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Using Performance Assessments to Connect Fractions and Rational Expressions: Noyce Scholars as Mentors to Pre-Service Elementary Teachers

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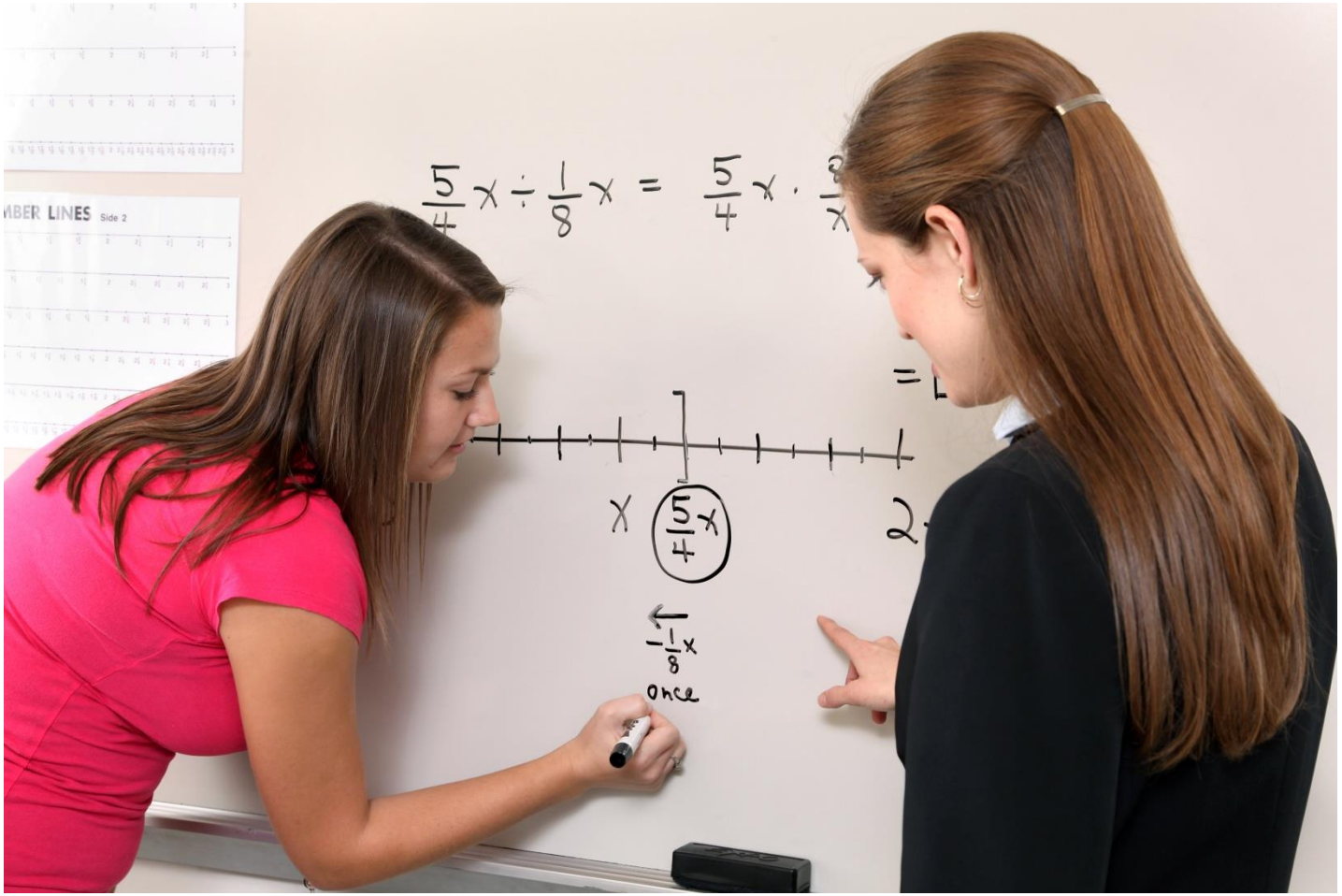
Using Performance Assessments to Connect Fractions and Rational Expressions:

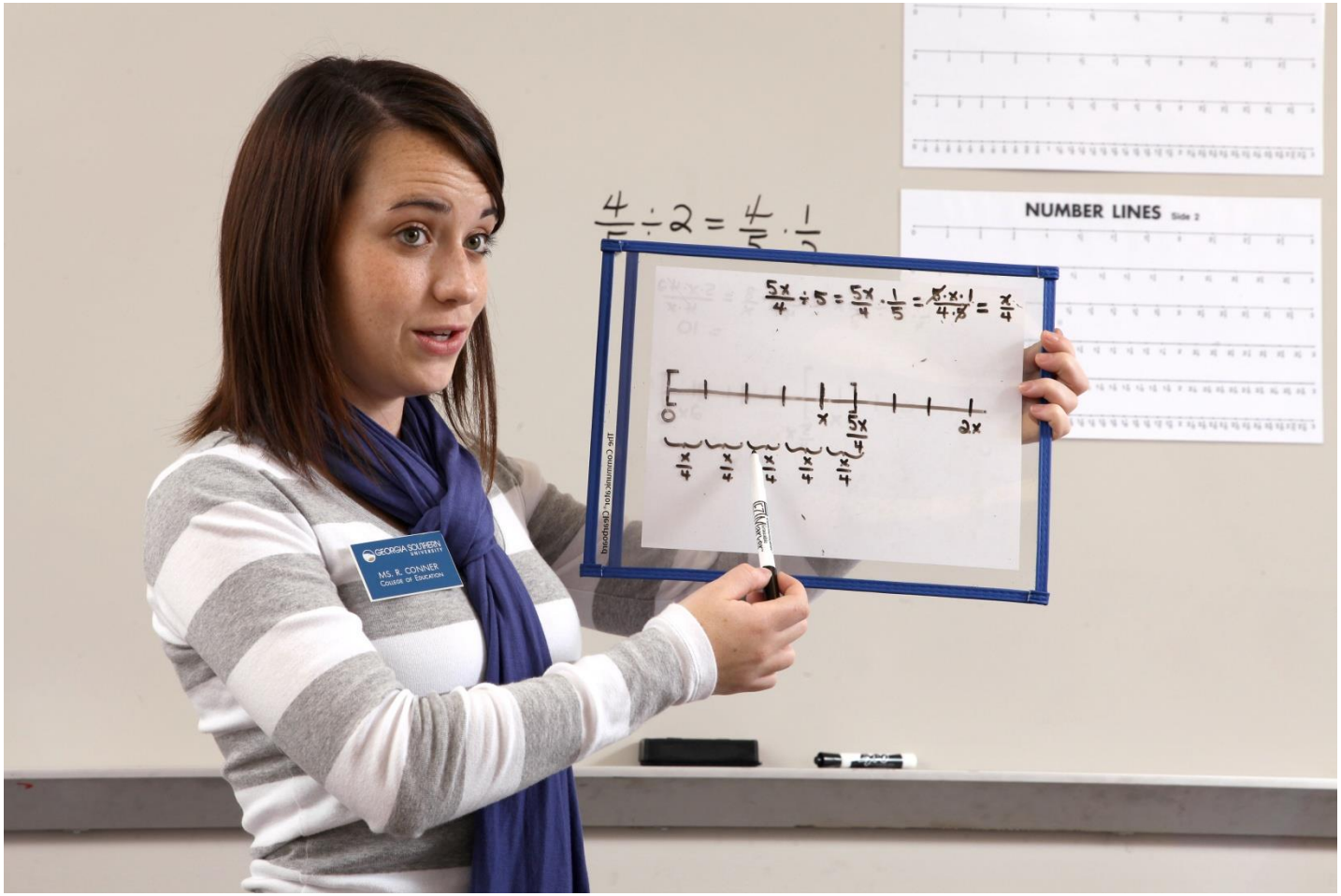
Noyce Scholars as Mentors to Pre-Service Elementary Teachers

Joy W. Darley Georgia Southern University

Noyce scholars are not only being mentored, but they are also serving as mentors to pre-service elementary teachers.


BER LINES Side 2
$\frac{5}{4}x \div \frac{1}{8}x = \frac{5}{4}x \cdot$
$\frac{5}{4}x$
$\frac{1}{8}x$
once

Since our Noyce scholars with mathematics degrees will be teaching algebra, it is important that they are fluent with the arithmetic to algebra connection. In addition, it is crucial that these mathematics majors become stakeholders in mathematics education at the elementary school level.

