Deflections of two-way slabs

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DEFLECTIONS

OF

TWO-WAY SLABS

BY

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A

THESIS

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ABSTRACT

A reinforced concrete member is by no means a truly elastic member, hence all ordinary deflection calculations relating to it, whatever method, are approximate. By the use of model study actual reinforced concrete slab models, of different end conditions, were built and tested for deflections under uniform loadings. Theoretical and actual deflection of reinforced concrete slabs are compared. The results of these tests indicate that the theoretical deflection equations may be 60 to 80 percent in error when applied to reinforced concrete slabs.

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INTRODUCTION

Since the beginning of the twentieth century, concrete has taken its place as one of the most useful and important materials in the field of construction. Reinforced concrete is widely used in building construction since it is fire-proof, durable, water proof, relatively low in cost, and comparatively easy to mold into any desired shape. Its structural uses are almost unlimited. This apparent ease with which concrete may be prepared has let to its being employed by anyone who feels that the material is suited to his particular purpose. In many instances, proper knowledge of the substance and skill in its manufacture are not available so that the resultant concrete is little more than a bulky, heavy material, lacking the strength and other properties which it should have attained, and often failing to fulfill that purpose for which it was intended.

The primary purpose of this study is to compare and arrive at a relationship between the theoretical and actual values of slab deflections in order to provide data which may aid in the understanding of actual existing conditions versus theoretical conditions.

There are many engineering structures in which slabs are used exten sively. Notable examples include the floors and roofs of reinforced concrete buildings. This thesis discusses only those problems of deflections in which one dimension of a body (the thickness of a slab) can be considered small as in comparison with the other dimensions.

The first step in making an analysis study is to determine what, in the engineering sense, is a model. A very common concept is that any representation of the prototype in an arbitrary scale is already a model. From the engineering point of view, a scalar representation of the prototype can be called a model only when, from a study of its behavior, conclusions can be derived that will disclose that a certain relationship is to exist between the behavior of the model and the prototype. This relationship must necessarily be of a rather simple nature and one which is brought about by the fulfillment of the similarity conditions. These similarity conditions are governed by the purpose for which the model study is to be made and may cover a wide range of different requirements. In a number of simpler problems the only similarity condition to be met is the proper scalar representation of the prototype. In more complicated,problems, additional requirements may be included in which the similarity conditions will require that the model is not a true scalar representation of the prototype. In many cases the relationship between the properties of the material or materials that are used in the prototype and in the model is covered by similarity requirements that may be of no less importance than requirements covering the geometrical relationship bet ween the shapes of the prototype and the model.

THEORETICAL DEFLECTIONS OF SLABS

Theoretical deflections of the square reinforced concrete slabs can be approached by several different methods. Solutions of differential equations of curvature together with the boundary con ditions is a very long and tedious task. The problem may be interpreted as seeking the functions which satisfy the boundary conditions and minimize the potential energy or the complementary energy. The method used here is the energy method and is carried out as follows: First, assume the solution in the form of a series which satisfies the boundary conditions but with undetermined coefficients a_{mn}. Second, insert these functions into the expression of the potential energy or the complementary energy, and carry out the required integration.

Energy is defined as the capacity to do work, while work is the product of a force and the distance in the direction the force moves. In solid deformable bodies, stresses multiplied by the areas on which they act are forces, and deformations are distances. The product of these two quantities is the internal work done by the externally applied forces. This work is stored in a body as the elastic strain energy. Using the principles of the conservation of energy, and equating the internal work to the analytically expressed external work, one can obtain total energy for which the deflections are obtainable.

When the thickness of an elastic body is small compared with the other dimensions, it can be called a thin body. The problem may be set up by choosing a coordinate axes so that the x and y axes are in the middle plane of the slab and the z axis is perpendicular to the middle plane.

If a thin slab is bent with a small deflection, i.e., when th e deflection of the middle plane is small compared with the thickness, t, the following assumptions can be made:

1. The normals of the middle plans before bending are deformed into the normals of the middle plane after bending.
2. The stress σ_z is small compared with the other stress components and may be neglected in the stress-strain relations.
3. The middle plane remains unstrained after bending.

Consider a section of the plate parallel to the xz plane, as sh own in Figure #1.

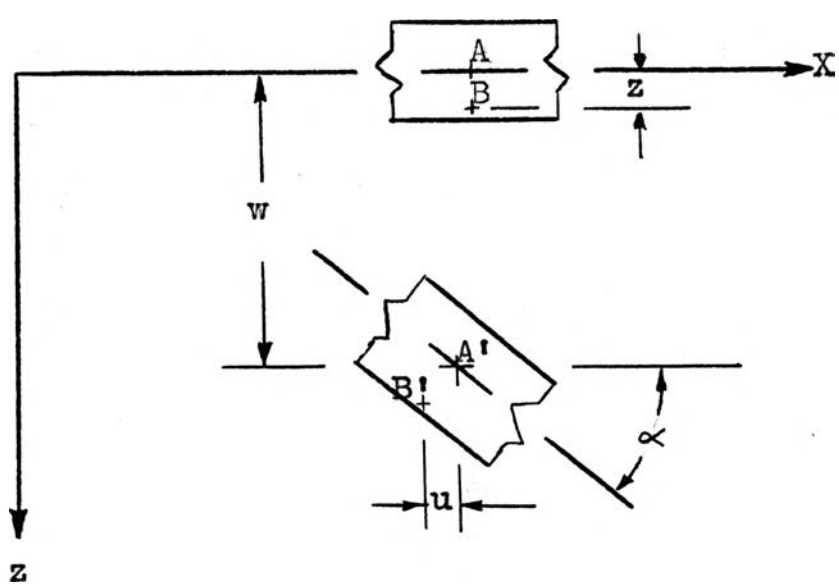


Figure #1

Bending of Slab Element

From figure #1 it is observed that the displacement of the point B' in the x direction is

$$\bar{u} = -z\,\alpha$$

Since the deflection is small, $\alpha \cong \tan = \frac{\partial w}{\partial x}$, and

$$\bar{u} = -z\,\frac{\partial w}{\partial x}$$

Similarly, the displacement of the point B' in the y direction is

$$v = -z\,\frac{\partial w}{\partial y}$$

From the definition of strain, i.e., elongation per unit length

$$\epsilon_x = \frac{\partial \bar{u}}{\partial x} \qquad \epsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{xy} = \frac{\partial \bar{u}}{\partial y} + \frac{\partial v}{\partial x}$$

From the derivatives of displacement

$$\epsilon_x = -z\,\frac{\partial^2 w}{\partial x^2} \qquad \epsilon_y = -z\,\frac{\partial^2 w}{\partial y^2} \qquad \gamma_{xy} = -2z\,\frac{\partial^2 w}{\partial x\,\partial y}$$

According to assumption 2, the stress-strain relations for a thin slab in bending are

$$\epsilon_x = \frac{1}{E}(\sigma_x - u\sigma_y) \qquad \epsilon_y = \frac{1}{E}(\sigma_y - u\sigma_x) \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

from which

$$\sigma_x = \frac{E}{1-u^2}(\epsilon_x + u\epsilon_y) \qquad \sigma_y = \frac{E}{1-u^2}(\epsilon_y + u\epsilon_x)$$

and

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1 + u)}\gamma_{xy}$$

Substituting the expressions for strain into the stress equation yields:

$$\sigma_x = -\frac{Ez}{1 - u^2}\left(\frac{\partial^2 w}{\partial x^2} + u\,\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_y = -\frac{Ez}{1 - u^2}\left(\frac{\partial^2 w}{\partial y^2} + u\,\frac{\partial^2 w}{\partial x^2}\right)$$

$$\tau_{xy} = -\frac{Ez}{(1 + u)}\,\frac{\partial^2 w}{\partial x\,\partial y}$$

When a system is in a position of stable equilibrium, its total energy is a minimmm. The total energy consists of two parts; the strain energy of bending, and the potential energy of the load distributed over the plate. The total energy of a system is

$$\Pi = U - W$$

where U is the strain energy and W is potential energy.

The strain energy stored in an element dxdydz under a general three dimensional stress system can be found to be

$$dU = \tfrac{1}{2}(\sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_z\epsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx})\,dxdydz$$

The total strain energy stored in a deformed elastic body, U, can be found by integrating dU over the whole volume, V, namely

$$U = \frac{1}{2}\iiint_V (\sigma_x \epsilon_x + \sigma_y \epsilon_y + \sigma_z \epsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx})\,dxdydz$$

The thickness of the slab is small in comparison to the lengths, therefore; σ_z, γ_{xz}, γ_{yz}, will be neglected. By neglecting terms containing σ_z, γ_{xz}, γ_{yz}, in the energy expression and eliminating the strain components by substituting in the stress-strain relations for thin slabs

$$U = \iiint_V \left[\frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2 - 2u\,\sigma_x\sigma_y\right) + \left(\frac{1+u}{E}\right)\tau_{xy}^2\right] dxdydz$$

where the relation $G = \frac{E}{2(1+u)}$ has been used.

Substituting into the formula above the expression for σ_x, σ_y, and τ_{xy} in terms of w and neglecting terms in z direction the general strain energy equation becomes

$$U = \frac{D}{2}\iint_A \left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2u\,\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2(1-u)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right] dxdy$$

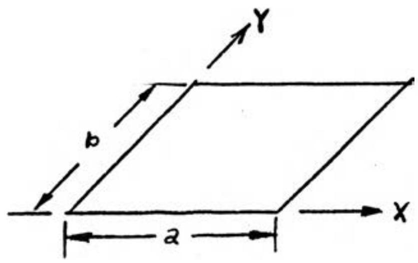
$$= \frac{D}{2}\iint_A \left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-u)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\} dxdy$$

where A is the area of the slab, D is the flexural rigidity of the slab, and $D = \frac{Et^3}{12(1-u^2)}$

If the plate is under the action of a uniformly distributed load of intensity, P_o, the potential energy of the external force is

$$-W = \int_0^a \int_0^b P_o \, w \, dxdy$$

Deflections of Simply Supported Slab



Boundary Conditions

$$W = 0, \frac{\partial^2 w}{\partial x^2} = 0 \quad @ \begin{matrix} x = 0 \\ x = a \end{matrix}$$

$$W = 0, \frac{\partial^2 w}{\partial y^2} = 0 \quad @ \begin{matrix} y = 0 \\ y = b \end{matrix}$$

Uniform load

Assuming the following general deflection equation in a Fourier series, the above boundary conditions are satisfied.

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

where A_{mn} is an undetermined coefficient dependent on the values of m and n.

Substituting the general deflection equation into the general strain energy equation, the first term under the integral sign in t he strain energy equation becomes

$$U = \frac{D}{2} \int_0^a \int_0^b \left[\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \left(\frac{m^2 \pi^2}{a^2} + \frac{n^2 \pi^2}{b^2} \right) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \right]^2 dxdy$$

To calculate the particular coefficient A_{mn}, multiply both sides of the general equation by $\sin \frac{m'\pi x}{a} \sin \frac{n'\pi y}{b} dxdy$, observing that if $m \neq m'$ and $n \neq n'$

$$\int_0^a \sin \frac{m\pi x}{a} \sin \frac{m'\pi x}{a} \, dx = 0$$

$$\int_0^b \sin \frac{n\pi y}{b} \sin \frac{n'\pi y}{b} \, dy = 0$$

and if $m = m'$ and $n = n'$

$$\int_0^a \sin \frac{m\pi x}{a} \sin \frac{m'\pi x}{a} \, dx = \frac{a}{2}$$

$$\int_0^b \sin \frac{n\pi y}{b} \sin \frac{n'\pi y}{b} \, dy = \frac{b}{2}$$

After integration the total strainenergy is

$$U = \frac{\pi^4 ab}{8} D \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2$$

for the second term under the integral sign is zero. The potential energy of the external force is

$$- W = - \int_0^a \int_0^b P_o \, w \, dxdy$$

where P_o is the uniform load.

Substituting the general deflection equation into the above expression

$$- w = - \int_0^a \int_0^b P_o \, A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \, dxdy$$

integrating

$$-w = -\frac{abP_o}{mn\pi^2} A_{mn} \cos \frac{m\pi x}{a}\Big]_0^a \cos \frac{n\pi y}{b}\Big]_0^b$$

$$= -\frac{4abP_o}{mn\pi^2} A_{mn}$$

for odd values of m or n only; A_{mn} will approach zero when even values of m or n are used.

The total energy of the system is obtained by adding the strain energy and the potential energy

$$\Pi = U - W = \frac{\pi^4 abD}{8} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} A_{mn}^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2 - \frac{4abP_o}{mn\pi^2} A_{mn}$$

To place the system in a position of stable equilibrium, take th e first derivative of the total energy which places it at a minimum. This condition, therefore, gives

$$\frac{\partial \Pi}{\partial A_{mn}} = 0 = \frac{\pi^4 abD}{4} A_{mn}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2 - \frac{4abP_o}{mn\pi^2}$$

Solving for A_{mn} gives

$$A_{mn} = \frac{16\ P_o}{\pi^6 Dmn\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}$$

Substituting A_{mn} into the general deflection equation, the final deflection equation is

$$w = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{16P_o \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{\pi^6 D mn \left(\frac{m^2}{a^2} + \frac{m^2}{b^2}\right)^2}$$

where m = 1, 3, 5, ... and n = 1, 3, 5, ... for if m or n or both are even numbers, $A_{mn} = 0$. The vanishing of all terms with even m or n in the final general deflection equation may be observed from the following physical reasoning: under a uniform load, the deflection surface of the slab must be symmetrical.

