Effective Use of Process Capability Indices for Supplier Management

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Abstract

Process capability indices were originally invented to enable an organization to make economically sound decisions for process management. Process capability is a comparison of the voice of the process with the voice of the customer. Current practice is to use C_p and C_{pk} regardless of the validity of the underlying assumptions necessary for their use. Even if all necessary assumptions are satisfied, important problems can be missed if these indices are the sole process evaluation examined. Customer-supplier axioms are introduced to motivate more useful process evaluations and foster long-term harmonious relationships. This paper explores the alternative capability indices C_{pm}, C_{pmk}, C_{jkp}, C_θ, and C_s and loss function approaches including Taguchi's unbounded quadratic loss function and the multivariate upside-down normal loss function. Illustrative case studies are presented.

Keywords
Process Capability, Voice of the Process, Voice of the Customer, Quadratic Loss Function

1. Introduction
Process capability is a measure of the repeatability of a process. A process must be in a state of statistical control prior to establishing process capability. Process capability has three main uses. First, process capability is used to determine the ability of a process to meet specifications. Process capability is also used to establish new specifications or modify existing specifications. Finally, process capability can be used as a basis for constructing process control charts.

Process capabilities should allow for a scale-free comparison of process performance quality. A process capability index should also be useful over a wide variety of process types and applications. Process capability should be easy to compute and require a minimum number of reasonable supporting assumptions. Finally, a process capability index should be easy to explain to non-technical decision makers.

2. Process Capability Indices
The two most commonly used capability indices include C_p and C_{pk}. C_p is a measure of the ability of a process to produce constituent results. It is the ratio between the permissible spread of the process (total tolerance) and the actual process spread. C_{pk} is intended to relate process performance to the likelihood of producing bad material. C_{pk} assumes that process data is normally distributed and the usefulness of C_{pk} depends directly on the validity of the specification limits. C_{pk} is a point estimate and, therefore, provides no indication of variability.

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

Where,
- LSL = lower specification limit
- USL = upper specification limit
- σ = process standard deviation

$$C_{pk} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right)$$ (2)

Where,
- LSL = lower specification limit
- USL = upper specification limit
- σ = process standard deviation
- μ = process mean

Kotz and Lovelace [1] give a catalog of indices:
- C_{pm} incorporates quadratic loss
- C_{pmk} combines C_{pk} and C_{pm} properties
- C_{jkp} accounts for a simple type of heteroscedastiscity
- C_θ applies to non-normally distributed data
- C_s applies to skewed data
- Clement's method for non-normal or skewed data

3. Loss Function Approaches

Loss functions relate the product quality distribution directly to economic loss. Loss function approaches are not widely applied because some quantification of loss is required. There are two main techniques for loss functions. The first is Taguchi's quadratic loss function which is unbounded. The second is the upside-down normal loss function (UDNLF). The Quadratic Loss Function (QLF), also known as the Quality Loss Function, is a metric developed by Genichi Taguchi which focuses on achieving the target value rather than of focusing on performance within the wider specification limits. Using the quadratic loss function allows the Six Sigma team to quantify improvement opportunities in monetary terms, the language of upper management. The quadratic loss function translates variability into economic terms by calculating the relationship between performance and financial outcome. The general quadratic loss function is shown in Equation 3.

Loss at any point (L) = (monetary constant) * (average – target)^2 (3)

The quadratic loss function is used to determine the average loss per product or encounter, and it enables Six Sigma teams to focus on performance relative to target and avoid the goalpost mentality. The loss function approximates the long term loss from performance failures and encourages continuous improvement. The quadratic loss function is helpful both as a philosophical approach and as a quantitative method. Figure 1 illustrates the quadratic loss function.
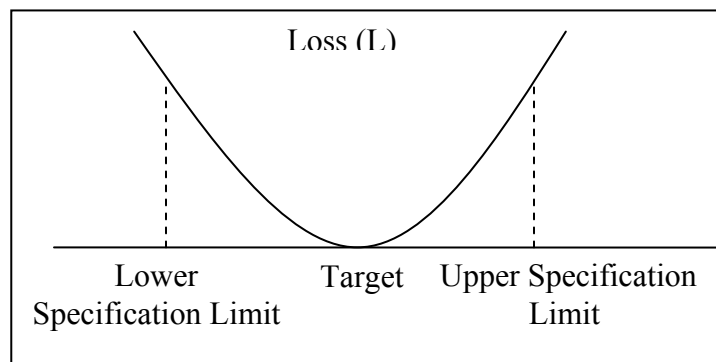

Figure 1: Quadratic Loss Function

Taguchi developed the quality loss function based on the economic consequences of not meeting target specifications. Taguchi defined quality as the "[avoidance of] loss a product causes to society after being shipped, other than any losses caused by its intrinsic functions" [2]. Losses to society encompass costs incurred by the manufacturer, the customer, and anyone affected by product deficiencies.

