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Effects of solids on friction factor in a sand-water mixture

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EFFECTS OF SOLIDS ON FRICTION FACTOR IN A SAND-WATER MIXTURE

BY

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A

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ABSTRACT

At the present time various industries in both the United States and Europe are very interested in using pipelines as a means of transporting solids. The pumping of solids in pipelines isn't new, however, many of the pipelines in use today were constructed on an empirical basis. This is primarily due to the lack of knowledge concerning the flow properties in fluid-solid mixtures and slurries. With this need for fundamental information in mind, the author made a study on the effect solids have on the friction factor in fluid-solid mixtures.

In this paper various percentages of fine and coarse sand were combined with water and the resulting change in friction factor noted. A correlation of percent deviation of friction factor with percent solids was presented. The stated percent deviation is a ratio of the difference between the friction factor of the mixture and the corresponding friction factor for water. Also a relationship between apparent viscosity and percent sand is shown.

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INTRODUCTION

The pumping of solids or the flow of fluid-solid mixtures through pipelines presents the hydraulics engineer with an interesting and challenging problem. With modern factories and plants being at widely diversified locations from their sources of raw materials, there is a demand for a better and more economical means of transporting the raw materials. Homogeneous fluids and liquids such as water, natural gas, steam, fuel oils, etc., have been pumped through pipelines for some time. The pumping of fluid-solid mixtures is gaining prominence in the United States and other countries as being a better and more economical means of transportation for many materials. The study of this method of transportation is presenting many new problems to the engineer.

Pumping solids or the transporting of fluid-solid mixtures through pipelines is not a new concept. Though not new, the pumping of solids is not as universal as the pumping of homogeneous fluids or solutions such as water, natural gas, steam, crude oil and various other products. The principal reason being, it is much more difficult to pump fluid-solid mixtures than those which are homogeneous due to the tendency the solid particles have to settle out. Until the last few years the other means of transporting solids have been more economical. The transporting of solids such as calcium chloride, coal and other bulk solids has proven to be somewhat more difficult than that of homogeneous liquids.

Many proposed pipelines for transporting solids, while technically feasible, were not sufficiently attractive economically to warrant the financial risk involved. In many instances, the transportation cost differential between pipelines and other transportation means was negligible. In other cases the threat of pipeline competition resulted in the lowering of prevailing rates on existing transportation facilities and in many instances the rates were reduced to such a point that the pipeline was no longer economically justified.

The earliest recorded date of fluid-solids transportation (1)* was in the hydraulic or placer mining operations in the late 1850's in California. In these early operations when there was not sufficient drop in elevation for washing the gold bearing gravel directly to the sluices to separate the gold from the sand, hydraulic elevators were used to raise the water and gravel to a higher elevation. The hydraulic elevators forced the material from 30 to 55 feet upwards under pressure through a pipe, discharging it in the flume or sluice at the desired elevation.

The hydraulic transportation of solids in pipes is currently being employed in many fields, especially that of Civil Engineering. Sand and gravel dredges are used to move large volumes of material from one point to another. An example of the magnitude of the dredging operations can be exemplified by the construction of the Fort Peck Dam on the Missouri River by the hydraulic fill method. According to Durand (2) this dam required the placing of 93 million cubic meters of earth at the rate of four million cubic meters per month. Another

*Numbers refer to references in bibliography.

example would be the improvements on the Delaware River in the last thirty years which have led to the dredging of over thirty million cubic meters of earth and continues to require maintenance dredging of seven and one-half million cubic meters per year.

In addition to dredging operations the pumping of fluid-solid mixtures has various other applications such as the pumping of coal, refuse, salt in the form of a saturated brine, borax slurries from mine to plant and many other commercial applications.

In November, 1951, Coal Age (3) in an article "Moving Coal to Market" compared the then current costs of what they called the "Old Methods: by rail, water or truck" with"New Methods: by Pipeline, Cross-country Belt or Pipeline Gas". Their conclusion was: "for a good many years to come, the bulk of the coal shipments doubtless will keep on moving by rail ... not because it is cheaper (water transportation is the cheapest, but is limited as to its availability) ... but becuase it would take a long time and lots of money to build another transportation network that would equal the railroads".

In the last few years many pipelines have been built such as one at Bonanza, Utah in which gilsonite is pumped at the rate of 50 tons per hour out of the mines and then 72 miles across a mountain range to a refinery in Colorado.

In August, 1957 the Soviets in their coal magazine UGOL (4) (pp. 38-40) claimed "the first long distance coal pipeline". This pipeline was 38 miles long, the diameter of the pipe was 12 inches and the line had a capacity of 220 tons per hour of minus 1/32-inch coal in a 38 percent solids by volume slurry. The velocity of the slurry was 4.8 feet/second and the maximum pump discharge pressure was 1,000 pounds per square inch gage. They claimed the pipeline would transport the coal three times cheaper than by railroad. Thus, it appears that the transportation cost by pipeline is about one-third that of railroad transportation. It was also stated that this one single pipeline will release 25,000 railroad coal cars for other services. To further illustrate the possible cost advantages of fluid solid transportation the relative costs of solids transported by various means can be compared (5); a cost of one-half cent per ton mile for pipeline transportation; truck haulage under the same conditions would be six to eight cents per ton mile, and if rail were available, the cost might be three or four cents per ton mile.

To date, most pipelines for fluid-solid mixtures have been designed by empirical means for individual cases rather than by a general solution of the problem. In most cases each proposed pipeline has some different and difficult engineering problems. These problems are not only associated with the pumping of the fluid-solid mixture but also with the inlet and exit characteristics. In most cases one of the greater problems is the processing of the raw material, and transforming it into a transportation slurry. In some instances the proper processing of the material adds enough value to the product to more than pay for the preparation cost. In other cases, the preparation of the slurry is an added expense similar to moving an empty truck, moving empty railroad cars or the added cost of a railroad's switching yard facilities.

It is known that most fluid-solid mixtures tend to act somewhat the same when being pumped through pipelines, but to date there is not a general solution for the design of pipelines used in the transportation of fluid-solid mixtures. The design of the pipeline is predominantly dependent upon the velocity of the fluid-solid mixture and the friction losses developed in the pipeline. The required velocity of a fluid-solid mixture is dependent upon the minimum carrying velocity of the fluid. The velocity for any one solid material is principally dependent upon the screen analysis of the solids used. From Stoke's law it is evident that a sand pebble will tend to fall faster than a small grain of sand. In a pipeline the turbulence of the fluid tends to keep the solids in suspension and as the particle size increases, the velocity of the fluid must also increase. The velocity must also be increased as relative specific gravity of the solids with respect to the specific gravity of the fluid is increased. It has been found that by adding slimes or colloidal materials or both to a fluid-solid mixture it will affect the required velocity by changing the relative specific gravities of the mixture. It has been determined that in order to have the same carrying capacity per unit area, the velocity of the slurry must be increased as the diameter of the pipe increases. However, these are not in a direct ratio and in most cases they seem to vary somewhat with the circumstances.

There are a number of conditions which must be favorable (6) prior to considering a pipeline for a source of transporting fluid-solid mixtures. The following are a few of the conditions to be considered:

1. An ample water supply to operate the pipeline.

2. A relatively large tonnage of dry solids to be transported.
3. The necessary right-of-way for the pipeline must be readily available.
4. The material or product must not be damaged by the water or transportating media.
5. Some degradation of the material must be permissible.
6. Freezing of the line must be unlikely.

To cite an example of a situation in which the feasibility of a fluid-solid pipeline should be studied is the Pea Ridge Iron District of Missouri. Mines in this area are remote from cheap water transportation and also have a dewatering problem. It is possible that a fluid-solid pipeline could help solve the water removal problem while providing a cheap means of transportation.

In spite of the many instances of usage of fluid-solid pipelines a recent request for data and friction losses involved in pumping of fluid-solid mixtures, by the Hydraulic Institute, New York, New York (7) indicated that the supply of published information on this subject was very limited. With this in mind the purpose of this thesis shall be to investigate the problem of friction factor variation with various water-sand mixtures.

REVIEW OF LITERATURE

The origin of the science of hydraulics dates back to Biblical times, but its growth was slow and spasmodic until the seventeenth century. During this period such men as Galileo, Guglielmini, Huygens, Newton, Pascal and Torricelli (8) contributed fragments of knowledge on the subject and helped develop a fundamental understanding of the basic hydraulic principles.

As a result of the work of Bernoulli, Lecchi, Pitot and Poleni, further developments in the field of hydraulics were made. It should be noted up to this time the bulk of the work in the hydraulics field was of a theoretical nature. In 1774 Bossut and Turin established as a fundamental principle, that empirical formulae must be deduced from experiment. Prior to this time little effort had been made to correlate the theoretical with the experimental.

From this early period to the present, the growth in the science of hydraulics has been phenomenal. Countless contributions to science were made by men, primarily of European extraction during the eighteenth Century. In about the middle of the nineteenth century the following equation to be used in pipe flow came into use:

$$H_f = f \frac{L}{D} \frac{V^2}{2g}$$

Credit for its origin is given to Darcy, Weisbach, Fanning or Eytelwein by present day authors and is most widely known as the Darcy-Weisbach equation and will be so referenced in this paper.

Following the latter part of the nineteenth century, during which time such men as Stokes and Reynolds were doing their work, there was a noticeable split in the manner in which hydraulic pipe problems were treated. On one hand there were individuals like John R. Freeman, Hiram Mills and Hamilton Smith, Jr. who were leaders in the determination of friction factors and coefficients from experimental data. The work of these men was widely accepted and much used by the practicing engineer. During the twentieth century, this work was continued by Schoder and Scobey of this country, and others. These men did not work solely on the determination of a friction factor or chezy coefficient, but instead used their experimental findings as a basis for the development of so called exact or exponential type formula. On the other hand, such men as Bakhmeteff, Blasius, Prandtl, Rouse and Schiller appeared to favor a theoretical treatment of hydraulics.

A parameter which has proven to be a lever in the further development of pipe flow theory and practice was developed by Osborne Reynolds from his classic experiments performed in 1883. The parameter carries his name and is defined by the equation:

$$N_R = \frac{DV\rho}{\mu}$$

Stanton and Pannell of the National Physical Laboratory in London, England utilized Reynolds number and put it in a usable form in 1914. They developed the much used curve found by plotting experimental data and correlating Reynolds number with friction factor. Since this time, scores of engineers have studied and written on this relationship. Engineers soon found that the relative roughness of the pipe also affected the friction factor determination. They plotted new curves from experimental data and found that most of these paralleled the Stanton and Pannell curve in the turbulent flow regions.

Since 1930 many laboratory experiments have been run on the roughness effect of pipes on fluid flow. Nukuradse, who published his findings in 1933, found that in the turbulent flow region an increase in the relative roughness of the pipe caused a corresponding increase in friction factor. Somewhat similar experiments were conducted by Streeter, who published his work in 1935, with similar findings.

Fluids are distinguished from solids by the fact that the latter possesses both rigidity and elasticity whereas the former possesses only elasticity. When a solid is acted upon by an external shear force, it yields to the force and will deform till such time that the internal forces establish equilibrium, and thereafter no further deformation occurs. On the other hand, when an external shear force acts on a fluid, it yields to the force, or flows and continues to flow as long as the force continues to act.

It has been found, however, that when a tangential force exists and flow takes place, the rate at which the fluid yields to force varies for different fluids. For example, water flows much more readily than tar. That property of fluids, by virtue of which they are able to offer resistance to a tangential force is called viscosity. Viscosity is that property or quality of gaseous or fluid bodies, resulting from molecular attraction, which makes them offer a resistance to flow. An indirect means of measuring the viscosity of liquids is with one of the many types of viscosimeters commercially available.

Prior to making any further analysis of the literature, the writer believes it prudent to define and summarize some of the terms and principles that will be encountered.

The absolute coefficient of viscosity, μ , is defined as the necessary force required to move a flat surface of unit area at a unit relative velocity parallel to another surface a unit distance away, with the space between the surface filled with the fluid. The units of viscosity are expressed in foot-pound-second units, is expressed in pound seconds/square foot or slugs/foot second.

The term kinematic viscosity, ν , is the recurring ratio of absolute viscosity of a fluid to its density, and is expressed as square feet per second.

The density, ρ , may be defined as the mass of fluid per unit volume. The dimensions of ρ are pound seconds squared per foot to the fourth power.

The specific weight, γ , is defined as the weight of fluid per unit volume and is expressed as pounds/cubic foot.

Specific Gravity, G, may be defined as the ratio of the density or specific weight of a substance to the density or the specific weight of pure water at some specified temperature.

Reynolds Number, N_R, is a dimensionless number that expresses a ratio of inertia to viscous forces.

The friction factor, f, may be described by the Darcy-Weisbach equation for head loss due to friction. The Darcy-Weisbach equation:

$$H_f = f \frac{L}{D} \frac{V^2}{2g}$$

where

H_f = Head loss in feet of flowing fluid

L = Length of section experiencing the loss in head, in feet

D = Diameter of the pipes in feet

V = Average velocity of the fluid in feet per second

g = Acceleration due to gravity, 32.2 feet per second squared

f = Friction factor of the pipe

It has been found from both theory and experimentation that the apparent viscosity of a fluid-solid system is increased over that of fluids alone. This fact has been used, at times, as the basis for the view that friction losses of fluid-solid mixtures will be greater than those of real fluids, at equivalent velocities. It is well to remember at this point, that when water is used as the fluid media, the flow is usually well up in the turbulent region, where viscosity has little affect on the friction factor. It is very possible that with sand or other materials in suspension, the laminar sublayer attached to the wall may tend to be broken up. These particles may also add to the turbulence of the fluid.