For center deflection $x = a/2$, $y = b/2$

$$w = \frac{16P_o}{\pi^6 D} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{(-1)^{\frac{m+n}{2}-1}}{mn\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}$$

For a square slab, a = b

$$w = \frac{16P_o a^4}{\pi^6 D} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{(-1)^{\frac{m+n}{2}-1}}{mn(m^2 + n^2)^2}$$

This series converges very rapidly, and the first few terms will, in general, give a satisfactory answer. For a square slab, using Poisson's Ratio of u = 0.15, the deflection given by the first few terms in the series, with the sign convention from figure #2

$$w = \frac{16P_o a^4}{\pi^6 D}(A_{11} - A_{13} - A_{31} + A_{15} + A_{51} - A_{35} \ldots .)$$

$$= \frac{16P_o a^4}{\pi^6 D}\left(\frac{1}{4} - \frac{1}{300} - \frac{1}{300} + \frac{1}{3,380} + \frac{1}{3,380} + \ldots .\right)$$

$$= \frac{16P_o a^4}{\pi^6 D}(0.244) = 0.00405\,\frac{P_o a^4}{D}$$

for $$D = \frac{Et^3}{12(1 - u^2)}$$

$$w = 0.0475\,\frac{P_o a^4}{Et^3} \quad @ \quad \begin{matrix} x = a/2 \\ y = a/2 \end{matrix}$$

<u>For quarter point deflection</u>

$$w = \frac{16P_o a^4}{\pi^6 D}\sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty}\frac{(\frac{1}{2})^{\frac{n^2+m^2}{8} + \frac{m+n}{2} + 2.75}}{mn(m^2 - n^2)^2}$$

$$= \frac{16P_o a^4}{\pi^6 D}(A_{11} + A_{13} + A_{31} - A_{15} - A_{51} + \ldots .)$$

$$= \frac{16P_o a^4}{\pi^6 D}\left(\frac{(\frac{1}{2})^4}{4} + \frac{(\frac{1}{2})^6}{300} + \frac{(\frac{1}{2})^6}{300} - \frac{(\frac{1}{2})^{8.75}}{3,380} - \ldots .\right)$$

$$= \frac{16P_o a^4}{\pi^6 D}(0.0157) = 0.00307\,\frac{P_o a^4}{Et^3} \quad @ \begin{matrix} x = a/4 \\ y = a/4 \end{matrix}$$

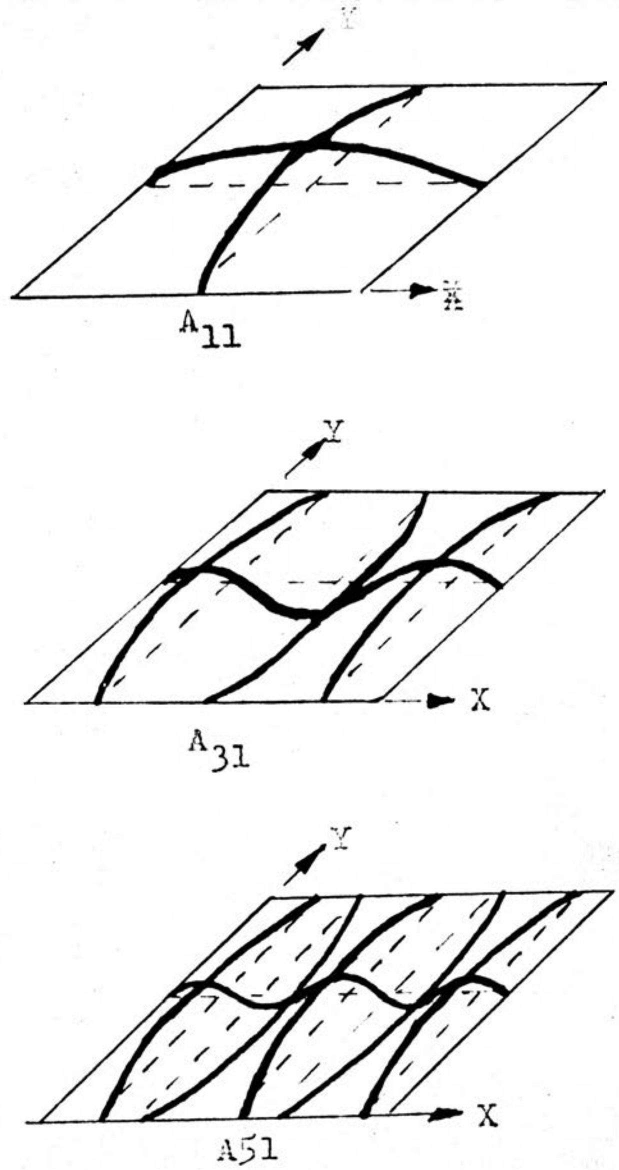
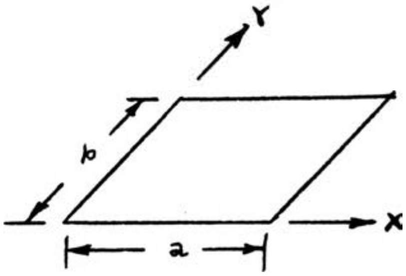


FIGURE # 2

Coefficient Signs for Simple Supported Slab

Deflections of Fixed-end Supported Slab



Boundary Conditions

$w = 0, \dfrac{\partial w}{\partial x} = 0$ @ $\begin{matrix} x = 0 \\ x = a \end{matrix}$

$w = 0, \dfrac{\partial w}{\partial y} = 0$ @ $\begin{matrix} y = 0 \\ y = b \end{matrix}$

Uniform load

Assuming the following general deflection equation in a Fourier series, the above boundary conditions are satisfied.

$$w = \sum_{m=1,2,3}^{\infty} \sum_{n=1,2,3}^{\infty} A_{mn} \left(1 - \cos \frac{2m\pi x}{a}\right)\left(1 - \cos \frac{2n\pi y}{b}\right)$$

where A_{mn} is an undetermined coefficient dependent on the values of m and n.

The general strain energy equation is

$$U = \frac{D}{2} \iint_A \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1 - u) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dxdy$$

where A is the area of the slab.

The above equation can be simplified for slabs with fixed edges where w = 0 along the edges. Integrating the last part of the above equation by parts, first with respect to y

$$\iint_A \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} dxdy = \int_S \frac{\partial^2 w}{\partial x \partial y} \frac{\partial w}{\partial x} dx - \iint_A \frac{\partial^3 w}{\partial x \partial y^2} \frac{\partial w}{\partial x} dxdy$$

then with respect to x

$$= \int \frac{\partial^2 w}{\partial x \partial y} \frac{\partial w}{\partial x} dx - \left[\int \frac{\partial^2 w}{\partial y^2} \frac{\partial w}{\partial x} dy - \iint_A \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} dxdy \right]$$

$$= \int \frac{\partial^2 w}{\partial x \partial y} \frac{\partial w}{\partial x} dx - \int \frac{\partial^2 w}{\partial y^2} \frac{\partial w}{\partial x} dy + \iint_A \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} dxdy$$

if $w = 0$ along the edges $\frac{\partial w}{\partial x} = 0$. Therefore, the first two line integrals in the above equation are zero, and the second half of the strain-energy equation may be written with the two terms cancelling one another.

$$\iint_A \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dxdy = 0$$

The general strain energy equation now becomes

$$U = \frac{D}{2} \iint_A \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 dxdy$$

Substituting the general deflection equation into the strain energy equation

$$U = \frac{D}{2} \int_0^a \int_0^b \left\{ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} 4 \pi^2 A_{mn} \left[\frac{m^2}{a^2} \cos \frac{2n \ x}{a} \left(1 - \cos \frac{2n\pi y}{b} \right) \right. \right.$$

$$\left. \left. + \frac{n^2}{b^2} \cos \frac{2n\pi y}{b} \left(1 - \cos \frac{2n\pi x}{a} \right) \right] \right\}^2 dxdy$$

and in tegrating

$$U = 2D\pi^4 ab \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[3\left(\frac{m^4}{a^4}\right) + 3\left(\frac{n^4}{b^4}\right) + 2\left(\frac{m^2}{a^2}\right)\left(\frac{n^2}{b^2}\right)\right] A_{mn}^2$$

Th e potential energy of the external force is

$$-W = -\int_0^a\int_0^b P_o w \, dxdy$$

where P_o is the uniform load.

Substituting the general deflection equation into the above expression

$$-W = -P_o \int_0^a\int_0^b \left[\sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty} A_{mn}\left(1 - \cos\frac{2m\pi x}{a}\right)\left(1 - \cos\frac{2n\pi y}{b}\right)\right] dxdy$$

$$= -P_o ab \sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty} A_{mn}$$

For odd values of m and n.

The total energy of the system is obtained by adding algebraically the strain energy and the potential energy.

$$\Pi = U - W = 2D\pi^4 ab \sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty}\left[3\left(\frac{m^4}{a^4}\right) + 3\left(\frac{n^4}{b^4}\right) + 2\left(\frac{m^2}{a^2}\right)\left(\frac{n^2}{b^2}\right)\right] A_{mn}^2 - P_o ab \sum_{m=1,3,5}^{\infty}\sum_{n=1,3,5}^{\infty} A_{mn}$$

To place a system in a position of stable equilibrium, take the first derivative of the total energy which places it at a minimum. This condition gives

$$\frac{\partial \Pi}{\partial A_{mn}} = 0 = 4Dab\pi^4 \left[3\left(\frac{m^4}{a^4}\right) + 3\left(\frac{n^4}{b^4}\right) + 2\left(\frac{m^2}{a^2}\right)\left(\frac{n^2}{b^2}\right)\right] A_{mn} - P_o ab$$

Solving for A_{mn}

$$A_{mn} = \frac{P_o}{4D\pi^4 \left[3\left(\frac{m^4}{a^4}\right) + 3\left(\frac{n^4}{b^4}\right) + 2\left(\frac{m^2}{a^2}\right)\left(\frac{n^2}{b^2}\right)\right]}$$

Substituting A_{mn} into the general deflection equation, the final deflection equation is

$$w = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{P_o \left(1 - \cos \frac{2m\pi x}{a}\right)\left(1 - \cos \frac{2n\pi y}{b}\right)}{4D\pi^4 \left[3\left(\frac{m^4}{a^4}\right) + 3\left(\frac{n^4}{b^4}\right) + 2\left(\frac{m^2}{a^2}\right)\left(\frac{n^2}{b^2}\right)\right]}$$

where m = 1, 3, 5, ... and n = 1, 3, 5, ..., for if m or n or both are even numbers, $A_{mn} = 0$. The vanishing of all terms with even m or n in series may be observed from the following physical reasoning; un der a uniform load, the deflection surface of the slab must be symmetrical.

For a square slab, a = b

$$w = \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{P_{\bar{o}}\, a^4 \left(1 - \cos \frac{2m\pi x}{a}\right)\left(1 - \cos \frac{2n\pi y}{b}\right)}{4D\pi^4 \left[3m^4 + 3n^4 + 2m^2n^2\right]}$$

<u>For center deflection,</u> $x = a/2$, $y = a/2$

$$w = \frac{P_o a^4}{4D\pi^4} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{4}{3m^4 + 3n^4 + 2m^2n^2}$$

The series converges rapidly, and the first few terms will, in general, give a satisfactory answer. Using sign convention from figure #3

$$w = \frac{P_o\, a^4}{4D\pi^4} (A_{11} + A_{13} + A_{31} + A_{15} + A_{51} + A_{35} + \dots\dots)$$

$$= \frac{P_o\, a^4}{4D\pi^4} \left(\frac{1}{2} + \frac{1}{66} + \frac{1}{66} + \frac{1}{482} + \frac{1}{482} + \dots\dots\right)$$

$$= \frac{P_o\, a^4}{4D\pi^4} (0.5344) = 0.00137 \frac{P_o a^4}{D}$$

$$= 0.01489 \frac{P_o a^4}{Et^3} \qquad @ \begin{matrix} x = a/2 \\ y = a/2 \end{matrix}$$

For quarter point deflection, $x = a/4, \quad y = a/4$

$$w = \frac{P_o a^4}{4D\pi^4} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{1}{3m^4 + 3n^4 + 2m^2n^2}$$

The series converges rapidly, and the first few terms will, in general, give a satisfactory answer.

$$w = \frac{P_o a^4}{4D\pi^4} (A_{11} + A_{13} + A_{31} + A_{15} + A_{51} + \cdots\cdots)$$

$$= \frac{P_o a^4}{4D\pi^4} \left(\frac{1}{8} + \frac{1}{264} + \frac{1}{264} + \frac{1}{1928} + \frac{1}{1928} + \cdots\cdots \right)$$

$$= \frac{P_o a^4}{4D\pi^4} (0.133612) = \frac{P_o a^4}{D} (0.000342)$$

$$= 0.003723 \frac{P_o a^4}{Et^3} \quad @ \begin{array}{l} x = a/4 \\ y = a/4 \end{array}$$

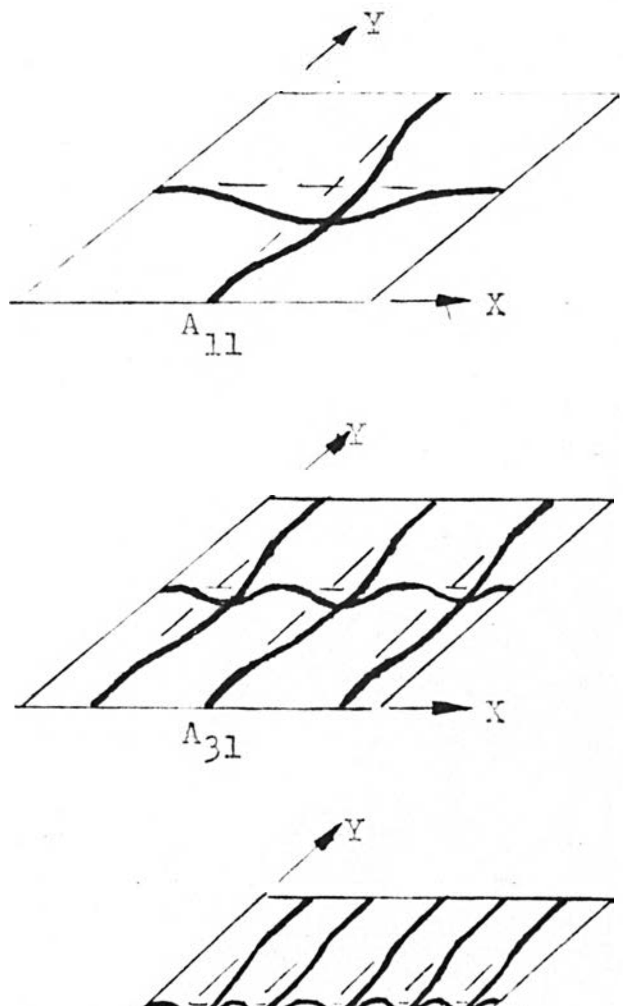


FIGURE # 3

Coefficient Signs for Fixed-end Supported Slab

SYMBOLS

A_{mn}	=	Undetermined coefficient
A	=	Area
a	=	Length
b	=	Length
D	=	Flexural rigidity of a slab
E	=	Modulus of elasticity in tension or compression
G	=	Modulus of elasticity in shear
m	=	A number, integer
n	=	A number, integer
P_o	=	Uniform load
t	=	Slab thickness
u	=	Poisson's ratio
U	=	Strain energy
v	=	Component of displacement
V	=	Volume
w	=	Component of displacement
W	=	Potential energy
x, y, z	=	Rectangular coordinates
$\sigma_x, \sigma_y, \sigma_z$	=	Normal components of stress
ϵ_x, ϵ_y	=	Unit elongations
γ_{xy}	=	Shearing strain components
τ	=	Shearing stress
Π	=	Total energy
α	=	An angle
$\bar{u}$	=	Displacement

SLAB DESIGN

The design of reinforced concrete two-way slabs could be approached by several methods; however, the most common method in use is Method #2 of the American Concrete Institute Building Code.[2] The slabs were designed to carry a live load of 100 psf and a dead load of 75 psf. The strength of the concrete was assumed at 3000 psi and an allowable steel stress of 40,000 psi. Two square slabs were designed; one of simply supports and the other with all four sides fixed.

The two-way slab with all four edges fixed could be a typical interior panel of 18 foot square. A thickness of 6 inches was used. The fixed ends were simulated by pouring an edge beam large enough to resist the bending couple of the slab.

The two-way slab with all four edges simply supported could be a single panel of 18 foot square. A thickness of 6 inches was used.

