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Z-and C-Purlins Connected with Corrugated Steel Sheeting

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by

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INTRODUCTION

It has long been known that thin-walled open sections can be used with great success as purlins, etc. If the beams shall be competitive it is necessary, however, to prevent or reduce the torsional stresses which accompany the plane bending stresses and sometimes are greater than these. This can be done by fixing the free flange, for instance, by means of sag-bars. That method is, however, rather expensive. This paper presents a simplified theory in which the stiffening effect of the roof or wall sheeting is utilized.

Since torsional stresses, normally reaching 10 to 30 per cent of the total stress, still occur, it has proved suitable to use steel with a yield strength of 32 to 40 kp/mm^2 (45 to 56 ksi). This is often impossible for traditional purlins because of the great deflections that occur in such construction.

NATURE OF THE PROBLEM

Z-and C-sections are characterized by having no or only one axis of symmetry. This implies that the section tends to rotate or to move also in another direction than that of the load. When such sections are used in beams and fastened to corrugated sheets they will be fixed along at least one axis. The problem is made more difficult by the fact that the amount of the restraint depends not only on the rigidity of the sheeting and the shape of the section but also on the loading direction. Furthermore, the position of the axis of rotation changes when the load alters direction, as shown in Fig. 1.

MOMENTS AND STRESSES

A vertical external force generates a vertical shear force in the web with the same value as the external force. It also generates in the free flange a horizontal shear force of the same value as caused by a fictitious external horizontal force of

$$q_h = q_v \cdot k_h \tag{1}$$

The constant k_h depends only on the shape of the cross section.

The "load" q_h is carried partly by the free flange which bends laterally, and partly by the web, which is restrained by the sheeting. The free flange may be regarded as a beam on an elastic foundation with the modulus of foundation expressed by $C = \frac{1}{\delta_2}$ [kp/mm^2], where δ_2 equals the deflection of the web caused by a unit force per unit length of the beam.

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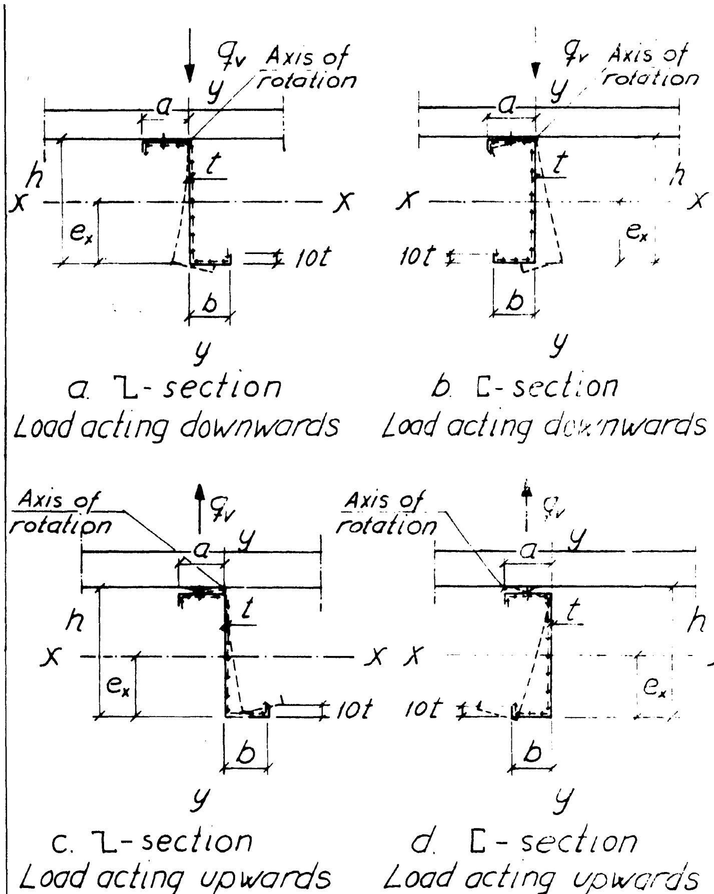


FIG. 1 The amount of the restraint effect provided by the sheeting depends on the shape of the profile and the direction of the load.

$$N_{z_1} = N_{z_2} = \frac{M_x}{I_x} \cdot A_{fl} \cdot (e_x - e^*_x) \tag{2}$$

Thus, N_{z_1} and N_{z_2} depend on the boundary conditions at the supports. For multi-span with C-section jointed web to web over the supports, M_y equals zero over the supports.