The quality loss function unifies quality and cost to drive on-target engineering. It also relates economic and engineering terms in one model. The function enables optimization of costs through the production and use of a product [3]. The loss to society includes costs incurred due to a product or service not meeting customer expectations, not meeting performance characteristics, and harmful side effects [4]. The quality loss function associates a dollar value to the current state of quality for a product or process. The quality loss function approximates losses due to scrap, rework, poor performance, lack of customer satisfaction, etc. This dollar amount can be used to identify areas for improvement and evaluate improvement efforts. The quality loss function focuses on target values rather than specifications for process output. The measured loss, *L(y)* for a single product is estimated as shown in Equation 4.

$$L(y) = k(y - T)^2 \tag{4}$$

where,
- *L(y)* is the loss in dollars,
- *y* is the measured response,
- *T* is the target value of the product's response, and
- *k* is the quality loss coefficient.

The quality loss coefficient, *k*, is determined using customer tolerance and the economic consequence. The economic consequence, A_0, is the cost to replace or repair the product. The associated costs include losses incurred by the manufacturer, customer, or a third party. The customer tolerance is the point at which the product reaches unacceptable performance.

$$k = \frac{A_0}{\Delta_0^2} \tag{5}$$

where,
- A_0 is the economic consequence of failure, and
- Δ_0 is the functional limits or customer tolerance for the measured response.

The quality loss function can also be used to determine the average loss per product. The expected loss, E(L), is used to depict the average loss. To reduce the estimated loss, the variability and deviation from target must be reduced [5]. Equation 6 shows the estimated loss.

$$E(L) = k\left(\sigma_y^2 + \left(\bar{y} - T\right)^2\right) \tag{6}$$

where,
- σ_y^2 is the process variance,
- $\bar{y}$ is the response average,
- *T* is the target value, and
- *k* is the quality loss coefficient.

One disadvantage of the Taguchi quadratic loss function is the fact that it is unbounded, but losses in the real world are bounded. This is a serious problem in high-tech industries where specifications are very tight and it is entirely possible to manufacture products several standard deviations from target. In these cases a quadratic loss function does not fit well: it would either have to give unrealistically low losses near target or unrealistically high losses far from target. The upside-down normal loss function (also known as the inverted-normal loss function) is a bounded

alternative to quadratic loss. It was introduced by Leung and Spiring [9] and further developed by Drain and Gough [8]. UDNLF is defined as follows:

$$U(x) = 1 - e^{-\frac{(x-\tau)^2}{2\lambda^2}} \tag{7}$$

Where,

- x=point where loss is evaluated
- μ=process mean
- τ=process target
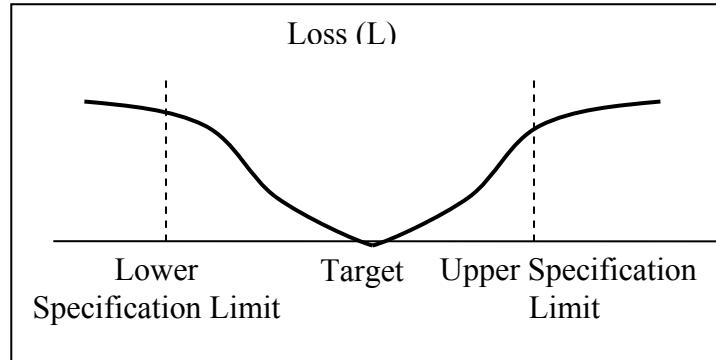- λ=loss function scaling parameter

Figure 2: Upside-down Normal Loss Function

The loss function scaling parameter can be chosen to match actual loss data: larger values mean the customer is more tolerant of variation from target. In the absence of actual loss data a reasonable default value for L is 42.5 percent of the specification range. In this case, the loss when a process is centered on a specification limit is 50% (corresponding to step-function loss in the same situation.) One convenient feature of the UDNLF is that, if the process is normally distributed, the expected loss due to variation from target can be calculated with the following formula:

$$E(U) = 1 - \frac{\lambda}{\sqrt{\lambda^2 + \sigma^2}} e^{-\frac{(\mu-\tau)^2}{2(\sigma^2+\lambda^2)}} \tag{8}$$

Where,

- μ=process mean
- σ=process standard deviation
- τ=process target
- λ=loss function scaling parameter

6.0 Supplier Practices

Current practice is to require a minimum C_p or C_{pk} value. This practice is based on the assumption that the specification limits are valid and a step-function loss is appropriate. Another implicit assumption is that the process data is normally distributed. Finally, there is an assumption that C_{pk} is immune to sampling error which is evidenced by the lack of a confidence level when the data is reported.