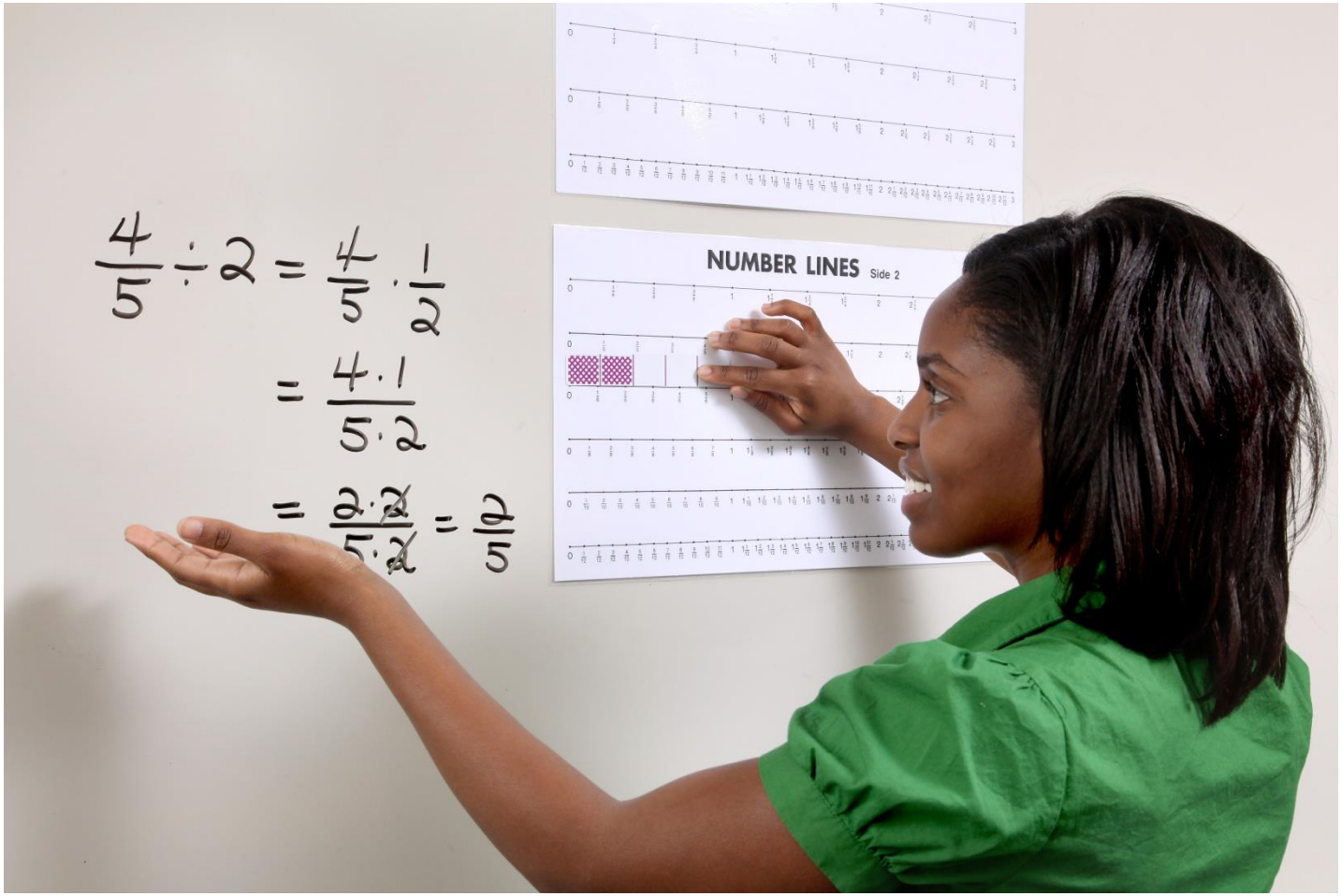

NUMBER LINES
GEORGIA SOUTHERN UNIVERSITY
MS. R. CONNER
College of Education

Benefits to SMART Scholars:

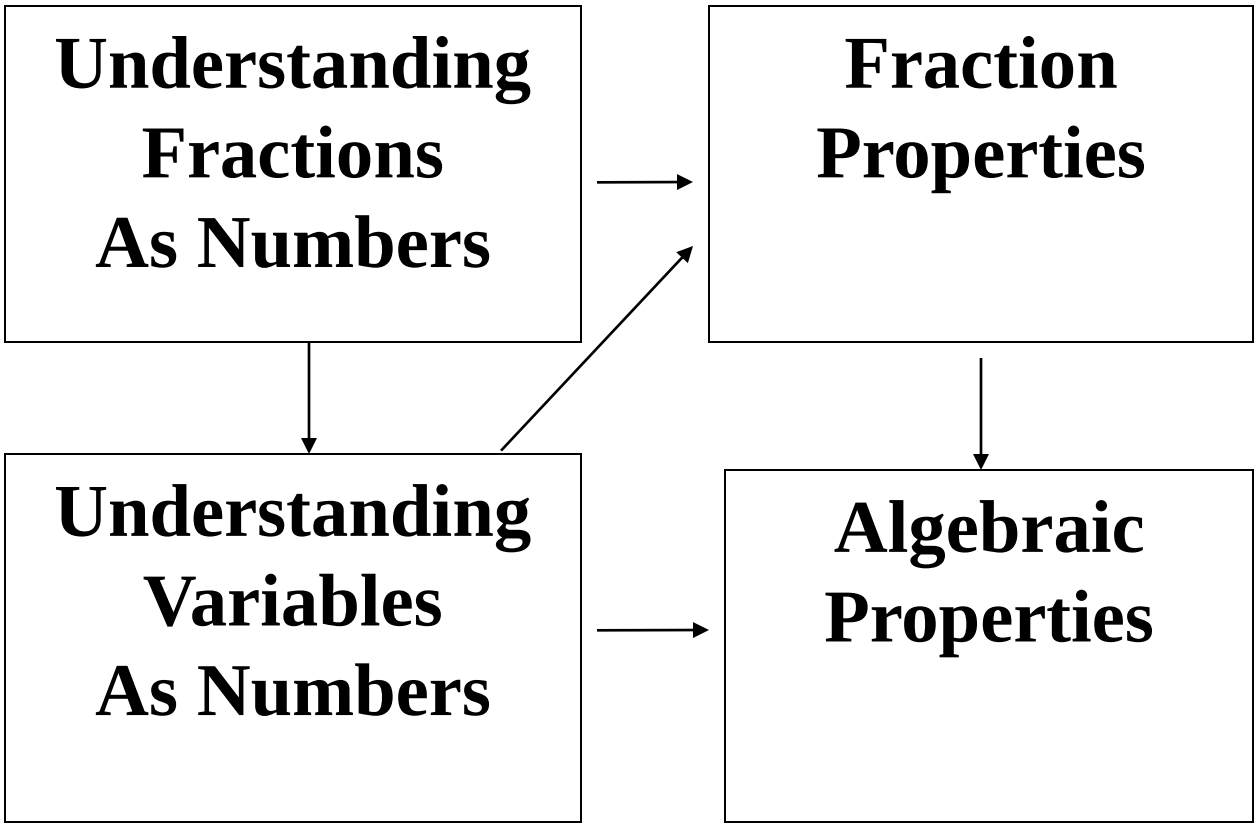
- SMART Scholars learn to evaluate pre-service teachers with their mathematics content knowledge and therefore strengthen their own knowledge.
- SMART Scholars gain strategies and resources needed to teach fractions in a way that leads to conceptual understanding.
- SMART Scholars become stakeholders in mathematics education at the elementary school level.

Benefits to Elementary Pre-service Teachers:

- Elementary pre-service teachers gain mathematics content knowledge & strategies that will provide a foundation so that their students are better able to connect arithmetic to algebra.
- Elementary pre-service teachers learn to use accurate communication concerning mathematics.
- Elementary pre-service teachers gain access to mathematics mentoring and support.

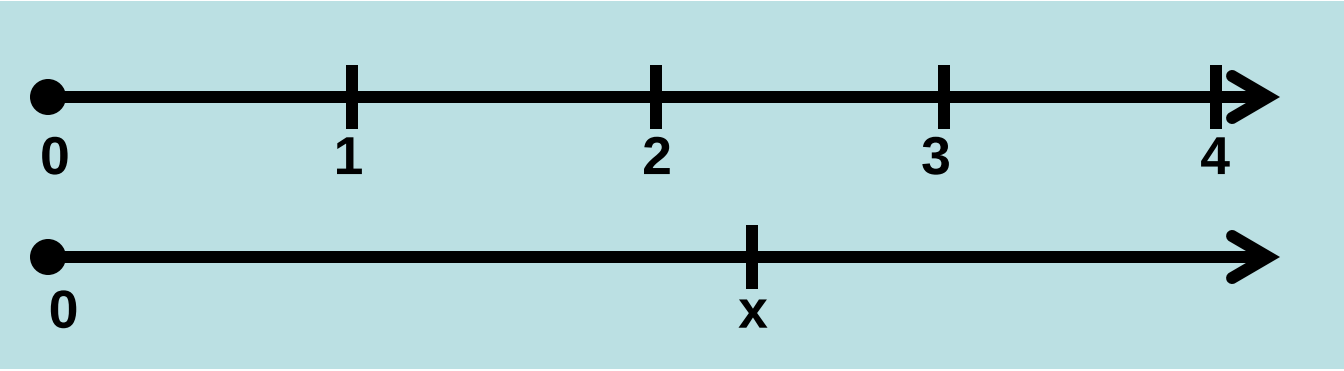

$\frac{4}{5} \div 2 = \frac{4}{5} \cdot \frac{1}{2}$
$= \frac{4 \cdot 1}{5 \cdot 2}$
$= \frac{2 \cdot 2}{5 \cdot 2} = \frac{2}{5}$
NUMBER LINES Side 2

