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RELIABILITY VERIFICATION USING SERVICE LOADS

by

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ABSTRACT

Reliability is analysed for existing structures using information on service loads. As an alternative to in situ proof testing to verify the strength of a structure, the service load estimate is used as a proof load with statistical uncertainty, and the estimate of strength is revised. Improved reliability estimates are obtained for service-proven structures that may allow, depending on the service load level, the costs of formal testing to be avoided. In an example, reliability is analyzed for spot-welded cable and conduit structures in nuclear power plants. Initial strength estimates are obtained from prototype tests and service loads can be estimated from data on cable and tray loads. Improvements in reliability are found to be greatest in designs with a large variance in strength rather than loads, owing to the screening effect of the successful service load on low strength values.

INTRODUCTION

In a complex engineering system, many components may be critical to the operation of the system, although in a structural context these same components might be considered secondary. Sometimes the potential consequences of failure of the whole system are much more important than the direct consequences of structural failure. In nuclear power plant systems, for example, cold-formed steel cable and conduit trays are an integral part of the total system. Figures 1 and 2 illustrate typical support systems and cross-sections for electrical cables and conduits. While these systems are peripheral to the actual power plant structure, their failure may precipitate not only the failure of other structural systems, but also the failure of critical non-structural systems that rely upon the cable network. An earlier study [3] found that the failure of these tray systems was governed by spot weld strength, and for some configurations [e.g., Figure 1(h)] prototype tests gave low strength values. The fact that these systems were apparently performing well in service led to the idea for this paper.

In this study, particular attention is paid to the reliability analysis of structural systems already in use, and known to be successfully resisting service loads from self-weight and other sources. A first estimate of strength, called the prior strength, is made on the basis of the knowledge of structural geometry, material strength, and any other information on the structure, such as test results. The successful service load is estimated and treated as a proof load on the strength of the structure. Allowance is made for uncertainty in the value of the service proof load, which is treated as a

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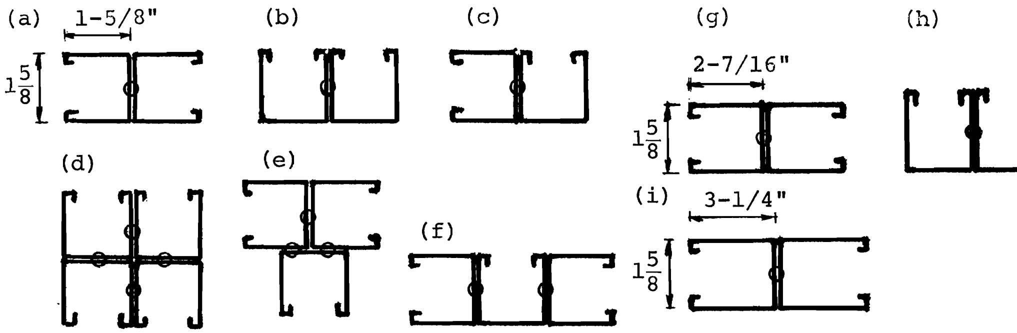


