, 97, and 156, where w is the flat width between the intermediate longitudinal stiffener and the supported edge of the compression flange. The specimens were hat sections with the stiffener cold-formed in the center of the compression flange, as shown in Fig. 5.

The stiffeners of each test series were designed such that the centroidal moment of inertia of the stiffener for some of the specimens was above and for others below the minimum value currently required by the American Iron and Steel Institute (AISI) Specification (Ref. 1). The specific stiffener centroidal moments of inertia are given in Table 1 along with the specimen dimensions. Table 2 gives the material properties, experimental ultimate moments, and buckling stress coefficients. The test set-up is shown schematically in Fig. 6.

Strain gages were placed in the central region of the beams to establish the approximate stress distribution across the section and to determine when local and/or stiffener buckling occurred.

The experimental buckling stress coefficients were obtained by the strain reversal method (Ref. 8) and are compared to those obtained from the buckling analysis in Fig. 7. Buckling coefficients for test series I-47 are not shown in this figure since the compression flanges of these specimens either did not buckle or buckled inelastically. All but one experimental buckling coefficient, shown in the figure, exceeded the theoretical value for the local buckling mode. A plausible reason for this is that the compression flanges were restrained rotationally by the webs. This was not accounted for in the buckling analysis. Nevertheless, agreement between experiment and theory is satisfactory.

By comparing the longitudinal strain at the stiffener-plate juncture with the strain at the web-plate juncture, the relative support provided by different stiffener rigidities could be assessed. The variation of strain at these locations is shown in Fig. 8 for a representative test series. It is apparent that the strain variations at the longitudinally supported edge and at the longitudinal stiffener are virtually the same for specimens with sufficiently large stiffeners. The strain variations of test I-97-313 (97 is the w/t ratio, and 313 is the nondimensional stiffener moment of inertia I_s/t^4) are representative of this type of behavior. In contrast, the strain at the longitudinal stiffener of a specimen with a relatively small stiffener shows a strain reversal tendency. This type of behavior, illustrated by test I-97-115, indicates a stiffener that buckles outwardly in a direction perpendicular to the plane of the plate.

ULTIMATE STRENGTH CONSIDERATIONS

In this section, procedures are formulated for predicting ultimate strengths of elements that have stiffeners of adequate and inadequate rigidity. In a subsequent section, formulas will be developed for establishing required stiffener rigidities. In the following development, it will be apparent that procedures for predicting ultimate strengths and for assessing adequate stiffener rigidities are interdependent.

An intermediate stiffener is defined as adequate if the intermediately stiffened plate can be treated as an assembly of two stiffened plate elements of width w, and a fully effective stiffener. When partially stiffened by an inadequate intermediate stiffener, the assembly can be treated as a partially stiffened plate of width b and a reduced or partially effective stiffener. Here b is the width between the supported edges of the intermediately stiffened plate and w is the width of each component plate. In the following development, it will be assumed that b is twice the width w. This is a valid assumption provided the width of the stiffener is small relative to b.

The ultimate strengths of the component plates will be predicted by an effective width approach. This is consistent with the design procedure developed in Ref. 5 for edge stiffened elements. The effective width equation in the current AISI specification (Ref. 1) will be employed. The following is a generalization of that equation for a plate of width w

$$w_{eff} = 0.95t\sqrt{\frac{k_w E}{\sigma_y}}\left(1 - 0.209(t/w)\sqrt{\frac{k_w E}{\sigma_y}}\right) \qquad (3)$$

which is valid for $w/t > 0.64\sqrt{\frac{k_w E}{\sigma_y}}$. (3a)

The effective width of the total plate of width b can also be expressed as

$$b_{eff} = 0.95t\sqrt{\frac{k_b E}{\sigma_y}}\left(1 - 0.209(t/b)\sqrt{\frac{k_b E}{\sigma_y}}\right). \quad (4)$$

Depending upon the degree of support, the intermediately stiffened element is categorized as adequately stiffened, partially stiffened, or stiffened. The latter is the limiting case, where there is no intermediate stiffener and the plate is supported by two webs on its longitudinal edges. These categories are defined below, and the assumptions for buckling stress coefficients of each are given.

Adequately Stiffened Element. An intermediately stiffened element is adequately stiffened if the intermediate stiffener is of sufficient flexural rigidity so that the ultimate strength of each component plate element (of width w) equals that of an identical plate stiffened by webs on both longitudinal edges.

The effective width of each component plate is predicted by Equation 3 with a buckling stress coefficient k_w equal to four. The intermediate stiffener is assumed to remain straight or unbuckled up to the ultimate failure load. Thus, it is assumed to remain fully effective for purposes of predicting its contribution to the ultimate strength of the assembly.

Alternatively, the ultimate strength of the total plate of width b can be predicted by Equation 4 where

$$b = 2 \cdot w \quad (5)$$

and

$$k_b = 4 \cdot k_w \quad (6)$$

Substituting Equations 5 and 6 into Equation 4

$$b_{eff} = 2 \cdot w_{eff} \quad (7)$$

which is equivalent to summing the predicted effective widths of each component element.

Effective widths of partially stiffened elements are discussed below and will be based on the effective width equation expressed in terms of the total assembly width b.

Partially Stiffened Element. An intermediately stiffened compression element is partially stiffened if the stiffener is inadequate, namely if the flexural rigidity of the stiffener is insufficient to permit the ultimate strength of each component plate to equal that of an identical plate stiffened by webs on both longitudinal edges.

Intuitively, the ultimate strength of a partially stiffened element is bounded by two extreme cases: (a) a stiffened plate of width b with an adequate intermediate stiffener and (b) a plate of similar dimensions but without an intermediate stiffener. Likewise, the buckling stress coefficient of a partially stiffened element $(k_b)_{p.s.}$ is bounded by the buckling stress coefficient of an adequately stiffened element $(k_b)_{a.s.}$ and by the coefficient of an element without an intermediate stiffener $(k_b)_{n.s.}$. The latter two buckling coefficients were shown to be 16 and 4 when expressed as functions of the total plate width.

Theoretical values for $(k_b)_{p.s.}$ require a detailed critical buckling analysis. However, for design purposes, $(k_b)_{p.s.}$ may be determined from the following expression, which is a close fit (Fig. 9) of the theoretical buckling stress coefficients obtained from a critical buckling analysis.

$$(k_b)_{p.s.} = \left[\frac{I_s}{I_s^*} \right]^{1/2} \left[(k_b)_{a.s.} - (k_b)_{n.s.} \right] + (k_b)_{n.s.} \qquad (8)$$

where

$(k_b)_{p.s.}$ = predicted buckling stress coefficient for partially stiffened elements;

I_s = moment of inertia of the stiffener about its centroidal axis parallel to the plate element;

I_s^* = stiffener centroidal moment of inertia at which critical buckling of the assembly is initiated by simultaneous stiffener and local plate buckling;

and $(k_b)_{a.s.}$ and $(k_b)_{n.s.}$ are equal to 16 and 4. Equation 8 was obtained by fitting a quadratic expression through the theoretical values for $(k_b)_{p.s.}$ at I_s/I_s^* equal to zero and one. I_s^* is the adequate stiffener rigidity based on a criterion of critical buckling; however, when the flat width ratio of the assembly exceeds some limiting value (to be determined in a following section), adequacy should be based on an ultimate strength criterion. Since the adequate stiffener rigidity for the latter criterion, $(I_s)_{adequate}$, can be considerably larger than I_s^* these criteria are not identical. Consequently, it would not be appropriate to replace I_s^* with $(I_s)_{adequate}$.

