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Elastoplastic Large Deflection Analysis of Cold-Formed Members Using Spline Finite Strip Method

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ABSTRACT: The elastoplastic large deflection behaviour of cold-formed members is analysed by a nonlinear spline finite strip method. The method is developed using the principle of virtual work, based on the total Lagrangian description. It is used to deal with problems of geometric and material nonlinearity. The displacement function of a strip is expressed as the product of transverse interpolation polynomials and longitudinal B3-splines. The effect of arbitrary initial imperfections is taken into consideration. The influence of cold-bending residual stress on the local and overall behaviour of cold-formed lipped angle columns is investigated especially. The numeric examples show that the method possesses such advantages as fewer degrees of freedom, fine continuity, good boundary adaptation, quick computation speed and high accuracy etc.

INTRODUCTION

The finite element method(FEM) is a powerful analytical approach that can be used to study almost any kind of problem in structures(Bathe and Wilson 1976; Zienkiewicz 1977; Cook 1981). However it is inconvenient to apply the method to some complex problems. For some problems, FEM may not be the best method. In such cases, classic semi-analytical finite strip method(CFSM) may be appropriate(Cheung 1976) because of its easy derivation, less number of degrees of freedom, etc. For example, CFSM is one of the most efficient methods for solving regular prismatic structures with simply supported ends and simple initial imperfections. However, it is difficult to treat arbitrary initial imperfections, complicate boundary and loading conditions as well as actual plastic development, etc., by CFSM. In order to both overcome such difficulties and retain the advantages of CFSM, a spline finite strip method(SFSM) is developed(Cheung and Fan 1982).

No matter what kind of analysis method is adopted, research into linear elastic and small deflection problems has been undertaken much more. The main new field of structural analysis is nonlinear structural behaviour(Bathe and Wilson 1976; Owen and Hinton 1980; Cook 1981). To develop a method that can be conveniently and effectively used to solve many complicate nonlinear problems of plates, shells and cold-formed thin-walled structures and members, based on the total Lagrangian description and using the principle of virtual work, the nonlinear spline finite strip method(NSFSM) for problems of large elastoplastic deflection is derived in this paper. The NSFSM can deal with arbitrary initial imperfections, such as initial deformation and residual stress, different loading patterns, real plastic development as well as special support and displacement restraining conditions. Its boundary adaptability is very satisfying,

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and the shape of displacement function can be rectified by itself corresponding to the development of buckling deformation and plasticity, so that the method can approach the actual situation much more.

The behaviour of practical cold-formed members is under the influence of initial geometric imperfections and residual stresses to different extent. The second type of buckling, nonlinear buckling always occurs in cold-formed members. It has been found that the distribution of cold-bending residual stresses along the thick direction is not uniform but nearly linear(Weng and Pekoz 1988; Fang et al. 1993). As a further example, the influence of cold-bending residual stresses on the local and overall behaviour of cold-formed lipped angle columns is analyzeded in this paper. The nonlinear spline finite strip method can be effectively used to study elastoplastic large deflection behaviour of plates, shells and cold-formed thin-walled members under various complicated conditions.

DISPLACEMENT FUNCTIONS OF SPLINE FINITE STRIP

The displacement functions of a classic semi-analytic finite strip are expressed in product form. Interpolation functions of simple polynomials are used in one direction, and continuous and smooth series in another direction. For a spline finite strip(as shown in Fig.1), interpolation functions of B3-splines are used in another direction, so that it is convenient and simple to treat various boundary conditions, and it is easy to converge for concentric loading conditions. B3-splines can be obtained from a general fourth order differential equation, it is a cubic polynomials in subsections and twice continuously differentiable (Ahlberg et al. 1967, Prenter 1981). For a dividing For a dividing π of region [a, b]:

$$x_{-1} < a = x_0 < x_1 < \cdots < x_{m-1} < x_m = b < x_{m+1}, \quad h = x_{j+1} - x_j = (b-a)/m \tag{1}$$

