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STRENGTH AND STIFFNESS CALCULATION PROCEDURES FOR COMPOSITE SLABS

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SUMMARY

Two procedures for calculating the strength and stiffness of composite slabs based on a partial interaction model are introduced. The procedures rely on elemental test results for interfacial and end-anchorage behavior, and thus offer an alternate solution to the m and k method that relies heavily on full scale slab tests. Strength calculations made using the new procedures along with calculations from the Steel Deck Institute procedure are compared to a series of full size composite slab test results.

INTRODUCTION

Cold-formed steel deck is widely used in composite slab systems, which are the prevalent floor system used in most steel framed buildings. This type of composite system has a unique mechanism of composite interaction that is provided by the anchorage systems and the shear bond between the steel deck and the concrete slab. Such shear bond capacity is typically very limited and generates a weak link in the chain of composite interaction within the system and thus raises a partial composite interaction type of problem. This action has received the attention of researchers for a long time. The early procedure introduced to handle the situation was a semi empirical procedure known as the m and k method (Porter & Ekberg 1975). The method relies heavily on full scale test results. Problems arise as to how to incorporate effects of additional parameters such as deck profile, thickness, shear bond, end anchorages, etc., without necessarily conducting many full scale tests. Therefore, an analytical procedure or formulation is needed that can sufficiently describe the physical behavior of the composite interaction, with less dependency on experimental tests.

This paper presents two analytical procedures that are very straight forward and simple in concept, but yet accurate in predicting the behavior of composite slabs. The first method has an iterative nature of analysis and thus will be referred to the iterative procedure in this paper. The second procedure does not require iterative calculations, because it constitutes a single point of analysis in the iterative method, namely the ultimate point. The later method will be referred to the direct method. Both methods can incorporate the effects of shear bond and end anchorages, provided constitutive law data is available. These data can be obtained from elemental tests with no slab tests required. Results of analyses using these two methods were compared to previously tested full scale composite slab specimens.

The iterative and direct procedures were also compared to the Steel Deck Institute (SDI)

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procedure (Heagler, et al. 1991; Terry & Easterling 1994). A description of the SDI procedure is given in the following section. Subsequent sections describe the iterative and direct procedures.

SDI PROCEDURE

The SDI procedure distinguishes between two different cases: studded and non-studded composite slabs. The procedures were based on research conducted at Virginia Polytechnic Institute and State University and West Virginia University. Analytical expressions for the studded slab procedure shows that simple analysis for a singly reinforced concrete section can adequately model the system (Easterling & Young 1992; Terry & Easterling 1994). The strength of slabs with arc spot welds, and no shear studs is calculated based on initiation of extreme fiber yielding in the steel deck.

Although the procedures do not incorporate shear bond action explicitly, test results showed that they predicted the strength of composite slabs very well (Easterling & Young 1992; Terry & Easterling 1994). One major draw back of the procedure is that it distinguishes the two cases (studded and non-studded) in a separate formulation.

ITERATIVE PROCEDURE

The iterative procedure is very simple in concept, using a singly reinforced concrete beam section as the basis of the approach. All the effects that help the concrete resist cracking in the positive moment area are considered as *reinforcement* as indicated in Fig. 1. Such effects come from shear bond action, end anchorages, reinforcing bars, etc.

There are two phases of the analysis: phase-1, analysis of the composite cross section in which the steel deck acts as a tensile member reinforcing the concrete slab, and phase-2, analysis of the steel deck as a flexural member. Phase-1 can be regarded as the *composite action* while phase-2 is the *non-composite* action of the system.

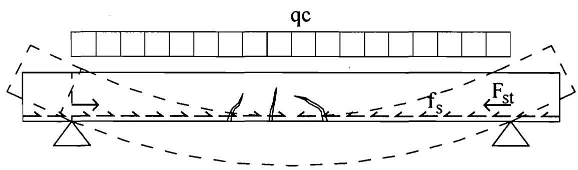


Figure 1. Reinforcing effects of some devices

In phase-1, the analysis is performed exactly in the same manner as one treats a singly reinforced concrete section. Two equilibrium equations are considered: equilibrium of forces and equilibrium of moments on the cross section.