For this reason, Equation 8 is restated in terms of $(I_s)_{adequate}$ and an exponent 1/n, where n will be chosen such that the following equation is a close approximation of the theoretical buckling coefficients.

$$(k_b)_{p.s.} = \left[\frac{I_s}{(I_s)_{adequate}}\right]^{1/n} \left[(k_b)_{a.s.} - (k_b)_{n.s.}\right] + (k_b)_{n.s.} \tag{9}$$

The effective width of a partially stiffened element of width b is then determined by Equation 4 with $(k_b)_{p.s.}$ given by Equation 9. The contribution of the partial intermediate stiffener to the ultimate strength of the assembly is discussed below.

The procedure for predicting the contribution of a partial or buckled stiffener to the ultimate strength of the assembly, is the same as outlined for edge stiffened elements in Ref. 5. The cross-sectional area of the partial stiffener is reduced by a simple linear expression to reflect its reduced effectiveness in resisting load.

$$(A_s)_{p.s.} = A_s \cdot I_s/(I_s)_{adequate} \tag{10}$$

where A_s is the unreduced cross-sectional area of the stiffener, I_s is its moment of intertia, and $(I_s)_{adequate}$ is the minimum required stiffener rigidity necessary to support the plate adequately.

Experimental justification for Equation 10 is given in Fig. 10, where the experimental data is shown to scatter about the linear relationship given by the above equation. The experimental partial stiffener areas for this figure were obtained by subtracting the contribution of the predicted effective areas of the webs and flange from the total effective section modulus of the beam specimen at failure.

In the limit, the partially stiffened element becomes adequately stiffened. That is,

$$(k_b)_{p.s.} = 16 \tag{11}$$

In this case, the effective widths for adequately and partially stiffened elements are equivalent, when predicted by Equation 4.

Stiffened Element. A stiffened element can be considered as the limiting case of an intermediately stiffened element. Therefore, the equation for predicting effective widths of elements partially stiffened by an intermediate stiffener must also be consistent with the effective width approach currently employed to predict effective widths of stiffened elements.

For this limiting case, I_s equals zero and Equation 9 reduces to

$$(k_b)_{p.s.} = (k_b)_{n.s.} \tag{12}$$

where $(k_b)_{n.s.} = 4$.

With this value for k_b, Equation 4 becomes the effective width equation used in the AISI specification for stiffened elements of width b.

INTERMEDIATE STIFFENER REQUIREMENT

The definition of an adequate stiffener could be based on either a critical buckling criterion (CBC) or an ultimate strength criterion (USC). With the CBC, the adequate stiffener is defined as that stiffener rigidity at which instability is initiated by simultaneous stiffener buckling and local plate buckling. With the USC, the adequate stiffener rigidity is defined as the minimum rigidity at which the ultimate strength of each component plate on either side of the stiffener equals that of an identical plate stiffened by webs on both edges.

Critical Buckling Criterion. To obtain a stiffener requirement based on a critical buckling criterion, the stiffener buckling equation is solved for those stiffener dimensions for which the minimum buckling stress coefficient k_b equals 16. This is the stiffener dimension for which stiffener buckling and local plate buckling occur simultaneously for all aspect ratios ϕ_b. The requirement is shown in Fig. 11 as $(I_s/t^4)_{req.}$, the required stiffener rigidity nondimensionalized with respect to the plate thickness t. Three requirements are shown in this figure. For the larger requirement, it is assumed that the neutral axis of bending of the stiffener coincides with its centroidal axis, and for the smaller it is assumed that the neutral axis of the stiffener coincides with the middle surface of the plate. Neither of these, however, is strictly correct. The stiffener attempts to bend about its centroidal axis, and the plate attempts to bend about its middle surface, as though each would buckle independently of the other. Because of compatibility of strains, shear stresses develop at the stiffener-plate juncture and the actual stiffener neutral axis is located somewhere between these two extremes. Based on a plane stress analysis, Seide (Ref. 7) has derived an expression for the stiffener moment of inertia that reflects this change in neutral axis. The third stiffener requirement, shown in Fig. 11, is based on a stiffener moment of inertia due to the adjusted neutral axis. The stiffener requirement based on an adjusted neutral axis is not significantly different from that which assumes that the neutral axis is located at the plate's middle surface. Some design specifications

(Ref. 1) implicitly make this assumption and for this case it appears to be valid. Nevertheless, the CBC stiffener requirement for intermediate stiffeners (and for edge stiffeners in Ref. 5) will be based on the adjusted neutral axis.

To assess the applicability of the CBC Requirement, the observed ultimate strengths of several series of longitudinally stiffened elements are compared in Fig. 12. Each series consists of specimens with identical plate dimensions but different stiffener rigidities. An increase in stiffener rigidity beyond that required by the CBC increases the ultimate strength of the assembly to varying degrees. For test series I-47, I-70, I-97, and I-156, they are about 9, 15, 12, and 20 percent. Admittedly, an increase in the stiffener cross-sectional area contributes to some increase in ultimate strength; however, this increase amounts to a few percent at most. The significant increase in ultimate strength is attributed to the fact that stiffeners with larger rigidities retard the post-buckling deflections of the plate at the stiffener-plate juncture. This was demonstrated experimentally in Fig. 8 and was demonstrated analytically in Ref. 5 for edge stiffened elements.

Also, it is apparent from Fig. 12 that for test series I-47 only small increases in ultimate strength are observed when the stiffener rigidity is increased beyond the CBC Requirement. For such relatively small w/t ratios, this implies that a stiffener requirement based on the CBC is not significantly different from one based on the USC; therefore, only for w/t ratios in the advanced post-buckling range is the CBC stiffener requirement excessively unconservative. The reason for this is discussed below.

In the range of w/t ratios for which the component plate of width w remains fully effective (hereafter called the fully effective range), the stiffener rigidity based on the CBC Requirement provides the minimum required support so that stiffener buckling occurs simultaneously with material yielding.

Consequently, the stiffener remains unbuckled up to the ultimate failure load. The CBC and USC stiffener requirements are identical in this range.

On the other hand, for w/t ratios larger than the limiting w/t ratio (the post-buckling range) the stiffener rigidity based on the CBC Requirement provides support such that simultaneous stiffener and local plate buckling occur before material yielding. For this range, the stiffener will deflect normal to the plane of the plate prior to the stiffener-plate assembly reaching its ultimate load. Consequently, the stiffener requirements based on the CBC and USC will differ substantially.

In the following section, stiffener requirements based primarily on experimental results will be developed for the USC. It will be shown that for w/t ratios in the post-buckling range a stiffener requirement based on the CBC Requirement is insufficient for purposes of predicting ultimate strength.