m is the number of subsections, h is the equal length of section. A standard B3-spline base ϕ_j as show in Fig.2 is

$$\phi_j(x) = \frac{1}{6h^3}\begin{cases} (x-x_{j-2})^3 & x_{j-2} \le x \le x_{j-1} \\ h^3 + 3h^2(x-x_{j-1}) + 3h(x-x_{j-1})^2 - 3(x-x_{j-1})^3 & x_{j-1} \le x \le x_j \\ h^3 + 3h^2(x_{j+1}-x) + 3h(x_{j+1}-x)^2 - 3(x_{j+1}-x)^3 & x_j \le x \le x_{j+1} \\ (x_{j+2}-x)^3 & x_{j+1} \le x \le x_{j+2} \\ 0 & otherwise \end{cases} \tag{2}$$

The B3-spline interpolation function for the dividing π is expressed as the summation of (m+3) local B3-spline by

$$Y(x) = \sum_{j=-1}^{m+1} a_j \phi_j(x) \qquad x \in [a,b] \tag{3}$$

where a_j is a coefficient to be determined, corresponding to local B3-spline ϕ_j. Because of the feature of local non-zero, the number of related non-zero terms at any point in the region is not more than four(Fig.3). In order to self adapt the end boundary conditions such as simply supported,clamped sliding, clamped and free, only require to locally amend the B3-spline base,i.e.,

$$[\Phi(x)] = \left[\tilde{\phi}_{-1},\ \tilde{\phi}_0,\ \tilde{\phi}_1,\ \phi_2,\ \ldots,\ \phi_{m-2},\ \tilde{\phi}_{m-1},\ \tilde{\phi}_m,\ \tilde{\phi}_{m+1}\right] \tag{4}$$

where $\tilde{\phi}_j$ is an amended boundary local spline(Fig.4).

The displacement functions of a spline finite strip are expressed as following

$$\{d\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = [N][\Phi_a] = \begin{bmatrix} [N_M] & \\ & [N_F] \end{bmatrix}\begin{bmatrix} [N_M] & \\ & [N_F] \end{bmatrix}\begin{Bmatrix} \{a_M\} \\ \{a_F\} \end{Bmatrix} \tag{5}$$

in which {a} is a general displacement vector for a spline strip, each section knot has four degrees of freedom corresponding to the two membrane displacement u,v and the two flexural deformations v, θ. Using subscript "M" and "F" denote the membrane and flexural parts respectively, if let $\xi=x/b$, $N_1=1-\xi$, $N_2=\xi$, $N_3=1-3\xi^2+2\xi^3$, $N_4=b\xi(1-2\xi+\xi^2)$, $N_5=3\xi^2-2\xi^3$, $N_6=b\xi(\xi^2-\xi)$, then

$$[N_M]=\begin{bmatrix} N_1 & 0 & N_2 & 0 \\ 0 & N_1 & 0 & N_2 \end{bmatrix};[\Phi_M]=\begin{bmatrix} [\Phi_{ui}] & & & \\ & [\Phi_{vi}] & & \\ & & [\Phi_{uj}] & \\ & & & [\Phi_{vj}] \end{bmatrix};\{a_M\}=\begin{Bmatrix} \{u_i\} \\ \{v_i\} \\ \{u_j\} \\ \{v_j\} \end{Bmatrix} \tag{6}$$

$$[N_F]=[N_3\ N_4\ N_5\ N_6];[\Phi_F]=\begin{bmatrix} [\Phi_{wi}] & & & \\ & [\Phi_{\theta i}] & & \\ & & [\Phi_{wj}] & \\ & & & [\Phi_{\theta j}] \end{bmatrix};\{a_F\}=\begin{Bmatrix} \{w_i\} \\ \{\theta_i\} \\ \{w_j\} \\ \{\theta_j\} \end{Bmatrix} \tag{7}$$