Symbols Used:

A_s = Area steel (in.^{2})

M = Moment Ft. Lb.

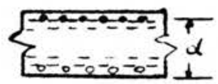
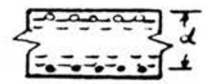
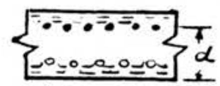
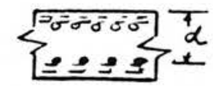
d = Depth

t = Thickness

P_oLL = Live load (psf)

P_oDL = Dead load (psf)

Depth and Steel Calculation for Slab of Simple Supports on Four Sides

	Short Span		Long Span	
				
Middle Strip	- M	+ M	- M	+ M
Moment Coefficient	0.033	0.050	0.033	0.050
Moment, ft.- lb.	1666.500	2525.000	1666.500	2525.000
Min. $d = \sqrt{M/230.9}$	2.690	3.300	2.690	3.300
Cover	1.500	1.500	1.500	1.500
For Dia. = 0.594" bars	0.297	0.297	0.891	0.891
Min. t	4.487	5.097	5.081	5.691
Actual d for t = 6"	4.203	4.203	3.509	3.509
$A_s = \frac{M \times 12}{40{,}000 \times 0.866\, d} = in.^2 / ft.$	0.137	0.208	0.161	0.247
0.0025 bd min.	0.126	0.126	0.107	0.107
Spcg. of dia. = 0.594" bars	18" c-c	18" c-c	18" c-c	12" c-c

Column Strips: The column strips require 2/3 as much steel as the middle strips.

For simplification use the same amount.

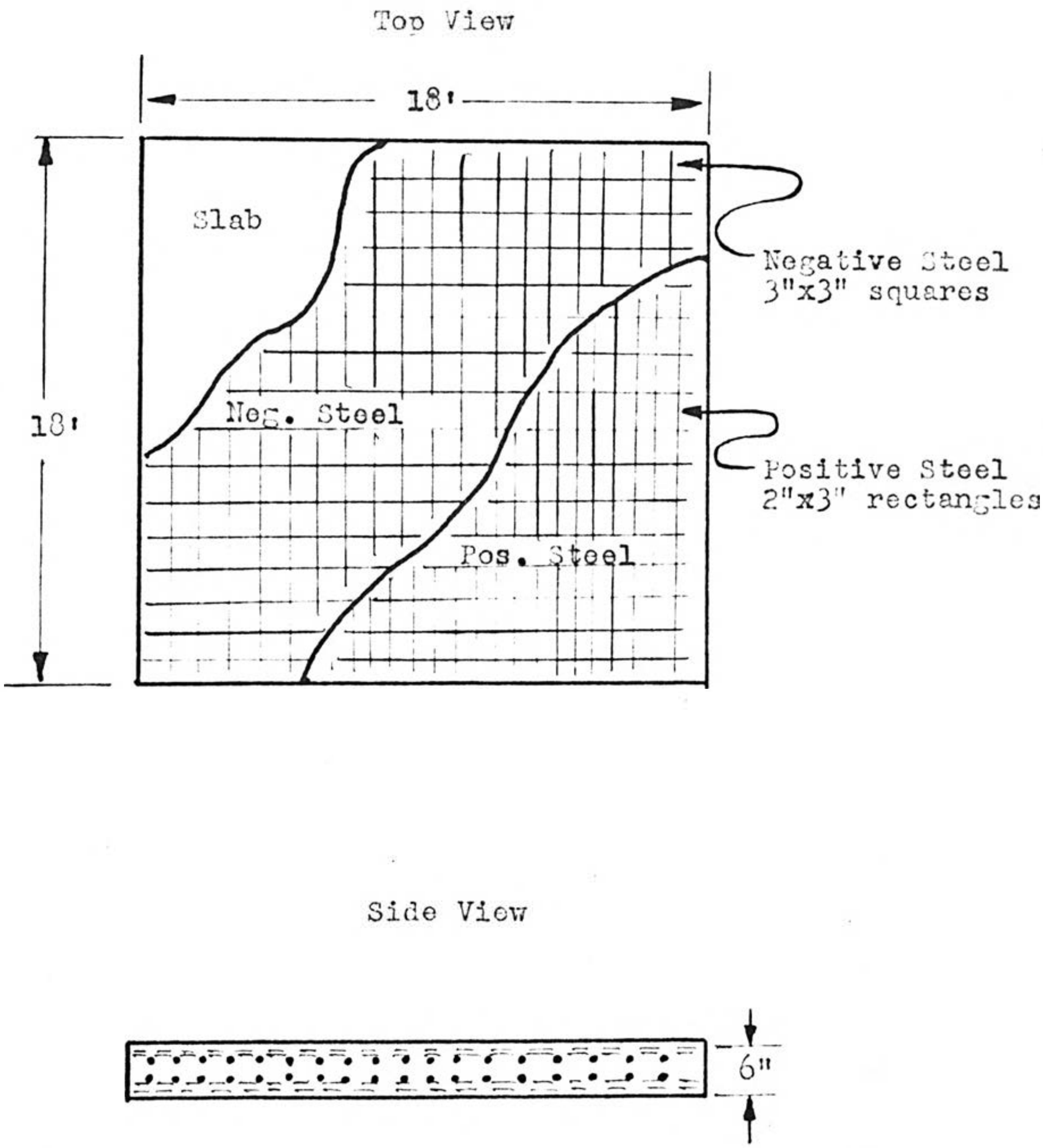


FIGURE # 4

Diagram of Simply Supported Slab

Depth and Steel Calculation for Slab Fixed on Four Sides

	Short Span		Long Span	
Middle Strip	- M	+ M	- M	+ M
Moment Coefficient	0.033	0.025	0.033	0.025
Moment, ft. - lb.	1666.500	1262.500	1666.500	1262.500
Min. $d = \sqrt{M / 230.9}$	2.690	2.340	2.690	2.340
Cover	1.500	1.500	1.500	1.500
For dia. = 0.594" bars	0.297	0.297	0.891	0.891
Min. t	4.487	4.137	5.081	4.731
Actual d for t = 6"	4.203	4.203	3.609	3.609
$A_s = \frac{M \times 12}{40{,}000 \times 0.866\ d} = in.^2 / ft.$	0.137	0.104	0.161	0.121
0.0025 bd min.	0.126	0.126	0.107	0.107
Spcg. of dia. = 0.594" bars	18" c-c	18" c-c	18" c-c	18" c-c

Column Strips: The column strips require 2/3 as much steel as the middle strips.

For simplification use the same amount.

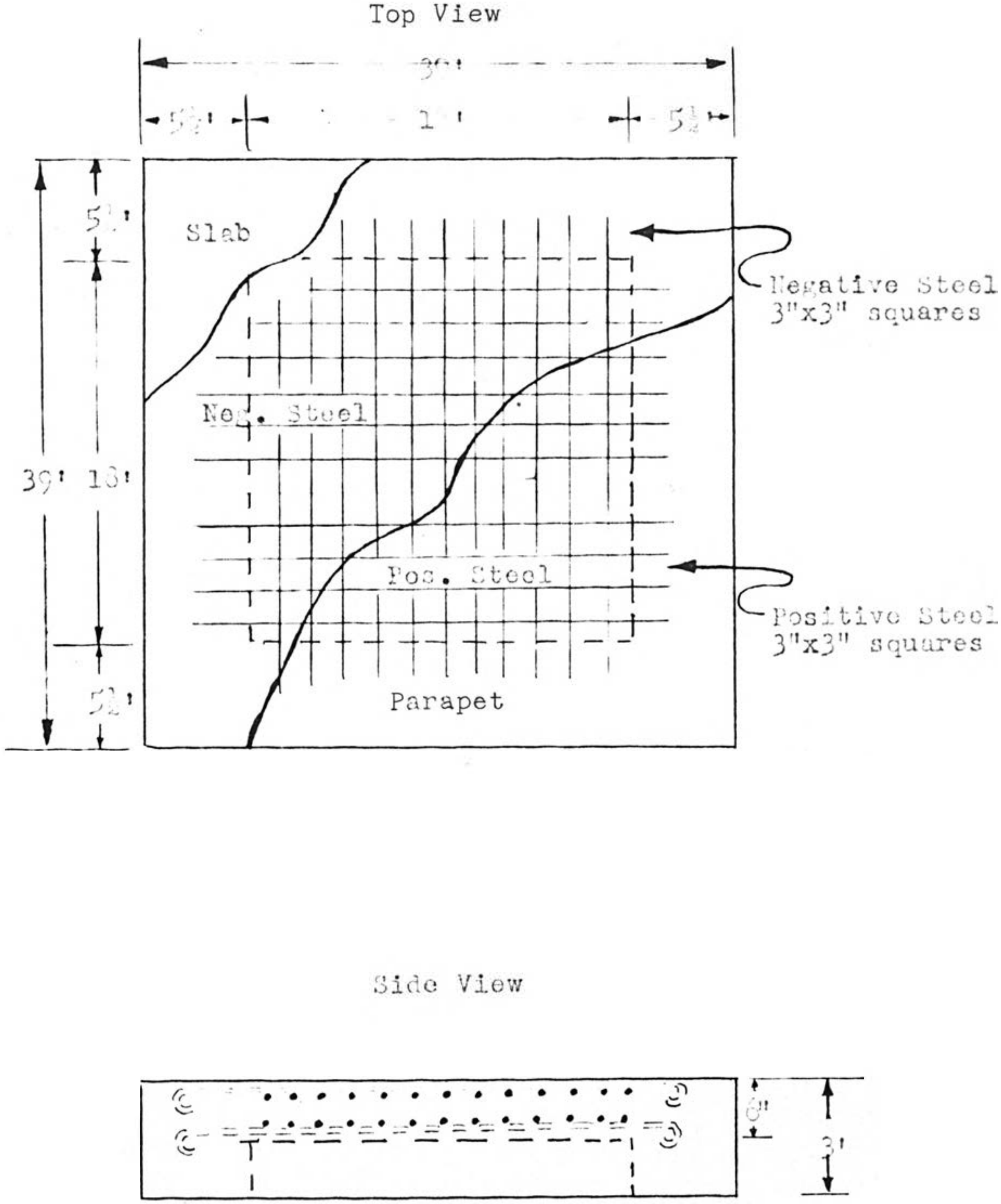


FIGURE # 5

Diagram of Fixed-end Slab

MODEL INVESTIGATION

The general theory of the model design will be developed by use of the Buckingham Pi Theorem. The model and prototype are to be loaded by uniform loads of such character and magnitude that the deflections due to the dead weight of the slab will be neglected and the similarity conditions will be formulated as follows:

a) The model is to be a true scalar representation of the prototype.

b) The material of the model is such that, in the range of stresses of the model, the variations of the modulus of elasticity of the material of the prototype in its corresponding range of stresses.

c) The loadings of the model are to be proportional to the loadings of the prototype, but no limitations apply to the value of the model loadings.

d) The ratio of the modulus of elasticity in shear and of the modulus of elasticity in flexure of the model material within the range of the stresses of the model is to be the same as that of the material of the prototype within its range of stresses. In other words, Poisson's ratios of the model and prototype are to be equal at corresponding stresses.

e) The deflections of the model are proportional to the linear scale multiplied by the ratio of the moduli of elasticity of the prototype and model materials.

The Buckingham Pi Theorem, from the reference by Glenn Murphy,[1] in a general term, states that the number of dimensionless and in dependent quantities required to express a relationship among the variables is equal to the number of quantities involved, minus the number of dimensions in which those quantities may be measured. In equation form, the Pi Theorem is:

$$s = n - m$$

in which s is the number of pi terms,

n is the total number of quantities involved,

m is the number of basic dimensions involved.

The first step is to determine the pertenent variables and indicate their dimensions.

w	deflection	L
a	span	L
b	width	L
t	thickness	L
q	coordinate of deflection	L
P_o	uniform load	FL^{-2}
E	modulus of elasticity	FL^{-2}
A_s	area of steel	L^2

The selection of these variables neglect the deflection of the slab under its own weight. It also assumes that shearing deflection is negligible and that the load is applied so that no twisting takes place.

With eight quantities and two dimensions involved, there must be six pi terms. The only restriction placed on the pi terms is that they be dimensionless and independent. A possible set of pi terms leads to the following general equation:

$$\frac{w}{a} = f\left(\frac{b}{a}, \frac{t}{a}, \frac{q}{a}, \frac{A_s}{a^2}, \frac{P_o}{E}\right)$$

A similar equation may be written for the model:

$$\frac{w_m}{a_m} = f\left(\frac{b_m}{a_m}, \frac{t_m}{a_m}, \frac{q_m}{a_m}, \frac{A_s}{a^2_m}, \frac{P_{o_m}}{E_m}\right)$$

Since each equation refers to the same type of system, the functions are identical in form. The design conditions will involve distances indicative of the size of model and prototype. The ratio of some pertinent distance or length of the prototype to the corresponding distance in the model is called the length scale, and is designated as n.

$$a_p = na_m$$

With the introduction of the length scale, the pi terms may be reduced, as indicated below:

Pi Terms	Pi Terms Reduced	
$\frac{b_m}{a_m} = \frac{b_p}{a_p}$	$b_m = \frac{b_p}{n}$	(a)
$\frac{t_m}{a_m} = \frac{t_p}{a_p}$	$t_m = \frac{t_p}{n}$	(b)

Pi Terms	Pi Terms Reduced	
$\frac{q_m}{a_m} = \frac{q_p}{a_p}$	$q_m = \frac{q_p}{n}$	(c)
$\frac{A_{s_m}}{a^2_m} = \frac{A_{s_p}}{a^2_p}$	$A_{s_m} = \frac{A_{s_p}}{n^2}$	(d)
$\frac{P_{o_m}}{E_m} = \frac{P_{o_p}}{E_p}$	$P_{o_m} = P_{o_p}\frac{E_m}{E_p}$	(e)

Con ditions indicate:

(a) and (b) that the model is to be geometrically similar to the prototype

(c) that the deflection is measured at a geometrically similar point in the model and prototype.

(d) that the areas of steel are proportional.

(e) that the magnitude of the load to be used in the model is established and completely independent of the length scale.

From the prototype slab design and using a desired length scale of six, from conditions (a), (b), and (d):

$$b_m = \frac{18'}{6} = 3' \qquad \text{width}$$

$$t_m = \frac{6''}{6} = 1'' \qquad \text{thickness}$$

and

$$A_{s_m} = \frac{0.277 \text{ in}^2}{36} = 0.00769 \text{ in}^2 \quad \text{steel area}$$

The load is determined from condition (e):

$$P_{o_m} = P_o \frac{E_m}{P_{E_p}}$$

If these conditions are satisfied, the prediction equation becomes:

$$w_p = nw_m = 6w_m$$

Figures 6 and 7 show the models of the fixed-end and simply supported slabs, respectively.