Ultimate Strength Criterion. In Ref. 5, the adequate edge stiffener requirement was expressed as a function of the w/t ratio normalized with respect to $(w/t)_\alpha$, where $(w/t)_\alpha$ is the ratio below which the plate of width w is fully effective as an adequately stiffened element. For intermediate stiffeners, from Equation 3a,

$$(w/t)_\alpha = 0.64 \sqrt{\frac{k_w E}{\sigma_y}} \tag{13}$$

which reduces to

$$(w/t)_\alpha = 221 \Big/ \sqrt{\sigma_y} \tag{14}$$

for steel members and with k_w equal to 4. By normalizing the stiffener requirement with respect to $(w/t)_\alpha$, the requirement can be expressed as one equation rather than a family of equations, one for each yield stress.

For edge stiffened elements, the transition between the fully effective range and the post-buckling range was taken as

$$(w/t)/(w/t)_\alpha = 1.0 \tag{15}$$

For the range $w/t \leq (w/t)_\alpha$ the critical buckling criterion was used to determine the value of the adequate stiffener rigidity. For the intermediate stiffener requirement, a slight modification of this is warranted. For the test series I-47, $w/t/(w/t)_\alpha = 1.4$, and it is seen in Fig. 12 that the CBC criterion is satisfactory. Therefore, the transition between the range of the CBC and the USC will be extended to $(w/t)/(w/t)_\alpha = 1.5$ instead of 1.0, and the required minimum moment of inertia can be adjusted to agree with the adequate rigidity based on test series I-47. For lack of experimental results in the range of $(w/t)/(w/t)_\alpha$ ratios below 1.4, the stiffener requirement is varied linearly in this range.

$$(I_s/t^4)_{adequate} = 100(w/t)/(w/t)_\alpha - 50 \tag{16}$$

and is valid for

$$(w/t)_\beta < (w/t) \leq 1.5(w/t)_\alpha \tag{17}$$

where $(w/t)_\alpha$ is defined by Equation 13, and $(w/t)_\beta$ is the flat width ratio below which a plate of width b (or 2w) is fully effective without an intermediate stiffener.

$$(b/t)_\beta = 221/\sqrt{\sigma_y} \tag{18}$$

Equation 18 can be expressed in terms of w if one substitutes $b = 2 \cdot w$.

$$(w/t)_\beta = 111/\sqrt{\sigma_y} \tag{19}$$

For the range $w/t > 1.5(w/t)_\alpha$, a procedure similar to that for the edge stiffened elements (Ref. 5) is employed. Eight hypothetical stiffener requirements, A through H (in Fig. 13), will be compared to the test results. These requirements are arbitrarily drawn straight lines of increasing slope having a common origin at $(w/t)/(w/t)_\alpha$ equals 1.5. The requirement for which the predicted and experimental ultimate strengths are in best agreement will be suggested as a possible design requirement.

COMPARISON OF PREDICTED AND EXPERIMENTAL ULTIMATE STRENGTHS

To facilitate a comparison of predicted and experimental ultimate strengths, two nondimensional ratios are used

$$R = \frac{(P_{ult})_1 - (P_{ult})_{n.s.}}{(P_{ult})_{a.s.} - (P_{ult})_{n.s.}} \tag{20}$$

and

$$R_o = \frac{(P_{ult})_t - (P_{ult})_{n.s.}}{(P_{ult})_{a.s.} - (P_{ult})_{n.s.}} \tag{21}$$

where

$(P_{ult})_1$ = predicted ultimate strength determined by the procedures outlined for adequately or partially stiffened elements (whichever is appropriate);

$(P_{ult})_t$ = experimental ultimate strength;

and $(P_{ult})_{a.s.}$ and $(P_{ult})_{n.s.}$ are predicted ultimate strengths for an adequately stiffened element and an element with no intermediate stiffener. All predicted ultimate strengths are determined by the effective width approach previously outlined.

These ratios normalize the predicted and experimental ultimate strengths such that they provide a measure of the degree to which the intermediate stiffener supports the flange. That is, if the ratios equal one, the ultimate strength of the intermediately stiffened flange equals that of an adequately stiffened flange; and if they equal zero, the ultimate strength equals that of a stiffened element without an intermediate stiffener.

In the following, experimental and predicted ultimate strengths are compared using the above defined ratios.

The Range $w/t_\beta < w/t \le 1.5(w/t)_\alpha$. For this range only one test series was conducted. For this series the ratios of experimental to predicted ultimate strengths R_o/R are given in Table 3 and are graphically compared in Fig. 14. In this range, the adequate stiffener requirement is given by Equation 16, and $(k_b)_{p.s.}$ is determined from Equation 9 with n equals 2.

Correlation between experimental and predicted ultimate strengths is satisfactory, which is to be expected since the required stiffener rigidity was adjusted based on this test series. It is noted, however, that the predicted ultimate strength of two adequately stiffened tests are overly conservative. This is attributed to the partial plastification of the webs of these beam specimens. It has been shown (Ref. 6) that compression flanges with w/t ratios of the order of, or significantly smaller than, $(w/t)_\alpha$ have an inelastic reserve capacity due to a partial plastification of the webs. Since this was not accounted for in the analysis, it is in all likelihood the reason for the predicted conservative ultimate strengths for several of these tests.

The Range $w/t > 1.5(w/t)_\alpha$. For this range, $(k_b)_{p.s.}$ is given by Equation 9 as a function of $(I_s)_{adequate}$ and n. In the following comparisons of experimental and predicted ultimate strengths, eight hypothetical stiffener requirements are assumed for $(I_s)_{adequate}$ and several values for n (1 through 5) are considered. The comparisons of R_o and R have been extended to determine both a satisfactory stiffener requirement and a good approximation of $(k_b)_{p.s.}$ for that requirement.

As in the comparison of ultimate strengths for the fully effective range, the ratios R_o/R are compared. To identify the value of n in these comparisons, R is subscripted by the appropriate integer n. Also, to determine the combination of buckling coefficient and stiffener requirement which best predicts the experimental ultimate strengths, it is arbitrarily assumed that the predicted ultimate strength agrees satisfactorily with the experimental if

$$0.85 \leq R_o/R \leq 1.15 \qquad (22)$$

The number of tests that satisfies this inequality is given in Table 4 for each combination of stiffener requirement and n.

From this comparison, the experimental ultimate strengths are best predicted by Requirement F, since the largest number of tests satisfies the above inequality for this requirement. The requirement is expressed as

$$(I_s/t^4) = 257(w/t)/(w/t)_\alpha - 285 \qquad (23)$$

and is valid for

$$w/t > 1.5(w/t)_\alpha \qquad (23a)$$

In Table 5 the R_o/R ratios are given for Stiffener Requirement F. The arithmetic mean and standard deviation of the ratios for adequately stiffened elements are 0.961 and .05. For partially stiffened elements n equals 3 is the best choice for n, since the predicted ultimate strengths are slightly more conservative for n equals 3 compared to n equals 4. The arithmetic mean and standard deviation for R_o/R_3 are 1.02 and .13, which are quite satisfactory.

The experimental and predicted R ratios are graphically compared for all the stiffener requirements in Fig. 15, which confirms that the chosen Requirement F is the most satisfactory of the eight.