Reynolds in 1901 first stated the basic principles of the modern theory of turbulent motion. G. I. Taylor, Karman and Prandtl have since amplified and made his work practically useful. The phenomenon of turbulence is one of the basic factors affecting the flow of solids in fluid-solid mixtures. The practical aspect of transporting fluid-solid mixtures requires that in most cases it should be transported by turbulent flow, and this is especially true when transporting materials of the size and weight used in this study.

In the flow of fluid-solid mixtures, the solid material continually tends to settle to the bottom and would probably remain there were it not for the upward or random motion of the fluid masses, which lifts the material upward and results in a random distribution of the solids.

When a homogeneous fluid such as water passes through a long pipe, there is a velocity (v) parallel to the axis of the pipe. This velocity may be measured by a device such as pitot tube or a current meter which records the average velocity. However, in general, there are two other components of the average velocity which go unrecorded since they average out to have zero velocity parallel to the pipe. These may be designated v and w, and are perpendicular to each other and also to the axis of the pipe or the flow is parallel to the axis of the pipe.

The phenomenon of turbulence is associated with the solid walls or boundaries of pipes or other containing elements. The presence of these solid boundaries appears to be the origin of the turbulent fluctuations in the flow of fluid and fluid-solid mixtures. The results of experiments which have been conducted on viscous liquids indicate there is no relative motion at solid boundaries. It is generally believed there is a laminar layer attached to the solid boundaries even under turbulent flow conditions. This laminar layer is considered to become somewhat thinner as the turbulence increases and probably changes continually. It has been found that energy losses are converted directly into heat in these layers.

In most cases as the size of the pipe increases, there appears to be less effect due to the solid boundaries conditions. Wilson (9) indicates that an increase in the pipe size will, up to a point, reduce the expenditure of energy per pound of solids transported.

TEST PROCEDURE

The primary purpose for conducting this research problem was to study the effects of various mixtures of sand and water upon the friction losses in the pipe. Two gradations of sand (fine and coarse) were used, thus enabling a partial comparison of the effects of grain size upon resulting friction factors. The coarse sand was material that passed a standard number twenty sieve and was retained on a standard number thirty sieve. The fine sand was the material that passed a standard number thirty sieve and was retained on a standard number forty sieve.

For this study a washed river sand was purchased from the Rolla Concrete Reddi-Mix Plant, who obtained it from Pacific, Missouri.

The fine sand was roughly 75 percent rounded and frosted quartz with the remainder being angular buff to dark chocolate brown chert (10). The coarse sand contained 5 percent rounded and frosted quartz and 95 percent angular buff to chocolate brown chert. The dry rodded unit weights of the fine and coarse sand were 98 and 99 pounds per cubic foot respectively. A further characteristic of each of these materials was the bulk specific gravity for the rodded sands was 2.60 and 2.53 for the fine and coarse sand respectively. A relative comparison of the fine and coarse sand is shown in Figure 1.

The definition of the sand percentages in the fluid-solid mixtures as used in this paper were determined by using the following procedure: A representative sample of each gradation of sand was

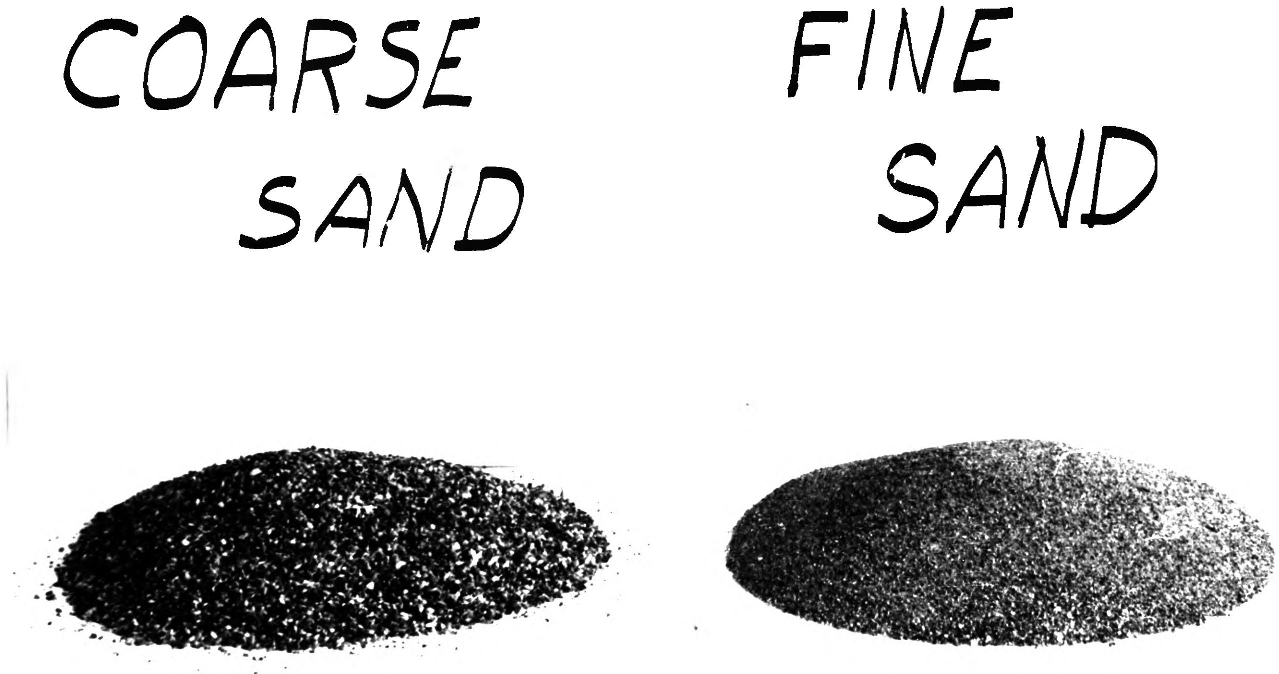

Figure 1, Fine and Coarse Sand Comparison taken and the specific gravity and unit weight were then determined. Assuming the specific gravity of the sand was uniform, one-tenth of a cubic foot of oven-dry rodded sand at a time was added to a cubic foot bucket. After each increment of sand was added the total volume was brought to one cubic foot with the addition of water filling the bucket to the top and the total weight recorded. Using this information a weight versus percentage sand curve was established for various percentages of sand, i.e., 10%, 20%, 30%, etc., as shown in Figure 2. Further study disclosed that the weight of either gradation of sand and water at the various percentage remained essentially constant, as illustrated in said Figure. The weight and volume of the sand and water collected in each test during the study were determined after each test. The weight of material collected was expanded to yield a weight in pounds per cubic foot, thus giving an equivalent weight per cubic foot. The percentage of sand in the mixture was then determined from Figure 2.

This method of determining sand percentages was chosen as the procedure for the determination of the dry-rodded volumes of sand is standard. These procedures can be duplicated and thus these experiments could be repeated. Other methods considered had various variables which made duplication of results difficult or impossible.

As the data was tabulated, it was noted that in a number of cases the first one or two runs of each series of tests produced results which were inconsistent. These runs were eliminated on the basis that the pipe had probably corroded slightly between runs. Although a quantity of sand and water was run through the pipe prior to any test runs, it was felt that the results of the first two test

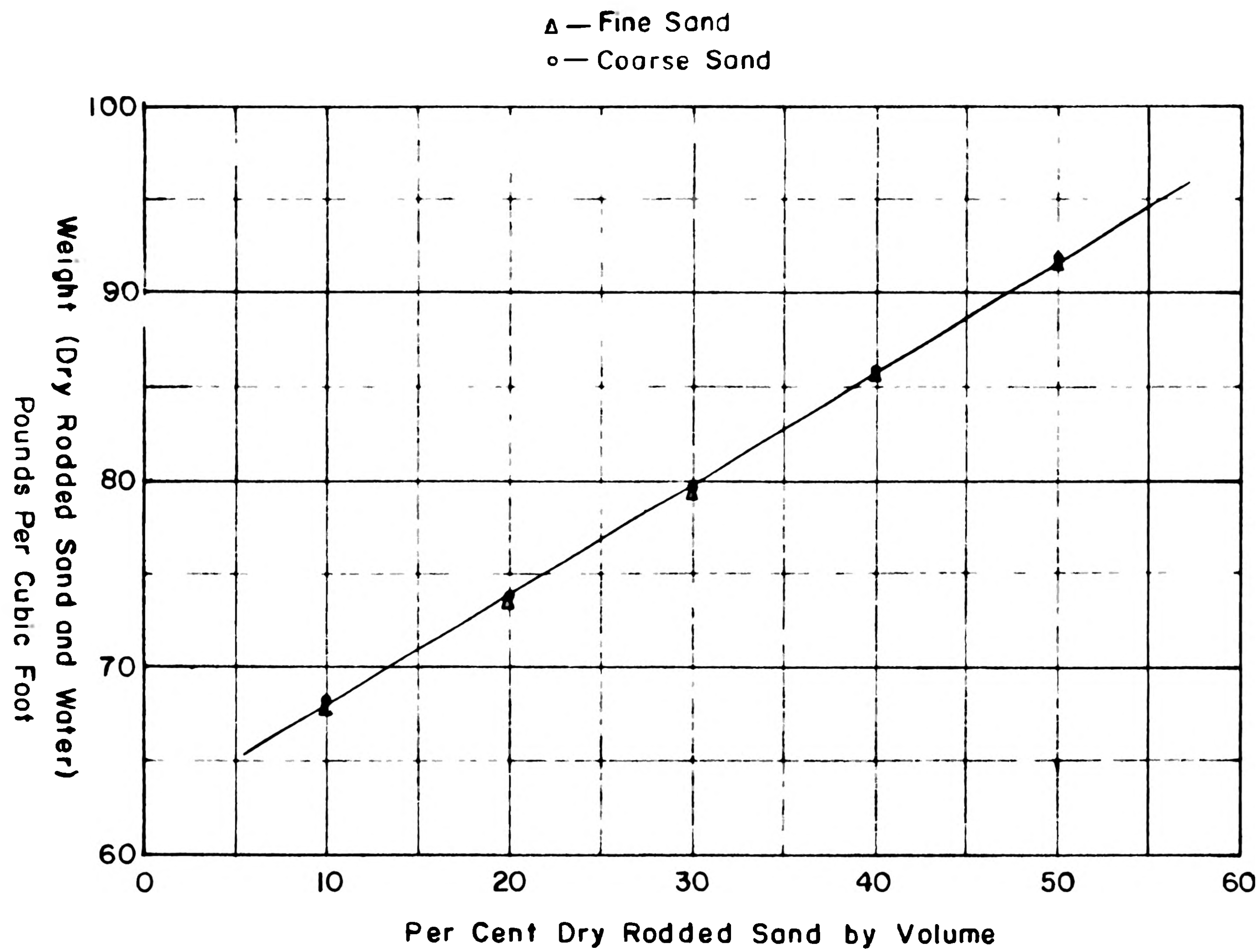

Figure 2 Weight of Mixture Correlated with Dry Rodded Sand

runs in any series should be disregarded. Other runs were eliminated due to pipe stoppages which produced obviously inconsistent results.

EQUIPMENT LAYOUT

The layout of the equipment as shown in Figure 3 consisted of a pump at one end, weight and volume determination equipment at the other end of the test apparatus. The centrifugal pump had an approximate maximum capacity of 36 gallons per minute under the operating conditions and was driven by a one-half horsepower 110 volt electric motor turning 1750 revolutions per minute. This unit was mounted in a vertical position on a steel base as shown in Figure 4 with the motor on the bottom.

A section of 3/4 inch pipe approximately 24 inches in length was connected to the pump outlet by means of a reducer. The test section of 3/4 inch nominal diameter black iron pipe was connected to the above section with a union. A mean inside diameter of the test pipe was measured and found to be 0.825 inches. The overall length of the pipe was 22.5 feet with a test section between manometer connections exactly 20.0 feet apart.

At the discharge end of the pipe a four foot section of 3/4 inch inside diameter rubber hose was slipped over the 3/4 inch test pipe. As the hose was flexible, a representative sample could quite easily be collected in a one cubic foot standard ASTM test bucket. A modified "C" clamp was used to control the flow rate of material through the pipe by flattening the hose. This method of control, rather than a standard valve, was used because it presented less pipe stoppage problems.