However, the assumptions required to calculate a valid C_{pk} may be unsatisfied in many common situations. This leads to spending time and money focusing on the unimportant problems. In turn, the important problems may be overlooked. This also leads to a strained supplier-customer relationship. There are appropriate alternatives in some cases do exist but these are not widely known. Some other situations do not have any easy solution. As Peter Nelson stated, ". . . the concept of attempting to characterize a process with a single number is fundamentally flawed." [6] Kitska stated, "I would prefer eliminating C_p and C_{pk} statistics. They grossly oversimplify process characteristics and, without adequate exploratory analysis, often lead to erroneous or meaningless conclusions." [7]

More effective methods for determining process capability must be established by agreeing upon some basic axioms of supplier-customer relationships. We here propose a set of such axioms:

- All businesses are in the business to make money.
- Everyone benefits from long-term, harmonious relationships.
- Customers want consistent materials meeting their actual needs at the lowest possible cost. Suppliers want a fair price for their time, effort, and goods.
- Additional burdens imposed on suppliers usually translate to increased customer costs or relationship erosion.

In addition to the supplier-customer axioms, there are general guidelines for success. Standard methods should only be used when appropriate and do not apply in all situations. Next, prior performance must be considered; in fact, all information available should be used. Every decision should be a cost-benefit decision. Prioritize efforts to focus on high risk items using a process failure modes and effects analysis (PFMEA).

7.0 Example

Polymer molecular weight is very stable as long as the monomer is from the same 60 gallon drum. Approximately six production runs per drum are produced. Also, in approximately every six production run, a shift in molecular weight is observed. Monomer variation is unavoidable; however, it is well-characterized and controlled. Current quality control techniques at the supplier assure no more than a 1,700 Dalton shift from target in the polymer. Also, process adjustment to compensate for monomer variation causes more harm than good. Figure 2 shows the batch averages and individual run values for 40 runs. Figure 3 shows the process capability of the molecular weight.
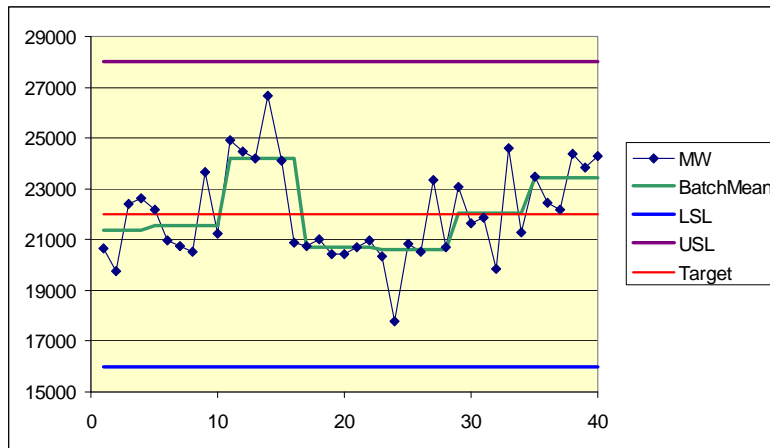

Figure 3: Batch Averages and Individual Run Values

The estimate of a C_{pk} value of 1.09 is not very useful because it over-estimates the percent of material out of specification due to inflation of the estimate of standard deviation which is in fact constrained by controlling the allowable molecular weight ranges. The estimation based on the moving range, i.e. the d2 method, yields a C_{pk} of 1.50. This is invalid because it underestimates the total process variation by ignoring batch-induced variation.