LIMITATIONS

The approach developed here is limited to intermediately stiffened elements whose stiffener width is small relative to the width of the assembly (i.e., w approximately equals b/2 in Fig. 5). When w is considerably smaller than b/2 the equations for adequate stiffener rigidity and effective widths may be inappropriate. Two reasons are given for this: (a) The critical buckling equation given in Ref. 2 for partially stiffened elements (and on which the above development is in part based) is not applicable to this case. In that derivation, it is assumed that w equals b/2. (b) The stiffener may itself be prone to local plate buckling, further complicating the behavior of the assembly. These influences are currently under study at Cornell University with the objective of extending the approach developed here.

SUMMARY AND CONCLUSIONS

An approach has been presented for predicting effective widths of intermediately stiffened compression elements that are either adequately or partially stiffened. A stiffener requirement that provides the minimum required stiffener rigidity to support these elements adequately is also presented.

Two criteria were considered for assessing the stiffener adequacy. For flanges which are in the range $(w/t)_{\beta} < w/t \leq 1.5(w/t)_{\alpha}$, a stiffener requirement based on a critical buckling criterion appears satisfactory. For flanges having larger w/t ratios, a requirement based on an ultimate strength criterion is necessary.

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APPENDIX II -- NOTATION

The following symbols are used in this paper:

A_s = cross-sectional area of intermediate stiffener;

A_s^* = characteristic stiffener cross-sectional area at which stiffener buckling and plate buckling occur simultaneously;

$(A_s)_{p.s.}$ = effective cross-sectional area of a partial intermediate stiffener;

E = Young's modulus, 30,000 ksi (210,000 NM/m^2);

I_s = moment of inertia of edge stiffener taken about its centroidal axis parallel to the flange;

I_s^* = characteristic stiffener centroidal moment of inertia at which local plate buckling of the flange and stiffener buckling occur simultaneously;

$(I_s)_{adequate}$ = required minimum stiffener moment of inertia necessary to adequately support the flange;

L = supported length of beam specimens;

LS = distance from concentrated load to support in beam tests;

M_{ult} = experimental ultimate bending moment of beam specimens;

$(P_{ult})_t$ = experimental ultimate strength;

$(P_{ult})_{a.s.}$ = predicted ultimate strength of an intermediately stiffened assembly with an adequate size stiffener;

$(P_{ult})_{n.s.}$ = predicted ultimate strength of a simple stiffened element (i.e., a stiffened element with no intermediate stiffener);

$(P_{ult})_1$ = predicted ultimate strength of adequately, partially, or simple stiffened element (whichever is appropriate);

R = normalized predicted ultimate strength;

R_o = normalized experimental ultimate strength;

b = total width of intermediately stiffened assembly;

b_{eff} = effective width of plate components of intermediately stiffened assembly of width b;

b_{α} = limiting width of a stiffened plate with an adequate size intermediate stiffener below which it is fully effective;

b_{β} = limiting width of a simple stiffened plate (i.e., a stiffened eleme with no intermediate stiffener) below which it is fully effective;

k_b = buckling coefficient for a plate of width b;

$(k_b)_{a.s.}$ = buckling coefficient of intermediately stiffened assembly that is adequately stiffened;

$(k_b)_{p.s.}$ = buckling coefficient of intermediately stiffened assembly that is partially stiffened;

$(k_b)_{n.s.}$ = buckling coefficient of simple stiffened element of width b;

k_w = buckling coefficient for a plate of width w;

n = integer;

t = thickness;

w = width of component plate of an intermediately stiffened assembly;

w_{eff} = effective width of plate of width w;

w_{α} = limiting width of a flange below which it is fully effective as a stiffened element of width w;

ν = Poisson's ratio;

$\sigma_{cr} = \dfrac{k_w \pi^2 E}{12(1-\nu^2)(w/t)^2}$, critical buckling stress;

σ_y = material yield stress, ksi (1 ksi = 6.9 NM/m^2);

ϕ_b = flange aspect ratio.

TABLE 1

DIMENSIONS OF INTERMEDIATELY STIFFENED BEAMS[a]

Test Series (1)	w/t (2)	b (in) (3)	web (in) (4)	bp (in) (5)	bl (in) (6)	L (in) (7)	LS (in) (8)	t (in) (9)	t_{bp} (in) (10)	r_s (in) (11)	d_s (in) (12)	$\frac{I_s}{t^4}$ (13)	$\frac{A_s}{t^2}$ (14)
I-47	48.0	6.10	5.00	9.35	1.50	88.	24.	.0565	.0575	.210	.145	60.0	16.3
	47.0	6.11	5.00	9.33	1.55	88.	24.	.0575	.0580	.213	.223	110.0	19.4
	47.4	6.10	5.00	9.35	1.55	88.	24.	.0570	.0588	.217	.259	142.	20.8
	47.0	6.10	5.00	9.38	1.54	88.	24.	.0578	.0572	.209	.350	217.	23.5
	46.8	6.00	5.00	9.38	1.50	88.	24.	.0577	.0590	.215	.535	499.	30.2
I-70	71.8	8.89	4.08	12.0	1.50	88.	24.	.0564	.0585	.192	.108	40.5	14.5
	70.4	8.83	4.09	12.0	1.50	88.	24.	.0571	.0585	.213	.230	118.	19.8
	70.1	8.80	4.00	12.0	1.50	88.	24.	.0571	.0593	.213	.230	118.	19.8
	69.5	8.95	4.10	12.0	1.30	112.	37.3	.0590	.0593	.217	.367	232.	24.0
	69.5	8.95	4.08	12.0	1.31	112.	37.3	.0587	.0592	.224	.458	367.	27.6
	69.8	8.95	4.10	12.0	1.34	112.	37.3	.0587	.0592	.224	.532	488.	30.1
	69.3	8.93	4.00	12.0	1.35	112.	37.3	.0590	.0598	.230	1.150	2650.	51.2
I-97	96.4	11.7	5.50	15.0	1.56	88.	24.	.0570	.0690	.219	.226	115.	19.6
	97.5	11.8	5.45	15.0	1.58	88.	24.	.0564	.0690	.214	.263	150.	21.2
	79.3	11.7	5.50	15.0	1.30	88.	24.	.0694	.0698	.236	.359	153.	21.0
	79.5	11.7	5.50	15.0	1.50	88.	24.	.0692	.0697	.224	.545	313.	25.9
	80.7	11.7	5.50	15.0	1.55	88.	24.	.0681	.0695	.228	.681	527.	30.5
I-156	155.	18.6	5.50	21.9	1.55	88.	14.	.0580	.0595	.217	.218	107.	19.3
	156.	18.6	5.49	21.9	1.50	88.	14.	.0580	.0573	.218	.268	147.	21.0
	159.	18.7	5.50	21.8	1.54	88.	14.	.0572	.0575	.218	.350	235.	24.1
	157.	18.6	5.50	21.8	1.57	88.	14.	.0573	.0575	.217	.514	477.	29.8
	157.	18.6	5.50	21.9	1.58	88.	14.	.0572	.0571	.218	1.130	2670.	51.3