Figure 3, General Layout

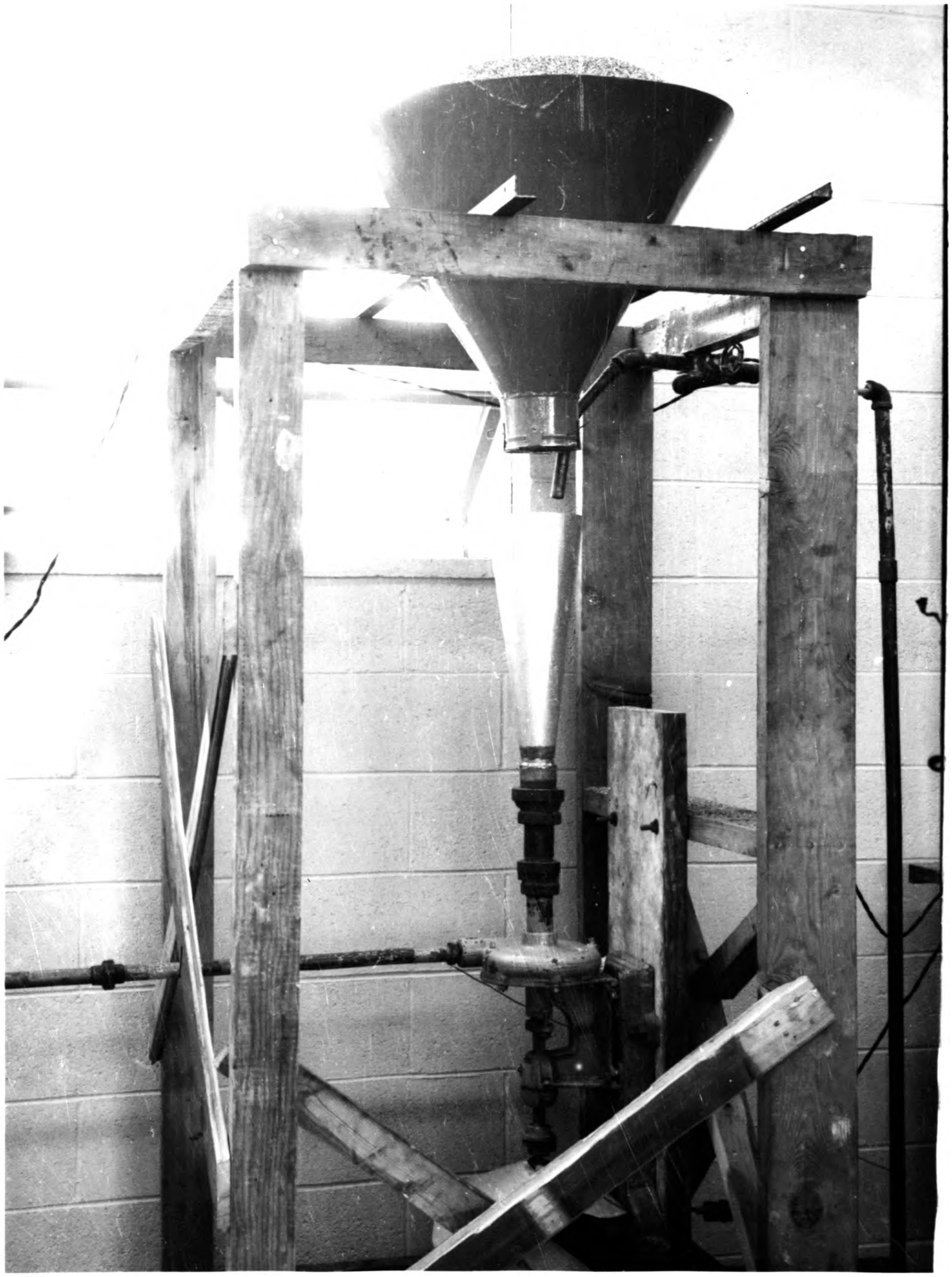

Figure 4, Side View of Pump Assembly

A Toledo scale reading directly to one-quarter pound was used at the discharge end to weigh the fluid-solid mixture collected during testing.

Above the pump the fluid was discharged into a galvanized sheet metal cone which was used as the mixing chamber for the sand and water. The top of the cone was six inches in diameter with the lower end welded to a two inch diameter steel pipe which was further reduced to fit the 1¼ inch inlet of the pump. The pump had a one inch outlet which was also reduced to fit the 3/4 inch horizontal test pipe section.

Above the mixing chamber there was a larger sheet-metal cone frustum with a top diameter of 25 inches and 2½ inches bottom diameter. A metal cylindrical sleeve was soldered to the bottom of this cone. Removable covers were placed on the sleeve with various sized openings from ½ inch diameter to a rectangular section 1½ inches by 2½ inches in size. The various sized openings served to control the volume of sand flowing into the mixing chamber. The different covers could be changed easily and quickly.

A sheet metal shield was mounted on the shaft below the pump to prevent water damage to the motor. This was necessary as the water was allowed to flow over the mixing cone to insure a uniform head on the pump. The writer noted that a great deal of sand flowed over the cone with the water. It was found that by mounting the pump in this manner a 90 degree bend in the intake pipe was eliminated. This prevented stoppages at this point due to higher-head losses in the 90 degree bend.

Once the equipment was set up, the next step was to calibrate the various parts. The zero reading of the mercury-water manometer was first checked. It was found that the plastic tubing had a tendency to develop leaks at the joints. These were eliminated by inserting copper tubing inside the plastic tubing. Leaks also occurred in the sand trap tops. The manometer bounced somewhat more at high velocities than at low velocities and consequently was somewhat more difficult to read. In all cases a mean reading was taken. The differences in height were read in centimeters and tenths of centimeters or millimeters and then reduced to feet of mercury by dividing by 2.54 times 12. This was converted into feet of water by multiplying by 12.56, since the specific gravity of mercury is 13.56 and that of water is unity giving a specific gravity differential of 12.56.

The cone was filled from the school water supply system by means of a 3/4 inch pipe with two gate valves. In most cases one valve was used to adjust the flow to the desired rate while the other was used to open and close the line. When high percentages of sand were run, it was found that in order to prevent clogging, the hose valve at the end of the test pipe must remain unclamped. During these periods the 3/4 inch water supply was inadequate and therefore was supplemented with a 5/8 inch hose supply. The intake water valve was then adjusted so that the mixing cone would just overflow, thus maintaining a constant head on the pump intake.

QUANTITIES MEASURED

Since the purpose of this research problem was to study the effects various percentages of sand had on the friction factor of the pipe, when sand quantities were being transported, the following quantities were measured: the weight of the fluid-solid mixture flowing through the pipe, the head-loss on the length of the test section, the time involved per unit weight of discharge, the discharge volume and the temperature of the fluid-solid mixture.

MEASUREMENT OF HEAD-LOSS

When making pipe flow determinations of various types, one of the simpler and more accurate means of securing data is through the medium of head-loss measurements. In this particular case the total length of the pipe used for testing was 22.5 feet. To more nearly eliminate the inlet and exit effects, such as the union immediately upstream of the test section, the first tap as illustrated in Figure 5 for measuring the head loss, was placed 18 inches or approximately 22 internal diameters from the union. There was also a section of three quarter inch pipe three feet in length between the union and the pump. The second tap was placed 20.00 feet from the first tap, and 12 inches or approximately 15 internal diameters from the end of the test pipe section. The taps were made by welding one-quarter inch brass fittings to the pipe at the above points. The distance was re-measured and holes 1/64 inch in diameter were drilled 20.00 feet apart. By mounting the taps in this manner any change in the pipe cross section was more nearly eliminated.

The fittings were then connected to a 60 centimeter mercury-water manometer by means of a combination of plastic, rubber and copper tubing. Copper tubing was connected to the fittings and

Figure 5, Front View of Pump Assembly

sections of copper tubing were also soldered to the tops of 64 oz. glass jars which were used as sand traps, as shown in Figure 6. Rubber tubing was slipped over the copper tube connected to the brass fitting and the copper tube on the jar and finally clamped with wire to prevent leakage. Plastic tubing one-quarter inch in diameter, was used to connect the sand traps to the manometer. It was found necessary to use short sections of one quarter inch outside diameter copper tubing inside the one quarter inch plastic tubing with a common brass sleeve and cap over the plastic in order to prevent leakage at the fittings. Brass caps were used to bleed air from either side of the manometer.

The manometer connections were made through the top of the pipe as indicated in Figure 6. Precautions were taken to remove all burrs inside the pipe and also to have the holes at right angles to the axis of the pipe.

DISCHARGE WEIGHT

Prior to collecting any one sample the fluid-solid mixture was allowed to become uniform by means of a bypass collection screen. The mixture was simply allowed to pile up on a drainage screen, a portion of which may be seen in Figure 7, until such time as the mix became uniform. By moving the discharge hose a slight amount a sample could be collected on the scale in the one cubic foot bucket. Before and after each run the zero reading on the scale was checked. A springless Toledo scale which was read to the nearest quarter pound was used and most samples weighed between fifty and sixty pounds. The sand had a tendency to stay in the bottom of the test bucket when

Figure 6, Manometer Connection and Sand Trap

Figure 7, Weighing Apparatus

it was emptied, therefore, it was necessary to wash the inside and outside of the bucket after each test. Immediately prior to each test the tare weight of the bucket was zeroed on the scale. The bucket was a standard one cubic foot bucket as described in A.S.T.M. C-29.

TIME MEASUREMENT

A stop watch with a least reading of two tenths second was used to record the duration of each test. The stop watch was started at the time the hose was switched from the bypass screen to the sample bucket and stopped when the hose was switched back to the bypass screen. It was found the hose helped materially in obtaining more consistent results than with the use of other methods. The hose was more flexible and thus could be moved more easily in accordance with starting and stopping of the stop watch. Other methods tried were to open the valve or move the bucket, neither method proves satisfactory.

MEASUREMENT OF DISCHARGE VOLUME

The discharge volume was measured with a calibrated brass rod with a hook on the end as shown in Figure 8. The one cubic foot bucket had vertical sides, 11.25 inches high. The rod had marks every 0.1125 inches which enabled a volume determination to the nearest one hundredth cubic foot. The square brass rod was tapped, threaded and a wing screw was inserted as shown in Figure 8 to hold the rod in any desired position. The zero point on the rod was equal to one cubic foot and volumes down to five tenths cubic foot could be read in this fashion. The volume of most samples was seven or eight tenths cubic foot.

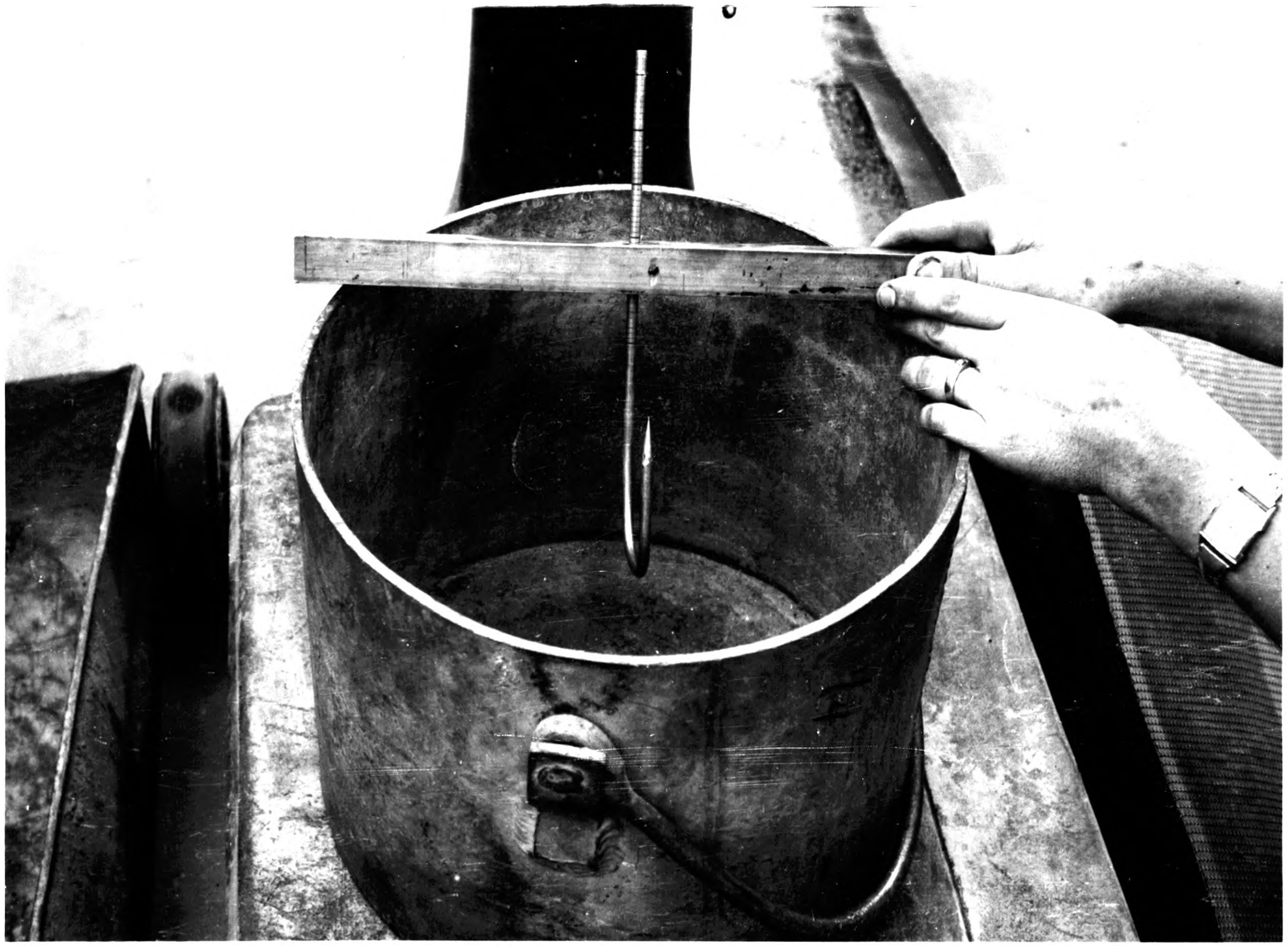

Figure 8, Volume Determination Apparatus

The temperature of the fluid-solid mixture was read immediately after the weight measurement. The temperature was read to the nearest degree fahrenheit.