Fig. 1 Typical Cross Sections of Support Members; Spot Welds are Denoted by O

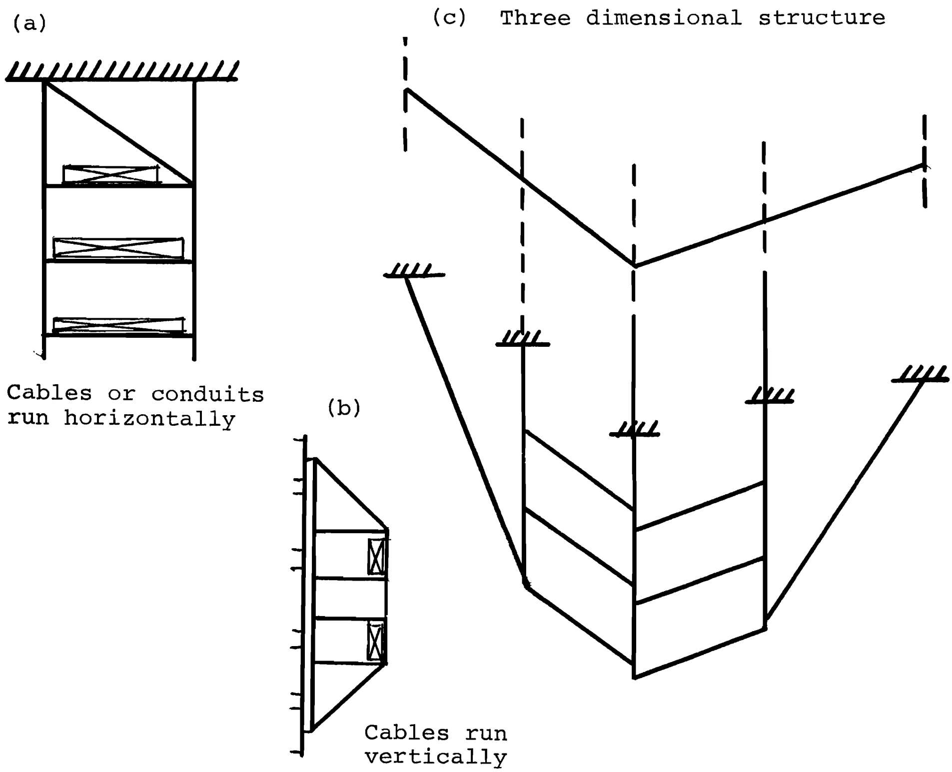


Fig. 2 Typical Configurations of Supports

random variable. The estimate of the strength is then revised, in light of this successful past performance, using probabilistic methods. The new, improved estimate of strength, called the posterior strength, is used for any subsequent analysis of the reliability or safety of the structure.

STRUCTURAL RELIABILITY

Let the strength of a structure be represented by the random variable R and the load effect by the random variable Q (see Figure 3). The performance of the structure can be described by

$$Z = R - Q \; . \tag{1}$$

Failure occurs if the load exceeds the strength. Thus, the probability of failure is

$$P_F = P(R-Q<0), \tag{2}$$

which can be calculated as

$$P_F = \int_{-\infty}^{\infty} F_R(q) f_Q(q) dq, \tag{3}$$

in which $F_R(r)$ is the cumulative probability distribution function of the strength estimate R and $f_Q(q)$ is the probability density function of the load effect Q. The probability of failure is related to (but is not equal to) the overlap of the two distributions as shown in Figure 3.

As an alternative to the integration in Equation 3, if the distribution function of Z is known, then

$$P_F = F_Z\left(-\frac{\bar{Z}}{\sigma_Z}\right) \tag{4}$$

for which the mean and standard deviation of Z are found from the mean and standard deviation of load and strength:

$$\bar{Z} = \bar{R} - \bar{Q} \tag{5}$$

$$\sigma_Z = (\sigma_R^2 + \sigma_Q^2)^{1/2} \; . \tag{6}$$

SAFETY MEASURE

The safety index is

$$\beta = \frac{\bar{Z}}{\sigma_Z} = \frac{\bar{R} - \bar{Q}}{(\sigma_R^2 + \sigma_Q^2)^{1/2}} \tag{7}$$

and is a measure of the safety of a design. If the load and strength are normally distributed, then

$$P_F = \Phi\,(-\beta) \tag{8}$$

in which Φ is the standard normal distribution function. Typical design values of β lie between 3.0 and 4.0 and correspond approximately to nominal probabilities of failure in the range of 10^{-3} to 10^{-4}.