[a]Refer to Figures 5 and 6

1 in. = 25.4 mm

TABLE 2

MATERIAL PROPERTIES AND TEST RESULTS

		MATERIAL PROPERTIES		TEST RESULTS	
Test Series	I_s/t^4	Yield Stress (ksi)	Ultimate Stress (ksi)	M_{ult} (kip-in)	k_b
(1)	(2)	(3)	(4)	(5)	(6)
I-47	60.	45.9	51.4	92.4	10.2[a]
	110.	43.2	50.6	109.	10.2[a]
	142.	43.6	51.2	98.4	10.3[a]
	217.	43.0	50.2	102.	*
	499.	44.9	56.6	119.	10.3[a]
I-70	40.5	43.9	51.4	64.6	12.3
	118.	42.1	50.7	77.5	16.4
	118.	43.0	50.7	69.6	14.9
	232.	43.6	56.7	82.1	18.4
	367.	43.8	56.8	86.6	16.0
	488.	44.9	52.1	89.6	16.8
	2650.	42.6	56.3	87.7	16.7
I-97	115.	42.9	51.3	103.	12.3
	150.	44.5	51.5	108.	16.8
	153.	44.9	56.7	150.	15.3
	313.	44.0	57.2	159.	16.7
	527.	44.9	56.1	168.	18.4
I-156	107.	42.4	50.9	104.	10.6
	147.	42.5	49.6	104.	12.8
	235.	42.5	50.2	106.	11.3
	477.	42.2	51.2	123.	15.1
	2670.	40.0	50.4	127.	17.2

*No observed stiffener or local plate buckling

[a]Inelastic buckling stress coefficient

1 ksi = 6.9 mN/m^2 ; 1 kip-in. = 111 Nm

TABLE 3

INTERMEDIATELY STIFFENED ELEMENTS--COMPARISON OF PREDICTED AND EXPERIMENTAL ULTIMATE STRENGTHS IN THE RANGE $(w/t)_\beta < w/t \leq 1.5(w/t)_\alpha$

w/t	I_s/t^4	$\frac{I_s}{(I_s)_{adequate}}$	w/w_α	R_o	Partially Stiffened R_o/R	Adequately Stiffened R_o/R
(1)	(2)	(3)	(4)	(5)	(6)	(7)
47.0	60.0	.619	1.40	.853	1.09	
	110.	1.23		1.40		1.4
	142.	1.55		1.06		1.06
	217.	2.43		1.10		1.10
	499.	5.44		1.37		1.37
			Arithmetic Mean		--	1.23
			Standard Deviation		--	.177

TABLE 4

NUMBER OF ULTIMATE STRENGTHS ACCURATELY PREDICTED--
THE RANGE $w/t \geq 1.5(w/t)_\alpha$

Stiffener Requirement \ n	1	2	3	4	5
(1)	(2)	(3)	(4)	(5)	(6)
A	9	9	8	8	8
B	10	10	8	8	8
C	12	12	10	9	8
D	10	15	11	8	8
E	8	14	15	12	10
F	7	10	16*	16*	14
G	7	8	15	15	13
H	6	8	13	15	15

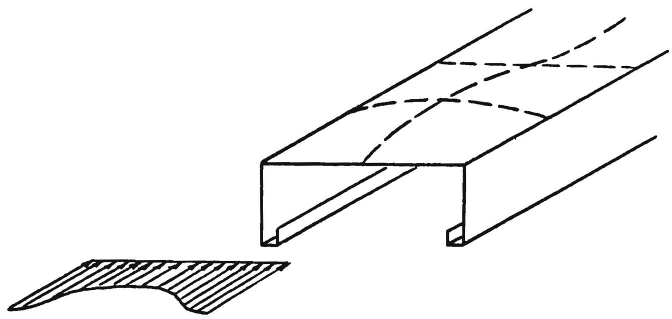
SAMPLE = 17

*The largest number of accurately predicted ultimate strengths.

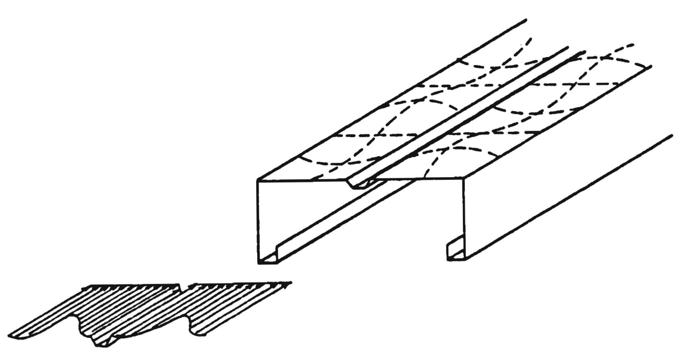
TABLE 5

COMPARISON OF ULTIMATE STRENGTHS IN THE RANGE $w/t \geq 1.5(w/t)_{\alpha}$--INTERMEDIATE STIFFENER REQUIREMENT F

Test Series	w/t	I_s/t^4	$\frac{I_s}{(I_s)_{adequate}}$	w/w_{α}	R_o	Partially Stiffened: $I_s < (I_s)_{adequate}$ R_o/R_1	R_o/R_2	R_o/R_3	R_o/R_4	R_o/R_5	Adequately Stiffened; $I_s \geq (I_s)_{adequate}$ R_o/R
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
I-70	70.	40.5	.151	2.10	.500	2.22	1.18	.952	.855	.806	
		118.	.480		.993	1.75	1.43	1.34	1.30	1.27	
		118.	.480		.682	1.22	.990	.926	.893	.877	
		232.	.937		.887	.935	.917	.917	.909	.909	
		367.	1.47		1.01						1.01
		488.	1.89		1.01						1.01
		2650.	11.0		.925						.925
I-97	97.	115.	.256	2.87	.593	1.77	1.16	1.00	.935	.893	
		150.	.320		.689	1.72	1.22	1.09	1.03	1.00	
	80.	153.	.461	2.37	.704	1.29	1.04	.971	.935	.917	
		313.	.956		.845	.877	.870	.862	.862	.862	
		527.	1.53		.962						.962
I-156	156.	107.	.121	4.57	.500	2.73	1.35	1.06	.935	.870	
		147.	.163		.486	2.10	1.16	.952	.855	.806	
		235.	.256		.563	1.70	1.12	.971	.901	.862	
		477.	.529		.852	1.42	1.21	1.14	1.11	1.09	
		2670.	3.06		.900						.900
			Arithmetic Mean			1.64	1.14	1.02	.960	.930	.961
			Standard Deviation			.539	.164	.129	.130	.133	.0495