Assumptions used in the procedure therefore follow directly from the concrete beam section procedure, with one exception. Because in this procedure we want to obtain the response of the slab through the entire loading history, the Whitney stress block (equivalent rectangular stress block) for the concrete is replaced by an elasto-plastic model of the stress distribution. This is illustrated in Fig. 2. F_s and F_{st} are forces resulting from the effect of shear bond and end anchorages respectively. Additional effects of welds or pour stop can be added in a way similar to F_s and F_{st}.

There are two independent variables that have to be solved to determine the equilibrium of forces and moment on the cross section. In Fig. 2, c and f_l are chosen as the independent variables. By using the two equilibrium equations, these two independent variables can be resolved. It can be noted that the magnitude of F_s and F_{st} depends upon the value of the slip between the concrete and the deck which in turn depends on the concrete strain at the location where these two forces are acting. Because of this and the nonlinear relation between F_s and F_{st} to the concrete strain, c and f_l are coupled together in a nonlinear system of equations. Therefore, an iterative procedure is required to solve for c and f_l. Iteration is performed for each cross section for a given load level. The greater the number of cross sections being investigated, the more accurate the prediction of the location of the critical section.

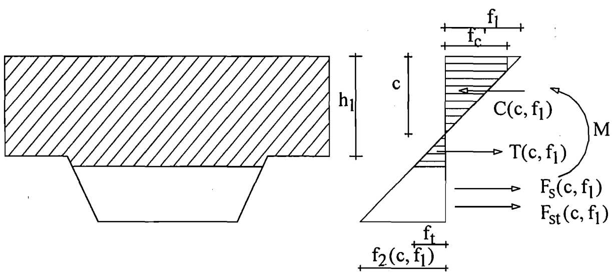


Figure 2. Forces acting on the cross section

The shear bond interaction is illustrated in Fig. 3. A typical relation is shown in Fig. 3a between the shear bond force per-unit length, f_s, versus the slip at the interface of the concrete and the deck. This relationship is obtained from elemental tests. In general, at a certain load level, the distribution of f_s along the slab is not uniform due to the difference in the amount of slip at different cross sections. This is illustrated by $f_{s,A}$ and $f_{s,B}$ in Fig. 3b. The shear bond force, F_s, acting on a cross section is the sum of f_s from the end of the slab to the particular cross section (represented by the shaded area in Fig. 3b). Figure 3c shows the distribution of F_s along the slab. In the case of high strength shear bond, F_s can not be greater than the strength of the steel deck.

The partial interaction between the deck and the concrete is accounted for by limiting the deck contribution to the capacity of the shear bond, such that after a certain phase, the steel deck and concrete no longer have the same amount of strain at the interface. At any point of the loading the strength contribution of the deck can not be greater than F_s as shown in Fig. 3c. Thus, the steel deck is considered as regular reinforcing steel in reinforced concrete sections so that we have:

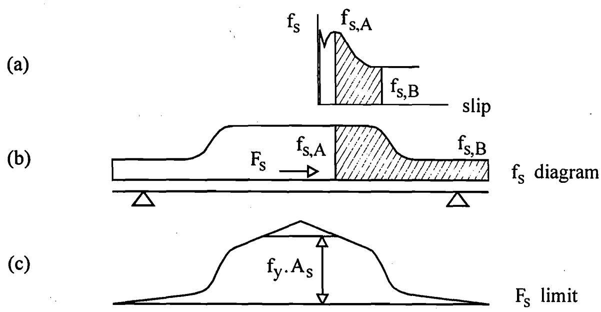


Figure 3. Shear bond interaction

$$F_s = \varepsilon_s.E_s.A_s \le F_{s,limit} \tag{1}$$

where, F_s = shear bond force shown in Fig. 3, ε_s, E_s and A_s are the strain, elastic modulus and cross sectional area of the steel deck respectively, $F_{s,limit}$ = limitation on the shear bond based on the shear bond vs. slip data obtained from the elemental tests. This limit is calculated based on the slip at the cross section being investigated. Note that $F_{s,limit}$ for a cross section does not have a constant value, but rather, forms a function of the slip at that location. Once the $F_{s,limit}$ is reached, the slip starts to occur. In an extreme condition with a very high shear bond strength, $F_{s,limit}$ can not exceed the strength of the steel deck, and hence we can state:

$$F_{s,limit} \le f_y.A_s \tag{2}$$

where, f_y is the steel deck yield stress.

The effect of the end anchorage, F_{st}, can be obtained upon the determination of slip of the slab relative to the beam at the location of the anchorages, i.e., at the support. Slip values can be obtained by summing the elongation of the bottom fiber of the concrete for each element or interval from the mid-span to the support, neglecting the axial deformation of the steel deck.

Both the shear bond force and the end anchorage force require the determination of the slip along the slab. This creates a problem because the slip is not known in advance. Two alternatives of approximation can be pursued to overcome the problem. One is to apply a *forward* iteration scheme, in which, the analysis proceeds by utilizing the values obtained from the last convergent state. These might not be correct for the current state, however, this forward scheme is very simple because it does not require additional iteration.

The second alternative is the *backward* iteration scheme. In this scheme an additional iteration loop is introduced inside the current iteration for which c and f_l are being computed. With this procedure, the computation becomes very tedious. To avoid this problem, a simplification technique can be introduced.

The simplification technique involves replacing the actual concrete elongation diagram, Fig. 4b, with the simplified Fig. 4d. By considering Fig. 4d, the elongation of the bottom fiber of a segment located at x_i from the support can be written as:

$$dL_i = \frac{x_i}{L/2} dL_c \tag{3}$$

where, L = the span of the slab, dL_i = elongation of the bottom fiber of segment-i and dL_c = elongation of the bottom fiber at the mid-span. Using Eq. (3), the total slip at the location x_i can be expressed as:

$$s_i = \sum_{i=1}^{n} dL_i = \left(x_i + x_{i+1} + \ldots + x_n\right)\frac{dL_c}{L/2} = \left(i + (i+1) + \ldots + n\right) d \frac{dL_c}{L/2} \tag{4}$$

where, s_i = the slip at the location x_i, n = total number of segments from the support to the mid-span, i = sequence number of segment counted from the support, and d = the length of each segment. Substituting Eq. (3) into Eq. (4) for dL_c, and replacing $\left(i + (i+1) + \ldots + n\right)$ in Eq. (4) by $\left[(1 + 2 + \ldots + n) - (1 + 2 + \ldots + (i-1))\right]$, the slip at a cross section can be expressed in terms of the elongation of that particular segment as is given by:

$$s_i = \left\{\frac{n(n+1)}{2} - \frac{i(i-1)}{2}\right\}\frac{1}{i} dL_i \tag{5}$$

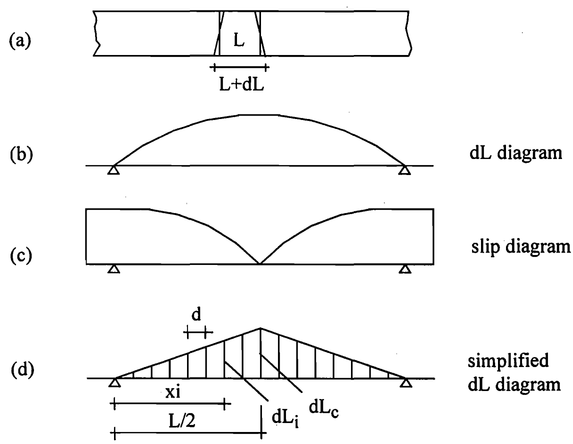
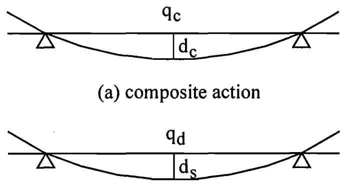