FIGURE # 6

Model of Fixed - End Supported Slab

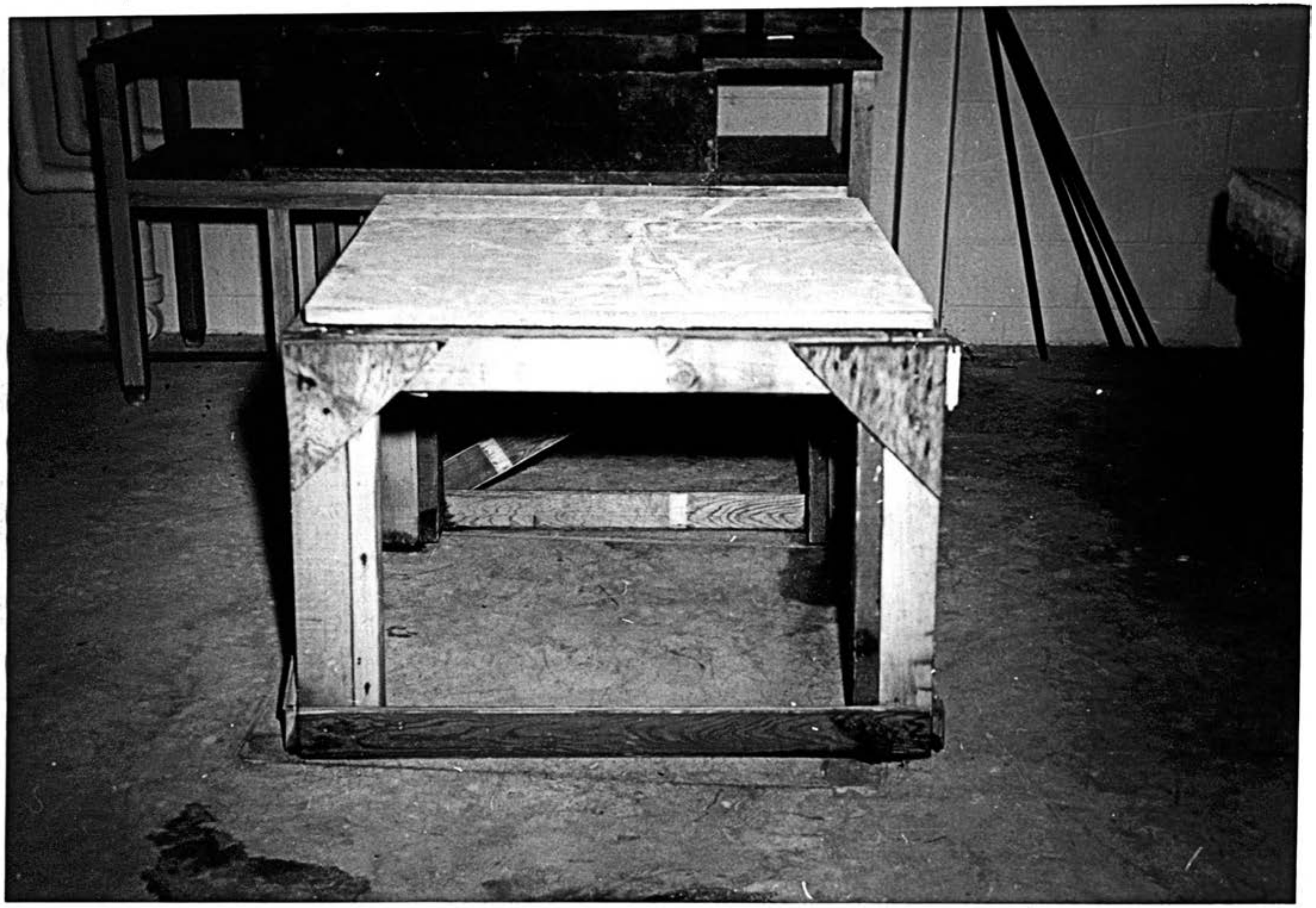


FIGURE # 7

Model of Simply Supported Slab

MATERIALS USED IN MODEL SLABS

Reinforcement:

The roll of cold-drawn steel was furnished by Laclede Steel Company of St. Louis, Missouri. The steel was placed in mat form and secured by binding the longitudinal and transverse bars together to the desired spacings. Both positive and negative steel spacing of the fixed-end supported slab was in 3" squares. The steel spacing of the simply supported slab required 3" squares for the negative steel and 2" x 3" rectangles for the positive steel. Figures 10 and 11 show the reinforcement properly oriented and placed. The correct depth of the steel was controlled by means of a small spacer grooved to the exact depth, as shown in figure 12.

The steel properties are as shown below:

Yield Stress	98,750 psi
Ultimate Stress	105,300 psi
Modulus of Elasticity	30×10^6 psi
Diameter	0.099 in.
Area	0.007693 in.^{2}

Curing:

The test slabs were cured for 14 days by covering them with burlap sacks and watering twice daily. Several test cylinders were made during the process of pouring the slabs and were moist cured for 21 days before testing. The results of the cylinder tests are shown in figure 8.

Forms:

The forms were made from 3/4" plywood, the base of which was braced underneath to avoid warping. The form of the simply supported slab, shown in figure 11, has grooved sides to hold the base securely and to set the depth of the form. The form of the fixed-end slab was made in two parts; the edge beam form which secured the slab, and the base form which floated in the edge beam form and was independently supported. This base form was removed by releasing the supports and removing it from the bottom, as shown in figure 10. The edge beam form also served as a testing frame. The forms were well oiled before pouring the concrete.

Concrete:

Concrete, more properly called mortar (since the size of the aggregate must be restricted), was used in the models to represent the concrete in the prototype. The simply supported slab required 1.5 cubic feet of mortar, and the fixed-end slab and its edge beam required 8.5 cubic feet. A rich mix of one part cement and two part sand by weight was used. The materials required are listed below:

	Fixed-end Slab	Simple-end Slab
Cement (Type I Portlant)	240#	45#
Sand	480#	90#
Water	84#	16#

Several test cylinders of the mortar used in the slabs were poured and tested. The average strength was 5,000 psi and the average Modulus of Elasticity for the mortar was 6×10^6 psi, as computed from the stress strain curve of figure 8. Poisson's ratio for the mortar was taken from the collection of data published by G. W. Washa and M. O. Withey[1] as 0.15.

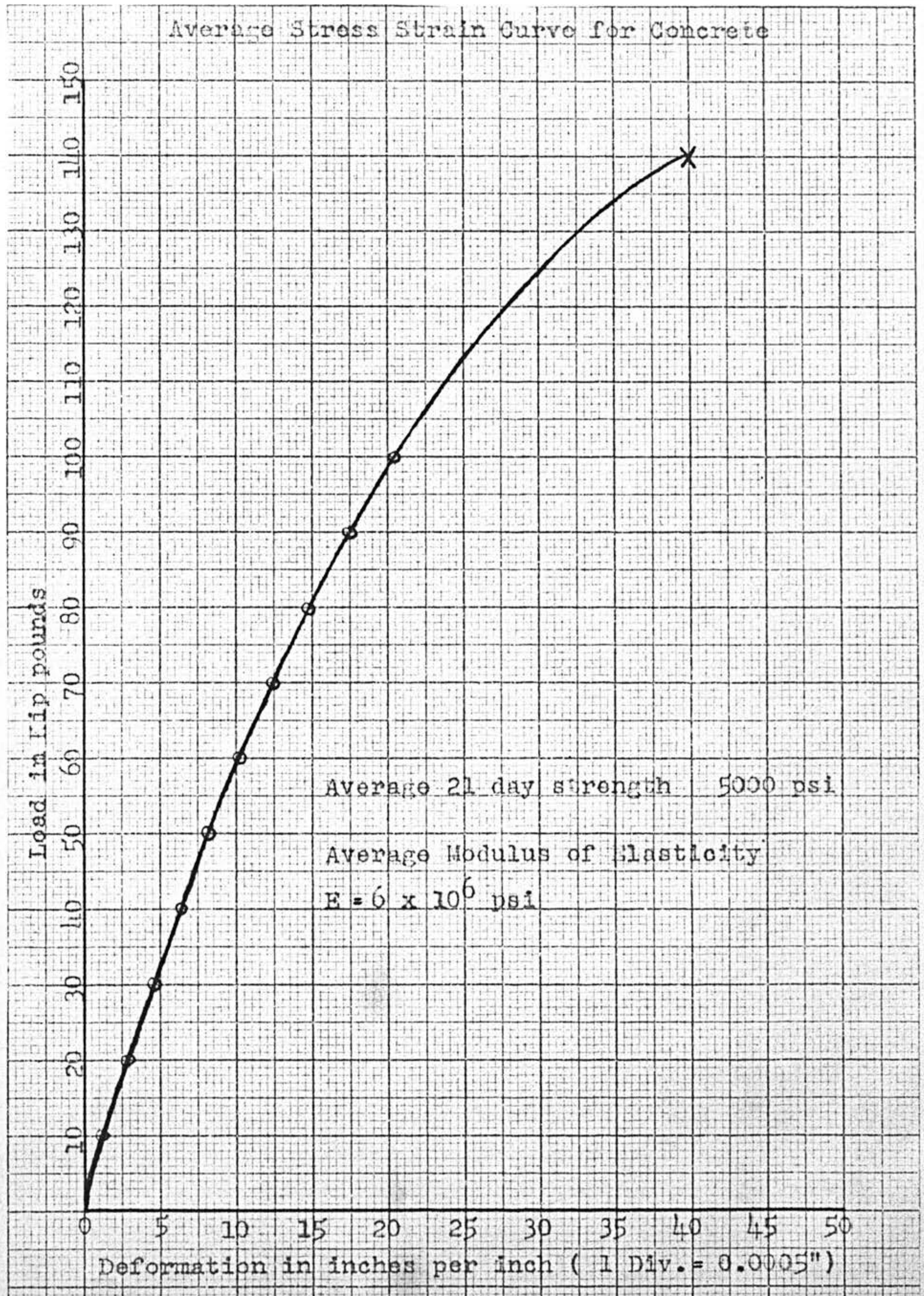

Average Stress Strain Curve for Concrete
Load in Kip pounds
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
0
5
10
15
20
25
30
35
40
45
50
Deformation in inches per inch (1 Div. = 0.0005")
Average 21 day strength 5000 psi
Average Modulus of Elasticity
E = 6 x 10^6 psi

Figure #8

The apparent modulus of elasticity in bending was obtained by testing the simply supported slab as a one-way beam, and computing the modulus of elasticity. The average modulus of elasticity in bending was 2.2×10^6 psi.

Apparent Modulus of Elasticity Curve

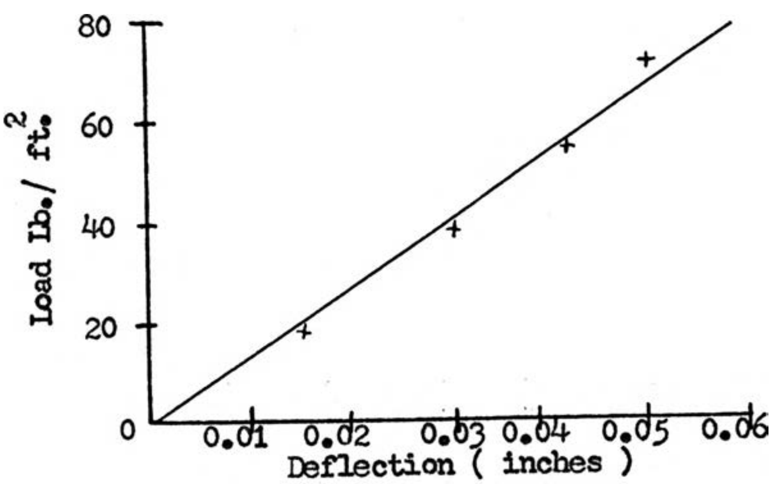


Figure 9

The equation for center deflections of a beam is

$$w = \frac{5}{384}\ \frac{P_oL^4}{D} = \frac{5(12)(1 - u^2)P_oL^4}{(384)(Et^3)}$$

Solving for modulus of elasticity for several points of load and averaging them:

$$E = \frac{5(12)(1-u^2)\ P_oL^4}{(384)(t^3)\ w} = 2.2 \times 10^6 \text{ psi}$$

where: u = Poisson's ratio = 0.15

P_o	=	Uniform load	=	lb/ft^2
L	=	Length	=	3 ft.
w	=	Deflection	=	Inches
t	=	Thickness	=	1 inch