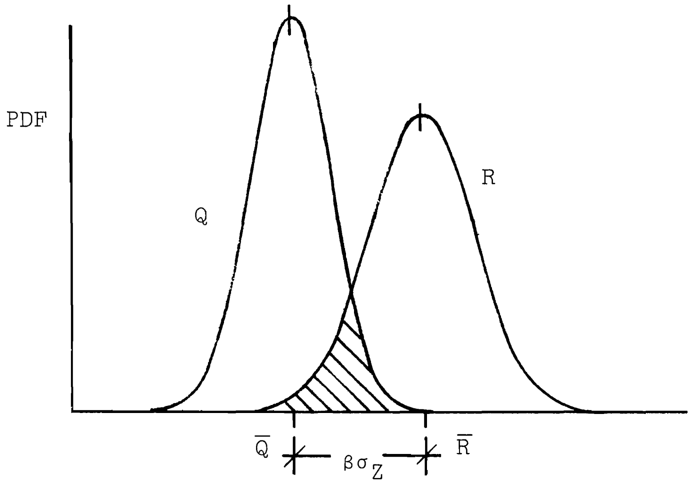


Fig. 3 Random Load Versus Random Strength

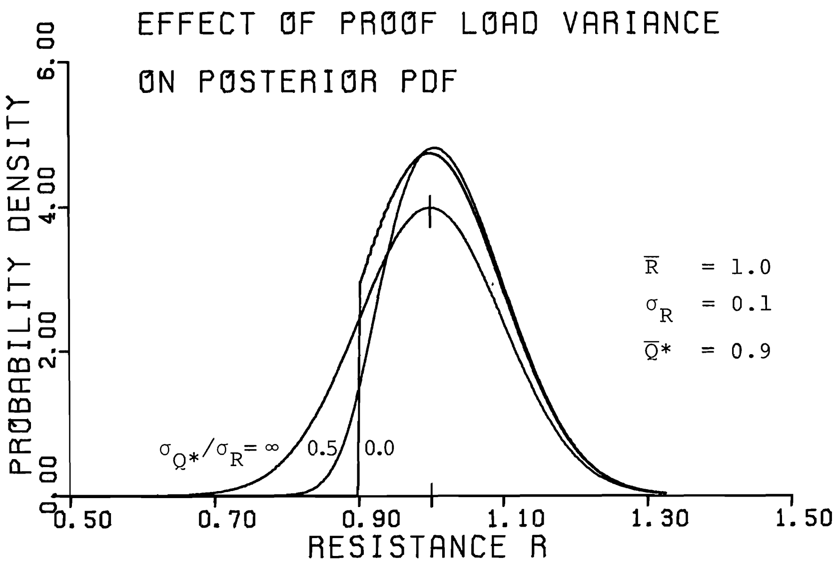


Fig. 4 Strength Distribution after Successful Loading

If the distributions of R and Q are not normal, and the necessary integration is possible, the probability of failure can be obtained from Equation 3 and the safety index can be defined as

$$\beta = \Phi^{-1} (P_F) . \qquad (9)$$

Alternatively, first order (normal) approximations of the real distributions of R and Q can be made at the so called "design point", a point at or near the point of maximum probability density on the failure boundary R-Q=0 (e.g., see [2, 4]). The design point is found iteratively and the approximating normal distributions (*) are then used to define the safety index:

$$\beta^* = (\bar{R}^* - \bar{Q}^*)/(\sigma^2_{R*} + \sigma^2_{Q*}) \qquad (10)$$

This method can also be applied to more complicated performance functions than Equation 1.

One of the objectives in reliability-based design is to provide a structure with strength estimate, R, that does not overlap too much with the load estimate, Q, as measured by the safety index, for example. Knowledge of the strength can be improved by testing, either destructively or nondestructively, and depending on the test results may provide an improved estimate of reliability. A classical non-destructive load test is described in the following section.

PROOF LOAD TESTING

The strength of a structure can sometimes be demonstrated non-destructively by proof load testing, in which a known proof load is applied to the structure [1]. A structure which survives the proof load belongs to that part of the population with strength greater than the test load. The effect of a proof load test is shown in Figure 4 (zero variance, known proof load). The revised distribution of strength for a structure that survives the proof load, $f''_R(r)$, is obtained from a truncation of the distribution prior to the test, $f'_R(r)$, at the proof load q*:

$$f''_R(r) = \frac{f'_R(r)}{1-F'_R(q^*)} \qquad r \geq q^* \qquad (11a)$$

$$f''_R(r) = 0, \qquad r < q^* . \qquad (11b)$$

The proof load test screens out low strength members of the population.

If the value of the proof load has uncertainty, that is, the proof load is a random variable, Q*, then the revised distribution of strength becomes

$$f''_R(r) = \frac{F_{Q*}(r) f'_R(r)}{\int_{-\infty}^{\infty} F_{Q*}(r) f'_R(r) dr} . \qquad (12)$$

This result reduces to Equation 11 when the proof load has zero variance, i.e., when it is known.