The flow-rate through the pipe for each run was computed by dividing the volume obtained from the hook gage reading divided by the elapsed time, which would give the flow rate in cubic feet per second. Since:

$$Q = \frac{\text{Volume}}{\text{Time}}$$

and

$$V = \frac{Q}{A} = \frac{4Q}{\pi d^2} = \frac{Q}{.00371}$$

where Q = flow rate in cubic feet per second, v = velocity in foot/second, A = area in square feet and d = diameter of the pipe in feet (0.825 inches).

DISCUSSION

To have a better understanding of fluid flow one should first review some of the basic relationships of both turbulent and laminar flow. It is desirable to obtain a dimensionless quantity to relate the constants in a particular fluid to variables such as percent solids. One of the most well known and more commonly used means of making a comparison of various types of flow is by the use of Reynolds number which is particularly adaptable in this study and therefore requires further consideration. In the special case of enclosed flow such as pipes, only inertia forces and viscous forces need to be taken into account for dynamic similarity.

The inertia force per unit volume is a measurement of mass times acceleration, or

$$\text{volume x density x } \frac{\text{velocity}}{\text{time}}$$

$$F_i = Ma$$

or

$$F_i = \rho \frac{dv}{dt}$$

since ds = vdt one finds that

$$F_i = \rho (v) \frac{dv}{ds}$$

To derive the action of friction or viscous forces on a unit volume one can use an element of volume, d, with an assumed horizontal direction of motion of unit length as shown in Figure 9.

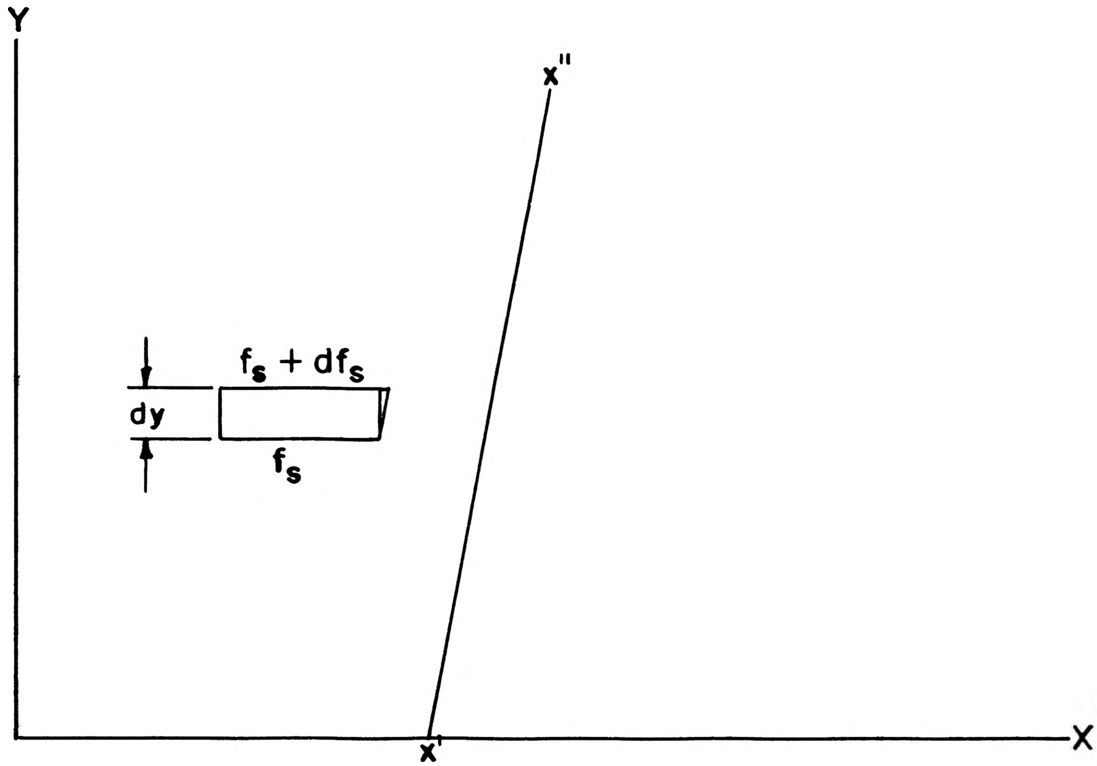

Figure 9

The element may be considered to have unit width and a differential height dy. The velocity of the fluid can be assumed to vary similar to the x values of the line x'-x'' in Figure 9. Thus the shearing force on the top surface is somewhat greater than that on the bottom surface. If f_s represents the shearing force on the bottom, the force on the top may be represented by $f_s + df_s$. Therefore, there is a resultant shear on the mass which may be represented by the difference in the two forces (df).

The length and width were considered unity and thus the volume may be represented as dy. The shearing intensity per unit volume may be expressed as:

$$F_f = \frac{df_s}{dy}$$

The basic equation for the coefficient of viscosity based upon Newton's Law is

$$f_s = \mu \frac{dv}{dy}$$

This expression indicates a relationship between shear rate and shear stress, where f_s is the shear stress and dv/dy is the shear rate. μ is called the coefficient of viscosity or more commonly the absolute viscosity.

Upon differentiating the above with respect to y one finds that

$$\frac{df_s}{dy} = \mu \frac{d^2v}{dy^2}$$

therefore:

$$F_f = \mu \frac{d^2y}{dy^2}$$

and since Reynolds number is the dimensionless ratio of inertia to friction forces, F_i/F_f or:

$$\frac{F_i}{F_f} = \frac{\rho v\ (dv/ds)}{\mu\ (d^2v/dy^2)}$$

Upon substituting dimensions for the derivatives, i.e., L/T for v and L for s and y one finds that:

$$\frac{F_i}{F_f} = \frac{\rho\ vL}{\mu}$$

In the case of pipe flow one may replace L with the pipe diameter d and the resultant is Reynolds number. As previously stated, Reynolds number is the ratio of inertia forces to friction forces and may be represented as:

$$N_R = \frac{Vd\rho}{\mu}$$

or

$$N_R = \frac{dV}{\nu}$$

when μ/ρ equals the kinematic viscosity ν. Substituting the units in Reynolds number one finds:

$$\frac{\text{ft./sec.} \quad \text{ft.} \quad \#\text{sec}^2/\text{ft.}^4}{\#\text{sec/ft.}} = \text{dimensionless number.}$$

There are several methods which may be used when making an analysis of data of this type. One method to consider would be the so-called modified Reynolds number. This method has been used successfully in the past with Bingham plastic type non-Newtonian fluids. These fluids are usually only considered in the laminar flow region. Under these conditions the modified Reynolds number has proved effective as it is based on corrections of the viscous force in terms of Reynolds number. A modified Reynolds number generally is used on fluids which require an initial shear stress to set the mixture in motion. These viscous forces tend to resist motion, thus requiring an initial shear stress of some magnitude prior to any shearing of the fluid as illustrated in Figure 10. This method concentrates on the modification of viscous forces rather than on the inertia forces of moving particles.

Newtonian fluids differ from non-Newtonian fluids in that they require no initial shear stress for motion as the viscous forces do not tend to resist motion. This is illustrated in Figure 11.

In this instance we are dealing with the newtonian fluid, water, which is carrying sand particles in any one of four ways or a

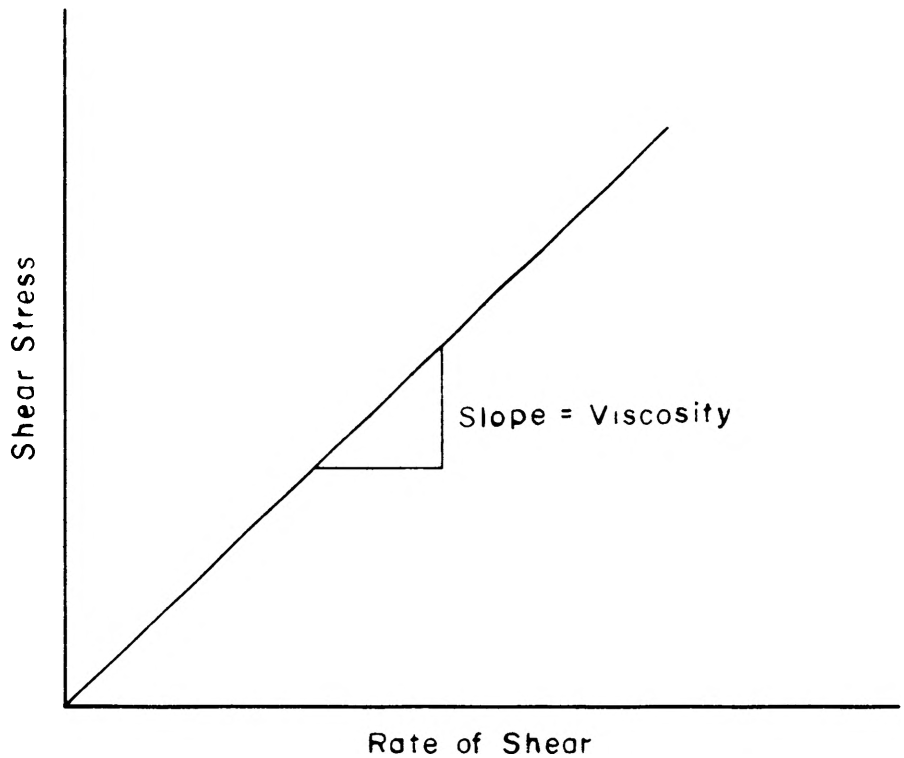

Figure 10 Newtonian Liquid Shear Diagram

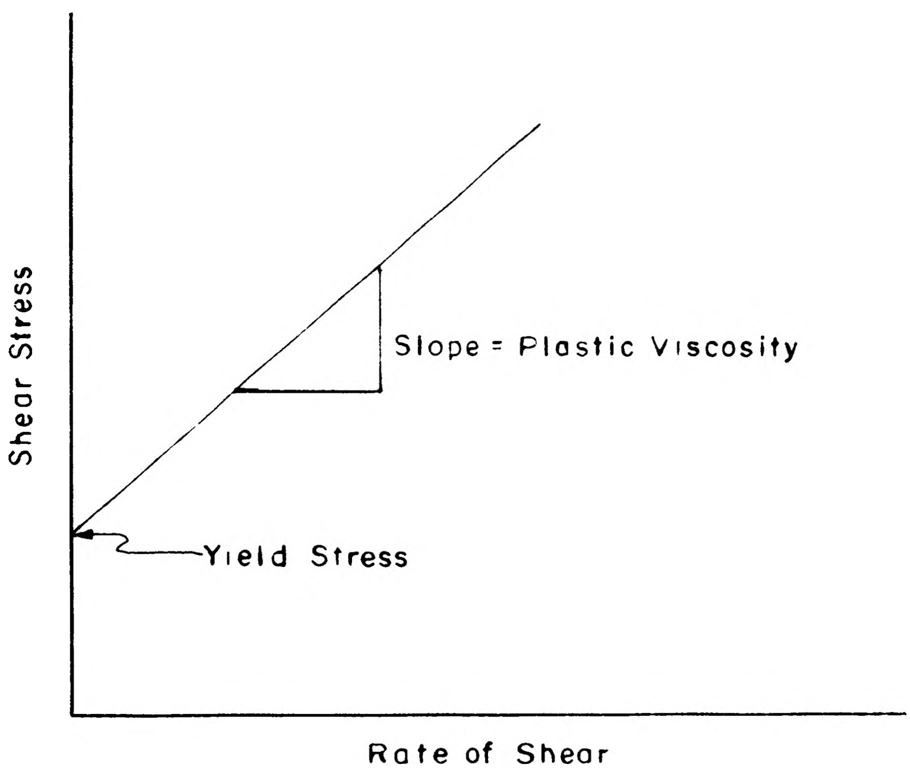

Figure 11 Non-Newtonian Liquid of the Bingham Plastic Type Shear Diagram

combination of the four. The solids may be suspended in the fluid and uniformly dispersed with the smaller particles near the top and the heavier particles near the bottom. This will usually occur at relatively high rates of flow. In other cases particles may become segregated and flow in dune type formations. Particles may flow as alternate intermittent slugs of water and solids, or in low velocity type flow there may be solid ripples that travel along the top of a stationary solid layer. The author believes in his case the flow is the first type in the form of a well dispersed mixture. In this case there was a combination of a mixing chamber, relatively small particle size and high velocities. The particles did not appear to be segregated and did not flow in slug type formations except when the pipe is nearly clogged at very high sand percentages.

In the computation of the results, the author arbitrarily chose to neglect the effects of the sand on Reynolds number and make a basic comparison on the basis of water. There are a number of advantages to this method. The losses in the line are known to vary directly in proportion to the friction factor. This afforded a very good means of making a comparison of the mixture with that of water.

Initially the friction factor of the pipe was determined for various velocities in the pipe using the fluid media by itself.