Inputs for Learning Algebraic Properties



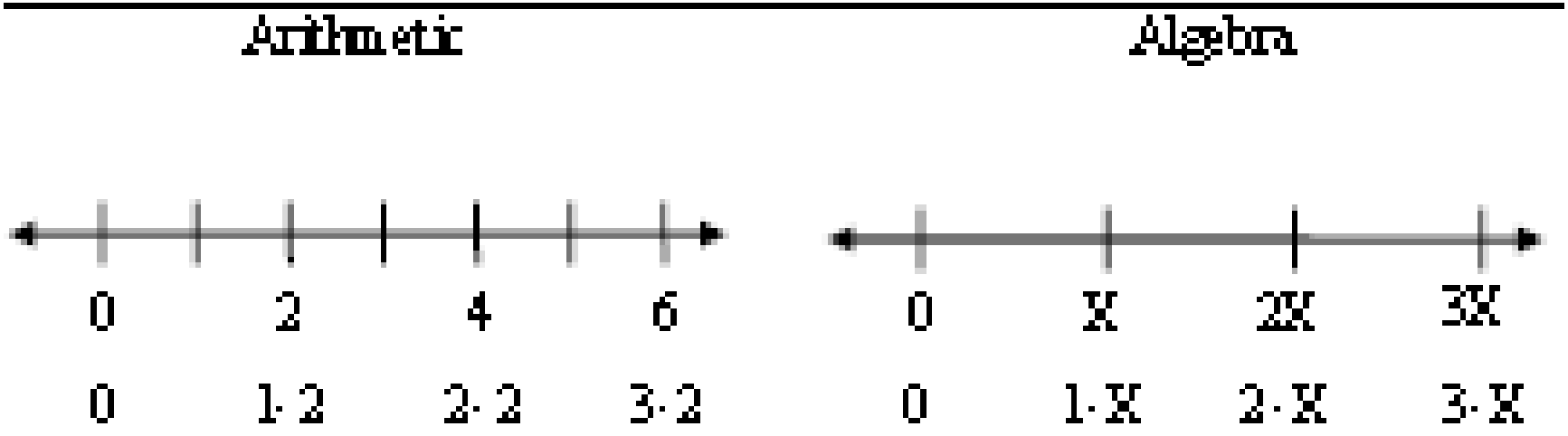
Conceptual Framework:

The number line is used as a unifying model for operations on whole numbers, fractions, and algebraic expressions.

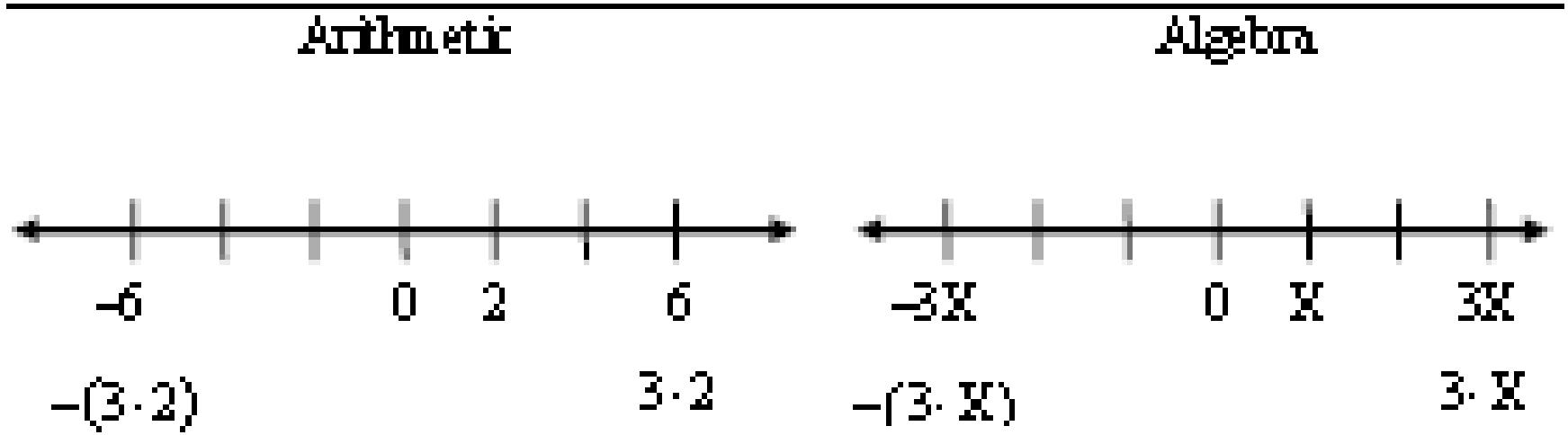


Since our goal is for students to better understand variables, we need to be certain that our students understand numbers. Once the connection is made between the two, the students will be more confident using variables.

Numbers / Variables



Numbers / Variables



Lesson 1

Definition of a Fraction

Definition.

Measurement:

$\frac{a}{b}$ is a out of b equal parts in the interval $[0,1]$.

Sharing:

$\frac{a}{b}$ is the size of the portion when a is divided into b equal parts.

Measurement Interpretation:

- Jack takes one candy bar and divides it into 3 equal parts and eats 2 of those parts. How much did Jack eat?

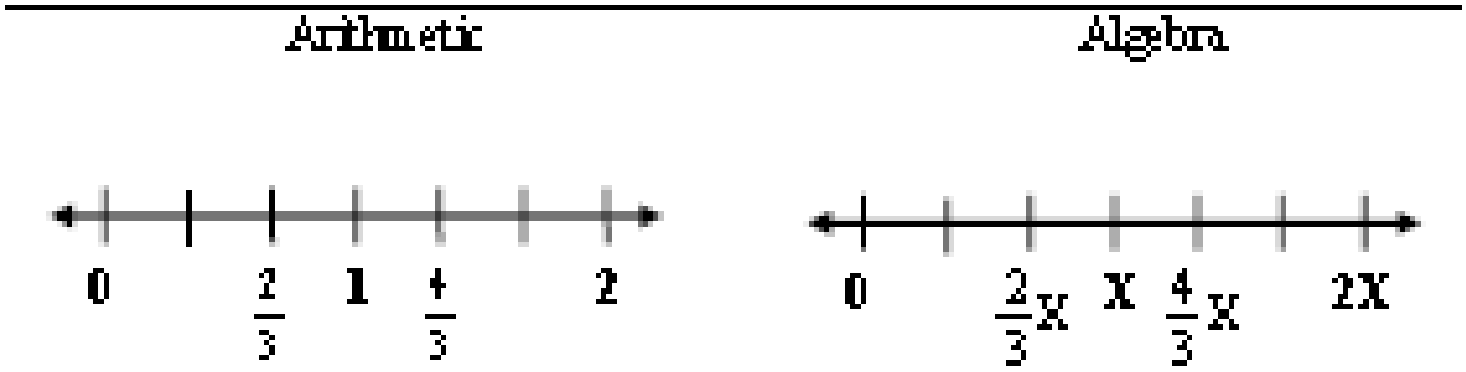
  (What if he eats 4 of those parts?)

- Jack takes X candy bars and divides them into 3 equal parts and eats 2 of those parts. How much did Jack eat?

  (What if he eats 4 of those parts?)

Measurement:

$\frac{a}{b}$ is a out of b equal parts in the interval $[0,1]$.