Figure 4. Concrete bottom fiber elongation, dL, and slip diagrams

In phase-2 of the analysis, we consider the strength of the deck. The deck contributes additional load carrying capacity and it is assumed that this action occurs through a *non-composite* type of action. A simple deflection compatibility condition is assumed between the deck and the concrete as illustrated in Fig. 5:



(a) composite action

(b) non-composite action

Figure 5. Additional load carrying capacity from the deck

$$d_s = d_c \tag{6}$$

where, d_s = deflection of the steel deck, and d_c = deflection of the partially composite section. The additional strength stemming from phase-2 of the analysis may be significant and therefore is considered. This additional resistance comes from the contribution of the flexural strength of the steel deck. The stress developed in the steel deck in conjunction with this additional resistance, however, can not be greater than the remaining strength available in the steel deck given by:

$$f_y^* = f_y - f_{cast} - f_{shore} - f_{bond} - f_{anchorage} - f_w \tag{7}$$

where, f_{cast}, f_{shore}, f_{bond}, $f_{anchorage}$, f_w = stress in the steel deck induced by concrete casting, shore removal, shear bond force, F_s, end anchorage force, F_{st}, and weld force respectively. If the additional load carrying capacity is denoted by q_d, then the total load carrying capacity is simply:

$$q = q_c + q_d \tag{8}$$

where, q_c = load carrying capacity from phase-1 of the analysis (*partially composite action*). Beyond this value, the deck is yielded and it deforms plastically without adding any contribution on the load capacity.

In addition to the strength formulation described above, the deflection of the slab can be computed simultaneously. In this part of analysis, however, there are additional assumptions required. The modulus of elasticity of the concrete is assumed unchanged and equal to its initial value, even though the concrete is inelastic in certain cross sections. Similar to the strength procedure, the portion of the concrete stressed beyond the tensile stress limit is considered to be ineffective. Therefore, the cross sectional inertia of the concrete varies along the slab. Contribution of the steel deck to the slab stiffness is proportional to the degree of interaction between the deck and the concrete. This degree of interaction is represented by the ratio of the portion of the deck strength active in the section analysis to the overall deck strength at the beginning of the analysis (after concrete casting and shore removal).

An alternative method to compute the deflection of the slab with non-prismatic cross sections is by utilizing the unit load method for which the integration can be performed numerically. The effective cross sectional inertia can be computed from:

$$\frac{1}{I_{eff}} = \frac{\Omega_1}{I_1} + \frac{\Omega_2}{I_2} + \ldots + \frac{\Omega_n}{I_n} \tag{9}$$

where, I_{eff} = effective cross sectional inertia of the slab, I_i = effective cross sectional inertia of segment-i, and,

$$\Omega_i = \frac{\int_i M\, m\, ds}{\int_L M\, m\, ds} \tag{10}$$

where, $\int_i$ = integration over the segment, $\int_L$ = integration over the entire length of the slab, M = bending moment function along the slab, and m = weighting function (bending moment caused by the unit load).

The iterative procedure results in the following advantages: full history of load vs. deflection of the composite slab is obtained, identification of important points along the loading history, such as first yield condition, location of the critical cross section, and mode of failure are obtained. Additionally, the procedure facilitates the incorporation of effects, such as shear stud, pour stop, etc., so long as the test data of the particular device is provided. This later test data can be obtained by performing small elemental tests, thus no large full-scale tests are necessary.

DIRECT METHOD

The direct method shares the same basis formulation as the iterative method. In fact, the direct method is just one point, namely the ultimate load condition, in the iterative analysis. In this case, a fully plastic condition of the cross section is assumed, and therefore, the Whitney stress block for the concrete is utilized. The stress condition is illustrated in Fig. 6.