With the simplifying assumption that the lateral deflection curve has the same shape as for a beam with C = 0, the following expression is obtained for the part of the load that is carried by the free flange acting as a beam

$$q_{h,dim} = \frac{1}{1 + \frac{\delta_1}{\delta_2} + k \cdot \frac{N \cdot s}{N_o}} = q_h \cdot \varepsilon = q_v \cdot \varepsilon \cdot k_h \tag{3}$$

where

$$N = \frac{M_x^{max}}{I_x} \cdot A_{fl} (e_x - e^*_x) \tag{4}$$

$$N_o = \frac{\pi^2 EI^*_y}{L^2} \tag{5}$$

s = safety factor

$$\delta_1 = K \frac{L^4}{EI^*_y} \qquad (6)$$

$\delta_2 = \frac{1}{C}$ = lateral deflection of the web for unit load at lower edge and without regard to the influence of the free flange.

This assumption concerning the shape of the lateral deflection curve is correct only when C approaches zero. In practice, however, this assumption is quite acceptable, except for downwards loading of Z-sections. In this case it may be assumed that the "load" q_h is carried by bending moments in the web. Next to the supports, however, q_h is carried by bending of the free flange as well. The bending moment in the free flange at the supports can be written as

$$M_s = 2b^2 \cdot q_h \qquad (7)$$

where

$$b = \sqrt[4]{\frac{EI^*_y}{4 \cdot C}}$$ see Reference No. 2.

This equation disregards the normal forces in the flange. The moment M_s is the same as the moment of a uniform load

$$q_{h,\,dim} = q_v \cdot \varepsilon_1 \cdot k_h \qquad (8)$$

where

$$\varepsilon_1 = 0.54\sqrt{\frac{\varepsilon}{1-\varepsilon}}$$ see Reference No. 1.

$$\varepsilon = \frac{1}{1 + \frac{\delta_1}{\delta_2}} \qquad (9)$$ compare this expression with Eq. 3.

The value of δ_2 depends on the shape of the cross section and the loading direction. Table 1 gives somewhat simplified expressions for different cases:

The values of the factors k and K depend on the shape of the cross section and the boundary conditions of the beam as given in Table 2.

The value of θh depends on the shape and the thickness of the corrugated sheeting and the spacing of the fasteners (screws). θh has been determined by tests for a series of sections produced by the steel mill of Domnarvets Jernverk in Sweden.

Its value was determined as 500 to 700 mm/kp for unfavorable sheetings with fasteners spaced at 400 mm.

The moments M_x and M_y now can be calculated in the usual way with q_v and $q_{h,\,dim}$ as external loads.

The maximum longitudinal stresses always occur at the web-to-flange junction at the free flange and are given by the expression

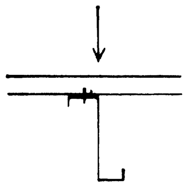
$$\sigma_{max} = \frac{M_x \cdot e_x}{I_x} + \frac{M_y \cdot e^*_y}{I^*_y} \qquad (10)$$

Allowable shear stresses and combined bending and shear stresses are given by the AISI Specification, Reference No. 4.

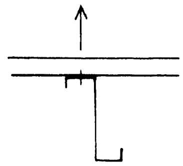
BUCKLING OF THE FLANGE

The free flange will buckle, for practical cases, in a single wave under upward loading. Putting the denominator

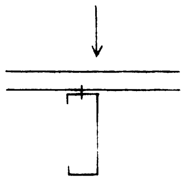
TABLE 1



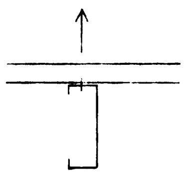
$$\delta_2 = \frac{4h^3}{Et^3} + \frac{h^2 d}{12\,EJ_{sh}} + \frac{h^3 k_h \cdot 6}{E t^3}$$



$$\delta_2 = \frac{4h^3}{Et^3} + \frac{h^2 d}{12\,EJ_{sh}}\left(1 - \frac{1}{k_h}\frac{a}{2h}\right) + \theta h + \frac{2h^2 a}{Et^3}$$



$$\delta_2 = \frac{4h^3}{Et^3} + \frac{h^2 d}{12\,EJ_{sh}} + \theta h + \frac{2h^2 a}{Et^3}$$



$$\delta_2 = \frac{4h^3}{Et^3} + \frac{h^2 d}{12\,EJ_{sh}}\left(1 + \frac{1}{k_h}\frac{a}{2h}\right) + \theta h\left(1 + \frac{1}{k_h}\frac{a}{h}\right) + \frac{8h^2 a}{Et^3} + \frac{5a^2 h}{2k_h E t^3}$$

Constants K and k

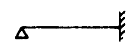
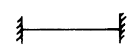
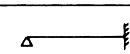
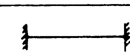
		K	k
Z-section		$\frac{1}{97}$	+ 0,46
—‖—		$\frac{1}{239}$	+ 0,14
—‖—		$\frac{1}{504}$	0
C-section		$\frac{1}{97}$	+ 0,46
—‖—		$\frac{1}{97}$	− 0,076
—‖—		$\frac{1}{97}$	− 0,63

Table 2

in Eq. 3 equal to zero gives

$$N_{kr} = \frac{N_o}{k}\left(1 + \frac{\delta_1}{\delta_2}\right) \qquad (11)$$

For downward loading the buckling of the flange near the supports is given by (see Reference No. 2),

$$N_{kr} = D\,\frac{EI^*_y}{L^2} + 2\sqrt{\frac{EI^*_y}{\delta_2}} \qquad (12)$$

According to Reference No. 3 D may be chosen equal to 200.