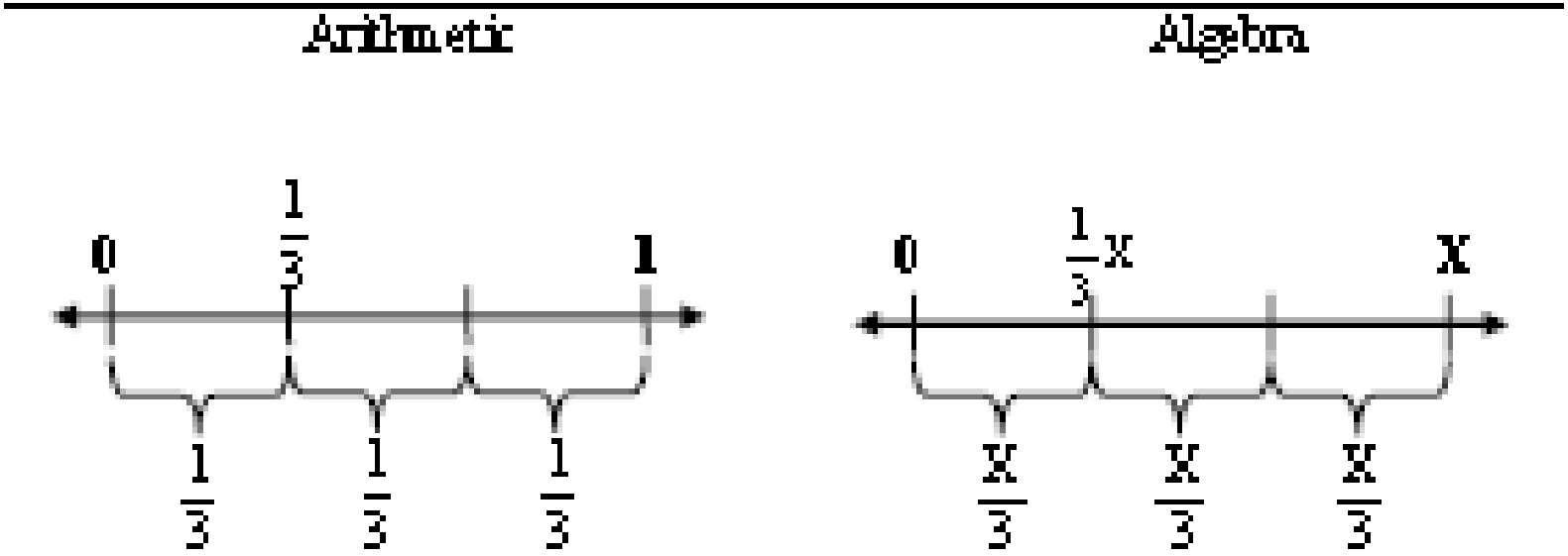
Sharing Interpretation:

- If Jack has one candy bar and wants to divide it equally among his three friends, how much would each friend get?

- What if Jack has X candy bars and wants to divide them equally among his three friends, how much would each friend get?

Sharing:

$\frac{a}{b}$ is the size of the portion when a is divided into b equal parts.



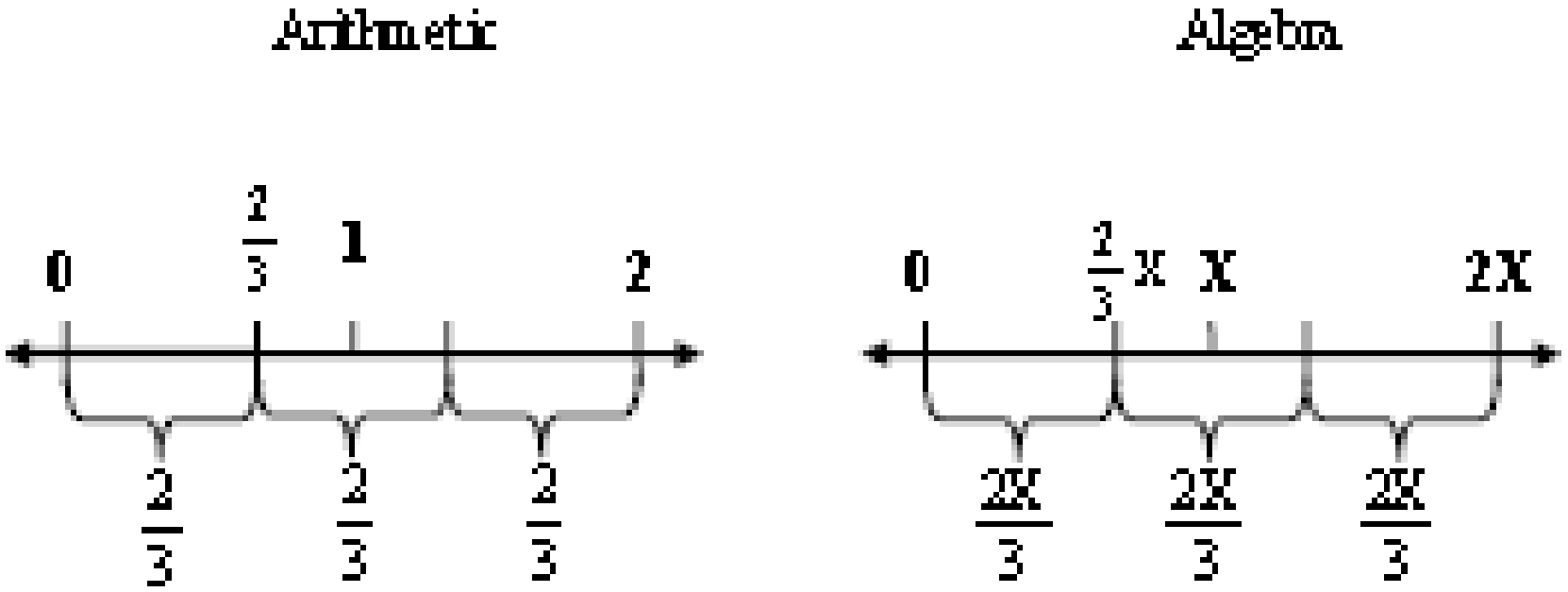
Sharing Interpretation:

- If Jack has two candy bars and wants to divide them equally among his three friends, how much would each friend get?

- What if Jack has 2X candy bars and wants to divide them equally among his three friends, how much would each friend get?

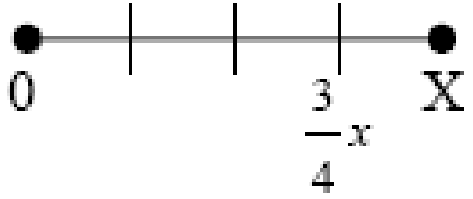
Sharing:

$\frac{a}{b}$ is the size of the portion when a is divided into b equal parts.

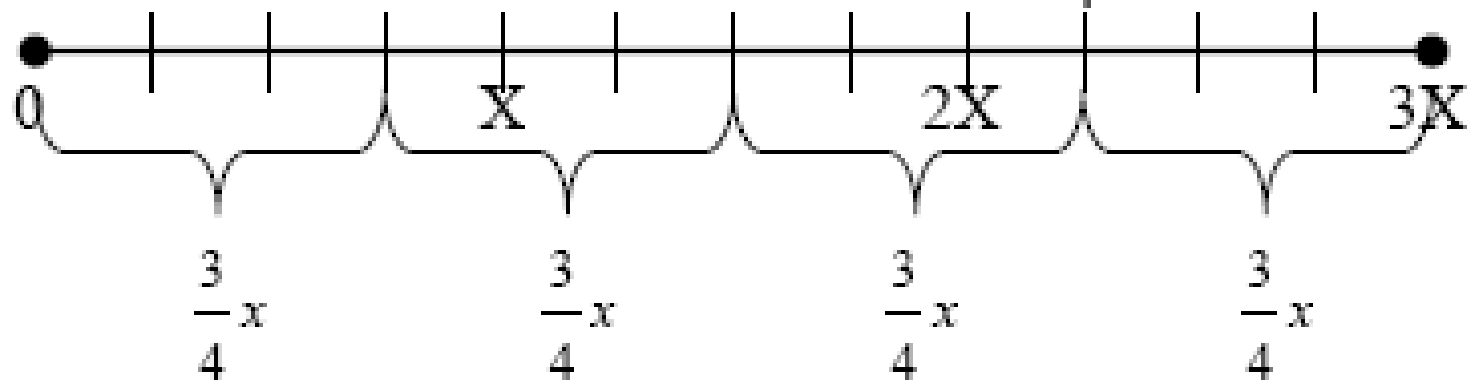


Algebra Number Line Instruction

A. Substitute $[0, x]$ in place of $[0,1]$ in 1st fraction definition to locate $\frac{3}{4}x$.



B. Use the 2nd fraction definition to locate $\frac{3x}{4}$.



Lesson 2

Renaming Fractions

Fraction Tasks

- Ed is responsible for putting up signs on the side of a road **two-thirds** of a mile leading to the school carnival. The signs are to be spaced **one-sixth** mile apart from each other. How many signs will he put up?
- The camp's water tank holds **six** gallons of water and **four** gallons are left. What fraction of the tank is full of water? Express this fraction in lowest terms.