(a) Stiffened Element



(b) Intermediately Stiffened Element

Fig. 1 Thin-walled Members

Stiffener Buckling Mode   Local Plate Buckling Mode

Fig. 2 Critical Buckling Modes

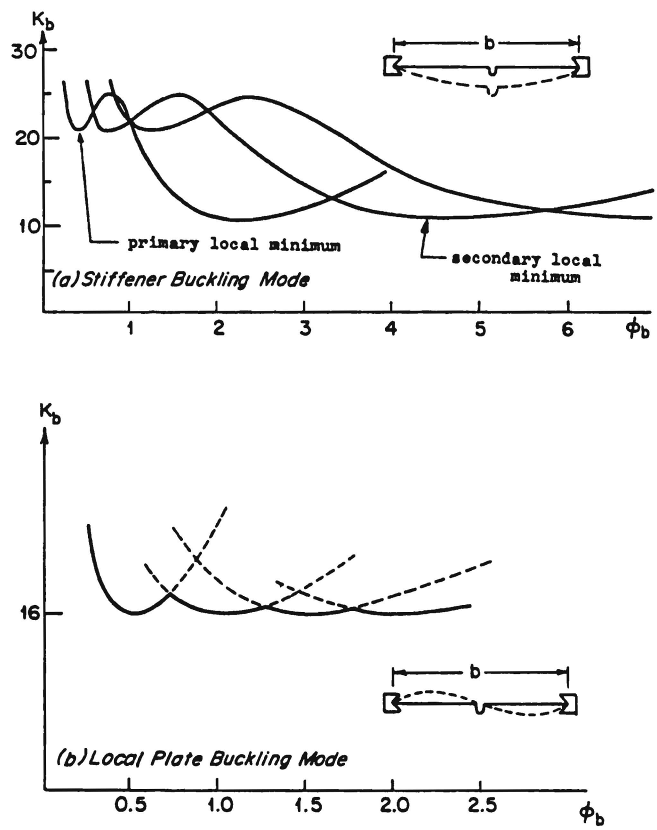


Fig. 3 Critical Buckling Coefficient Curves

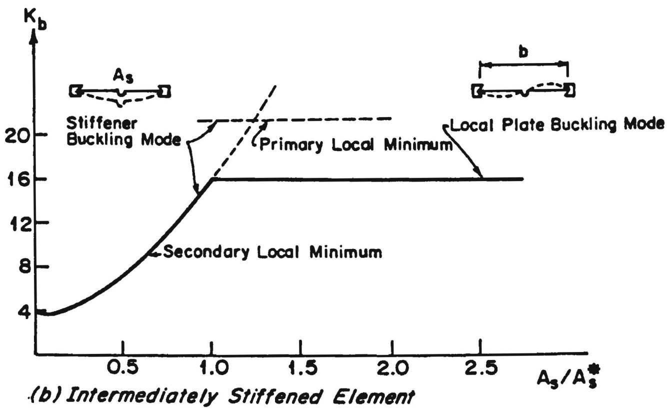


Fig. 4 Minimum Critical Buckling Coefficients

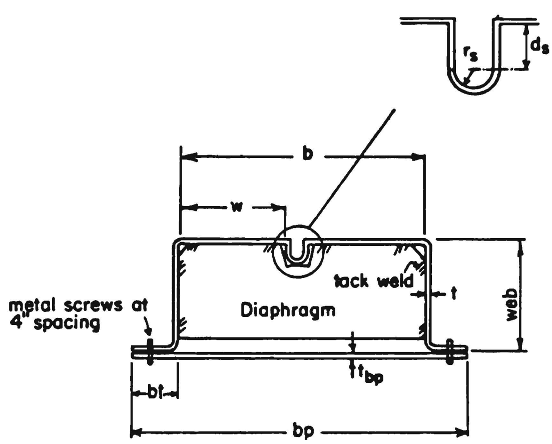


Fig. 5 Cross-sectional Geometry of Specimens

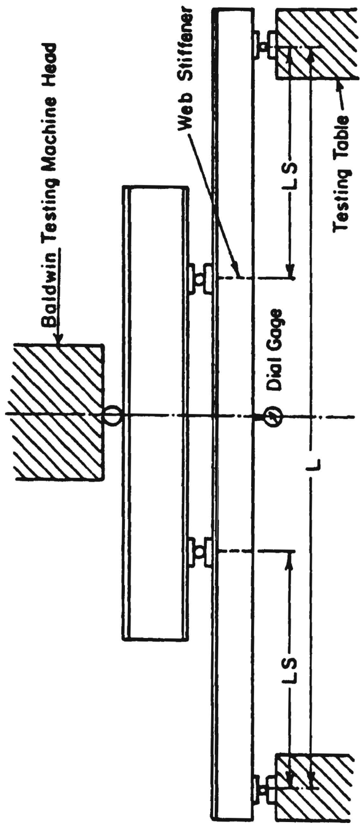


Fig. 6 Beam Test Set-up