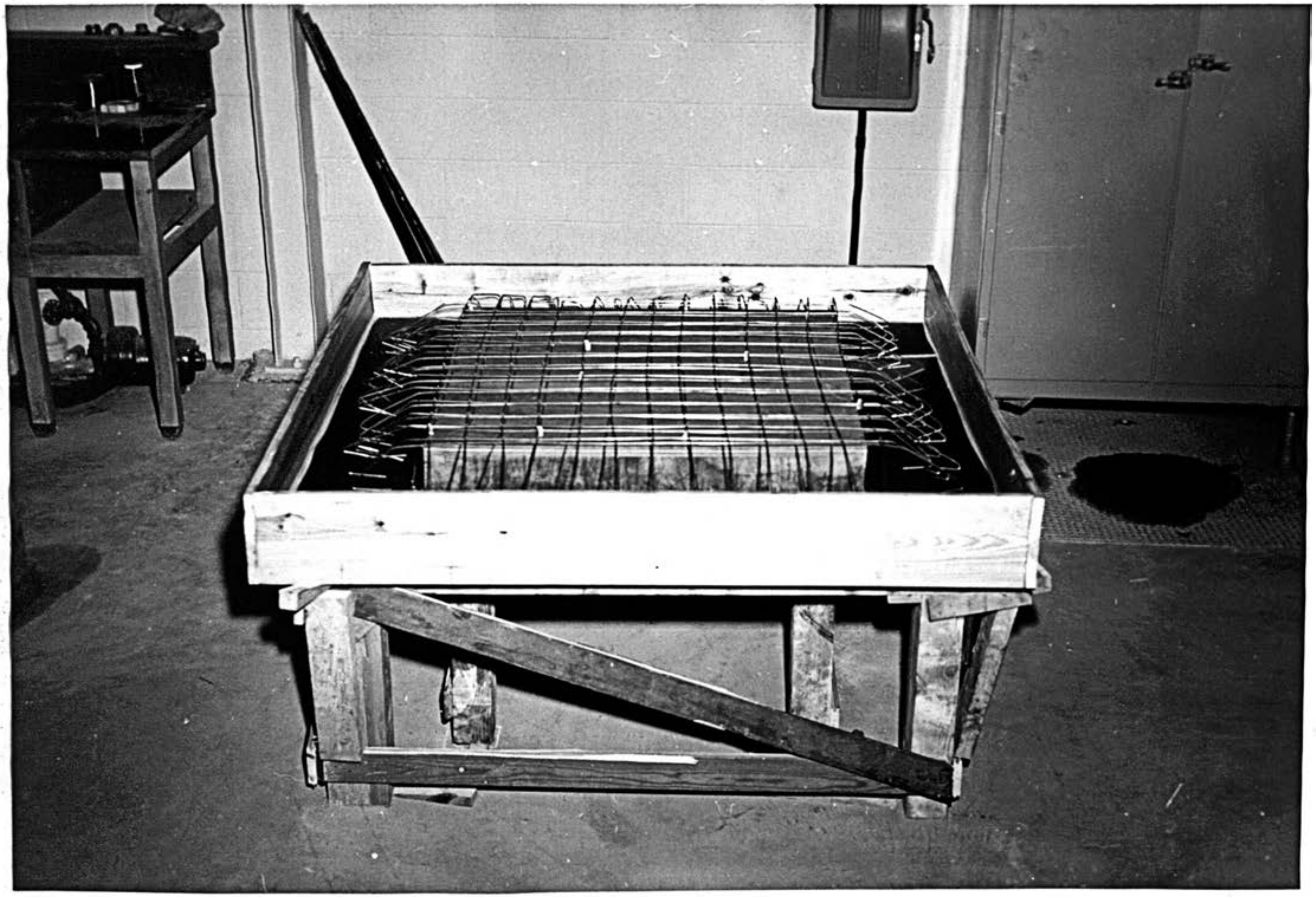


FIGURE # 10

Fixed - End Slab Form with Reinforcing Steel in Place

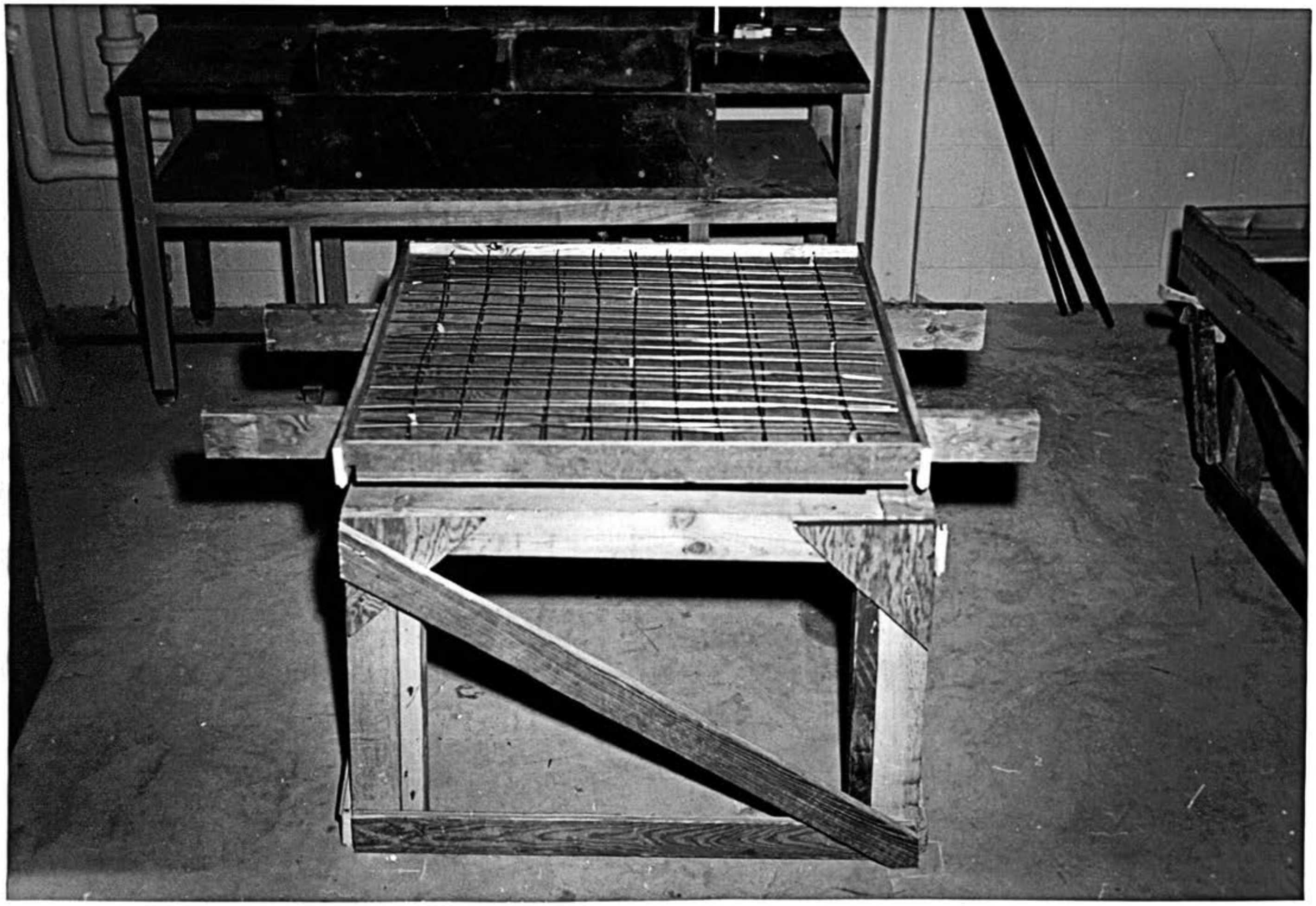


FIGURE # 11

Simply Supported Slab Form with Reinforcing Steel in Place

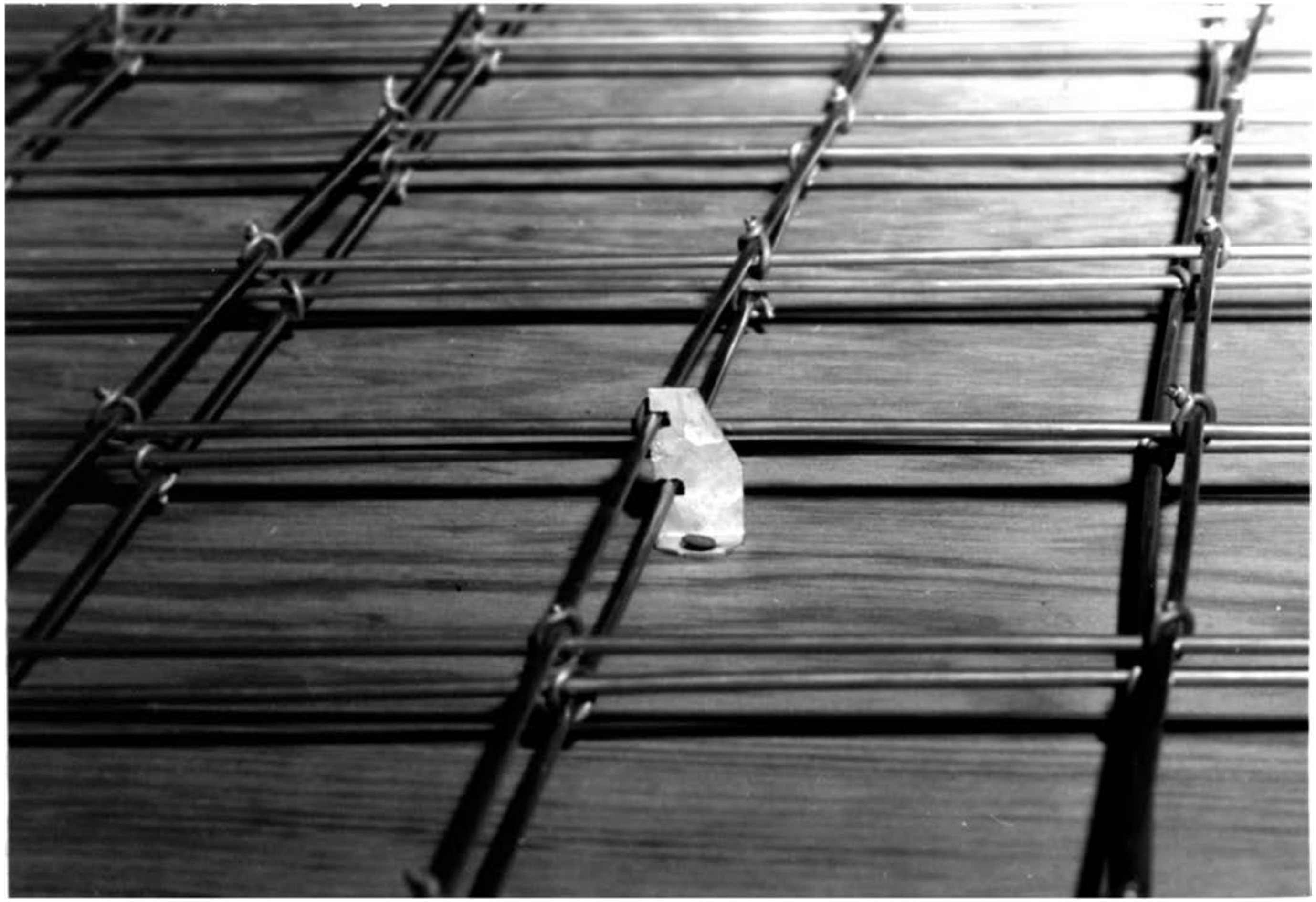


FIGURE # 12

Spacer for Reinforcing Steel Depth Control

FIGURE # 13

Curing of Slabs

TEST PROCEDURE

The model slabs were poured on May 31, 1960, and tested on June 21, 1960. The test was made on twenty-one day old slabs, after the test cylinders reached the desired strength.

Instrumentation:

The deflections were read directly from the model by placing dial indicators directly below the slab at the quarter points and the center point, as shown in figure 14. The dial indicators shown in figure 14 read to 0.001 of an inch. The deflections were measured by the dials and recorded at various loadings.

Method of Loading:

The slabs were tested for a uniform load, which was provided by a uniform sand weighing 94.3 pounds per cubic foot. The sand was poured into the testing box and leveled by means of a leveling jig at various increasing depths.

Description of Testing Frame and Box:

The testing frame for the simply supported slab is shown in figure 7. The slab was supported on steel bars which were secured firmly to the test frame to simulate simple supports.

The testing frame for the fixed-end slab was an integral part of the edge beam form.

The testing box consisted of vertical wooden sides around the slab to confine the loading sand to the testing area. The testing box was placed on the slabs to allow full freedom of the slabs. A lining of thin plastic membrane was placed between the slabs and forms to prevent the sand from sifting between the slab and box. A leveling jig with movable rails adjustable to any desired depth was placed on the inside of the testing box. A leveling rake rode on the rails to strike off sand at the desired depth.

The testing box and jig were used on both slabs, and are shown in figure 15.

Description of Typical Test:

The actual loading could begin when the dial indicators beneath the slab where zeroed. The sand was poured and leveled at various heig hts. The depth of the sand and the quarter and center point deflections were recorded. This process was repeated for sand depths varying from 0 to 26 inches at increments indicated in tables 1 and 2. Instantaneous deflection readings and loadings could not be taken. The sand was loaded and deflections recorded as quickly as possible to avoid interference due to creep.