the vectors of spline function bases as well as displacement coefficients are defined by

$$[\Phi_{ui}]=\left[\tilde{\phi}_{-1},\ \tilde{\phi}_0,\ \tilde{\phi}_1,\ \phi_2,\ \ldots,\ \phi_{m-2},\ \tilde{\phi}_{m-1},\ \tilde{\phi}_m,\ \tilde{\phi}_{m+1}\right]_{ui} \tag{8}$$

$$\{u_i\}=\left[u_{-1},\ u_0,\ u_1,\ u_2,\ \ldots,\ u_{m-2},\ u_{m-1},\ u_m,\ u_{m+1}\right]_i^T \tag{9}$$

INITIAL IMPERFECTIONS

It is important to study the influence of initial imperfections such as initial geometric imperfection and residual stresses. Initial geometric imperfection in structures is treated as a known state to start the process of incremental solution. It can be considered as the displacement or deformation from ideal perfect structures. It is a smooth, continuous field. If actual data are short of or some approximate assumption can be made, certain simple function will be used in description. If it is required to analyze structures as close as possible to the actual situation, the actual data of initial deformation should be measured. Because initial deformation is in a field distribution, it can only be expressed by interpolation based on the data.

Residual stresses always exit in actual structures for plastic deformation caused by welding, cutting, rolling, punching and cold-bending, etc. The actual distribution of residual stress may be very complicated, such as it is nonlinear in the wide direction and non-uniform in whether longitudinal or thick direction, but it is simplified through assumption some times. There is a requirement to develop a new method that can deal with arbitrary residual stress. In this paper, the residual stress is treated as initial stress field corresponding to the initial known state(i.e., initial deformation).

NONLINEAR SPLINE FINITE STRIP METHOD

Incremental Equilibrium Equations

In the total Lagrangian description, the state without any deformation is taken as the reference state and the state of initial imperfection is the next known state. The principle of virtual work expressing the equilibrium conditions of structure corresponding to state $\Omega^{(n+1)}$ is writen as following

$$\int_{V^{(0)}}\delta(\{\varepsilon\}+\{\Delta\varepsilon\})^T(\{\sigma\}+\{\Delta\sigma\})dV-\int_{V^{(0)}}\delta(\{d\}+\{\Delta d\})^T(\{p\}+\{\Delta p\})dV$$
$$-\int_{S_\sigma^{(0)}}\delta(\{d\}+\{\Delta d\})^T(\{q\}+\{\Delta q\})dS=0 \tag{10}$$

in which {d}, {ε}, {σ} are the displacement, Green strain, Kirchhoff stress vectors at known state $\Omega^{(n)}$, {p}, {q} are the volume and surface distributing force respectively, "Δ" denotes increment from $\Omega^{(n)}$ to $\Omega^{(n+1)}$, "δ" denotes variation. In the following description {ε}, {σ} are general values, defined for a spline strip as

$$\{\varepsilon\} = [\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy} \quad \rho_x \quad \rho_y \quad \rho_{xy}]^T \tag{11}$$

$$\begin{aligned}\{\sigma\} &= [N_x \quad N_y \quad N_{xy} \quad M_x \quad M_y \quad M_{xy}]^T \\ &= \int_{-t/2}^{t/2} [\sigma_x \quad \sigma_y \quad \tau_{xy} \quad z\sigma_x \quad z\sigma_y \quad z\tau_{xy}]^T dz\end{aligned} \tag{12}$$