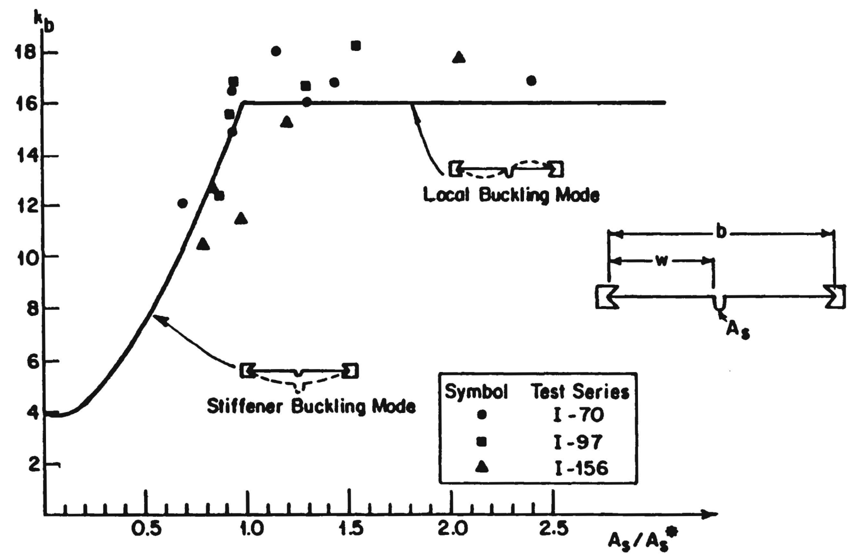


Fig. 7 Comparison of Experimental and Analytical Buckling Coefficients

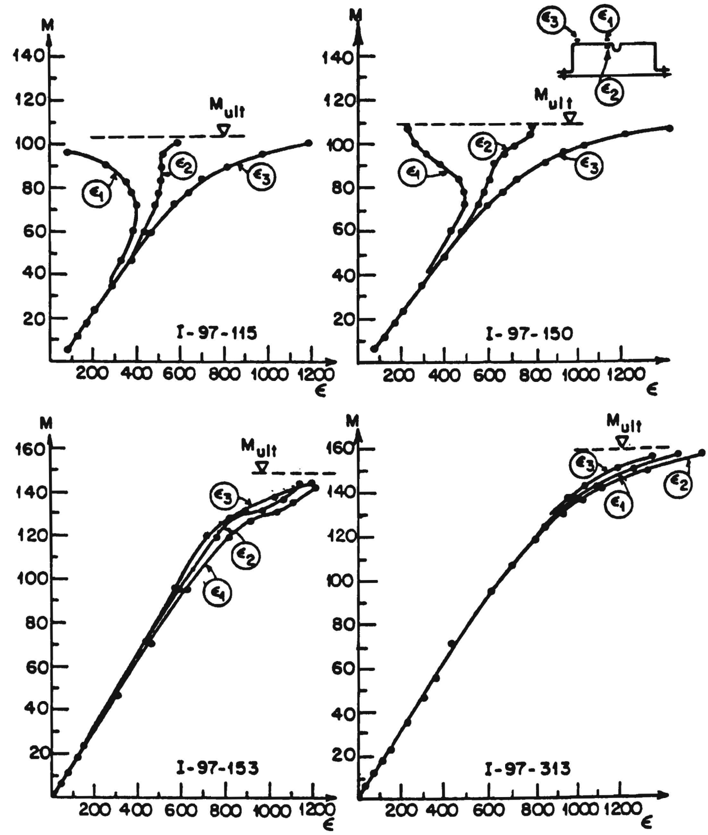


Fig. 8 Post-buckling Behavior of Intermediately Stiffened Assembly

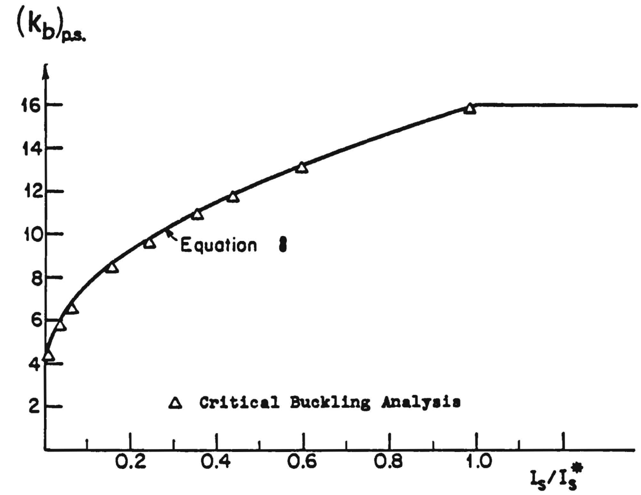


Fig. 9 Variation of $(k_b)_{p.s.}$ versus Stiffener Rigidity

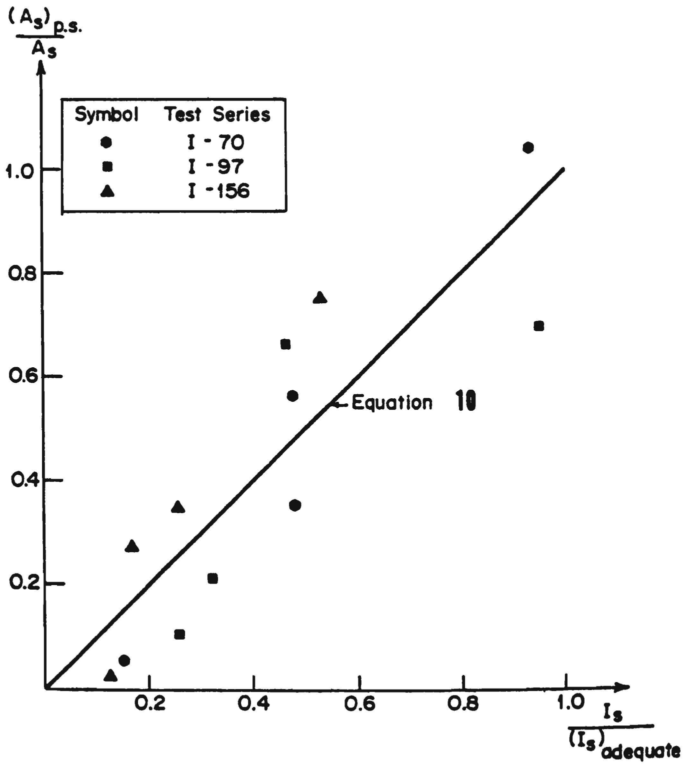


Fig. 10 Effective Stiffener Cross-sectional Areas

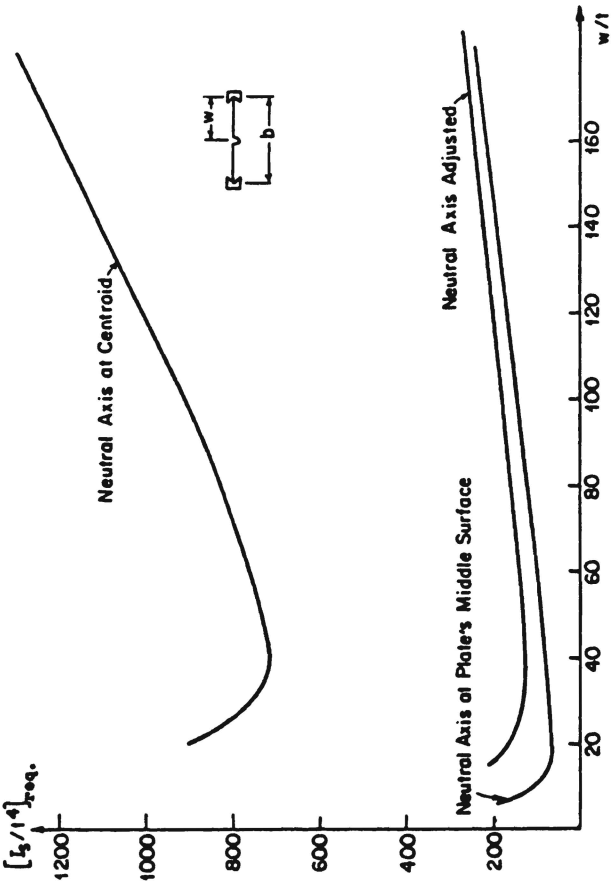


Fig. 11 Stiffener Requirement Based on Critical Buckling Criterion