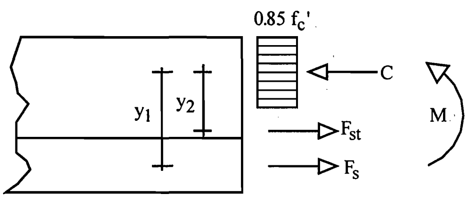


Figure 6. Forces acting on the cross section considered in the direct method

Because this method is basically the same as the final point of analysis in the iterative method, all assumptions of the iterative method are applicable. The main advantage of the direct method is that the procedure of computation is not iterative, thus it is convenient for hand computation. In addition to that, the effects of shear bond and end anchorages can also be taken into account. Partial interaction between the deck and concrete is also considered as in the iterative procedure. The ultimate moment capacity provided by the composite action of the steel deck and the concrete is given by:

$$M_{nc} = F_s y_1 + F_{st} y_2 \qquad (11)$$

where y_1, y_2 = the arm length of F_s and F_{st} respectively to the center of the compressive stress block. The depth of the stress block is obtained by:

$$a = \frac{F_s + F_{st}}{0.85 f_c' b} \qquad (12)$$

Equation (11) constitutes phase-1 of the analysis. Phase-2 of the analysis, the effect of the flexural deck strength is given by:

$$M_{nd} = f_y^* S \qquad (13)$$

where, f_y^* = the remaining deck strength, defined in Eq. (7), and S = plastic section modulus of the steel deck. One can not obtain the first yield condition, the location of failure, or the plot of load vs. deflection using the direct method.

COMPARISON OF CALCULATED AND TEST RESULTS

Predicted values of the slab strength were made by using the iterative, direct and SDI methods. They were compared to experimental test results. The experimental tests were performed using several different deck profiles, embossment patterns and steel thicknesses. Different span lengths, total slab depths, end anchorages and concrete strengths were used in the tests. The width of the specimens was 6 ft. Loading was applied through an air-bag to the top surface of the concrete slab to produce a uniformly distributed load. The test setup is shown in Fig. 7. Table 1 lists the main parameters of the specimens and the computed values using previously described methods. The embossment types listed in Table 1 are illustrated in Fig. 8.

From Table 1, one can see that the SDI, direct and iterative methods all predicted the load capacity of the slab very well. The SDI method, while not as accurate, gives generally conservative results that are acceptable for design. A graphical comparison of the test vs. predicted strengths using the iterative, the direct and the SDI methods are shown in Fig. 9.