Nonlinear Strain and Displacement Relationship

For large deformation problems, membrane strain $\overline{\varepsilon}_x$ of state $\Omega^{(n+1)}$ is defined by

$$\overline{\varepsilon}_x = \frac{\partial(u+\Delta u)}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial(u+\Delta u)}{\partial x}\right)^2 + \left(\frac{\partial(v+\Delta v)}{\partial x}\right)^2 + \left(\frac{\partial(w+\Delta w)}{\partial x}\right)^2\right] \tag{13}$$

therefore the incremental relationship between strain and displacement can be obtained

$$\Delta\varepsilon_x = \Delta\varepsilon_x^{L0} + \Delta\varepsilon_x^{L1} + \Delta\varepsilon_x^{N} \tag{14}$$

in which

$$\Delta\varepsilon_x^{L0} = \frac{\partial u}{\partial x}; \qquad \Delta\varepsilon_x^{L1} = \frac{\partial u}{\partial x}\frac{\partial \Delta u}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial \Delta v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial \Delta w}{\partial x};$$

$$\Delta\varepsilon_x^{N} = \frac{1}{2}\left[\left(\frac{\partial \Delta u}{\partial x}\right)^2 + \left(\frac{\partial \Delta v}{\partial x}\right)^2 + \left(\frac{\partial \Delta w}{\partial x}\right)^2\right]$$

the following expression can also be obtained similarly

$$\Delta\varepsilon_y = \Delta\varepsilon_y^{L0} + \Delta\varepsilon_y^{L1} + \Delta\varepsilon_y^{N} \tag{15}$$

$$\Delta\gamma_{xy} = \Delta\gamma_{xy}^{L0} + \Delta\gamma_{xy}^{L1} + \Delta\gamma_{xy}^{N} \tag{16}$$

Flexural strain and its increment are

$$\overline{\rho}_x = -\frac{\partial^2(w+\Delta w)}{\partial x^2}; \qquad \Delta\rho_x = -\frac{\partial^2 \Delta w}{\partial x^2}; \tag{17}$$

similarly

$$\Delta\rho_y = -\frac{\partial^2 \Delta w}{\partial y^2}; \qquad \Delta\rho_{xy} = -2\frac{\partial^2 \Delta w}{\partial x \partial y}; \tag{18}$$

So the incremental relationship between strain and displacement of large deformation can be written as follows

$$\{\Delta\varepsilon\} = \{\Delta\varepsilon^{L0}\} + \{\Delta\varepsilon^{L1}\} + \{\Delta\varepsilon^{N}\} \tag{19}$$

$$\delta\{\Delta\varepsilon\} = [B]\delta\{\Delta a\} \tag{20}$$

in which "L0" denotes the part independent of any displacement variable, "L1" and "N" denote the parts linearly depending on {a} and {Δa} respectively, $[B]=[B_{L0}]+[B_{L1}]+[B_N]$ is called incremental strain matrix, and

$$[B] = \begin{bmatrix} [B_M^{L0}] & \\ & [B_F^{L0}] \end{bmatrix} + \begin{bmatrix} [B_M^{L1}] & [B_F^{L1}] \\ [0] & [0] \end{bmatrix} + \begin{bmatrix} [B_M^{N}] & [B_F^{N}] \\ [0] & [0] \end{bmatrix} \tag{21}$$

in which

$$[B_M^{L0}] = \begin{bmatrix} -\frac{1}{b}[\Phi_{ui}] & [0] & \frac{1}{b}[\Phi_{uj}] & [0] \\ [0] & (1-\xi)[\Phi'_{vi}] & [0] & \xi[\Phi'_{vj}] \\ (1-\xi)[\Phi'_{ui}] & -\frac{1}{b}[\Phi_{vi}] & \xi[\Phi'_{uj}] & \frac{1}{b}[\Phi_{vj}] \end{bmatrix};$$