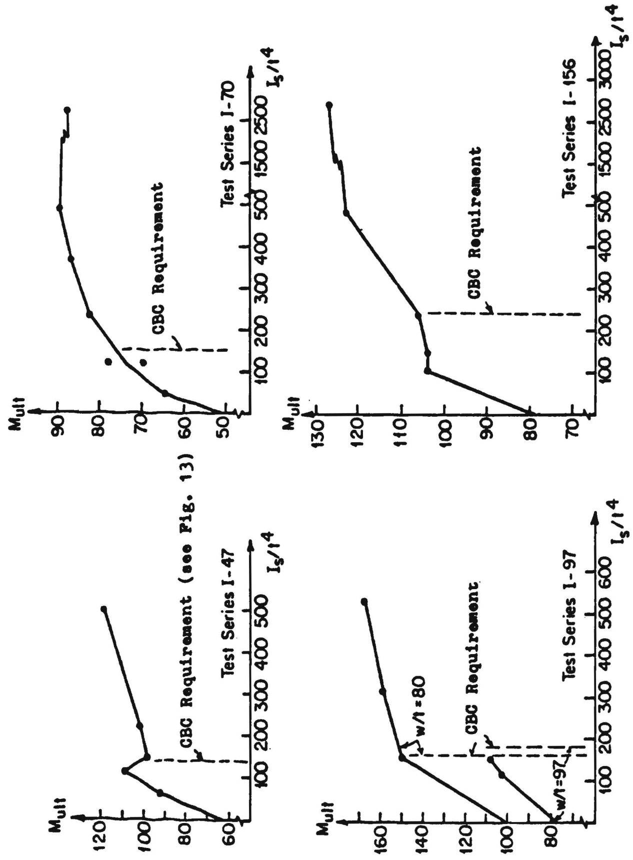


Fig. 12 Variation of Ultimate Strength Versus Stiffener Rigidity

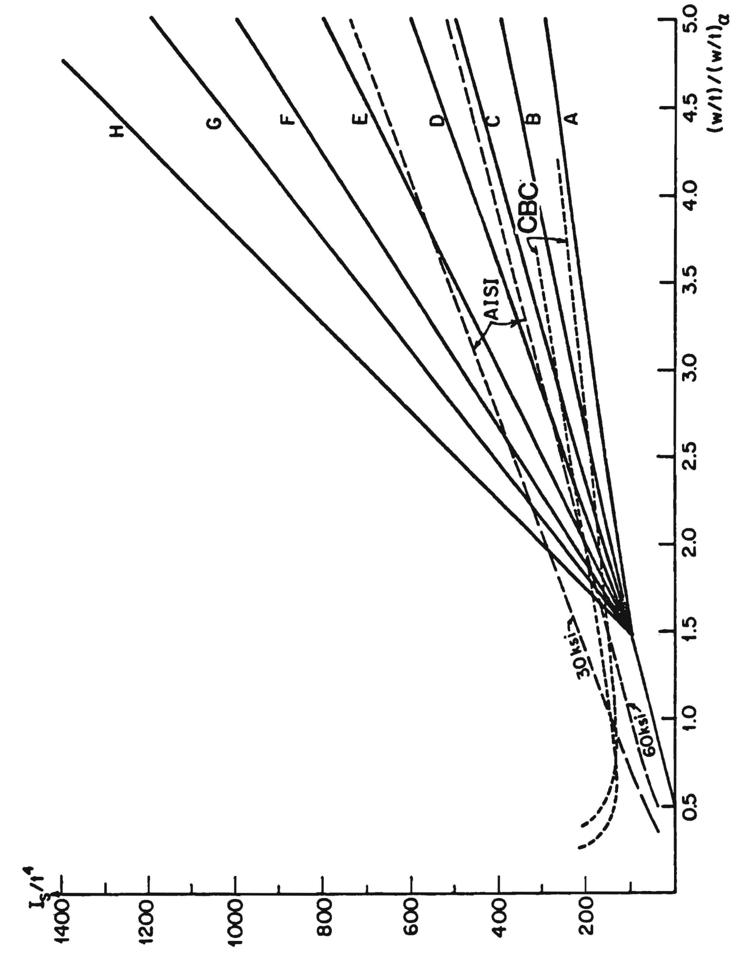


Fig. 13 Hypothetical Stiffener Requirements: Ultimate Strength Criterion (USC)

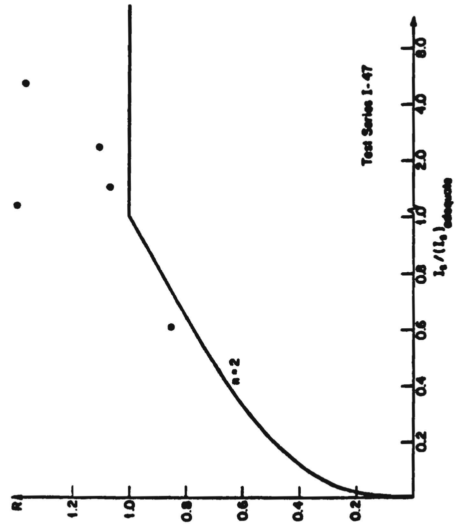


Fig. 14 Comparison of Experimental and Predicted Ultimate Strengths: $(w/t)_\beta < w/t \le 1.5(w/t)_\alpha$

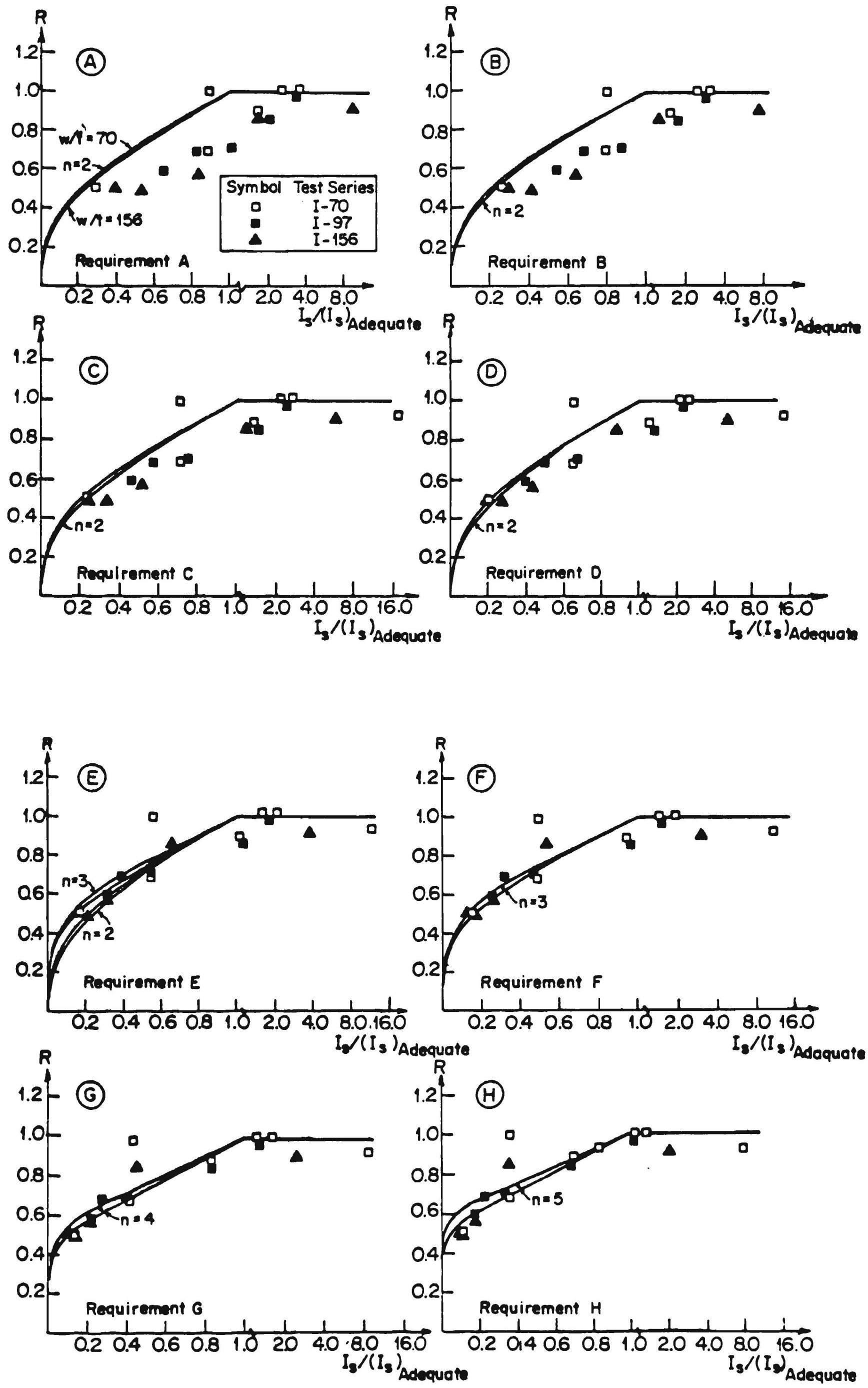


Fig. 15 Comparison of Experimental and Predicted Ultimate Strengths: $w/t \geq 1.5(w/t)_\alpha$