DEFLECTION

According to Maxwell's equation we get the following expression for the deflection in the direction of the web

$$\delta = \delta_o \, (1 + k_h^2 \cdot \varepsilon \cdot \frac{I_x}{I^*_y}) \qquad (13)$$

where

δ_o = deflection calculated without regard to the lateral deflection of the free flange.

TESTS

A number of full scale tests have been carried out. These tests have shown satisfactory agreement with the numerical analysis, see Figs. 4 and 5.

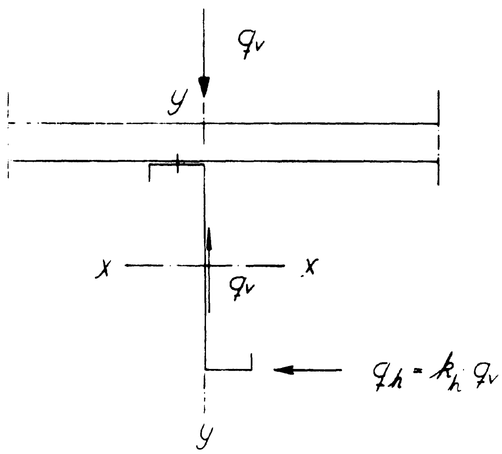
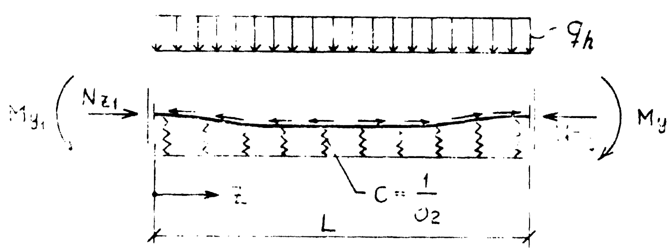
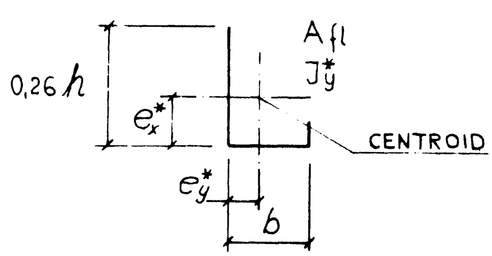


FIG. 2



a. FORCES ACTING ON THE FREE FLANGE.



b. PART OF THE SECTION THAT FORMS THE FREE FLANGE.

FIG. 3

PRACTICAL DESIGN

It is evident that the presented method of design, although elementary and simplified, is too tedious for practical use by the design engineer. It is therefore necessary to prepare a number of design diagrams, for instance, as is shown on Fig. 6, where each curve represents one shape. Diagrams must be drawn for all boundary conditions and for downward as well as for upward loading. Since the supports are subject to vertical as well as lateral forces and the diagrams do not give any information about the values of lateral forces, it is convenient to apply standardized fixings for each shape.