The effect of variance in the proof load is illustrated in Figure 4, assuming a normal prior strength distribution. For low variance in the proof load, the probability of low strength components is much reduced, similar to the screening action of a classical proof load test. If the proof load has high variance then little change in the strength distribution occurs.

PROOF LOAD EFFECTS ON RELIABILITY

Successful past resistance of a structure to a service load, Q*, acts as a proof test on its strength. In some applications it may be possible that a successful service load and its variance can be estimated from data on existing or past loads, inspections, or other information. Equation 12 can then be used to revise the initial strength estimate without a formal proof load test. The improved strength estimate is then used to evaluate the structural reliability of the service-proven system for the anticipated design load.

Figure 5 illustrates a conceptual structure of strength R, which has experienced a maximum load, Q*, in service. Given the probability distributions of the initial strength estimate and service load, the reliability of the same structure under a different load, Q, is required. This situation could arise in many different ways. For example, it may be that the design load has to be revised because of a change in use of the structure, or a different grade of steel than intended may have been used in service. Whatever the reason, it is required to take a second look at the safety of the structure, taking into account its successful past resistance to service loads.

Consider the case in which the design load, Q, and strength estimate, R, are normally distributed with mean values of 100 and 200 units, respectively. Suppose that the apparent safety index is $\beta'=2.0$; that is,

$$\bar{Z} = \bar{R} - \bar{Q} = 100 \qquad (13)$$

and from Equation 7,

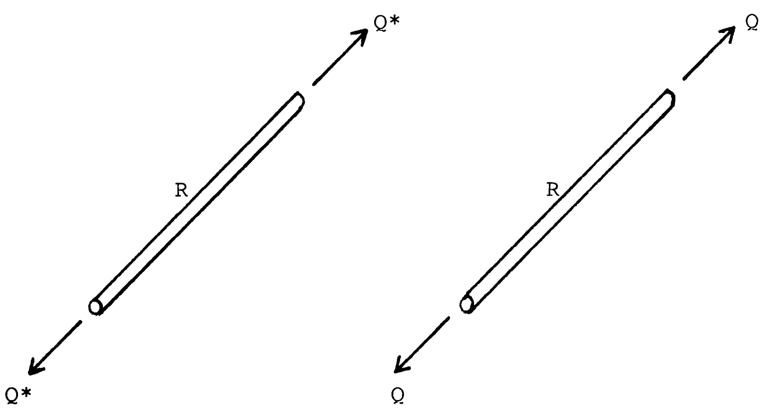
$$\sigma_z = 50 = (\sigma_R^2 + \sigma_Q^2)^{1/2}. \qquad (14)$$

The observed service load, Q*, is normally distributed with mean $\bar{Q}^*$ and an assumed coefficient of variation of 5%. Equation 12 is used to revise the strength distribution and Equations 3 and 9 are then used to evaluate safety for various levels of the mean service load $\bar{Q}^*$ and strength variance σ_R. Table 1 summarizes the results. The results for probability of failure and safety index, after service, show significant improvements when the design uncertainty, as measured by σ_z, comes primarily from the strength,(i.e., from σ_R). There is little benefit from the screening effect on low strengths of the service proof load when uncertainty is predominantly in the design load (i.e., when σ_Q is high relative to σ_R.) In this case low strength components already have small probabilities and little is gained from the proof load effect. Of course, the higher the observed service load, the greater is the improvement in safety levels.

Table 1. - EFFECT OF SUCCESSFUL SERVICE LOADS ON RELIABILITY

Mean Service Proof Load $\overline{Q}^*$	Strength Uncertainty σ_R	Design Load Uncertainty σ_Q	Nominal Prob. of Failure P_F	Safety Index β''
100	50	0	0.00198	2.88
	49	10	0.00564	2.54
	40	30	0.0187	2.38
	10	49	0.0228	2.00
	0	50	0.0228	2.00
150	50	0	$<10^{-7}$	>4.00
	49	10	3.39×10^{-7}	>4.00
	40	30	0.00436	2.62
	10	49	0.0228	2.00
	0	50	0.0228	2.00

$\overline{R} = 200$, $\overline{Q} = 100$, $\sigma_Z = 50$; $\beta' = 2.0$, $V_{Q*} = 0.05$.