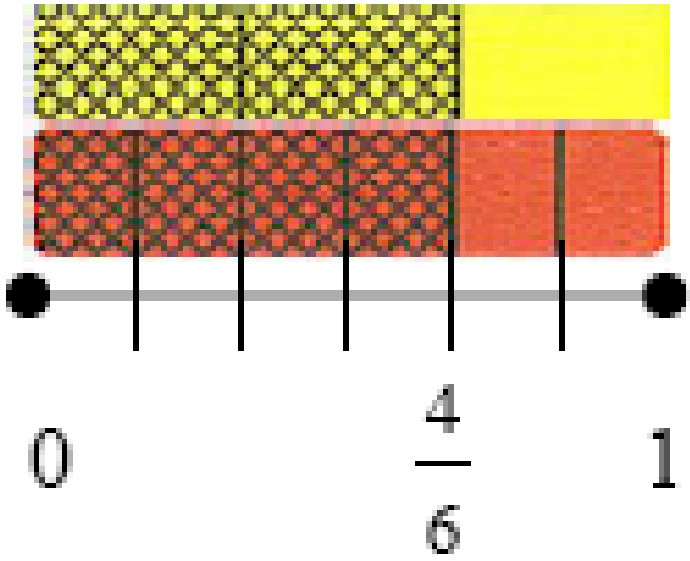
Fraction Bar and Number Line Instruction

(a) Rename $\frac{2}{3}$ as a fraction having a denominator of 6.

$$\frac{2}{3} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6}$$

Fraction Bar and Number Line Instruction

(b) Simplify $\frac{4}{6}$



$$\frac{4}{6} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{2}{3}$$

Algebra Instruction

Rename $\frac{2}{3}$ as a fraction having a denominator of 3x.

$$\frac{2}{3} = \frac{2 \cdot x}{3 \cdot x} = \frac{2x}{3x}$$

Note: $x \neq 0$

Algebra Instruction

Rename $\frac{x}{3}$ as a fraction having a denominator of $6x(x+y)$.

$$\frac{x}{3}=\frac{x\cdot 2x(x+y)}{3\cdot 2x(x+y)}=\frac{2x^2(x+y)}{6x(x+y)}$$

Note: $x \neq 0,\ x+y \neq 0$

Algebra Instruction

Simplify $\frac{4x}{5x}$.

$$\frac{4x}{5x} = \frac{4}{5}$$

Note: $x \neq 0$

Algebra Instruction

Simplify $\dfrac{6x-8}{9x-12}$.

$$\frac{6x-8}{9x-12}=\frac{2(3x-4)}{3(3x-4)}=\frac{2}{3}$$

Note: $x \neq \dfrac{4}{3}$

Lesson 3

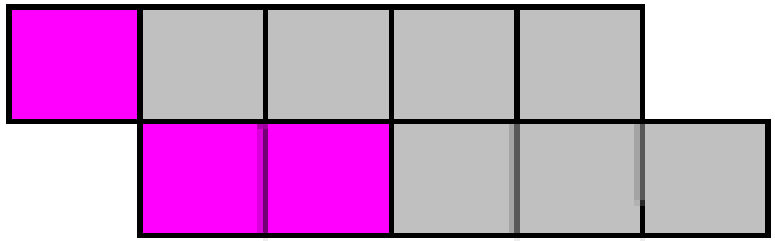
Adding Fractions with Like Denominators

Fraction Tasks

- Albert mowed **one-fifth** of the lawn while Hung-Hsi mowed **two-fifths**. Together, what part did they mow?
- Bryan drove **one-third** of a mile and then drove **two-thirds** more. How far did he travel?

Fraction Bar and Number Line Instruction

$$\frac{1}{5}+\frac{2}{5}=\frac{1+2}{5}=\frac{3}{5}$$



$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$$

Algebra Instruction

$$\frac{1}{x}+\frac{2}{x}=\frac{1+2}{x}=\frac{3}{x}$$ Note: $x \neq 0$

Algebra Instruction

$$\frac{2x}{x-y}+\frac{x-3y}{x-y}=\frac{2x+x-3y}{x-y}=$$

$$\frac{3x-3y}{x-y}=\frac{3(x-y)}{x-y}=3$$

Note: $x \neq y$

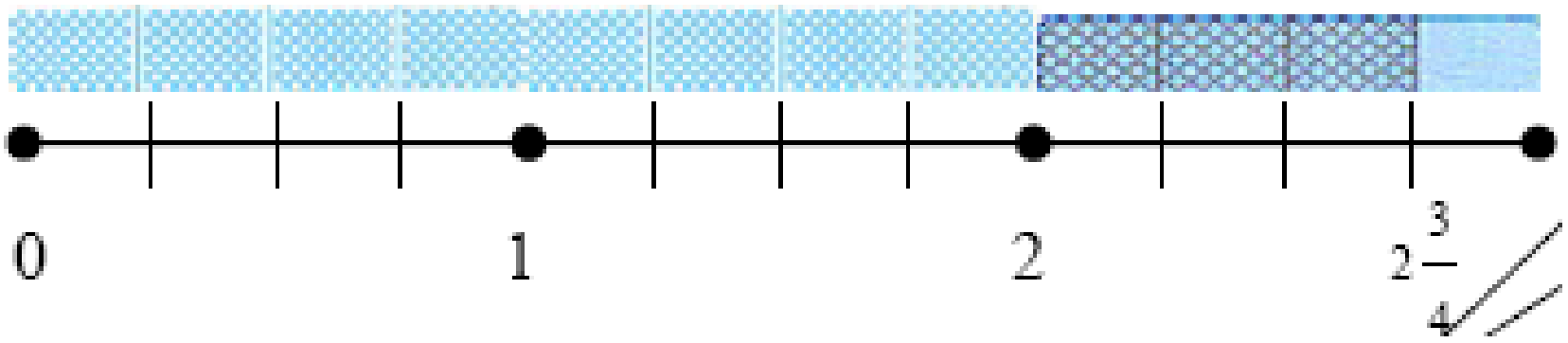
Fraction Tasks

- Hilton has a board that is **two and three-fourths** feet long. How many pieces that are **one-fourth** foot long can he cut from this board?
- After expanding a recipe, Noelle discovers that she needs **eleven-fourths** sticks of butter. How much butter does she need? (Convert to a mixed number.)

Fraction Bar and Number Line Instruction

Change $2\frac{3}{4}$ to an improper fraction.

$$2\frac{3}{4} = 2 + \frac{3}{4} = \frac{2 \cdot 4}{4} + \frac{3}{4} = \frac{2 \cdot 4 + 3}{4} = \frac{11}{4}$$



Algebra Instruction

Rename $x\frac{y}{z}$ as an improper fraction.

$$x\frac{y}{z} \text{ or } x+\frac{y}{z}=\frac{xz}{z}+\frac{y}{z}=\frac{xz+y}{z}$$

Lesson 4

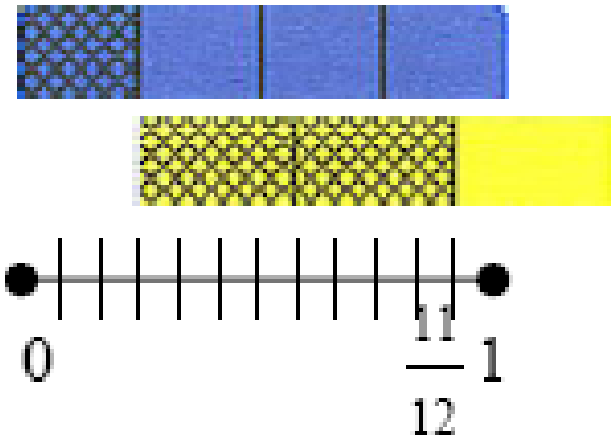
Adding Fractions with Unlike Denominators

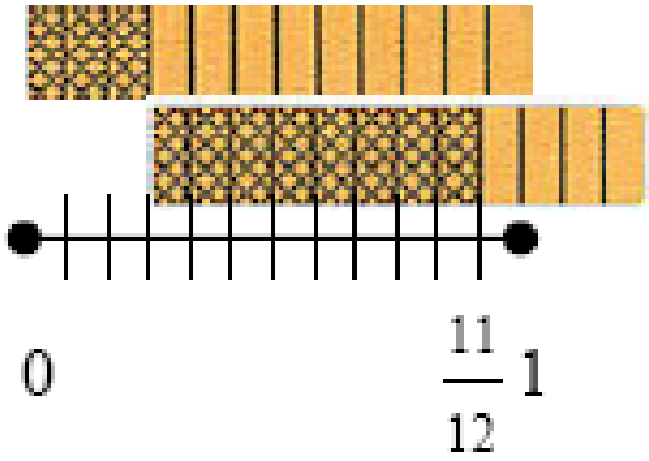
Fraction Tasks

- A local program has **one-fourth** hour of commercials and **two-thirds** hour of show time. How long is the entire show?
- Jack lives **one-sixth** of a mile due west from the Student Union. Bill lives **two-thirds** of a mile due east from the Student Union. How far does Bill live from Jack?