BEAM 1-C. DEFLECTIONS

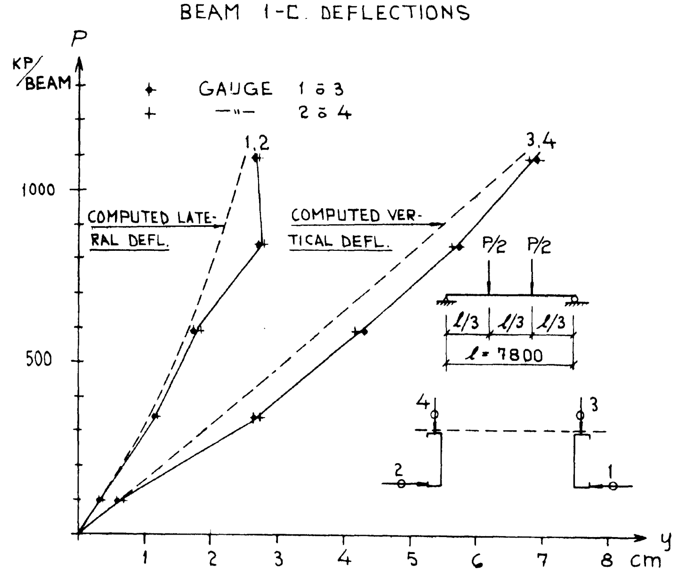


FIG. 4

BEAM 2-Z. DEFLECTIONS

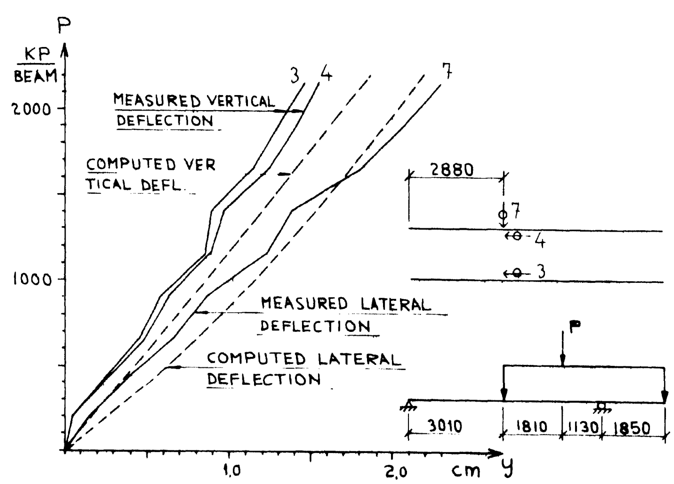


FIG. 5

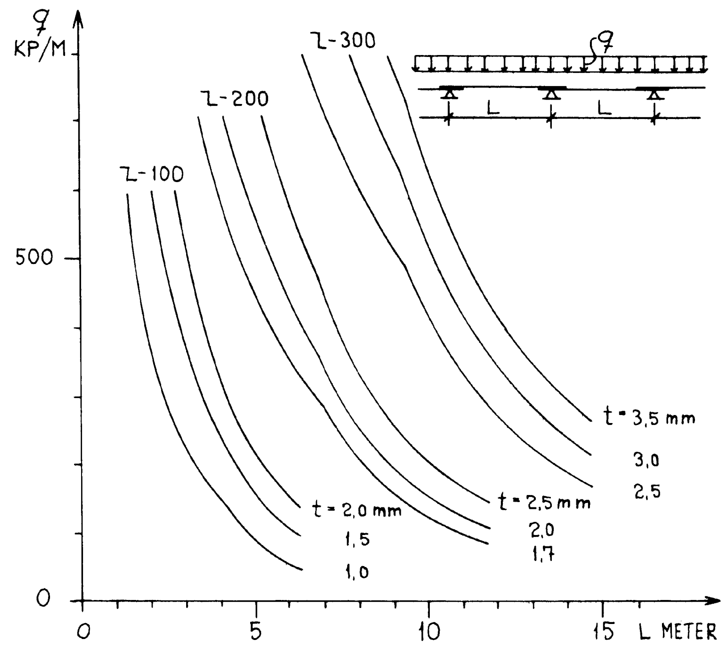


FIG. 6

CONCLUSIONS

Z-and C-shaped beams are competitive if the torsional stresses are reduced by taking into account the restraint effect of the roof or wall sheeting.

Deflections and load bearing capacity depend not only on the geometry of the section but also on the flexural rigidity of the sheeting and on the loading direction.

The approximations involved in the design method presented appear to be acceptable.

ACKNOWLEDGEMENTS

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APPENDIX I - REFERENCES

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2. M. Hetényi: Beams on elastic foundation, 1967.
3. Henrik Nylander: Stabilitet av kontinuerlig I-balk med sidostagad överfläns (Stability of continuous I-beam with laterally supported upper flange), Stockholm 1964.
4. AISI: Specification for the design of cold formed steel structural members, 1968 Edition.

APPENDIX II - NOTATION

x, y, z = Cartesian coordinates

E = Young's modulus

I_x = Moment of inertia about x-axis for the complete section

I^*_y = Moment of inertia about y-axis for the free flange

t = Wall thickness

a, b = Width of flanges

h = Depth of web

k_h = Shear force in flange generated by a unit force in direction of the web

A_{fl} = Area of the free flange, in which $0.26h \cdot t$ is included

e_x = Distance from neutral axis of the complete beam to the lowest fiber of the web

e^*_x = Distance from neutral axis of the free flange to the lowest fiber of the web

e^*_y = Distance from neutral axis of the free flange to web

q = Load per unit length

M_x = Moment about x-axis

M_y = Moment about y-axis

L = Beam span

d = Distance between purlins

I_{sh} = Second moment of area of the corrugated sheeting per unit width

$\delta_1 = K\frac{L^4}{EI^*_y}$

$\delta_2 = \frac{1}{C}$ = Lateral deflection of web for unit load at lower edge and without regard to the influence of the free flange

k, K = Constants