(a) (b)

Fig. 5 Structure under (a) Service, (b) Design Loads

EXAMPLE: ELECTRICAL CABLE SUPPORT SYSTEMS

Two of the spot welded cable support systems in Figure 1 are investigated, namely the back-to-back case (a) and side-to-side case (h). These were found to have low safety levels in the earlier study [3]. Figures 6 and 7 show the strength distributions obtained from prototype tests of these systems, and Figure 8 shows typical welds.

The strength of single spot welds for the back-to-back case is represented by a normal distribution with $\bar{R}$ = 3,400 lb (15.1 kN) and σ_R = 1,100 lb (4.90 kN). A hypothetical design load is assumed to be normal with $\bar{Q}$ = 1,200 lb (5.34 kN) and σ_Q = 600 lb (2.67 kN). This approximates the design situation of the systems, giving a safety index, before service, of β' = 1.76.(P_F' = 0.03955). The actual design load is a combination of the sustained load from self-weight and cable loads, D, and earthquake loads, E, and is not normally distributed (see [3]). The observed service load is assumed to be equal to the above-mentioned sustained load, giving Q* = 900 lb (4.01 kN) in this example, with a coefficient of variation of 5%. The resulting after-service safety index was found to be β'' = 1.88 (P''_F = 0.03022), a modest increase in safety. The estimate of fifth percentile strength increased from 1590 lb (7.08 kN) to 1670 lb (7.43 kN).

An analysis of the side-to-side case was made with $\bar{R}$ = 3,800 lb (16.9 kN), σ_R = 2,000 lb (8.90 kN), $\bar{Q}$ = 1,200 lb (5.34 kN) and σ_Q = 600 lb (2.67 kN), giving β' = 1.25 (P_F' = 0.1065). For a service load of Q* = 1,000 lb (4.45 kN) with 5% coefficient of variation, the safety index, after service, increased to β'' = 1.68 (P_F'' = 0.03976). The estimate of fifth percentile strength increased from 510 lb (2.27 kN) to 1,460 lb (6.50 kN) and the failure probability improved by a factor of almost three.

In both of these hypothetical cases, although improvements in failure probability were obtained, and obtained without the costs of formal proof testing, the revised safety levels still appear to be low. The reason for so modest an improvement appears to be that the design load (earthquake) is not well represented by the service loads (cable and element weights). In other design situations with more similarity between working loads and design loads, the proof load effect of service loads on reliability is likely to be more pronounced.

Options which can be used to further improve the reliability estimates for these systems include the use of system reliability for weld groups, the possible use of lognormal strength distributions, and the use of a superimposed proof load, q*, in addition to the sustained load already acting on the structure. In the side-to-side case, for example, a superimposed load of about 1,100 lb (4.90 kN), in combination with the existing cable loads, would verify the safety level at approximately β'' = 3.0.

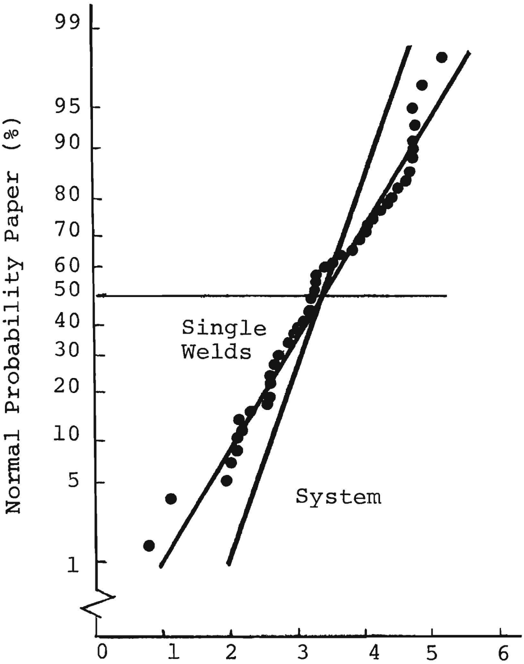


Fig. 6 Test Results and CDF for Back-to-Back Spot Welds, Fig. 1(a)