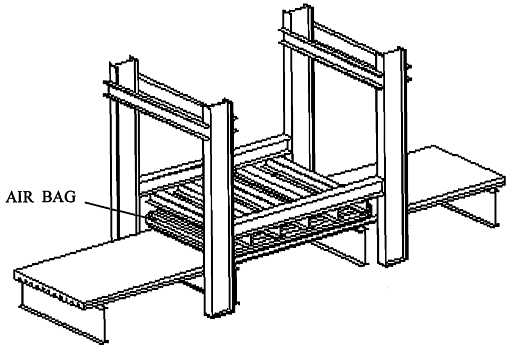


Figure 7. Test Setup

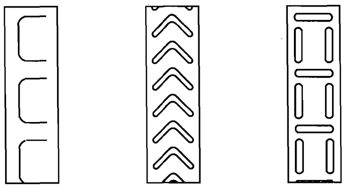


Figure 8. Embossment types

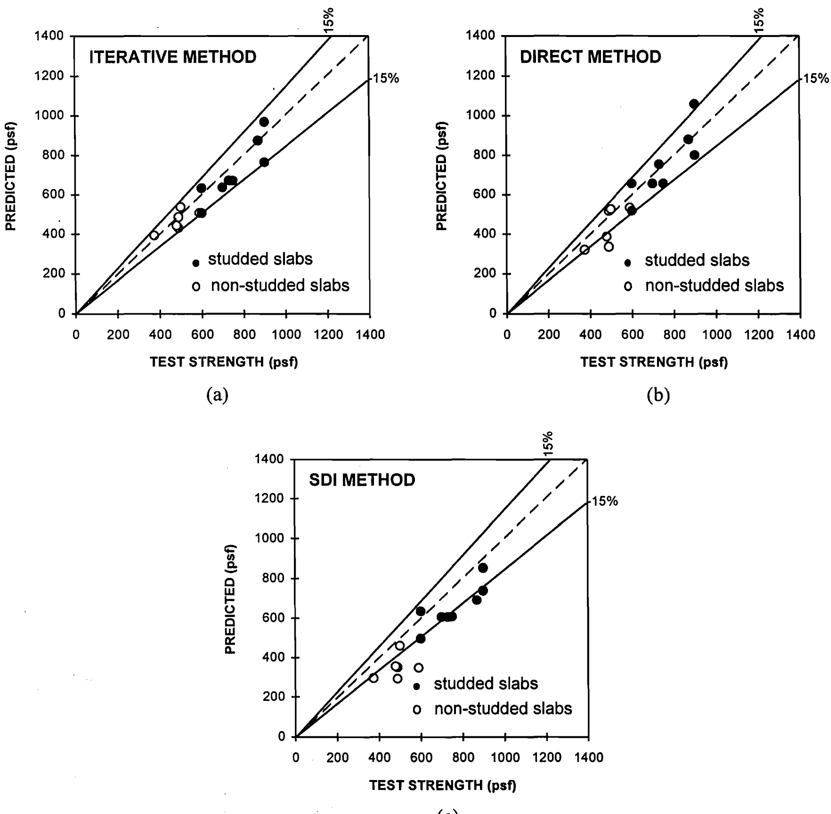
Table 1. Test parameters & comparison of the predicted strength

SLAB #	RIB HT. (in)	THCK. (in)	EMBSM. TYPE	OVER-HANG (ft)	END ANCHR. TYPE	TOTAL DEPTH (in)	DECK CONT.	CONCR fc'	ULTIMATE LOAD CAPACITY SDI psf	DIREC psf	ITER. psf	TEST psf	PREDICTED / TEST SDI	DIREC	ITER.
1	2	0.0345	1	1	S-5	4.5	C	3180	608	755	673	730	0.83	1.03	0.92
2	2	0.0345	1	1	S-4	4.5	C	3180	608	657	637	700	0.87	0.94	0.91
3	2	0.0345	1	1	S-3	4.5	C	5170	635	657	635	600	1.06	1.10	1.06
4	2	0.0345	1	1	S-2	4.5	C	5170	498	519	507	600	0.83	0.87	0.85
5	2	0.0345	1	1	W-7	4.5	C	3340	351**	337	431	490	0.72	0.69	0.88
6	2	0.0345	1	_	W-7,P	4.5	C	3340	349**	534	510	590	0.59	0.91	0.86
7	2	0.0345	1	1	W-7	4.5	D	3770	297**	321	393	375	0.79	0.86	1.05
8	2	0.0345	1	_	W-7,P	4.5	D	3770	293**	519	487	490	0.60	1.06	0.99
9	2	0.0470	2	1	S-3	4.5	C	5300	740	802	766	900	0.82	0.89	0.85
10	2	0.0470	2	1	S-5	4.5	C	5300	853	1060	970	900	0.95	1.18	1.08
11	3	0.0355	3	1	S-3	5.5	C	3750	610	658	672	750	0.81	0.88	0.90
12	3	0.0355	3	1	S-5	5.5	C	3750	693	881	876	870	0.80	1.01	1.01
13	3	0.0355	3	1	W-7	5.5	D	3370	357**	388	443	480	0.74	0.81	0.92
14	2	0.0470	2	1	W-7	4.5	D	3370	461**	528	538	500	0.92	1.06	1.08

Note

* End anchorages: S=stud, P=pour stop, W=puddle weld
* Numbers following S and W are the number of studs or welds installed
* Deck continuity: C=continuous over the support, D=discontinuous