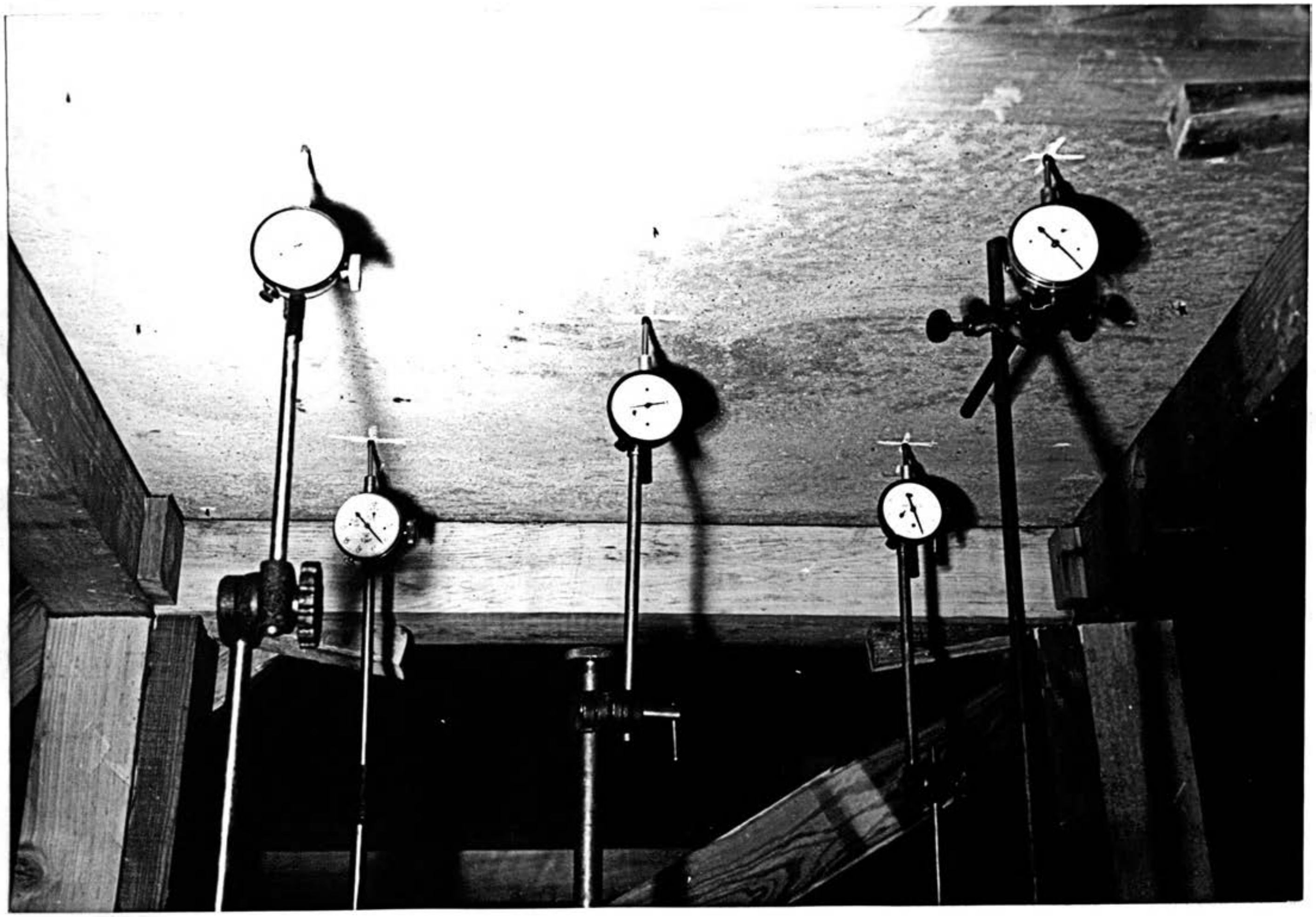


FIGURE # 14

Dial Indicators in Place

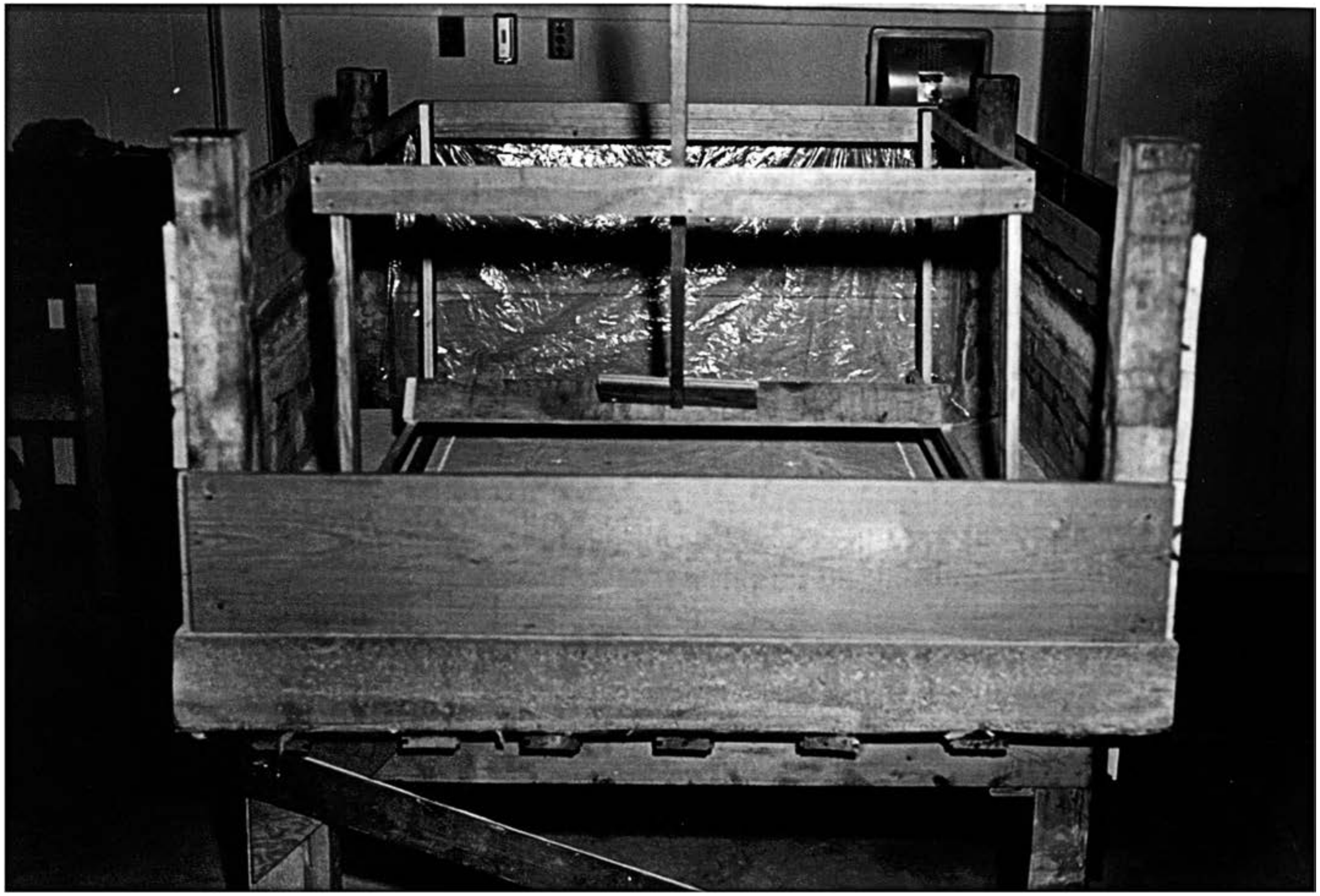


FIGURE # 15

Testing Box and Jig

FIGURE # 16

Fixed - End Slab Model Being Tested

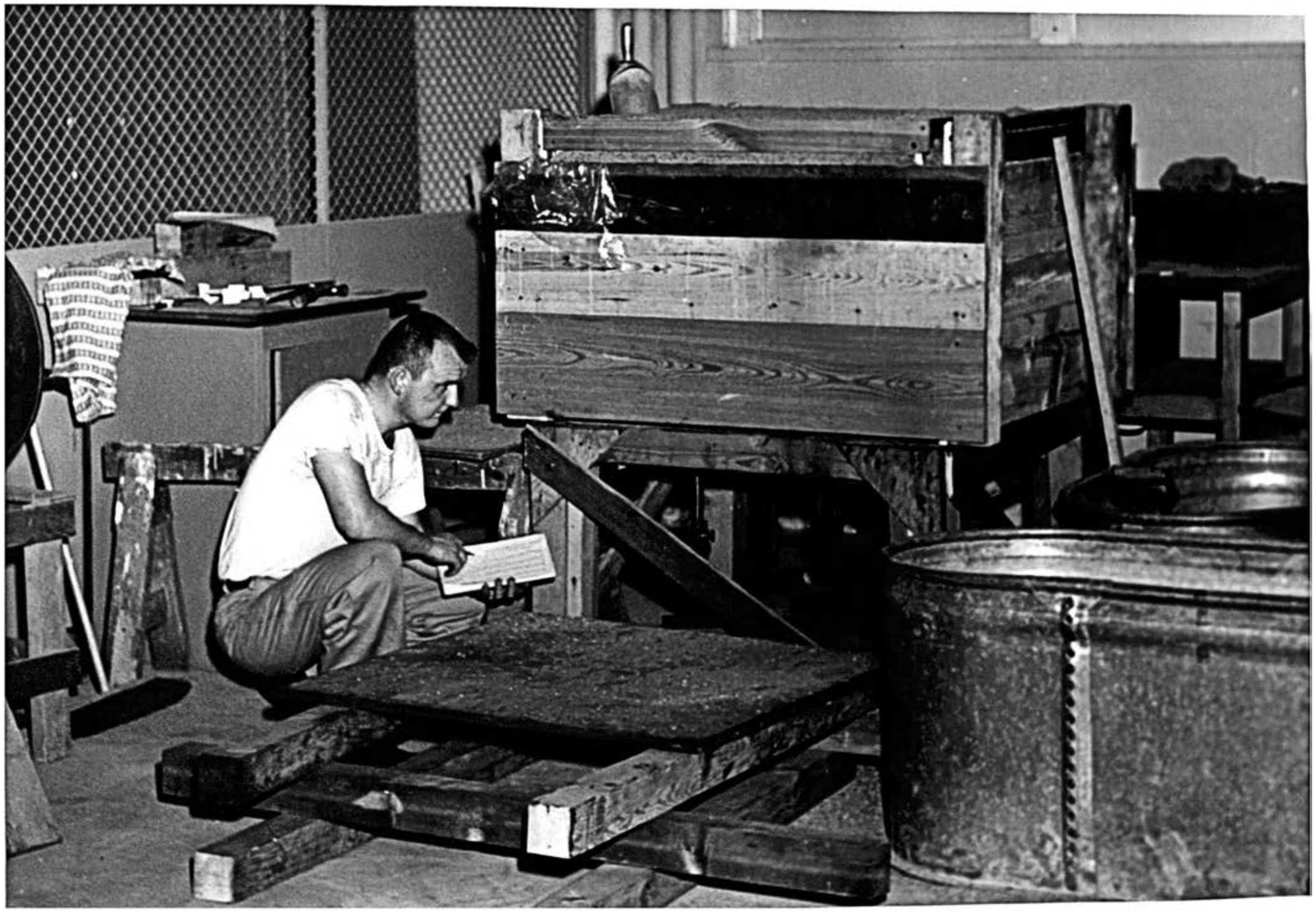


FIGURE # 17

Simply Supported Slab Model Being Tested

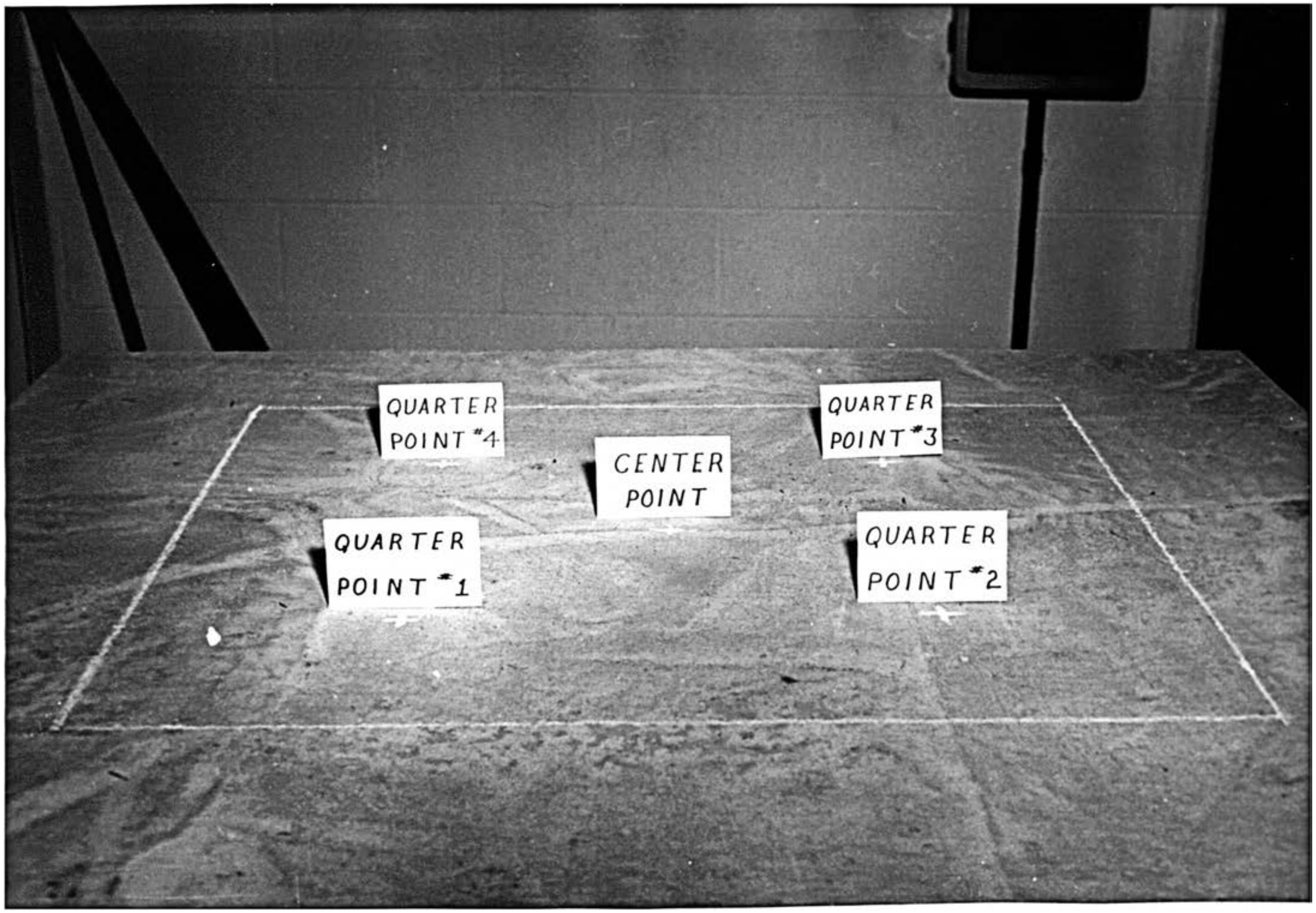


FIGURE # 18

Deflection Point Markings

EXPLANATION OF DATA TABLES

The following tables contain the compiled data taken during the slab test, plus the conversion of model slab deflections to prototype deflections.

Load:

The load was recorded as height of sand and later calculated into pounds per square foot. Uniform sand was used weighing 94.3 pounds per cubic foot.

Model Deflections:

The deflections were read directly from dial indicators with 0.001 of an inch least count. The dials were placed at the center and four quarter points. The average of the quarter points deflection readings were used as the final quarter point deflection. Data from dial #1, table #1, of the simply supported slab was disregarded for the recorded deflections were larger than the center point deflections.

Prototype Deflections:

The model deflections were converted directly to prototype deflections by the prediction equation.

Table #1

Data of Simple Supported Slab

Load		Model Deflections (Inches)				
Height	Lb/ft²	Center Point	Quarter Points			
			#1	#2	#3	#4
0	0.00	0.0000	0.0000	0.0000	0.0000	0.0000
1	7.86	0.0079	0.0046	0.0069	0.0095	0.0050
2	15.72	0.0149	0.0136	0.0119	0.0156	0.0116
3	23.58	0.0202	0.0210	0.0165	0.0199	0.0168
4	31.44	0.0270	0.0298	0.0210	0.0241	0.0230
5	39.30	0.0320	0.0345	0.0240	0.0281	0.0280
6	47.16	0.0361	0.0380	0.0280	0.0320	0.0312
7	55.02	0.0416	0.0438	0.0314	0.0360	0.0370
8	62.88	0.0466	0.0480	0.0350	0.0410	0.0418
9	70.74	0.0527	0.0530	0.0389	0.0459	0.0471
10	78.60	0.0589	0.0590	0.0438	0.0511	0.0531
11	86.46	0.0627	0.0620	0.0464	0.0547	0.0570
12	94.32	0.0688	0.0684	0.0502	0.0593	0.0638
13	102.18	0.0730	0.0724	0.0538	0.0629	0.0679
14	110.04	0.0799	0.0791	0.0582	0.0672	0.0743
15	117.90	0.0843	0.0832	0.0811	0.0711	0.0793
16	125.76	0.0895	0.0870	0.0640	0.0748	0.0839
17	133.62	0.0950	0.0912	0.0671	0.0790	0.0888
18	141.48	0.0990	0.0942	0.0701	0.0823	0.0920
19	149.34	0.1023	0.0970	0.0730	0.0860	0.0949
20	157.20	0.1062	0.0999	0.0758	0.0892	0.0979
21	165.06	0.1110	0.1031	0.0798	0.0935	0.1019
22	172.92	0.1149	0.1160	0.0822	0.0969	0.1049
23	180.78	0.1188	0.1188	0.0851	0.1001	0.1079
24	188.64	0.1233	0.1110	0.0880	0.1030	0.1101
25	196.50	0.1258	0.1129	0.0900	0.1049	0.1120
26	204.36	0.1288	0.1150	0.0922	0.1070	0.1140

Table #2

Data of Fixed-end Supported Slab

Load Height Inches	Lb/ft^2	Center Point	Model Deflections (Inches) Quarter Points #1	#2	#3	#4
0	0.00	0.0000	0.0000	0.0000	0.0000	0.0000
1	7.86	0.0012	0.0010	0.0010	0.0008	0.0008
2	15.72	0.0030	0.0023	0.0026	0.0018	0.0019
3	23.58	0.0050	0.0040	0.0039	0.0030	0.0040
4	31.44	0.0067	0.0058	0.0050	0.0045	0.0051
5	39.30	0.0083	0.0071	0.0065	0.0059	0.0069
6	47.16	0.0101	0.0090	0.0084	0.0085	0.0089
7	55.02	0.0131	0.0112	0.0109	0.0095	0.0110
8	62.88	0.0158	0.0132	0.0129	0.0111	0.0131
9	70.74	0.0182	0.0165	0.0153	0.0136	0.0160
10	78.60	0.0211	0.0190	0.0182	0.0160	0.0180
11	86.46	0.0252	0.0225	0.0221	0.0190	0.0210
12	94.32	0.0290	0.0255	0.0256	0.0220	0.0237
13	102.18	0.0320	0.0285	0.0288	0.0251	0.0267
14	110.04	0.0348	0.0315	0.0315	0.0271	0.0290
15	117.90	0.0371	0.0341	0.0330	0.0288	0.0310
16	125.76	0.0400	0.0364	0.0350	0.0310	0.0335
17	133.62	0.0410	0.0385	0.0370	0.0320	0.0350
18	141.48	0.0430	0.0400	0.0390	0.0338	0.0370
19	149.34	0.0450	0.0422	0.0411	0.0356	0.0390
20	157.20	0.0476	0.0448	0.0435	0.0370	0.0408
21	165.06	0.0499	0.0470	0.0460	0.0390	0.0425
22	172.92	0.0517	0.0493	0.0485	0.0410	0.0441
23	180.78	0.0539	0.0512	0.0506	0.0428	0.0460
24	188.64	0.0552	0.0530	0.0521	0.0440	0.0470
25	196.50	0.0572	0.0552	0.0541	0.0453	0.0489
26	204.36	0.0600	0.0585	0.0570	0.0480	0.0515