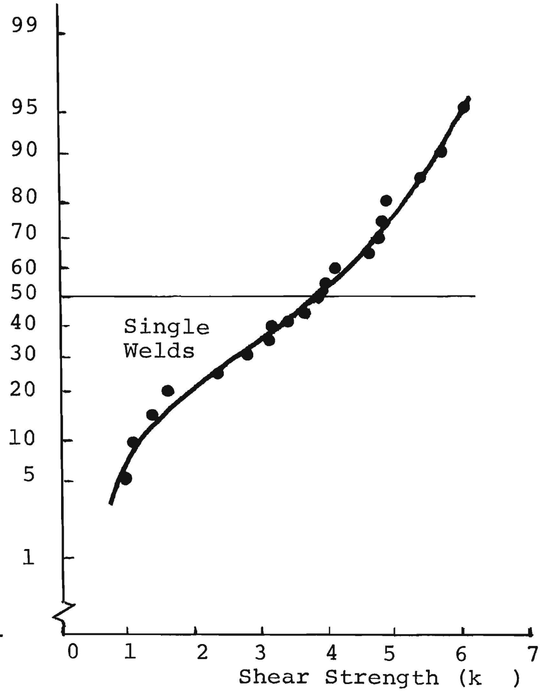


Fig. 7 Test Results and CDF for Side-to-Side Spot Welds, Fig. 1(h)

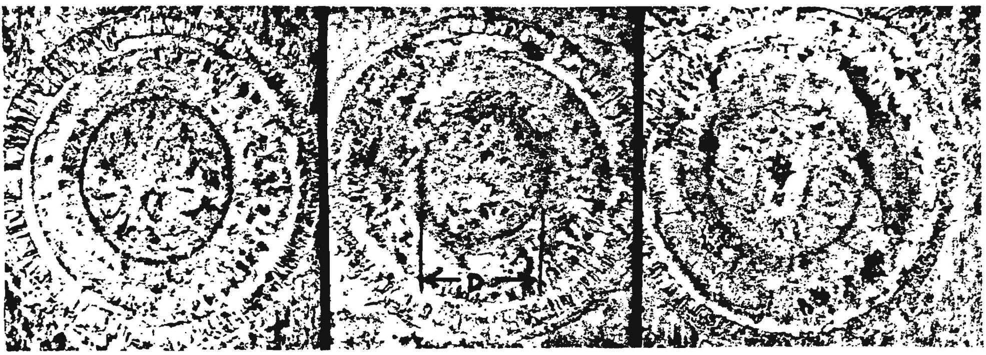


Fig. 8 Photographs of Back-to-Back Spot Welds. D is Diameter of the Fused Area

CONCLUSIONS

The reliability of existing structures which have survived service loads can be analysed by treating the service load as a proof load on the strength of the structure. The proof load is represented as a random variable to account for uncertainty in the value of past service loads.

Structures in service have smaller probabilities of low strength components owing to the proof load effect of successful resistance to past loads. Reliability of these structures is higher than that of new, unproven structures from the same population. The greatest improvements in reliability are found in structures which have a high variance in initial strength estimates and which have survived relatively high service loads.

Reliability analysis using service loads can be combined with destructive testing, conventional proof testing and other methods of analysis, as part of a safety verification program for existing structures.

APPENDIX - REFERENCES

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3. Nowak, A. S., and Regupathy, P. V., "Reliability of Electric Cables and Conduit Supports", Sixth International Specialty Conference on Cold-Formed Steel Structures, St. Louis, MO, November, 1982.
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APPENDIX - NOTATION

D	=	sustained load,
E	=	earthquake load,
F	=	cumulative distribution function,
f	=	probability density function,
P_F	=	probability of failure,
Q	=	design load,
Q*	=	service load,
R	=	resistance,
v	=	coefficent of variation,
Z	=	R - Q, performance function,
β	=	reliability index,
Φ	=	standard normal distribution function,
σ	=	standard deviation,

Superscripts:

-	=	mean value,
*	=	design point estimate,
'	=	before-service estimate
"	=	after-service estimate.