The data presented in Tables I, II and III represents the data accumulated in a series of test runs on water, fine sand and coarse sand through the pipeline. In comparing the first series of preliminary tests after the new pipe was put in place to similar tests after a number of sand-water mixtures had been pumped through

TABLE I

OBSERVED DATA FOR WATER FLOW

Run Number	Manometer Reading Centimeters Left	Manometer Reading Centimeters Right	Temperature Degrees Fahrenheit	Weight of Water Pounds	Volume of Water Cubic Feet	Time of Run Seconds
100	49.4	19.4	52.0	44.0	0.706	17.8
101	49.4	19.4	52.0	49.2	0.789	19.9
102	49.4	19.4	52.0	46.0	0.738	18.8
103	48.1	20.7	52.0	46.7	0.749	19.8
104	48.0	20.7	53.0	45.4	0.729	19.1
105	46.0	22.7	53.0	47.0	0.754	21.5
106	46.0	22.6	53.0	46.5	0.745	21.3
107	43.8	24.8	53.0	42.7	0.685	22.0
108	43.8	24.7	54.0	43.2	0.693	22.3
109	41.5	27.0	54.0	46.5	0.745	27.9
110	41.4	27.0	54.0	44.1	0.707	26.4
111	39.4	28.9	54.0	45.6	0.731	32.5
112	39.4	28.9	54.0	44.2	0.710	31.7
113	37.0	31.2	54.0	43.7	0.701	43.7
114	37.0	31.2	54.0	45.2	0.725	44.7
115	47.5	21.2	53.0	48.5	0.777	20.5
116	47.6	21.1	52.0	48.0	0.768	20.7
117	47.6	21.2	52.0	51.9	0.832	22.0
118	44.3	24.3	52.0	48.4	0.777	24.0
119	44.3	24.3	52.0	44.1	0.706	21.8
120	44.3	24.3	52.0	42.3	0.678	20.7
121	40.1	28.3	53.0	40.2	0.643	26.2
122	40.1	28.3	53.0	47.5	0.757	31.4
123	40.1	28.3	54.0	41.5	0.664	26.8
124	38.0	30.3	54.0	44.3	0.710	36.6
125	38.0	30.3	54.0	41.2	0.660	34.0

126	38.0	30.3	54.0	38.0	0.610	31.1
127	36.6	31.6	54.0	46.3	0.740	49.3
128	36.6	31.6	54.0	41.9	0.671	44.3
129	36.6	31.6	54.0	42.6	0.681	45.1
130	35.2	32.9	54.0	45.0	0.720	72.8
131	35.2	32.9	55.0	40.9	0.656	66.5
132	34.6	33.4	55.0	43.2	0.693	100.8
133	34.6	33.4	55.0	46.9	0.748	107.9
134	34.6	33.4	55.0	45.6	0.730	106.1
135	34.2	33.8	55.0	41.8	0.670	181.4
136	34.2	33.8	56.0	45.5	0.729	196.4
137	34.2	33.8	56.0	31.4	0.503	135.3
138	43.9	24.7	55.0	44.8	0.717	22.9
139	43.9	24.7	55.0	44.6	0.714	22.4
140	43.9	24.7	54.0	47.5	0.757	23.8
141	39.6	28.8	54.0	48.5	0.773	33.9
142	39.6	28.8	54.0	42.8	0.687	30.3
143	39.5	28.8	54.0	45.1	0.721	32.3
144	37.0	31.1	54.0	44.7	0.717	45.5
145	37.0	31.2	54.0	44.5	0.713	45.0
146	37.0	31.2	54.0	45.2	0.722	45.6
147	48.6	11.7	61.0	50.5	0.810	16.8
148	48.6	11.7	61.0	49.5	0.794	16.2
149	48.7	11.7	61.0	45.1	0.723	14.5
150	48.6	11.7	61.0	47.9	0.768	15.6
151	47.2	13.2	61.0	49.4	0.792	16.9
152	47.2	13.2	61.0	46.0	0.738	15.7
153	47.2	13.2	61.0	50.0	0.802	17.1
154	44.1	16.2	61.0	46.7	0.749	18.3
155	44.2	16.0	61.0	49.1	0.787	18.7
156	44.1	16.2	61.0	44.0	0.707	16.9
157	40.7	19.4	61.0	45.8	0.734	20.3
158	40.7	19.5	61.0	51.7	0.829	22.8
159	37.7	22.4	61.0	50.2	0.805	26.3
160	37.7	22.4	61.0	50.4	0.808	26.8
161	33.6	26.3	61.0	51.5	0.826	40.7
162	33.5	26.4	61.0	46.7	0.749	37.1
163	33.5	26.4	61.0	46.3	0.742	36.9

TABLE II
OBSERVED DATA OF FINE SAND MIXTURE

Run Number	Manometer Reading Centimeters		Temperature Degrees Fahrenheit	Weight of Sand-Water Mixture Pounds	Volume of Sand-Water Mixture Cubic Feet	Time of Run Seconds
	Left	Right				
200	44.4	24.4	58.0	54.9	0.730	22.0
201	46.9	22.1	58.0	57.5	0.738	20.3
202	46.4	22.7	58.0	53.9	0.703	19.3
203	48.1	20.8	58.5	61.8	0.738	19.0
204	49.1	20.2	59.0	59.3	0.685	17.2
205	48.1	21.1	59.0	64.0	0.738	19.5
206	46.5	22.7	59.0	55.9	0.730	19.9
207	44.8	24.4	59.0	48.3	0.694	20.0
208	49.4	19.9	61.0	57.7	0.685	17.5
209	49.4	19.5	61.0	59.0	0.720	17.4
210	50.5	18.7	62.0	57.5	0.807	18.3
211	50.5	18.4	62.0	50.6	0.700	16.1
212	49.5	19.5	61.0	55.9	0.660	16.4
213	49.3	19.7	61.0	58.0	0.681	17.2
214	50.5	18.5	61.0	47.5	0.595	14.3
215	49.5	19.5	61.0	44.2	0.530	13.0
216	49.4	17.5	61.5	47.2	0.563	13.5
217	49.2	17.7	61.5	67.4	0.811	19.4
218	49.2	17.7	62.0	61.2	0.764	18.2
219	47.7	19.1	62.0	58.4	0.781	19.1
220	49.3	17.7	62.0	61.8	0.816	18.9'
221	48.0	18.9	62.0	55.6	0.779	18.9
222	50.1	16.8	62.0	60.0	0.828	18.7
223	49.7	17.2	62.0	61.6	0.842	19.0
224	49.0	17.9	62.0	57.3	0.772	18.1
225	48.0	18.9	62.0	55.8	0.807	19.3
226	47.5	19.4	62.0	53.4	0.783	19.1

227	44.0	23.0	60.0	47.1	0.693	19.9
228	45.0	21.4	60.0	53.3	0.760	21.1
229	49.6	17.9	60.0	59.3	0.788	18.5
230	49.0	18.1	60.5	39.9	0.502	12.0
231	44.7	15.8	60.5	51.5	0.710	17.7
232	43.3	17.2	60.5	54.9	0.716	19.0
233	45.2	15.3	61.0	55.4	0.719	17.9
234	45.8	14.7	60.5	50.3	0.613	14.8
235	43.2	17.3	60.5	52.3	0.738	19.5
236	43.6	16.9	60.5	53.6	0.768	19.6
237	43.6	16.9	60.5	54.7	0.769	19.6
238	43.8	16.7	60.5	55.5	0.787	20.1
239	43.4	18.6	63.0	50.5	0.745	19.4
240	42.3	18.6	63.0	53.7	0.800	21.6
241	41.1	18.6	64.0	52.2	0.777	21.2
242	46.2	14.5	64.0	68.2	0.823	19.9
243	47.0	14.8	64.0	49.6	0.601	14.5
244	47.2	14.4	64.0	58.2	0.713	17.0

TABLE III

OBSERVED DATA FOR COARSE SAND MIXTURE

Run Number	Manometer Reading Centimeters Left	Manometer Reading Centimeters Right	Temperature Degrees Fahrenheit	Weight of Sand-Water Mixture Pounds	Volume of Sand-Water Mixture Cubic Feet	Time of Run Seconds
300	54.7	14.2	54.0	67.4	0.866	18.1
301	47.6	21.3	54.0	52.1	0.698	18.8
302	45.8	23.1	57.0	47.2	0.712	19.4
303	48.5	20.4	57.0	52.0	0.747	18.6
304	47.5	21.4	58.0	51.6	0.712	19.0
305	47.2	21.7	58.0	53.0	0.738	19.8
306	48.6	20.3	58.0	57.3	0.720	19.0
307	48.2	20.7	58.0	55.7	0.685	19.0
308	45.6	21.1	63.0	52.5	0.735	19.7
309	46.9	20.1	62.0	50.3	0.696	18.2
310	44.0	22.7	62.0	48.0	0.694	20.3
311	48.9	17.9	62.0	58.0	0.672	18.2
312	48.7	18.2	62.0	63.8	0.736	20.2
313	48.8	17.7	62.0	45.4	0.528	16.0
314	46.7	20.0	62.0	54.0	0.710	18.9
315	48.5	18.2	62.0	56.7	0.730	18.2
316	46.6	20.1	62.0	52.3	0.729	19.0
317	47.2	19.7	62.0	55.0	0.775	19.9
318	45.8	20.9	62.0	56.5	0.819	21.9
319	46.5	20.3	62.0	51.7	0.755	19.7
320	48.7	13.2	64.0	52.2	0.780	17.1
321	48.9	13.0	64.0	54.0	0.807	17.1
322	48.9	13.2	64.0	55.0	0.818	17.5
323	48.7	13.4	65.0	55.0	0.796	17.3
324	47.5	14.5	65.0	63.1	0.832	19.5

325	47.5	14.5	65.0	64.7	0.835	19.9
326	47.0	14.9	65.0	59.5	0.707	18.7
327	46.7	15.1	65.0	65.3	0.750	20.7
328	46.7	15.4	65.0	54.0	0.588	18.9
329	46.9	15.6	65.0	62.2	0.765	19.7
330	46.0	16.2	65.0	71.7	0.837	23.7
331	46.6	15.9	66.0	66.0	0.795	20.9
332	46.1	16.2	66.0	63.6	0.732	20.7
333	46.9	15.6	66.0	58.2	0.722	18.0
334	46.5	15.3	66.0	50.7	0.742	19.2
335	46.3	15.5	66.0	60.4	0.738	19.4
336	46.6	15.1	66.0	62.7	0.734	20.9
337	46.2	15.4	66.0	60.3	0.722	19.1
338	46.1	15.5	66.0	66.8	0.742	22.6
339	48.4	13.2	66.0	54.3	0.786	16.9
340	49.2	12.4	66.0	58.6	0.885	18.4
341	46.6	15.6	66.0	59.8	0.701	20.0
342	46.8	15.4	66.0	63.6	0.741	20.6
343	46.1	15.7	66.0	74.8	0.838	25.4
344	49.9	15.3	66.0	64.5	0.748	21.0
345	46.7	15.6	66.0	65.4	0.758	22.0
346	47.8	14.3	66.0	58.4	0.806	18.5
347	47.6	14.5	66.0	62.6	0.848	19.8
348	48.8	13.3	66.0	57.0	0.840	18.0
349	48.6	13.4	66.0	55.9	0.818	17.8
350	48.2	13.8	66.0	58.7	0.838	18.6

the pipe, there appeared to be a slight drop in friction factor in the latter series after sand cleaning. This reduction was probably due to the cleaning and polishing action on the pipe by the sand-water mixture. The Reynolds number of the equivalent flows were also determined. A plot of friction factor versus Reynolds number was then made as shown in Figure 12.

Upon making an examination of various friction factor versus Reynolds number curves the friction factor compared favorably to the smooth or extremely smooth type surfaces. It was concluded this was probably a result of the combination of the use of a new pipe and the polishing action produced by the movement of the fluid-solid mixtures. An example of the calculations involved is shown in Appendix I.

In a similar fashion the friction factors and Reynolds numbers were determined for fluid-solid mixtures with various percentages of sand. Since the head loss and friction factor in lines vary with Reynolds number there was a basis for a comparison of the Reynolds number of the fluid-solid mixture noted for any Reynolds number in the range of velocities and flows tested. The difference in the friction factors and the percent deviation of the friction factor of the fluid-solid mixture was determined with respect to the friction factor of the water. A tabulation of the results is shown in Tables IV, V and VI. A plot of percent deviation versus percent sand was made as illustrated in Figures 13 and 14 for both the fine and coarse sand.