Table #3

Conversion of Data of Simple Supported Slab

Load	Deflections (Inches)		
Lb/ft^2	Quarter Points		Center
	Model Aver.	Prototype	Prototype
0.00	0.0000	0.0000	0.0000
7.86	0.0071	0.0426	0.0474
15.72	0.0130	0.0780	0.0894
23.58	0.0177	0.1062	0.1212
31.44	0.0227	0.1362	0.1620
39.30	0.0267	0.1402	0.1920
47.16	0.0304	0.1824	0.2166
55.02	0.0348	0.2088	0.2496
62.88	0.0393	0.2358	0.2796
70.74	0.0439	0.2634	0.3162
78.60	0.0493	0.2958	0.3534
86.46	0.0527	0.3162	0.3762
94.32	0.0578	0.3468	0.4128
102.18	0.0615	0.3690	0.4380
110.04	0.0666	0.3996	0.4794
117.90	0.0705	0.4230	0.5058
125.76	0.0742	0.4452	0.5370
133.62	0.0783	0.4698	0.5700
141.48	0.0815	0.4890	0.5940
149.34	0.0846	0.5076	0.6138
157.20	0.0876	0.5256	0.6372
165.06	0.0917	0.5502	0.6660
172.92	0.0947	0.5682	0.6894
180.78	0.0977	0.5872	0.7128
188.64	0.1004	0.6024	0.7398
196.50	0.1023	0.6138	0.7548
204.36	0.1044	0.6264	0.7728

Table #4

Conversion of Data of Fixed-end Supported Slab

Load	Deflections (Inches)		
	Quarter Points		Center
Lb/ft^2	Model Aver.	Prototype	Prototype
0.00	0.0000	0.0000	0.0000
7.86	0.0009	0.0054	0.0072
15.72	0.0022	0.0132	0.0180
23.58	0.0037	0.0222	0.0300
31.44	0.0051	0.0306	0.0402
39.30	0.0066	0.0396	0.0498
47.16	0.0085	0.0510	0.0606
55.02	0.0107	0.0642	0.0786
62.88	0.0126	0.0756	0.0948
70.74	0.0153	0.0918	0.1092
78.60	0.0178	0.1068	0.1266
86.46	0.0212	0.1272	0.1512
94.32	0.0242	0.1452	0.1740
102.18	0.0273	0.1638	0.1920
110.04	0.0298	0.1788	0.2088
117.90	0.0317	0.1902	0.2226
125.76	0.0339	0.2034	0.2400
133.62	0.0356	0.2136	0.2460
141.48	0.0374	0.2244	0.2580
149.34	0.0295	0.2370	0.2700
157.20	0.0415	0.2490	0.2856
165.06	0.0436	0.2616	0.2994
172.92	0.0457	0.2742	0.3102
180.78	0.0476	0.2856	0.3234
188.64	0.0490	0.2940	0.3312
196.50	0.0509	0.3054	0.3432
204.36	0.0537	0.3222	0.3600

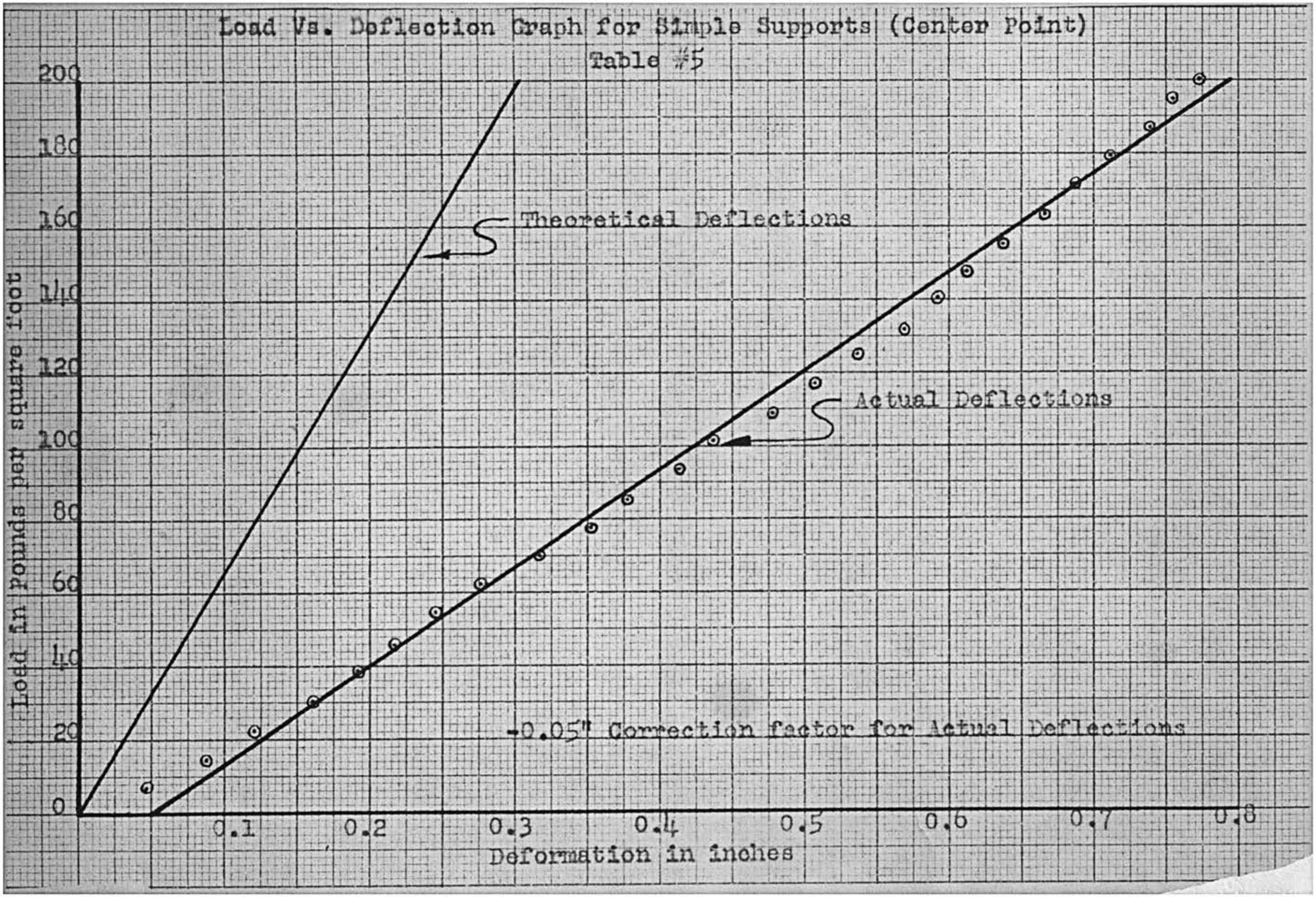
EXPLANATION OF GRAPHS

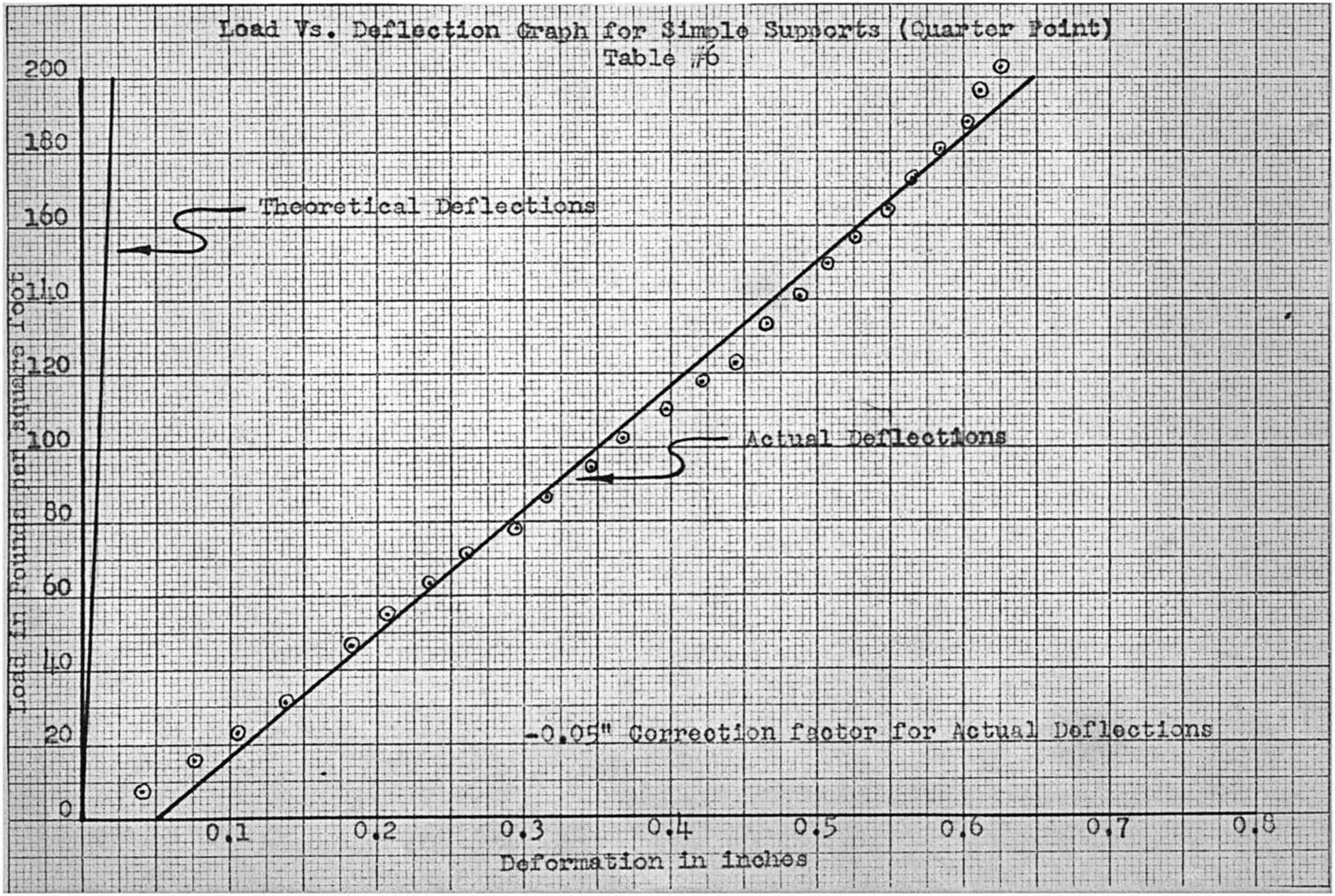
The following graphs have been drawn from data obtained in the actual tests and theoretical calculations. The graphs of load versus deflection were used to several advantages:

a) The gross errors of observations have been detected and eliminated.

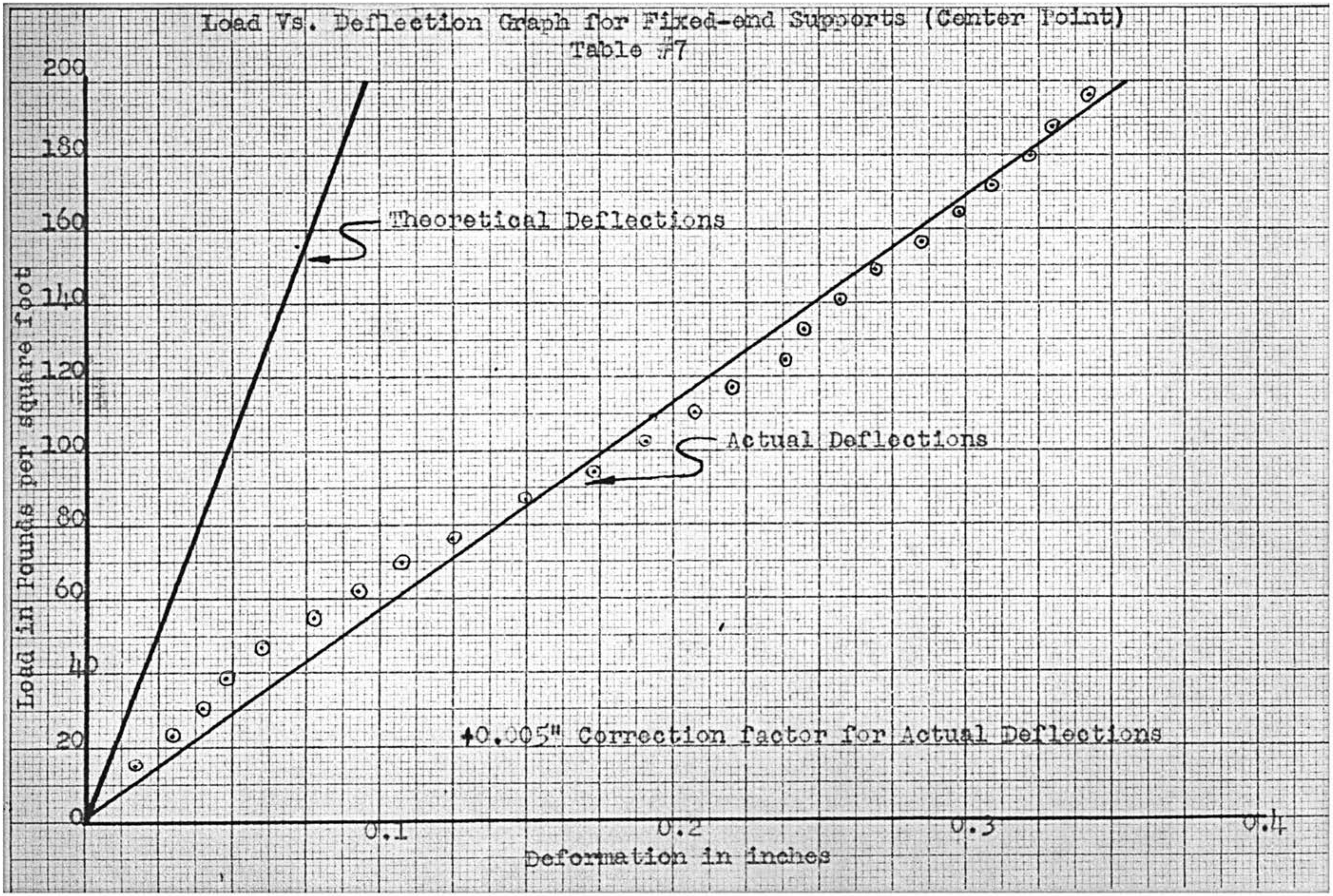
b) The errors due to an initial lag or slippage of dial indicators were detected and eliminated. Each graph has a correction factor for actual deflections, which has to be taken into consideration on each reading of the actual deflections.