Fraction Bar and Number Line Instruction

$$\frac{1}{4}+\frac{2}{3}=\frac{1\cdot 3}{4\cdot 3}+\frac{2\cdot 4}{3\cdot 4}==\frac{1\cdot 3+2\cdot 4}{12}=\frac{11}{12}$$





Algebra Instruction

$$\frac{3}{x}+\frac{5}{4y}=\frac{3\cdot 4y}{x\cdot 4y}+\frac{5\cdot x}{4y\cdot x}=\cdot$$

$$\frac{12y}{4xy}+\frac{5x}{4xy}=\frac{12y+5x}{4xy}$$

Lesson 5

Multiplying a Whole Number by a Fraction

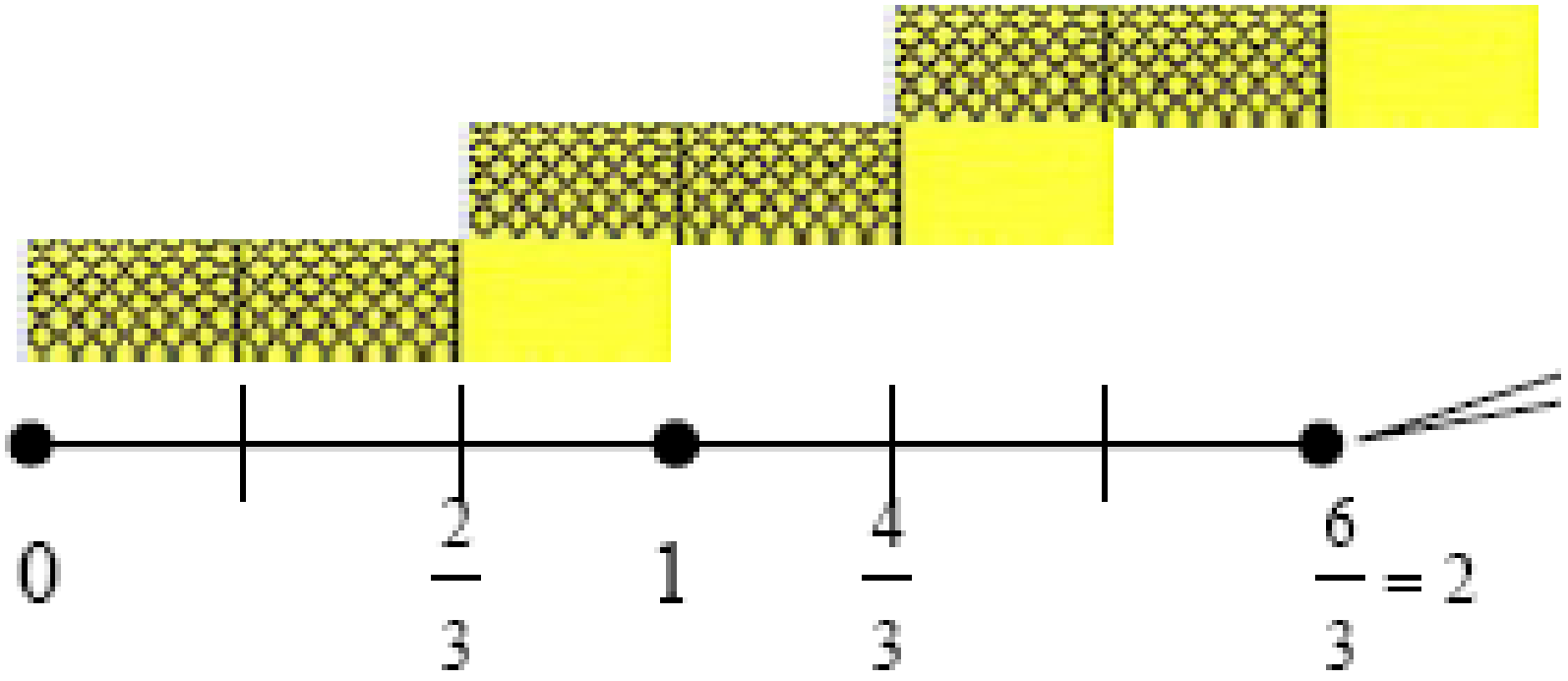
Fraction Tasks

- Amy **triples** a recipe that normally takes **two-thirds** of a stick of butter. How much butter should she use?

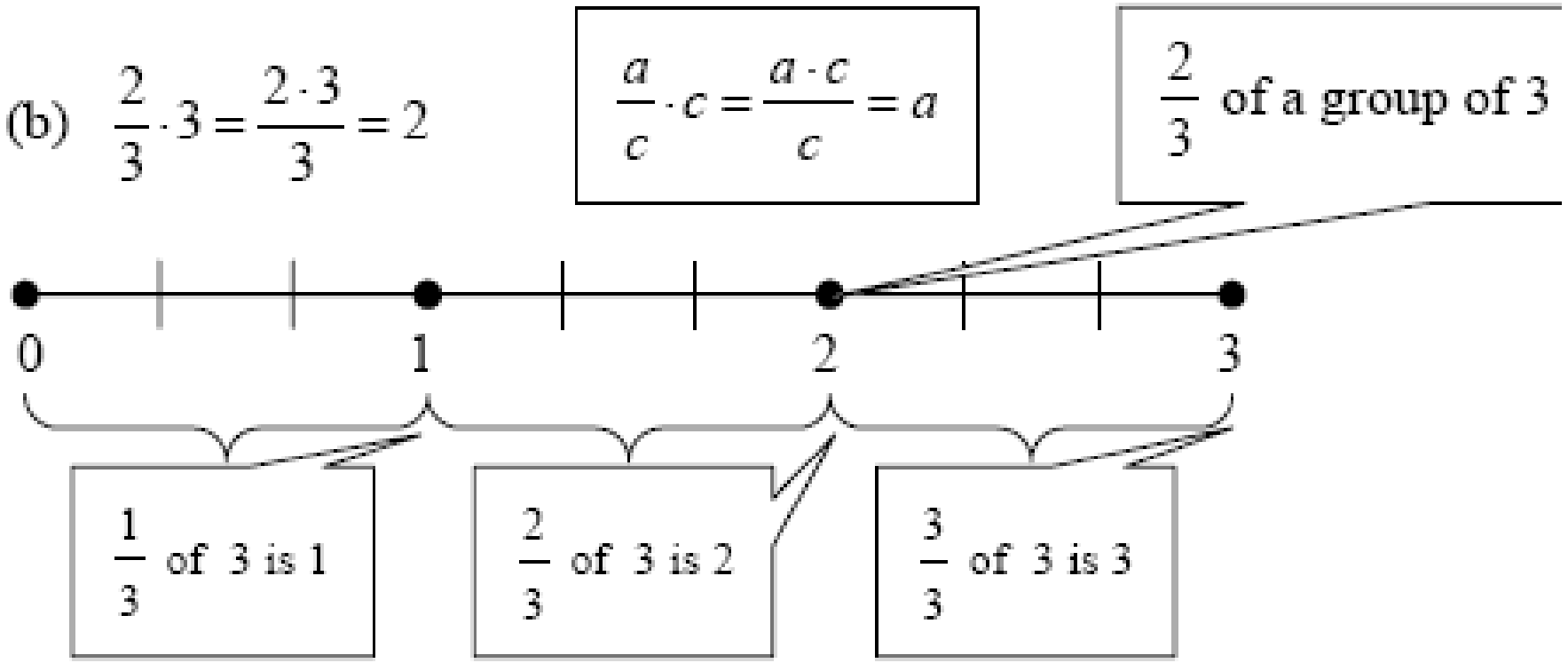
- A recipe for brownies calls for **three-fourths** cup of sugar. Peaches wants to **double** the recipe. How much sugar does she need?