Using the coordinates of the points plotted the best curve was computed to fit the points by themethod of least squares on the Royal

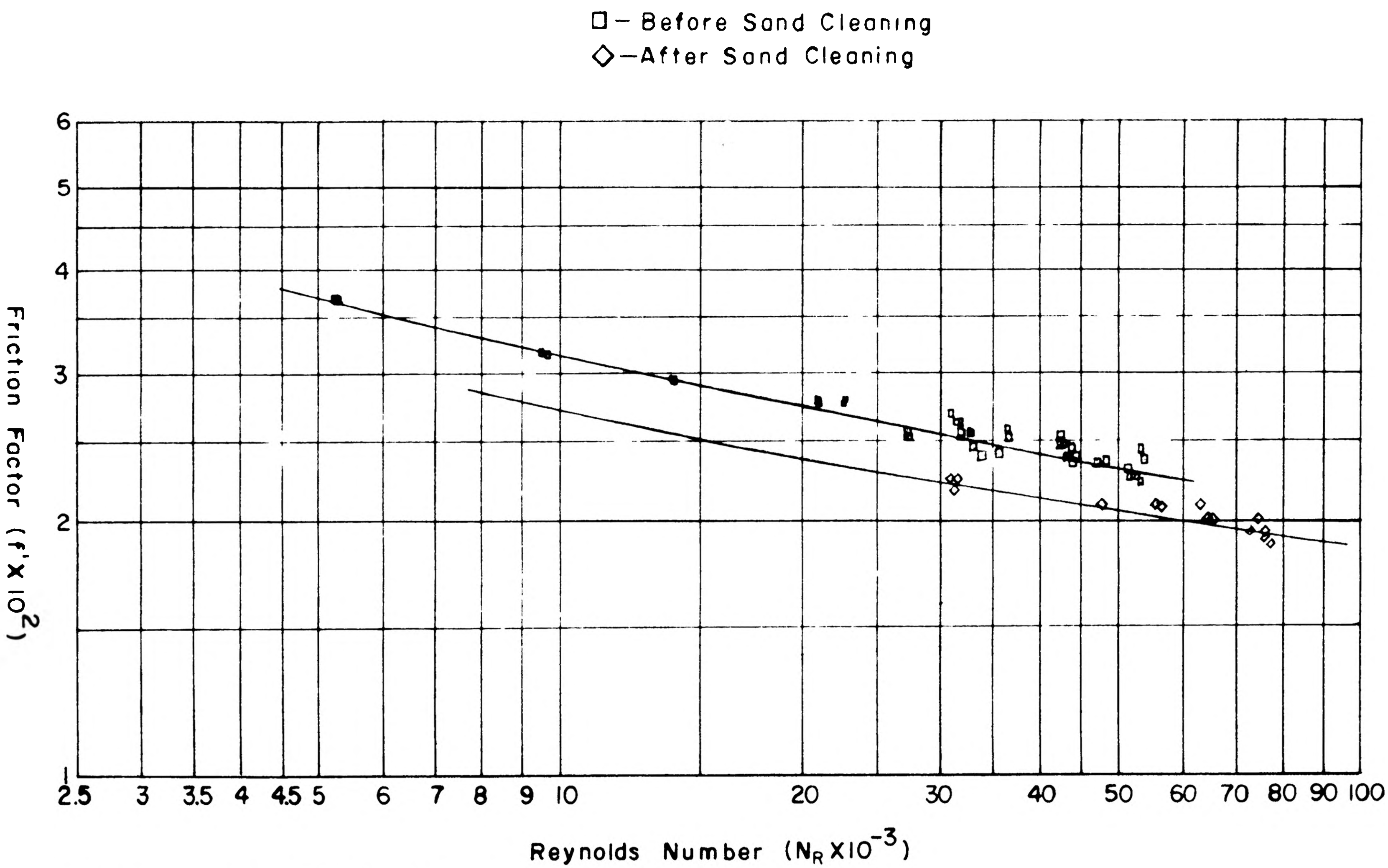

Figure 12 Friction Factor Versus Reynolds Number for Water

TABLE IV

CALCULATED DATA FOR WATER FLOW

*Run Number	$Q \times 10^2$	$N_R \times 10^{-4}$	Friction Factor $\times 10^2$
100	3.97	5.37	2.39
101	3.97	5.37	2.39
102	3.93	5.31	2.44
103	3.79	5.12	2.40
104	3.82	5.23	2.35
105	3.51	4.81	2.38
106	3.49	4.78	2.41
107	3.11	4.28	2.49
108	3.11	4.31	2.51
109	2.67	3.70	2.56
110	2.68	3.71	2.52
111	2.25	3.13	2.61
112	2.24	3.12	2.63
113	1.61	2.24	2.81
114	1.62	2.25	2.78
115	3.79	5.18	2.30
116	3.71	3.59	2.42
117	3.78	5.10	2.32
118	3.24	4.41	2.39
119	3.24	4.41	2.39
120	3.27	4.44	2.35
121	2.45	3.37	2.47
122	2.41	3.31	2.55
123	2.48	3.44	2.41
124	1.94	2.70	2.57
125	1.94	2.69	2.57
126	1.96	2.73	2.52
127	1.50	2.09	2.79
128	1.51	2.10	2.75
129	1.51	2.09	2.75
130	0.990	1.39	2.94
131	0.986	1.38	2.97
132	0.688	0.967	3.18
133	0.693	0.976	3.14
134	0.688	0.968	3.18
135	0.369	0.526	3.69
136	0.371	0.532	3.65
137	0.372	0.535	3.63
138	0.313	0.440	2.46
139	0.319	0.448	2.37
140	0.318	0.444	2.39
141	0.228	0.317	2.61
142	0.227	0.316	2.63

TABLE IV (Continued)

143	0.223	0.310	2.63
144	0.158	0.221	2.97
145	0.159	0.222	2.88
146	0.158	0.221	2.92
147	4.82	7.45	2.00
148	4.90	7.57	1.93
149	4.98	7.70	1.87
150	4.92	7.60	1.91
151	4.68	7.23	1.95
152	4.70	7.26	1.93
153	4.68	7.23	1.95
154	4.09	6.33	2.10
155	4.21	6.50	2.00
156	4.18	6.47	2.01
157	3.61	5.58	2.05
158	3.64	5.63	2.01
159	3.06	4.73	2.05
160	3.01	4.65	2.12
161	2.03	3.14	2.23
162	2.02	3.12	2.18
163	2.01	3.11	2.21

*Runs numbered 100-146 were made prior to fluid-solid tests
*Runs numbered 147-163 were made after fluid-solid tests.

TABLE V

CALCULATED DATA FOR FINE SAND MIXTURE

Run Number	$Q \times 10^2$	$N_R \times 10^{-4}$	Percent Sand	Friction Factor $\times 10^2$ Sand-water	Friction Factor $\times 10^2$ Water	$\Delta f \times 10^2$	Percent Deviation	$\mu' \times 10^5$
200	3.32	4.91	22.7	2.27	2.06	.21	10.2	5.05
201	3.64	5.38	27.0	2.36	2.03	.33	16.2	7.43
202	3.65	5.40	25.3	2.24	2.03	.21	10.3	5.30
203	3.89	5.80	36.6	2.27	2.00	.27	13.5	6.58
204	3.99	5.98	41.5	2.28	1.99	.29	14.6	7.12
205	3.79	5.68	41.5	2.36	2.01	.35	17.4	8.61
206	3.67	5.51	24.9	2.22	2.02	.20	9.9	5.07
207	3.47	5.21	13.5	2.13	2.04	.09	4.4	3.41
208	3.92	6.05	37.5	2.41	1.98	.43	21.7	9.52
209	4.13	6.38	33.6	2.20	1.97	.23	11.7	5.57
210	4.41	6.92	16.2	2.06	1.95	.11	5.6	3.55
211	4.35	6.83	17.7	2.13	1.95	.18	9.2	4.83
212	4.02	6.20	38.2	2.33	1.98	.35	17.7	8.16
213	3.96	6.12	39.1	2.37	1.98	.39	19.7	9.26
214	4.16	6.43	30.1	2.32	1.97	.35	17.8	7.80
215	4.08	6.31	36.1	2.26	1.98	.28	14.1	6.77
216	4.17	6.48	36.8	2.30	1.97	.33	16.8	7.74
217	4.18	6.50	35.6	2.26	1.97	.29	14.7	6.92
218	4.20	6.58	30.8	2.24	1.97	.27	13.7	6.37
219	4.09	6.41	22.0	2.15	1.97	.18	9.1	4.42
220	4.32	6.77	23.5	2.13	1.96	.17	8.7	4.60
221	4.12	6.45	16.2	2.16	1.97	.19	9.6	4.34
222	4.43	6.94	18.2	2.13	1.95	.18	9.2	4.53
223	4.43	6.94	19.4	2.08	1.95	.13	6.7	3.94
224	4.27	6.68	21.0	2.14	1.96	.18	9.2	4.46
225	4.18	6.55	12.5	2.09	1.97	.12	6.1	3.55

226	4.10	6.42	11.0	2.10	1.97	.13	6.6	3.30
227	3.48	5.31	10.6	2.18	2.04	.14	6.9	3.78
228	3.60	5.49	14.3	2.29	2.02	.27	13.4	5.49
229	4.26	6.50	23.1	2.19	1.97	.22	11.2	5.22
230	4.18	6.42	29.8	2.22	1.97	.25	12.7	5.98
231	4.02	6.17	18.1	2.25	1.98	.27	13.6	5.58
232	3.77	5.78	25.1	2.31	2.00	.31	15.5	6.53
233	4.02	6.20	25.7	2.32	1.98	.34	17.2	7.12
234	4.14	6.35	33.9	2.28	1.97	.31	15.7	7.00
235	3.79	5.82	15.5	2.27	2.00	.27	13.5	5.43
236	3.92	6.02	13.4	2.18	1.99	.19	9.5	6.05
237	3.93	6.03	15.7	2.17	1.99	.18	9.0	4.35
238	3.91	6.00	15.0	2.23	1.99	.24	12.0	4.97
239	3.84	6.10	10.3	2.11	1.98	.13	6.6	3.41
240	3.71	5.89	9.0	2.16	2.00	.16	8.00	3.67
241	3.67	5.92	9.1	2.10	2.00	.10	5.0	3.13
242	4.13	6.67	35.3	2.33	1.96	.37	18.9	7.60
243	4.13	6.67	34.6	2.37	1.96	.41	20.9	9.35
244	4.19	6.76	33.1	2.35	1.95	.40	20.5	8.57

TABLE VI

CALCULATED DATA FOR COARSE SAND MIXTURE

Run Number	$Q \times 10^2$	$N_R \times 10^{-4}$	Percent Sand	Friction Factor $\times 10^2$ Sand-Water	Friction Factor $\times 10^2$ Water	$\Delta f \times 10^2$	Percent Deviation	$\mu \times 10^5$
300	4.78	6.64	26.8	2.23	1.96	.27	13.8	6.81
301	3.72	5.18	21.7	2.38	2.04	.34	16.7	7.62
302	3.67	5.35	8.0	2.12	2.03	.09	4.4	3.34
303	4.02	5.86	13.5	2.19	2.00	.19	9.5	4.62
304	3.75	5.54	18.1	2.33	2.02	.31	15.4	6.67
305	3.73	5.52	17.0	2.30	2.02	.28	13.9	6.06
306	3.79	5.61	29.8	2.48	2.01	.47	23.4	10.52
307	3.61	5.34	32.8	2.65	2.03	.62	30.5	14.74
308	3.73	5.93	16.3	2.22	2.00	.22	11.0	4.79
309	3.83	6.01	17.8	2.30	1.99	.31	15.6	6.27
310	3.42	5.37	12.5	2.29	2.03	.26	12.8	5.23
311	3.69	5.78	41.0	2.86	2.01	.85	42.3	25.2
312	3.64	5.71	41.5	2.90	2.01	.89	44.3	25.9
313	3.30	5.17	40.5	3.59	2.04	1.55	76.0	45.4
314	3.76	5.89	24.0	2.38	2.00	.38	19.0	7.84
315	4.02	6.29	26.7	2.36	1.98	.38	19.2	7.98
316	3.84	6.02	17.0	2.26	1.99	.27	13.6	5.48
317	3.89	6.10	15.6	2.28	1.99	.29	14.6	6.02
318	3.74	5.87	12.3	2.24	2.00	.24	12.0	4.97
319	3.83	6.00	11.4	2.25	2.00	.25	12.5	5.13
320	4.57	7.37	8.9	2.14	1.93	.21	10.9	4.36
321	4.72	7.60	8.9	2.03	1.92	.11	5.7	3.31
322	4.68	7.53	9.3	2.05	1.93	.12	6.2	3.40
323	4.60	7.52	12.6	2.10	1.92	.18	9.4	3.99
324	4.27	6.97	23.7	2.27	1.95	.32	16.4	6.67
325	4.19	6.84	26.3	2.36	1.95	.41	21.0	8.30

326	3.78	6.18	37.5	2.82	1.98	.84	42.4	22.7
327	3.63	5.93	42.4	3.01	2.00	1.01	50.5	29.9
328	3.11	5.08	50.0	4.07	2.05	2.02	98.5	60.8
329	3.89	6.35	32.8	2.60	1.97	.63	32.0	14.6
330	3.53	5.84	40.2	3.01	2.00	1.01	50.5	29.1
331	3.81	6.30	35.5	2.66	1.98	.68	34.4	16.3
332	3.53	5.83	41.8	3.02	2.00	1.02	51.0	30.5
333	4.02	6.65	31.7	2.44	1.96	0.48	24.5	10.37
334	3.87	6.40	33.6	2.62	1.97	.65	33.0	16.3
335	3.81	6.30	33.6	2.67	1.98	.69	34.9	16.34
336	3.51	5.81	39.5	3.21	2.01	1.20	59.7	35.3
337	3.78	6.25	36.3	2.71	1.98	.73	36.9	18.2
338	3.29	5.44	47.1	3.55	2.02	1.53	75.7	47.4
339	4.65	7.69	12.6	2.05	1.92	.13	6.8	3.50
340	4.81	7.96	7.6	2.00	1.91	.09	4.7	3.05
341	3.51	5.81	39.3	3.16	2.01	1.15	57.2	33.9
342	3.60	5.96	40.3	3.05	2.00	1.05	52.5	31.3
343	3.30	5.47	46.0	3.51	2.02	1.49	73.8	45.3
344	3.57	5.91	40.8	3.41	2.00	1.41	70.5	43.8
345	3.45	5.71	40.9	3.28	2.01	1.27	63.2	37.7
346	4.36	7.21	18.2	2.22	1.94	.28	14.4	5.68
347	4.28	7.08	20.3	2.27	1.94	.33	17.0	6.63
348	4.67	7.73	10.4	2.05	1.91	.14	7.3	3.44
349	4.60	7.62	11.2	2.09	1.92	.17	8.9	3.93
350	4.51	7.47	13.9	2.13	1.93	.20	10.4	4.42