A more appropriate solution is to use the knowledge of the bounds on monomer variation together with historical estimates of standard deviation to calculate the percent of material that would fall inside the specification limits. In this case, this value is approximately 99.975 percent. The C_{pk} value can then be back-calculated to produce 1.22. This value actually falls between the two previous estimates. The percent of product out of specification (% OOS) is calculated using Equation 9.

$$\%OOS = 1 - \int_{-1700}^{1700} \left\{ \Phi\left(\frac{USL - y - \tau}{\sigma_{MW}} \right) - \Phi\left(\frac{LSL - y - \tau}{\sigma_{MW}} \right) \right\} f(y) dy \qquad (9)$$

where,

- σ_{MW} is an estimate of the MW component variance
- τ is the process target (22,000 Daltons)
- $f(y)$ is the uniform distribution (conservative)

The history of batch variation is essential in this example. We did not follow the usual rule, but took advantage of a related precedent. The information about the cause of batch to batch variation could come only via a good supplier-customer relationship.
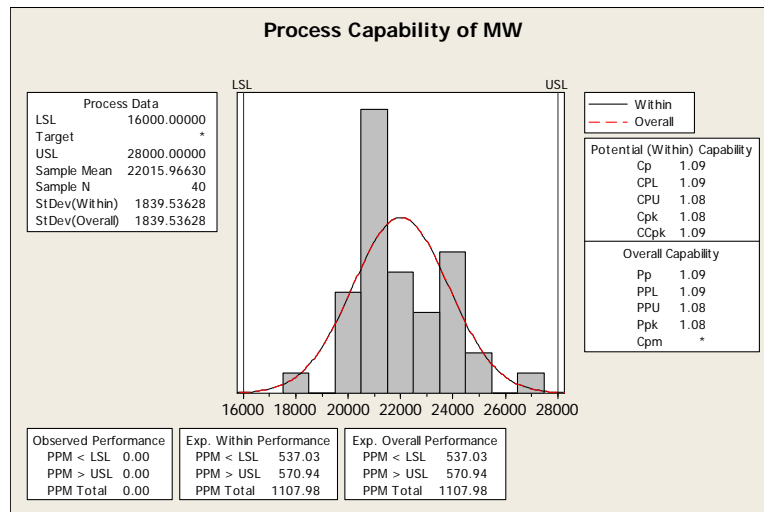

Process Capability of MW

Process Data	
LSL	16000.00000
Target	*
USL	28000.00000
Sample Mean	22015.96630
Sample N	40
StDev(Within)	1839.53628
StDev(Overall)	1839.53628

Potential (Within) Capability
Cp	1.09
CPL	1.09
CPU	1.08
Cpk	1.08
CCpk	1.09

Overall Capability
Pp	1.09
PPL	1.09
PPU	1.08
Ppk	1.08
Cpm	*

Observed Performance		Exp. Within Performance		Exp. Overall Performance	
PPM < LSL	0.00	PPM < LSL	537.03	PPM < LSL	537.03
PPM > USL	0.00	PPM > USL	570.94	PPM > USL	570.94
PPM Total	0.00	PPM Total	1107.98	PPM Total	1107.98

Figure 4: Process Capability of the Molecular Weight

8.0 Suggested Research
In cases where specification limits exist and percent out of specification is the primary concern there are three avenues for further research. First, applying Bayesian process capability assessment should be investigated. C_{pk} can be adapted to account for batch effects. Finally, further research on an index for processes with known time-series effects can be explored. In this instance, C_{pmr} can be a good starting place for the research. In cases where specification limits may be less relevant, but loss is observable a generic and easily implemented loss function methodology should be developed. In addition, loss function approaches for multivariate quality parameters can be surveyed.

References
[1] Kotz, S., and Lovelace, C., Process Capability Indices in Theory and Practice, Oxford University Press, NY, 1998.

[2] Evans, J., and Lindsay, W., The Management and Control of Quality, South-Western College Publishing, Cincinnati, Ohio, 1999, 399-405.

[3] Fowlkes, W., and Creveling, C., Engineering Methods for Robust Product Design, Addison-Wesley, Reading, Massachusetts, 1995, 34-52.

[4] Breyfogle, F., Implementing Six Sigma, John Wiley & Sons, Inc., Hoboken, New Jersey, 2003, 632-634.

[5] Kiemele, M., Basic Statistics, Air Academy Press, LLC, Colorado Springs, Colorado, 1999, 8-6, 8-7.

[6] Nelson, P., Editorial, Journal of Quality Technology, 24, 175.

[7] Kitska, D., Letters: "Cp and Cpk Statistics Should be Discontinued," Quality Progress, 24(3), 188-195.

[8] Drain, D., and Gough, A., "Applications of the Multivariate Upside-Down Normal Loss Function", IEEE Transactions on Semiconductor Manufacturing, 9(1), 143-145.

[9] Spring, F.A., "The inverted normal loss function", Canadian Journal of Statistics, V24, 1993, pp 321-330.