$$[B_F^{L0}] = \begin{bmatrix} \frac{6}{b^2}(1-2\xi)[\Phi_{wi}] & \frac{2}{b}(2-3\xi)[\Phi_{\theta i}] & \frac{6}{b^2}(2\xi-1)[\Phi_{wj}] & \frac{2}{b}(1-3\xi)[\Phi_{\theta j}] \\ -(1-3\xi^2+2\xi^3)[\Phi''_{wi}] & -b\xi(1-2\xi+\xi^2)[\Phi''_{\theta i}] & (2\xi^3-3\xi^2)[\Phi''_{wj}] & b\xi(\xi-\xi^2)[\Phi''_{\theta j}] \\ -\frac{12}{b}(\xi^2-\xi)[\Phi'_{wi}] & -2(1-4\xi+3\xi^2)[\Phi'_{\theta i}] & \frac{12}{b}(\xi^2-\xi)[\Phi'_{wj}] & 2(2\xi-3\xi^2)[\Phi'_{\theta j}] \end{bmatrix};$$

$$[[B_M^{L1}][B_F^{L1}]] = \begin{bmatrix} \{a_M\}^T[\Phi_M]^T[N'_M]^T[N'_M][\Phi_M] & \{a_F\}^T[\Phi_F]^T[N'_F]^T[N'_F][\Phi_F] \\ \{a_M\}^T[\Phi'_M]^T[N_M]^T[N_M][\Phi'_M] & \{a_F\}^T[\Phi'_F]^T[N_F]^T[N_F][\Phi'_F] \\ \{a_M\}^T([\Phi_M]^T[N'_M]^T[N_M][\Phi'_M] & \{a_F\}^T([\Phi_F]^T[N'_F]^T[N_F][\Phi'_F] \\ +[\Phi'_M]^T[N_M]^T[N'_M][\Phi_M]) & +[\Phi'_F]^T[N_F]^T[N'_F][\Phi_F]) \end{bmatrix};$$

the expression of $[[B_M^N][B_F^N]]$ is similar to the above one except that $\{a_M\},\{a_F\}$ is replaced by $\{\Delta a_M\},\{\Delta a_F\}$ respectively.

Nolinear General Stress and Strain Relationship

The stress and strain vectors are related by the constitutive relation of material. During the elastic loading period the stress and strain relationship remains constant. When material is loaded into plastic range, the Prantl-Reuss plastic flow rule, von Mises yielding criterion with isotropic hardening and the incremental theory of plasticity are adopted. The strip elements are divided into layers to consider plastic zone spreading due to the yield of material at integral points. The Simpson formulation is used for the mathematical integration in the thick direction. The real constructive relation at any point can be written in

$$d\{S\} = [\overline{D}_{ep}]d\{E\} \tag{22}$$

in which $\{S\}=[\sigma_x\ \sigma_y\ \tau_{xy}]^T$, $\{E\}=\{\varepsilon_M\}+z\{\varepsilon_F\}$ are real stress and strain at the point,$[\overline{D}_{ep}]$ is the constructive matrix depending on the level of stress and history of plastic development.

In the spline finite strip analysis, an increment general constructive relationship as follows is built through integration along the thick direction, and the Simpson integration formulation is used.

$$\{\Delta\sigma\} = [D_T]\{\Delta\varepsilon\} \tag{23}$$

$$[D_T] = \begin{bmatrix} [C] & [cd] \\ [cd] & [D] \end{bmatrix} \tag{24}$$

in which

$$[C] = \int_{-t/2}^{t/2}[\overline{D}_{ep}]dz; \qquad [D] = \int_{-t/2}^{t/2} z^2[\overline{D}_{ep}]dz; \qquad [cd] = \int_{-t/2}^{t/2} z[\overline{D}_{ep}]dz \tag{25}$$

Incremental Equilibrium Equations

During the period of incremental solving, the displacement and strain of state $\Omega^{(n)}$ is known, therefore

$$\delta(\{d\}+\{\Delta d\}) = [N][\Phi_a]\delta\{\Delta a\} \tag{26}$$

$$\delta(\{\varepsilon\}+\{\Delta\varepsilon\}) = [B]\delta\{\Delta a\} \tag{27}$$

introduce them into the virtual work equation and notice the arbitrary feature of $\delta\{\Delta a\}$, then

$$\int_{A^{(0)}}[B]^T\{\Delta\sigma\}dA + \int_{A^{(0)}}[B_N]^T\{\sigma\}dA + \int_{A^{(0)}}([B_{L0}]^T+[B_{L1}]^T)\{\sigma\}dA - \{\overline{R}\} = 0 \tag{28}$$

in which $\{\overline{R}\}$ is the equivalent knot force vector of state $\Omega^{(n+1)}$ corresponding to external load, i.e.