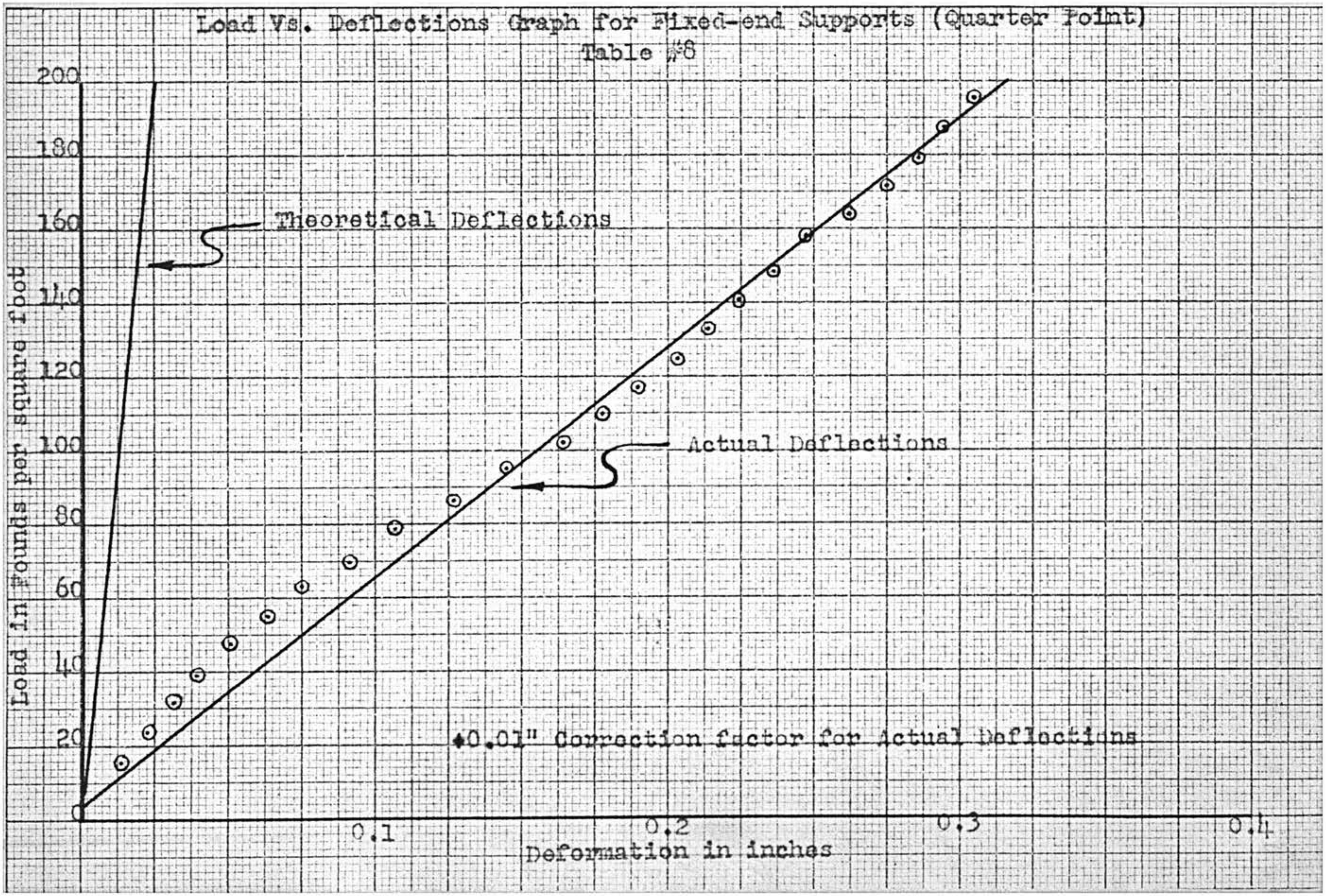
c) It was not necessary to apply the exact required load to the model, for any required deflection could be found by interpolation of the graphs.


Load Vs. Deflection Graph for Simple Supports (Center Point)
Table #5
Theoretical Deflections
Actual Deflections
-0.05" Correction factor for Actual Deflections
Load in Pounds per square foot
Deformation in inches
0
20
40
60
80
100
120
140
160
180
200
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8



Load Vs. Deflection Graph for Simple Supports (Quarter Point)
Table #6
Theoretical Deflections
Actual Deflections
-0.05" Correction factor for Actual Deflections
Load in Pounds per Square Foot
200
180
160
140
120
100
80
60
40
20
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Deformation in inches


Load Vs. Deflection Graph for Fixed-end Supports (Center Point)
Table #7
Theoretical Deflections
Actual Deflections
+0.005" Correction factor for Actual Deflections
Load in Pounds per square foot
Deformation in inches
0
20
40
60
80
100
120
140
160
180
200
0.1
0.2
0.3
0.4


Load Vs. Deflections Graph for Fixed-end Supports (Quarter Point)
Table #8
Theoretical Deflections
Actual Deflections
+0.01" Correction factor for Actual Deflections
Load in Pounds per square foot
200
180
160
140
120
100
80
60
40
20
0
0.1
0.2
0.3
0.4
Deformation in inches

DISCUSSION OF RESULTS

The results of comparing the theoretical and the actual deflections brought a large range of difference in the center and quarter point deflections for each slab. These differences will first be discussed for each individual slab and then the list of theoretical, model study, slab design, material, and instrumentations reasons will be brought forth. These reasons will be qualitative rather than quantitative for it would be impossible to list numerically the effect of each.

Simply Supported Slab:

The results of the simply supported slab test agreed the closest to the theoretical results of the two tests made. The actual center deflections and the theoretical center deflections had an average difference of 59 percent and the quarter points differed by 67 percent average. The test procedure on this slab ran rather well with the exception of dial #1 fluctuating too high and low, and had to be disregarded in averaging the quarter point readings. The differences per unit load for center and quarter point deflections can be obtained from table #9.

Fixed-end Supported Slab:

The results of the fixed=end supported slab varied from the theoretical results largely. The difference of actual and theoretical center point deflections had an averate of 77.5 percent difference, and the quarter points differed by 90.8 percent average. The reason for this large difference is obviously due to the means of fixing the slab in order to avoid rotation at the edges. This rotation obviously took place since the quarter point indicators recorded a large rotation, or deflection. After the initial test was made for deflections, a second test was made with the dials set up on the edge beam to record any rotation of the edge beam; however, no rotation was observed. The test procedure ran rather well with the quarter point dial indicators agreeing fairly closely. The differences per unit load for the center point and quarter point can be found on table #10.

Theoretical Formulas:

In deriving the theoretical deflections several assumptions were made which are difficult to satisfy. One assumption was that the middle of the slab is the neutral axis. If the slab is bent to shift this middle surface, the middle undergoes some stretching during bending and the theory of pure bending developed previously will be accurate only if the middle remains neutral. If this neutral axis shifts, the deflections diverge from the theoretical calculations.

Model Study:

When a model has a simple form the number of geometrical proportions needed for its description is small, but in order to accurately specify the shape of these dimensional models a large number of proportions are needed. If these proportions are chosen at random, without any guiding theory or experience, it is impossible to say which of them will be dominant. A model made with the aid of a template may seem to be of the right shape with a fair degree of accuracy; however, the errors in some proportions, needed for the accurate description of the shape, may be large while those with others may be small. If it happens that great accuracy is attained in the case of important proportions and low accuracy in the case of proportions which are really dominant, tests with the model will give misleading results. Every error of measurement is enlarged proportionally to the prediction equation.

Slab

The effective flexural rigidity of a slab varies along the length of a slab. Near the ends the concrete is uncracked and the effective area is the entire concrete cross section and the thickness is the total thickness (including cover under and over the steel). At the center of the slab the concrete on the tension side will be cracked to about mid-height, and the thickness and area decreases by this amount.

The arbitrary procedure in common use is to treat the slab as a member of constant cross section. The entire concrete section is used as though uncracked throughout and steel is not counted except as offsetting the cracking effect.

Material:

For a given specimen in the working stress range the value of the apparent modulus of elasticity is not constant and between different samples of the same mix, there appears to be more variation in the apparent modulus of elasticity than in the strength. The concrete strength increases in the length of time under favorable conditions with a resultant increase in the modulus of elasticity that is somewhat obscured by the creep effect.

Uniform shrinkage of plain concrete will not produce warping or curvature but the usual reinforced concrete member is reinforced unsymmetrically on the two faces. Since the reinforcement resists shrinkage, the effect of positive moment steel is to reduce this shortening on the bottom of the member and its eccentric action causes extra shortening on the top of the member. Thus, shrinkage causes deflection in the same direction as external moments.

Instrumentation and Loading:

As a whole, the instrumentation of both tests were satisfactory, with the exception of dial indicator #1 on the simply supported slab. As mentioned previously, measurements of the dial indicators are within 0.001 of an inch and estimates were made to one additional unit. Since simultaneous readings could not be taken, the deflection recorded could vary due to creep; however, creep is a factor of time measured in large quantities, as days and months, this could not effect the readings a great deal. The dials were zeroed at the beginning of the runs, the deflections due to dead load could be disregarded.

Interpretation of Data:

Tables 9 and 10 show the deflections and differences for the simple and fixed-end supported slabs, respectively. The deflections of the experimental slabs ran higher than the deflections obtained by theoretical calculations.

Table #9

Data Interpretations for Simple Supported Slab

Load	Deflections (Inches)					
	Actual		Theoretical		Percent Difference	
Lb/ft^2	Center	Quarter	Center	Quarter	Center	Quarter
0	0.0000	0.0000	0.0000	0.0000	00.0	00.0
10	0.0350	0.0280	0.0151	0.0098	56.9	65.0
20	0.0700	0.0600	0.0302	0.0195	56.9	67.5
30	0.1100	0.0880	0.0453	0.0293	58.8	66.7
40	0.1450	0.1180	0.0604	0.0390	58.4	67.0
50	0.1830	0.1470	0.0755	0.0488	58.7	66.9
60	0.2200	0.1770	0.0906	0.0586	58.8	66.9
70	0.2590	0.2080	0.1057	0.0683	59.2	67.2
80	0.2950	0.2370	0.1208	0.0781	59.1	67.0
90	0.3330	0.2670	0.1359	0.0878	59.3	67.2
100	0.3710	0.2970	0.1510	0.0976	59.3	67.1
110	0.4080	0.3260	0.1661	0.1074	59.3	67.1
120	0.4460	0.3570	0.1812	0.1171	59.4	67.2
130	0.4830	0.3870	0.1963	0.1269	59.4	67.2
140	0.5210	0.4160	0.2114	0.1366	59.4	67.2
150	0.5580	0.4450	0.2265	0.1464	59.4	67.1
160	0.5940	0.4750	0.2416	0.1562	59.3	67.1
170	0.6320	0.5050	0.2567	0.1657	59.4	67.1
180	0.6680	0.5360	0.2718	0.1757	59.4	67.2
190	0.7050	0.5650	0.2869	0.1854	59.3	67.2
200	0.7440	0.5950	0.3020	0.1952	59.4	67.2

Table #10

Data Interpretations for Fixed-end Supported Slab

Load	Deflections (Inches)					
	Actual		Theoretical		Percent Difference	
Lb/ft^2	Center	Quarter	Center	Quarter	Center	Quarter
0	0.0000	0.0000	0.0000	0.0000	00.0	00.0
10	0.0130	0.0070	0.0047	0.0013	63.8	81.4
20	0.0320	0.0230	0.0095	0.0026	70.3	88.7
30	0.0500	0.0380	0.0142	0.0038	71.6	90.0
40	0.0675	0.0550	0.0190	0.0051	71.8	90.7
50	0.0850	0.0720	0.0237	0.0064	72.1	91.1
60	0.1030	0.0875	0.0284	0.0077	78.1	91.2
70	0.1200	0.1030	0.0332	0.0090	72.2	91.3
80	0.1390	0.1190	0.0379	0.0102	72.8	91.4
90	0.1570	0.1360	0.0427	0.0116	72.8	91.5
100	0.1750	0.1515	0.0474	0.0128	72.9	91.5
110	0.1920	0.1675	0.0521	0.0141	72.8	91.6
120	0.2100	0.1840	0.0569	0.0154	72.9	91.6
130	0.2280	0.2000	0.0616	0.0166	72.9	91.7
140	0.2470	0.2160	0.0664	0.0179	73.1	91.7
150	0.2640	0.2320	0.0711	0.0192	73.1	91.7
160	0.2825	0.2475	0.0758	0.0205	73.2	91.7
170	0.3000	0.2630	0.0806	0.0218	73.1	91.7
180	0.3180	0.2800	0.0853	0.0230	73.2	91.8
190	0.3360	0.2970	0.0901	0.0243	73.2	91.8
200	0.3540	0.3120	0.0948	0.0256	73.2	91.8

RECOMMENDATIONS

The author believes that more tests should be run in order to bridge the gap of theoretical results with actual existing conditions. With a large, long series of results the average, or the mean deficit, c ould be found and used to modify theoretical deflection to fit the experimental results obtained from reinforced concrete slabs.

One of the few improvements that can be employed is a more balanced design between the materials of the slab (mortar and steel) for this perfect design is assumed in the theoretical calculations.

Better means of measurement should be obtained in the construction of the model and the placement of the steel in the form. Each error is magnified by the conversion factor, which in this case was six.

A better means of clamping the edges of the fixed-end supported slab should be used, rather than a bulky edge beam. In the authors opinion this edge beam simulated actual practicing conditions. Many designed reinforced concrete slabs are tied into smaller edge beams and the slabs are considered fixed.

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VITA

Gerald C. Godzwon, son of Mr. and Mrs. Theodore P. Godzwon, was born September 1, 1934, in Chicago, Illinois.

He received his early education in various schools throughout the United States and South America. His high schools days were spent at Kemper Military School in Boonville, Missouri.

In September, 1953, he entered the Missouri School of Mines and Metallurgy as a freshman in Civil Engineering, and received his B. S. C. E. in May, 1957.

He accepted employment with Boeing Airplane Company of Seattle, Washington, in their Structural Test Unit in August, 1957. He left Boeing in October, 1957, to report for Active Duty in the U. S. Army at Fort Belvoir, Virginia, as a Second Lieutenant. His next two y ears were spent at Fort Leonard Wood, Missouri, as a First Lieutenant (1/Lt. final rating) as Company Commander of an Engineer Basic Training Unit. While in the Army, he married Miss Yvonne Ellis of Fredericktown, Missouri.

Following his discharge from Military Service in October, 1959, he re-entered the School of Mines for work toward his M. S. degree in Civil Engineering.