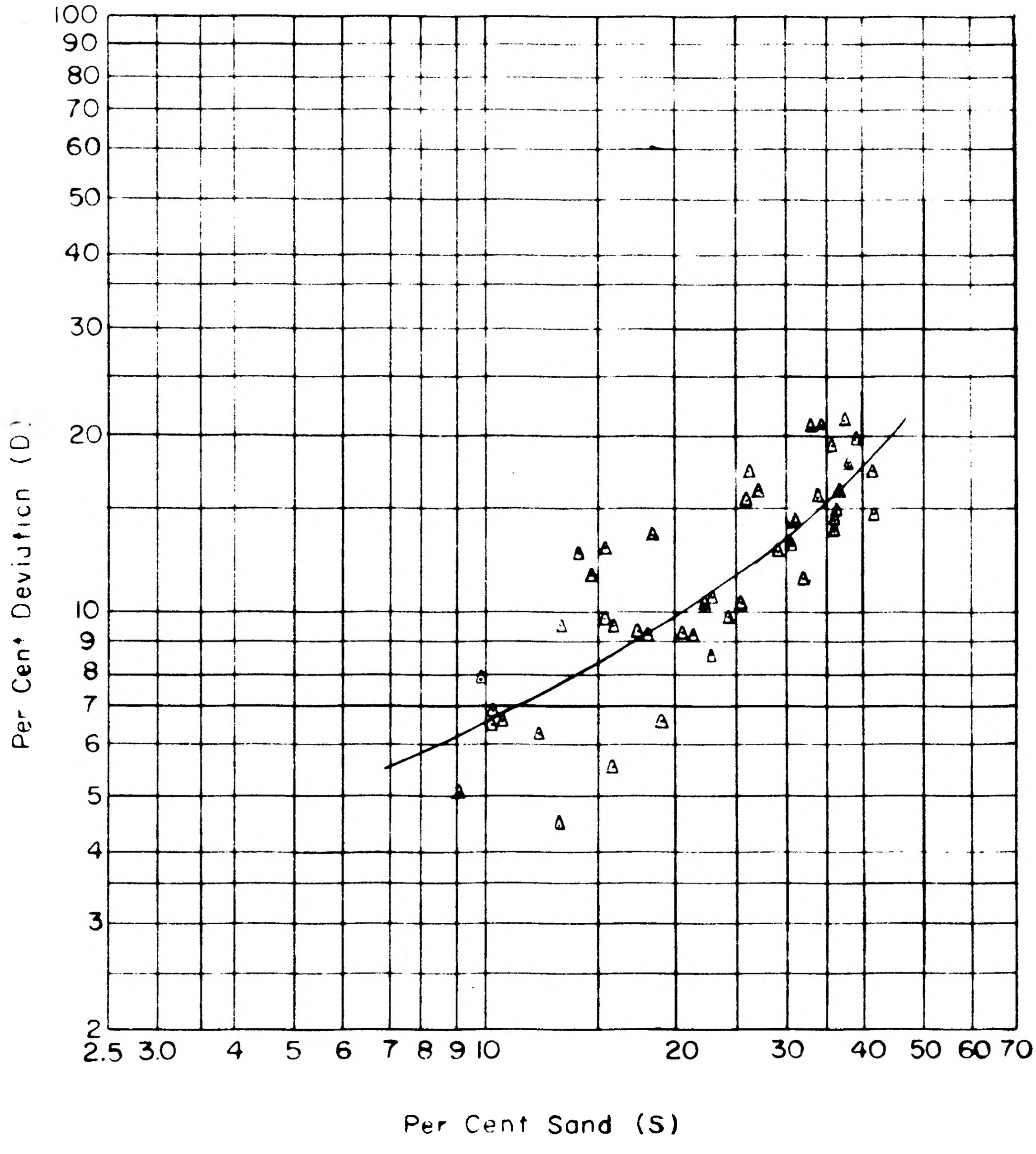

Figure 13 Per Cent Deviation Versus Per Cent Sand for a Fine Sand Mixture.

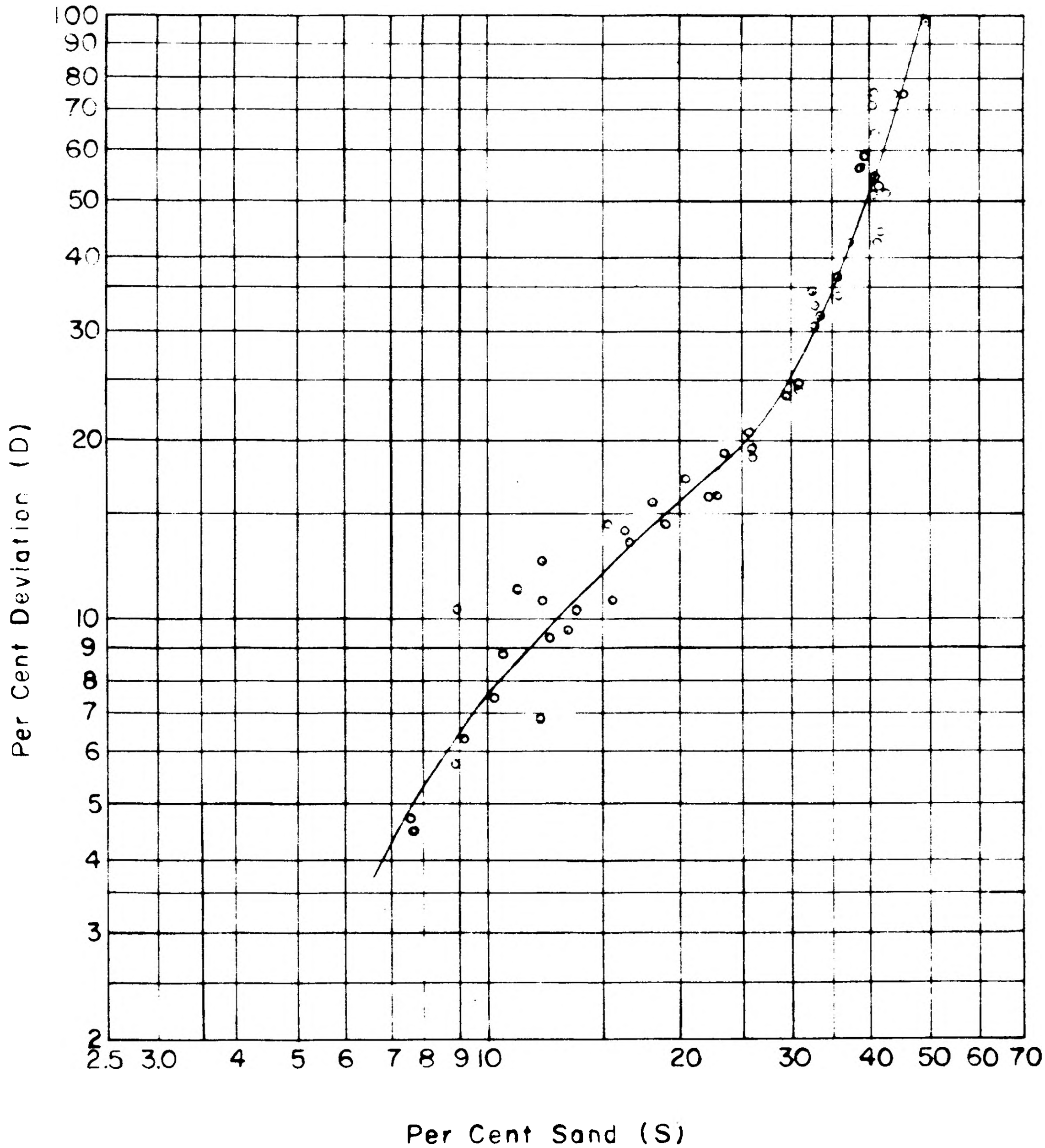

Figure 14 Per Cent Deviation Versus Per Cent Sand for a Coarse Sand Mixture.

McBee Computer in the computer center. Third degree curves were computed for both the fine and coarse sand mixtures. The equation of the curves was in the following form:

$$\ln D = A + B \ln S + C (\ln S)^2 + D' (\ln S)^3$$

In the case of the fine sand the equation became

$$\ln D = 3.14 - 1.98 \ln S + 0.80 (\ln S)^2 - 0.0766 (\ln S)^3$$

The resultant equation for the curve of the coarse sand was:

$$\ln D = -18.9 + 21.5 \ln S - 7.42 (\ln S)^2 + 0.886 (\ln S)^3$$

In evaluating the curves a uniformly higher percent deviation was noted for the coarse sand as the percent sand increased as compared to that of the fine sand.

Another method of comparison, which has been commonly accepted among researchers of slurries or mixture type studies, is on the basis of apparent viscosity. This method is somewhat incorrect because both inertia and viscous forces change with the addition of materials to the mixture. In the author's case the material added was sand. Thus the apparent viscosity is not a correct term but it is often used as a means of comparison.

Previously we found:

$$N_R = \frac{vd}{\nu}$$

If you have a true Reynolds number, for any value of Reynolds number which you have, the friction factor should remain constant. This is true since

$$f = \phi \left(N_R, \frac{e}{d}\right)$$

or $$f = \phi' (N_R)$$

when $\frac{e}{d}$ is constant

and thus Reynolds number and friction factor are a function of the velocity assuming everything else is constant. The term e/d may be termed the "relative roughness" of the pipe. "Relative roughness" expresses the size of the roughness protuberances relative to the diameter of the pipe.

The above approach to the problem is somewhat similar to the method followed in adding the "eddy viscosity", to determine the shear stress in a situation where both viscous and turbulent action are present in a flow.

The author determined the apparent viscosity that would make both Reynolds number and friction factor theoretically correspond. In this way the apparent viscosity could be used as a basic means of comparing the test results. Another means of determining the absolute viscosity would have been to do it experimentally, however, the necessary equipment was not available to perform an experimental analysis.

A tabulation of the apparent viscosities for both the fine and coarse sand is shown in Tables V and VI. Plots were made of the percent sand versus viscosity as illustrated in Figures 15 and 16 for both the fine and coarse sand. The points appeared to be grouped a little better than those of the previous plots of Percent Deviation versus Percent Sand.

As in the case of the plots of Percent Deviation versus Percent Sand the best curve fit was computed on the Royal McBee Computer using a similar type curve. A fourth degree curve was computed for the coarse sand as it appeared to fit the data somewhat better than either third or fifth degree curves. The computed curve for the coarse sand was:

$$\ln S = 12.24 + 2.43 (\ln u') + 0.740 (\ln u')^2 - 0.315 (\ln u')^3 - 0.0243 (\ln u')^4$$

A third degree curve was computed for the fine sand graph as $\ln S = -11.87 - 2.07 (\ln u') + 0.172 (\ln u')^2 + 0.0232 (\ln u')^3$

In the evaluation of the curves, as was expected for both the fine and coarse sand the apparent viscosity increased as the percent sand increased. At low percentages of sand the apparent viscosity for the coarse sand appeared to be somewhat less than for the fine sand.

The apparent viscosity of high percentages of fine sand is much less than those indicated for the coarse sand. It is felt the apparent viscosity of the fine sand would level out in a manner somewhat similar to the coarse sand at high sand percentages if larger pressures were available to move the fluid-solid mixture. In this particular case the equipment was inadequate to pump larger quantities of sand through the line and thus was a limiting factor. The author feels that the apparent viscosity would increase very rapidly with a rather small increase in sand content.

A study of the apparent viscosity curves in Figures 15 and 16 strengthens the author's belief in the type flow analysis presented in the previous section. As better defined curves would be produced the author feels this method would be a very acceptable basis for use in further studies.

At this time it should be noted once more the percent sand is on a dry rodded basis. As illustrated in Figure 2, the weight of the

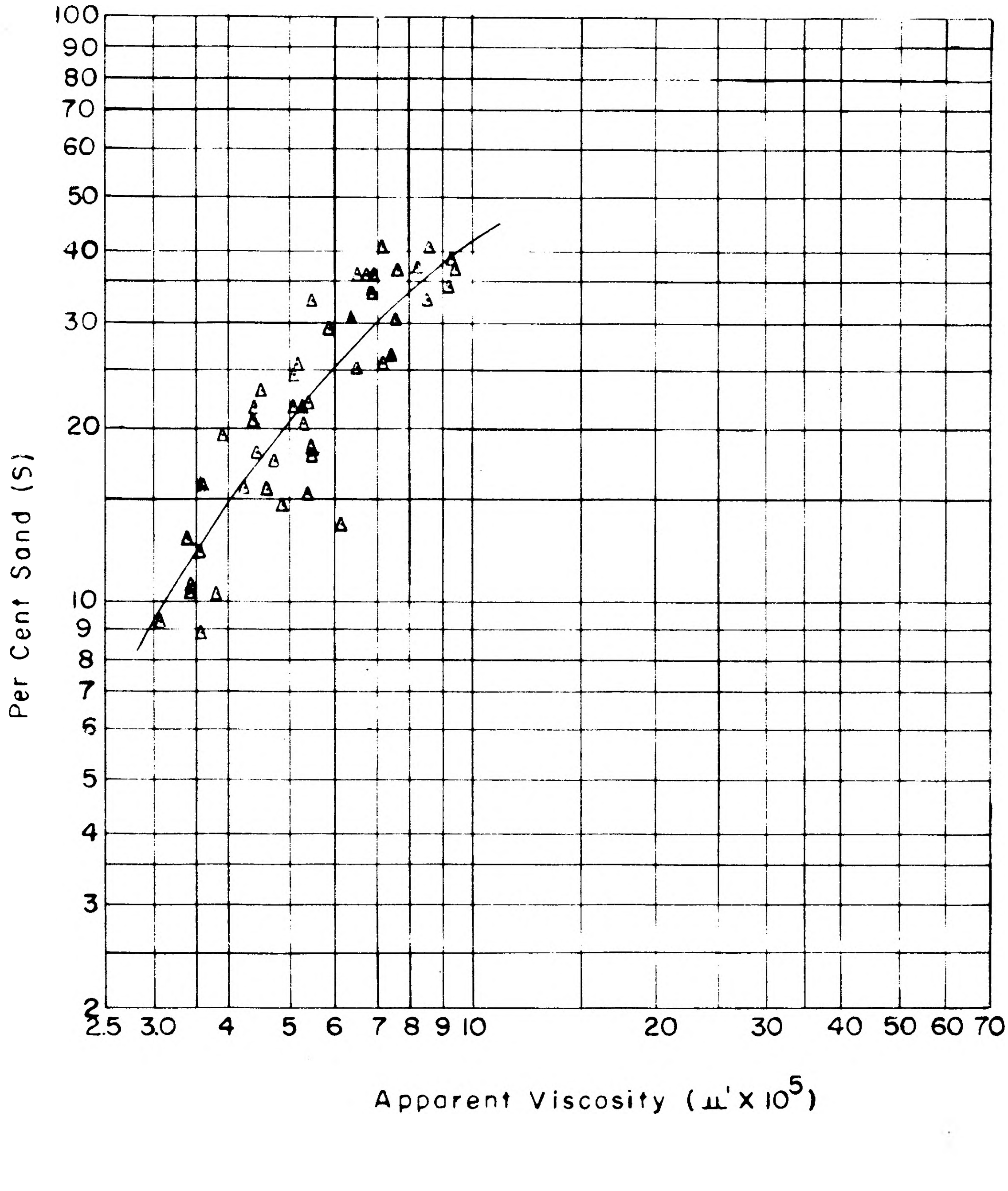

Figure 15 Per Cent Sand Versus Apparent Viscosity for a Fine Sand Mixture.