Fraction Bar and Number Line Instruction

(a) $3 \cdot \frac{2}{3} = \frac{3 \cdot 2}{3} = 2$



Fraction Bar and Number Line Instruction



Algebra Instruction

$$\frac{5}{x} \cdot x = \frac{5x}{x} = 5 \qquad \text{Note: } x \neq 0$$

$$5 \cdot \frac{2x+4}{5} = \frac{5(2x+4)}{5} = 2x+4$$

$$3 \cdot \frac{x}{4} = \frac{3x}{4}$$

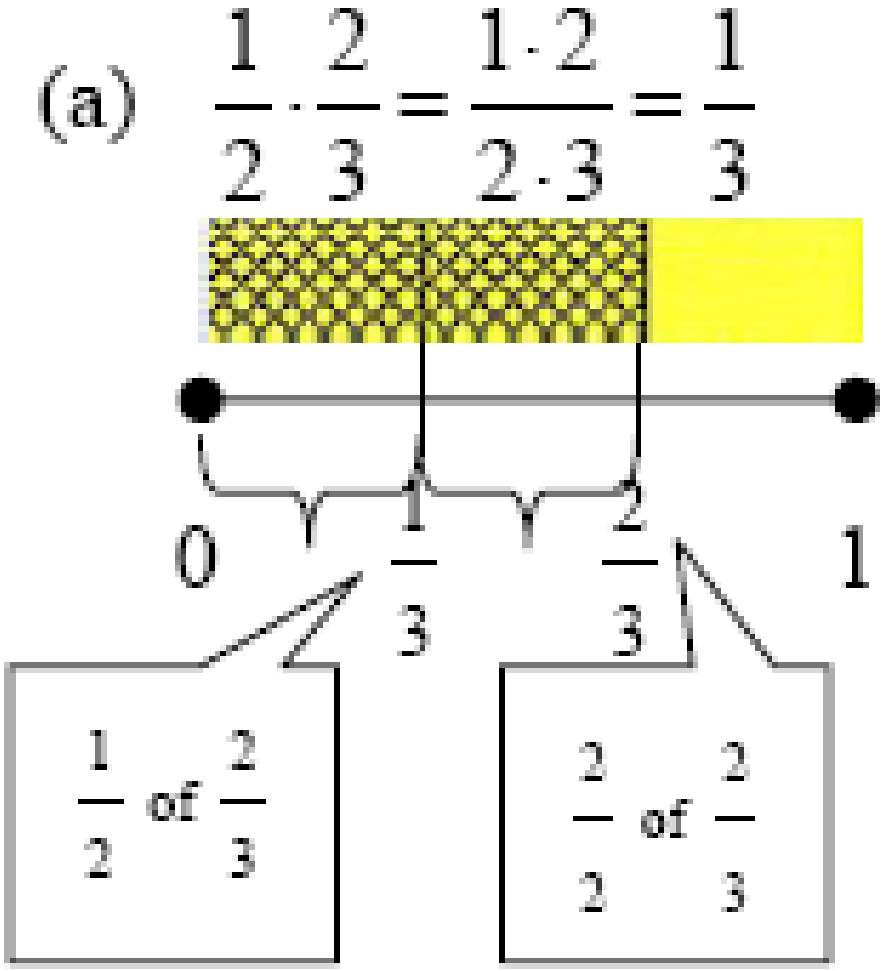
Lesson 6

Multiplying a Fraction by a Fraction

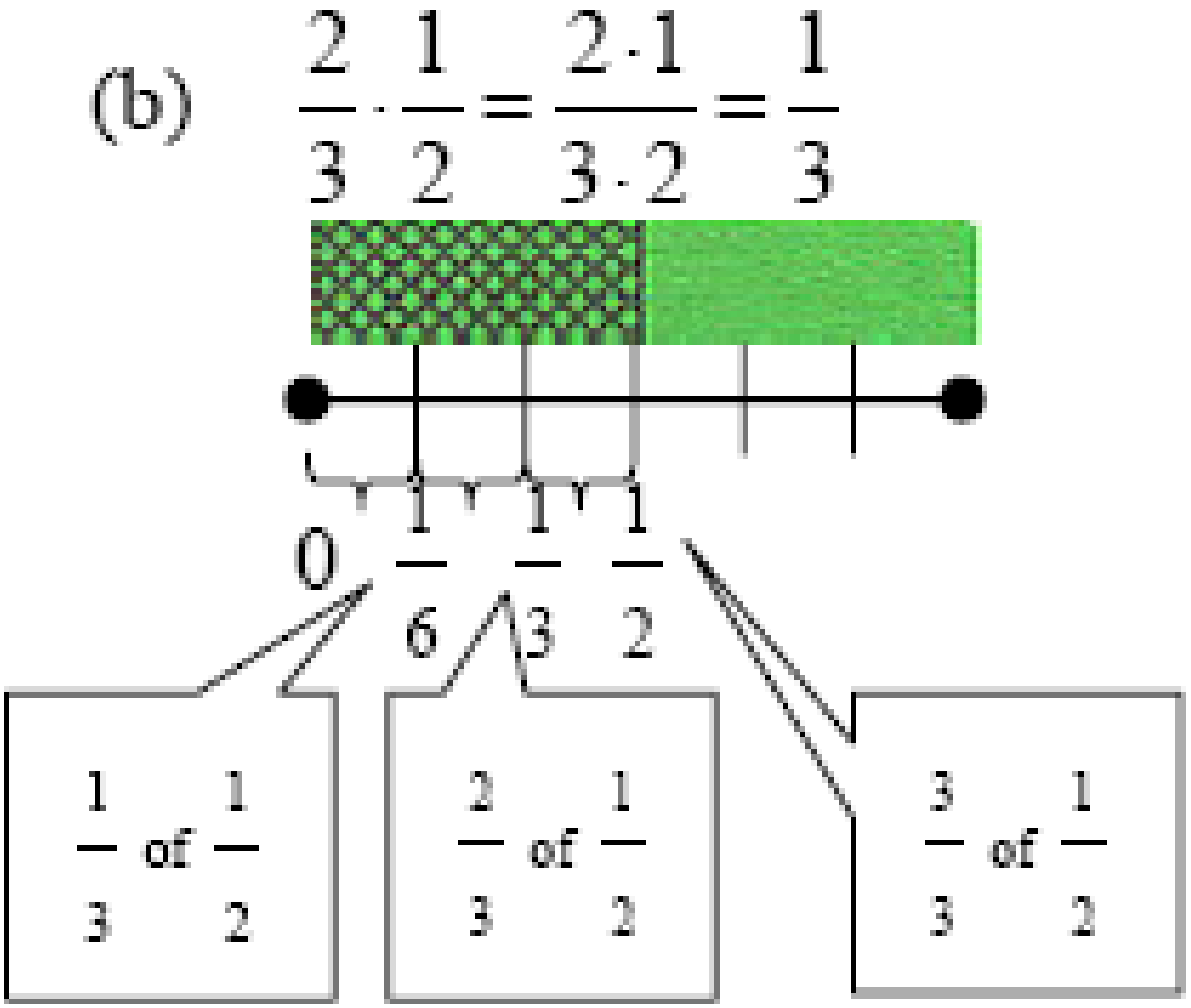
Fraction Tasks

- Marcia found **two-thirds** of a sub sandwich on the table. She decides to eat **one-half** of it. How much of the original sandwich did she eat?
- John lives **one-half** of a mile from his school. When he has walked **two-third** of the distance, how far has he walked?

Fraction Bar and Number Line Instruction



Fraction Bar and Number Line Instruction



Algebra Instruction

$$\frac{2x}{5y}\cdot\frac{3}{4}=\frac{2x\cdot 3}{5y\cdot 4}=\frac{2\cdot 3\cdot x}{2\cdot 2\cdot 5\cdot y}=\frac{3x}{10y}$$

$$\frac{5x+2}{3}\cdot\frac{1}{5x+2}=\frac{(5x+2)1}{3(5x+2)}=\frac{1}{3}$$

Lesson 7

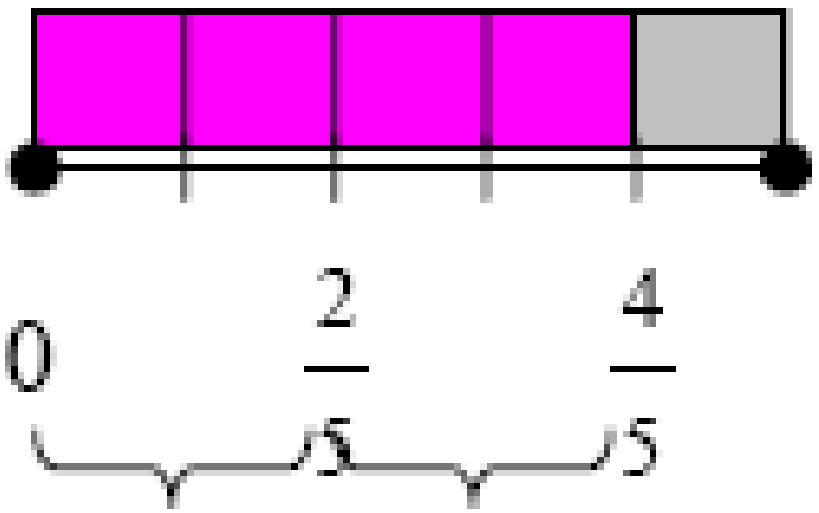
Dividing Fractions

Fraction Tasks

- Blake and David are looking at **four-fifths** of a pizza. How much should each boy get if they cut the pizza in **two** equal slices?
- Josh wants to make **one-fourth** pound hamburger patties. How many of these can he make from **1** pound of hamburger meat?
- A certain track is **two-thirds** of a mile. How many times will Julie need to run this distance if she needs to run **2** miles?