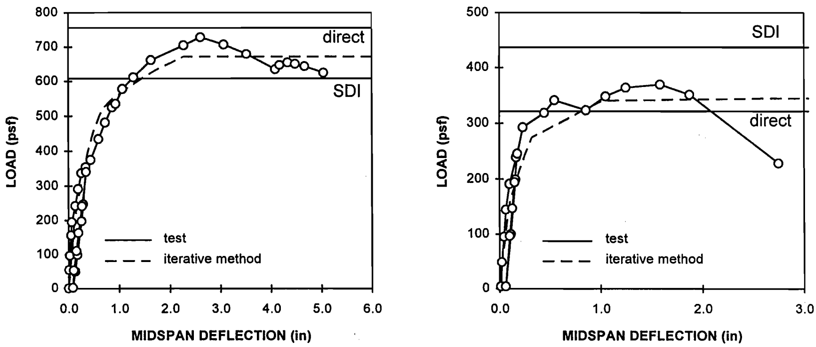
** Values were calculated based on the first yield condition



(c)

Figure 9. Test vs. predicted strength

A comparison of the experimental and iterative method response histories for slabs no. 1 (studded slab) and 7 (welded slab) are shown in Fig. 10. The prediction using the iterative analysis agree reasonably well with most tests. The comparison are generally better for slabs with studs as compared to slabs with welds.



(a) Slab #1 (b) Slab #7

Figure 10. Load vs. deflection responses of slabs #1 and #7

CONCLUSIONS

From the comparison and discussion presented, it can be concluded that the iterative method generally predicts the slab strength and behavior well. Both the direct and the iterative procedures offer an alternate solution to performing many full size slab tests. Moreover, because the procedures have a mechanics based model, they are able to take into account parameters such as shear bond, end-anchorages, etc. The SDI method, while more conservative than the other two methods is a good tool for design.

ACKNOWLEDGMENT

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Appendix.--References

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Appendix.--Notation

A_s	= steel deck cross sectional area
a	= depth of concrete stress block
b	= section width
C	= resultant of concrete compressive force
c	= depth of the neutral axis of composite section
d	= distance of the steel deck centroid to the top surface of the slab
	= length of each segment
d_c	= deflection of the partially composite section
d_s	= deflection of the steel deck
dL, dL_i	= elongation of the bottom fiber of concrete slab of segment i
dL_c	= elongation of the segment at the mid-span
E_s	= elastic modulus of steel deck
F_s, F_{st}	= tensile force in the steel deck resulted from the effect of shear bond and end anchorages respectively
$F_{s,limit}$	= upper limit of F_s
$f_{anchorage}$	= stress in the steel deck induced by end anchorages
f_s	= shear bond force per unit length
f_{bond}	= stress in the steel deck induced by shear bond force, f_b
f_c'	= concrete compressive strength
f_{cast}	= stress in the steel deck induced by concrete casting
f_{shore}	= stress in the steel deck induced by shore removal
f_t	= concrete tensile strength
f_w	= stress in the steel deck induced by puddle welds
f_y	= steel deck yield stress
f_y^*	= remaining strength of the steel deck
f_1, f_2	= elastic concrete compressive and tensile stress at the extreme fiber
h_1	= depth of the concrete flange
I_{eff}	= effective cross sectional inertia of the slab
I_i	= effective cross sectional inertia of a segment
i	= sequence number of a segment
L	= span and cantilever length of the slab respectively
M	= bending moment, general
m	= bending moment caused by the unit load

M_n, M_{nc}, M_{nd}
= ultimate moment capacity: total, phase-1 and phase-2, respectively

n = number of segment from the support to the mid-span

q, q_c, q_d = load carrying capacity: total, phase-1, phase-2, respectively

S = steel deck section modulus

s_i = total slip at a section

T = resultant of concrete tensile force

x, x_i = distance from the support to the section being investigated

y_1, y_2 = moment arm of F_s and F_{st}, respectively

ε_s = steel deck strain