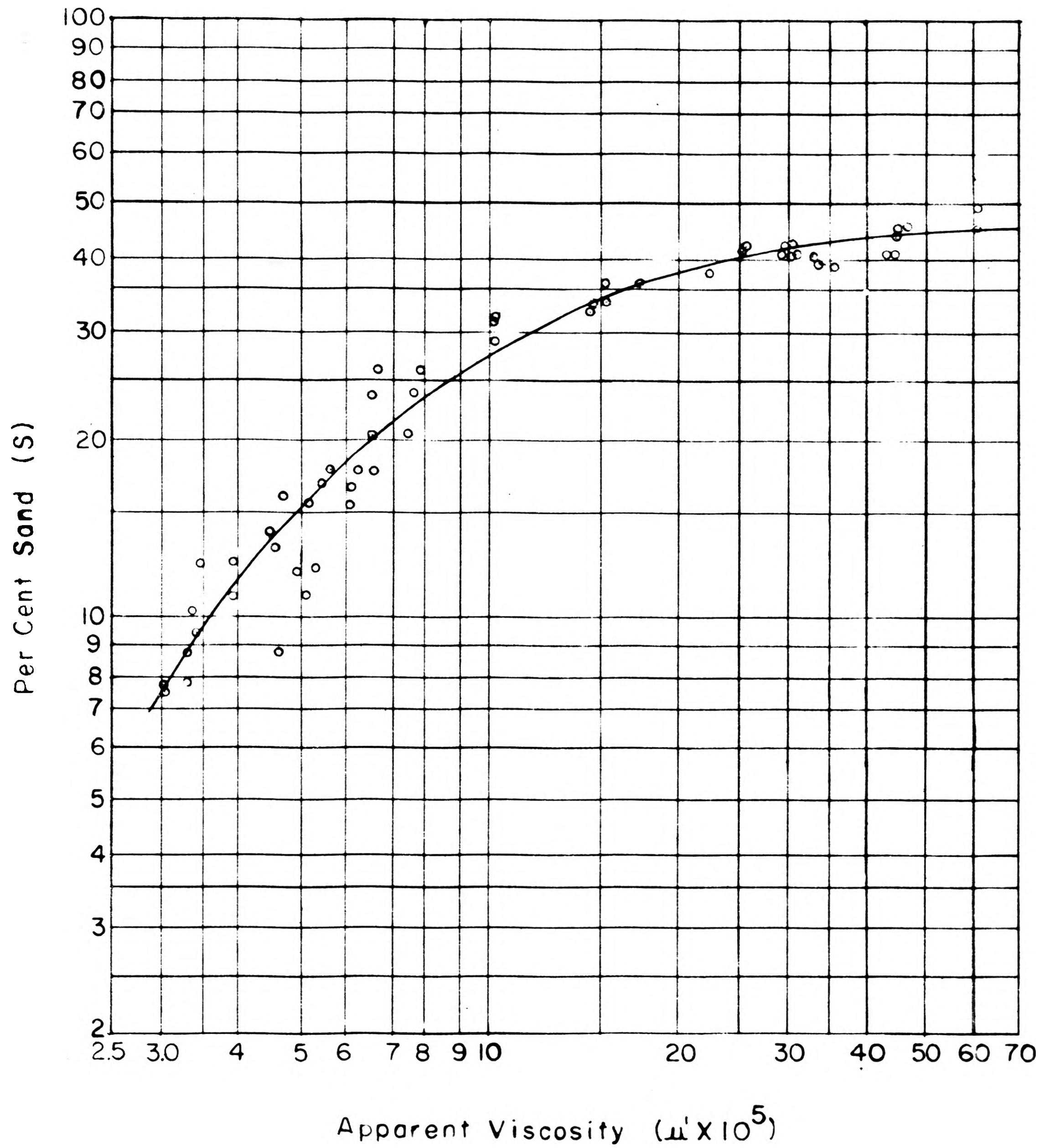

Figure 16 Per Cent Sand Versus Apparent Viscosity for a Coarse Sand Mixture

sand-water mixture is nearly identical for the fine and coarse sand.

The points on the fine sand plot of percent deviation versus percent sand are somewhat more scattered than that of the coarse sand. This is probably due to the fine sand expending more energy in random tubulent type motion at low percentages. This greater freedom of movement may be one of the factors contributing to greater variation in friction factor. The assumption of greater movement and greater random movement is strengthened when one considers that the net change in drag forces varies directly as a function of the diameter. Also it should be noted that as the percentage of sand increases the point grouping becomes more consistent. A reason for this is that as a greater mass needs to be transported by the same total available energy from the system, the flow will tend to become more efficient. A more efficient flow would have less turbulent action and would more closely follow a laminar type pattern. In the author's opinion,test observations and a study of Figures 15 and 16, indicate that maximum sand percentage type flow will act as a viscous mass.

The flow will continue until the water movement within this mass reaches a point that it will no longer support the particles. At this time settlement followed by clogging action in the pipe occurs. It was observed that the transition from normal flow to clogging occurred more rapidly with the fine sand than the coarse sand. This is logical when one considers the flow of water around the sand to be similar to the flow of water through a series of pipes; while the same flow area may be available, the smaller set of openings will have greater head loss and therefore less flow.

If we then consider a section of pipe undergoing clogging, it has been observed that a pressure build up caused by flow stoppage can more easily force enough water through a coarse sand plug to maintain a slug type flow. A similar condition with fine sand would result in a permanent stoppage. This statement applies to the data which indicates that higher percentages of coarse sand-water mixtures can be pumped with the same equipment as compared to a somewhat lower percentage for fine type particles.

CONCLUSIONS

The author believes that as a result of this study the following conclusions may be stated:

1. Contrary to an opinion poll of Engineers conducted by the Hydraulic Institute, New York, New York, the friction factor for sand-water mixtures increased with the percentage of sand throughout the range studied. The majority opinion obtained by the Institute's poll relative to the friction factor of fluid-solid mixtures was, "Up to about 30 to 40 percent solids by weight, assuming a carrying velocity exists, the friction losses will be about equal to that of water in a clean pipe".

2. Coarse sand has higher friction losses than fine sand. This was especially true in the higher sand percentage ranges.

3. In pumping fluid-solid mixtures a point was reached when an increase in the percent solids produced a radical change in the flow characteristics of the mixture. In most cases this occurred just prior to or concurrently with a pipe stoppage.

4. The coarse particle size mixture produces more consistent flow characteristics than the fine sand mixture.

5. The apparent viscosity increases as the percent sand increases for both fine and coarse sand mixtures.

6. The use of apparent viscosity proved successful in indicating a relationship between friction factor and percent sand.

7. With a great enough pressure differential it would be possible to transport one-hundred percent solids, however, from an

economic viewpoint there is a limiting maximum percent solids which a fluid is capable of carrying. In the case of sand this appears to be in the neighborhood of forty percent.

8. The condition of the interior surface of a pipe has a definite bearing on the friction losses when pumping fluid-solid mixtures in the turbulent range.

9. In the future pipelines will be used much more frequently as a means of transporting solids. The use of pipelines will enable such materials as minerals to be easily transported from areas which are now inaccessible.

RECOMMENDATIONS

The author would like to make the following recommendations for further study:

1. A study be made using a series of different sized spheres with the same specific gravity.

2. These spheres should be studied with both the sand and different types of carrying fluids.

3. A study using material which has the same size and shape but with a different specific gravity.

4. A method of relating flow characterists for various fluid-solid mixtures. This need could be satisfied by the development of a relationship similar to Reynolds number. This method should include some method of relating inertia and drag forces which would be dependent upon the material size.

5. A friction factor correlation with a number of various constant percentages of solids pumped in a vertical pipe.

6. After the previous studies have been completed an attempt should be made to correlate the data to develop a function defining a friction factor of the Reynolds number type.

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VITA

Frank J. Capek was born on February 9, 1934 at Milligan, Nebraska, the son of Mr. and Mrs. Frank F. Capek.

He received his early education in the Rolla Public Schools graduating from Rolla High School in May, 1951. After graduation he worked for the United States Geological Survey as a rodman until February 1952. He entered the Missouri School of Mines and Metallurgy in 1952 and received the Degree of Bachelor of Science in Civil Engineering in June, 1956.

Mr. Capek has gained practical experience through employment with the Missouri State Highway Commission, Howard, Needles, Tammen and Bergendoff and the U. S. Forest Service.

In September 1956 he accepted an Instructorship in Civil Engineering at the Missouri School of Mines and Metallurgy and has served in that capacity to date.

Having been commissioned a 2nd Lt. in the U. S. Army Corps of Engineers at the time of graduation from college, he entered active duty for six months in June, 1957.

In August 1957, he married Arah M. Keeney of Rolla, Missouri.

During the summer of 1960 Mr. Capek attended the University of Minnesota having received a grant-in-aid from the Asphalt Institute.

APPENDIX I

SAMPLE CALCULATION

Test Number 315

(a) Friction Factor

$$H_f = f \frac{L}{d} \frac{V^2}{2g}$$

$$f = \frac{2H_f \; dg}{L \; V^2} \qquad \text{but } V = \frac{Q}{A} = \frac{4Q}{\pi d^2}$$

Combining

$$f = \frac{g \; \pi^2 \; d^5 \; H_f}{8 \; L \; Q^2}$$

$$H_f \text{ (ft. of water)} = MR \frac{\text{(cm. of Hg.) } (13.56 - 1.00)}{2.54 \text{ (cm/in.) } 12 \text{ (in/ft.)}}$$

$g = 32.2$ ft./sec.^{2} $\quad$ $d = 0.0688$ ft. $\quad$ $L = 20.0$ ft.

Substituting

$$f = 1.257 \times 10^{-6} \frac{(MR)}{Q^2}$$

$$Q^2 = \left[\frac{\text{Volume}}{\text{Time}}\right]^2 = \left[\frac{0.730}{18.2}\right]^2 = 1.62 \times 10^{-3} \text{ cfs}$$

$$f = 1.257 \times 10^{-6} \frac{30.3}{1.62 \times 10^{-3}} = 2.36 \times 10^{-2}$$

(b) Percent Deviation

Reynolds number assuming only water

$$N_R = \frac{4\,Q}{\pi \, \nu \, d}$$

ν at 62°F is 1.185×10^{-5}

from Elementary Fluid Mechanics by J. K. Vennard

$$N_R = \frac{4\ (4.02 \times 10^{-2})}{\pi\ (0.0688)\ 1.185 \times 10^{-5}} = 6.29 \times 10^4$$

Friction factor of coarse sand 0.0236

Friction factor of water (Figure 12) 0.0198

$$\text{Percent deviation} = \frac{(0.0236 - 0.0198)\ 100}{0.0192} = 19.2$$

(c) Percent Sand

$$\text{Weight of Mixture (lbs./ft.}^3) = \frac{\text{Weight}}{\text{Volume}} = \frac{56.7}{0.730} = 77.7 \text{ lbs./ft.}^3$$

From Figure 2, Percent Sand is equal to 26.7%

d. Apparent Viscosity

Since friction factor equals 0.0236

N_R from (Figure 12)-2.25 x 10^4

$$\mu' = \frac{VD\rho}{N_R} \qquad V = \frac{Q}{A} \quad \frac{0.0402 \text{ (cfs)}}{.0037 \text{ (ft.}^2)}$$

$$\mu' = \frac{0.0402(0.0688)77.7}{0.00371(2.25 \times 10^4)32.2} \qquad \rho = \frac{77.7 \text{ (pcf}}{32.2 \text{ (ft./sec.}^2)}$$

$$\mu' = 7.98 \times 10^{-5} \text{ lb. sec./ft.}^2$$