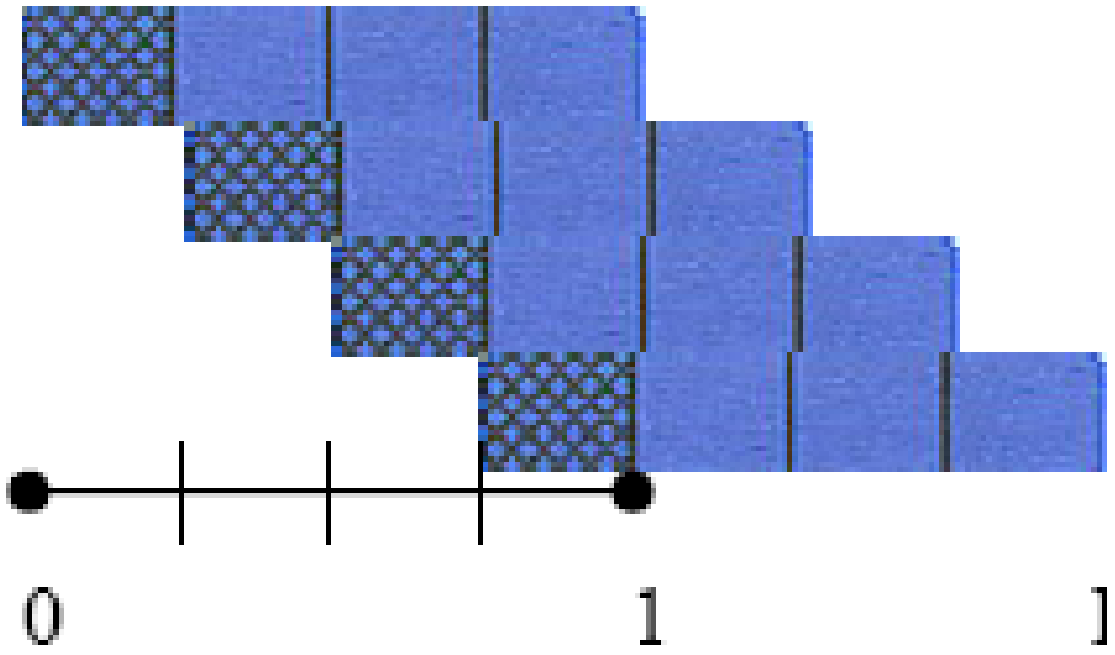
Fraction Bar and Number Line Instruction

(a) $\frac{4}{5} \div 2 = \frac{4}{5} \div \frac{2}{1} = \frac{4}{5} \cdot \frac{1}{2} = \frac{4 \cdot 1}{5 \cdot 2} = \frac{2 \cdot 2 \cdot 1}{5 \cdot 2} = \frac{2}{5}$

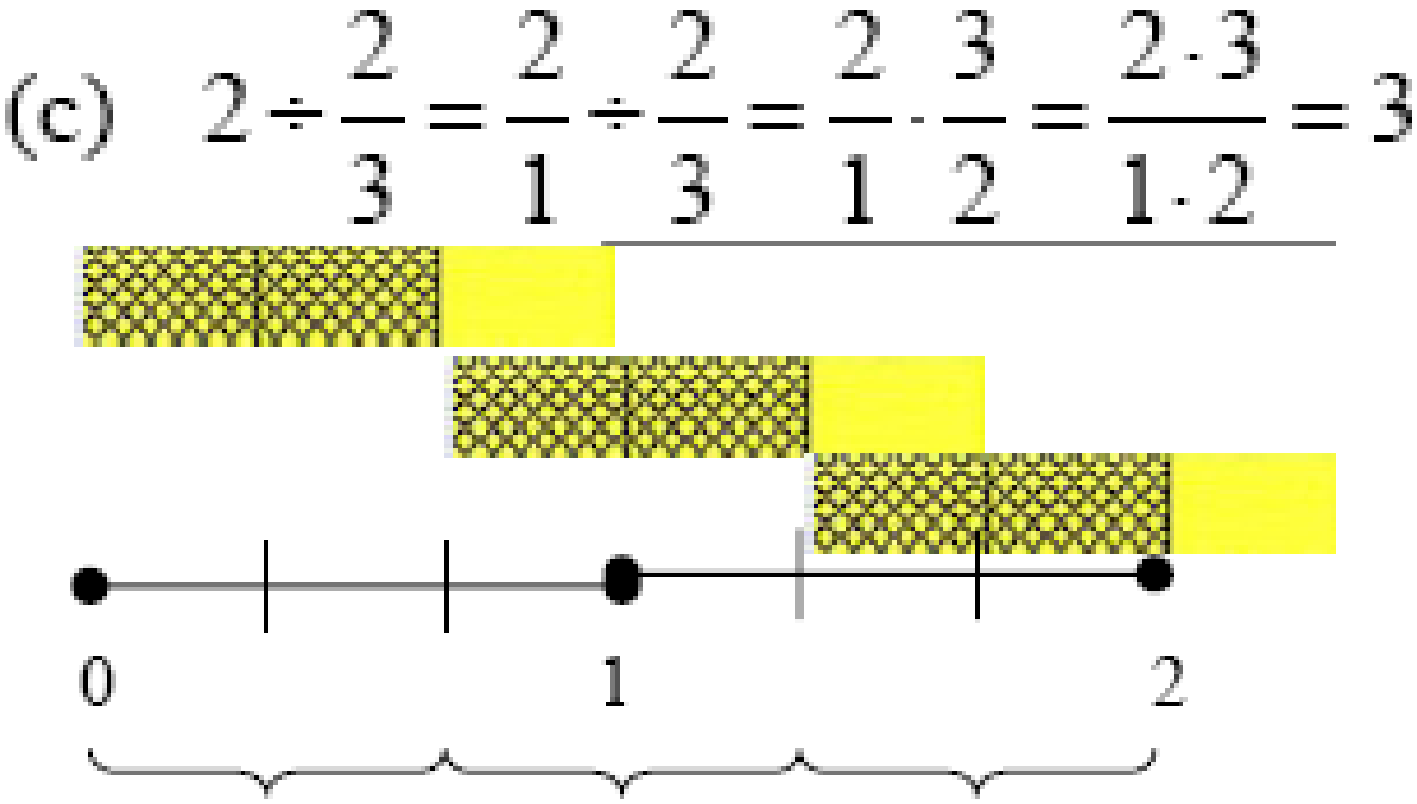


Fraction Bar and Number Line Instruction

(b) $1 \div \frac{1}{4} = \frac{1}{1} \div \frac{1}{4} = \frac{1}{1} \cdot \frac{4}{1} = 4$



Fraction Bar and Number Line Instruction



Algebra Instruction

$$\frac{6x}{5} \div 3 = \frac{6x}{5} \div \frac{3}{1} = \frac{6x}{5} \cdot \frac{1}{3} = \cdot$$

$$\frac{2 \cdot 3 \cdot x}{3 \cdot 5} = \frac{2x}{5}$$

Algebra Instruction

$$2 \div \frac{2}{5x} = \frac{2}{1} \div \frac{2}{5x} = \frac{2}{1} \cdot \frac{5x}{2} =$$

$$\frac{2 \cdot 5x}{1 \cdot 2} = \frac{5x}{1} = 5x$$

Actual Performance Assessment Tasks for Establishing the Arithmetic to Algebra Connection

1.A.

Fraction Definition: Measurement

Pair #1	Pair #2
$\frac{4}{5}$ and $\frac{6}{5}$	$\frac{5}{6}$ and $\frac{7}{6}$

1.B

Fraction Definition: Sharing

Pair #1	Pair #2
$\frac{3}{4}$	$\frac{2}{3}$

2.A

Simplify. Show algorithm, fraction bars and explain.

Pair #1	Pair #2
$\frac{10}{12}$	$\frac{8}{12}$

Add.
Show algorithm & fraction bars.

Pair #1	Pair #2
$\frac{1}{5}+\frac{2}{5}$ and $\frac{x}{z}+\frac{y}{z}$	$\frac{2}{5}+\frac{3}{5}$ and $\frac{x}{z}+\frac{y}{z}$

3.A

Show <u>all</u> steps in order to change each mixed number into its equivalent improper fraction. Show fraction bars.

Pair #1	Pair #2
$2\frac{3}{4}$ and $X\frac{y}{z}$	$2\frac{1}{3}$ and $A\frac{b}{c}$

3.B

Multiply.
Show algorithm & fraction bars.

Pair #1	Pair #2
$3 \cdot \frac{2}{3}$	$4 \cdot \frac{3}{4}$

4.A

Multiply.
Show number line.

Pair #1	Pair #2
$\frac{2}{3} \cdot 3$	$\frac{3}{4} \cdot 4$

4.B

Multiply.
Show algorithm & number line.

Pair #1	Pair #2
$\frac{1}{4} \cdot \frac{2}{3}$	$\frac{2}{3} \cdot \frac{1}{4}$

5.A

Divide.
Show algorithm & number line.

Pair #1	Pair #2
$\frac{8}{5} \div 4$	$\frac{6}{5} \div 3$

5.B

Divide.
Show algorithm & fraction bars.

Pair #1	Pair #2
$3 \div \frac{3}{4}$	$2 \div \frac{2}{3}$

6.A

Contrast. Model each using fraction bars & number line.

Pair #1	Pair #2
$2 \div 3$ vs. $2 \div \frac{1}{3}$	$3 \div 4$ vs. $3 \div \frac{1}{4}$

Science & Math Achievement through Research and Teaching

Start Date: January 1, 2009

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