$$\{\overline{R}\} = \int_{V^{(0)}} [\Phi_a]^T [N]^T \{\overline{p}\} dV + \int_{S_\sigma^{(0)}} [\Phi_a]^T [N]^T \{\overline{q}\} dV \tag{29}$$

after a series of derivations, it is easy to obtain such equation as

$$\int_{A^{(0)}} [B_N]^T \{\sigma\} dA = [K_\sigma]\{\Delta a\} \tag{30}$$

$$[K_\sigma] = \int_{A^{(0)}} [G]^T [M][G] dA \tag{31}$$

$[K_\sigma]$ is called as initial stress matrix or nonlinear strain stiffness matrix, in which

$$[G] = \begin{bmatrix} \partial[N]/\partial x[\Phi_a] \\ [N]\partial[\Phi_a]/\partial y \end{bmatrix}; [M] = \begin{bmatrix} N_x[I_3] & N_{xy}[I_3] \\ N_{xy}[I_3] & N_y[I_3] \end{bmatrix}; [I_3] = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \tag{32}$$

and let

$$\{R_S\} = \int_{A^{(0)}} ([B_{L0}]^T + [B_{L1}]^T)\{\sigma\} dA \tag{33}$$

$\{R_S\}$ is the equivalent knot force vector of state $\Omega(n)$ corresponding to internal stress field.

Introducing Eq.(23) and Eq.(30) into Eq.(28), and ignoring the high order items, after further derivation we can obtain the increment equilibrium equations as following

$$[K_T]\{\Delta a\} = \{\overline{R}\} - \{R_S\} \tag{34}$$

in which

$$[K_T] = [K_0] + [K_u] + [K_\sigma] \tag{35}$$

$$[K_0] = \int_{A^{(0)}} [B_{L0}]^T [D_T][B_{L0}] dA \tag{36}$$

$$[K_u] = \int_{A^{(0)}} ([B_{L0}]^T [D_T][B_{L1}] + [B_{L1}]^T [D_T][B_{L0}] + [B_{L1}]^T [D_T][B_{L1}]) dA \tag{37}$$

Solution to the Equilibrium Equations

The nonlinear equilibrium equations Eq.(34)) are solved by mixed methods, which combine the incremental method with various iterative modifying techniques, for example, Newton-Raphson method. In order to overcome the limit points of plastic or post-buckling problems, reducing load increment and the method restraining the length of displacement vector is applied(Crisfield 1981). For slow convergence or divergence due to "hardening", small incremental step or low loosing method is used. After the linearization of nonlinear equations, the solution is by means of Cholesky method with one-dimentional blocking store. Besides, the iterative criterion is mainly based on the convergence of modulus of displacement vector. For the efficiency of computation and convenience of programming, some beneficial techniques about coordinates transformation, stiffness matrix formulation, boundary conditions and so on are used.

APPLICATIONS

Elastic Large Deflection Analysis of Hinged Cylindrical Shell

The cylindrical shell shown in Fig.5 is subjected to point load at the centre. The two straight edges are hinged supported, but the two other two curve edges are free to move. L=20in(508 mm), R=100in(2540mm), t=0.5in(12.7mm), θ=0.1rad, E=4.5ksi (3102.75 N/mm^2), μ=0.3. Elastic large deflection analysis is performed on the shell by the nonlinear spline finite strip method. According to symmetrical condition, one fourth part of the shell is modelled with 6 strips and 4 sections. Let the load increment ΔP =0.1124kip(0.5kN) to begin the analysis. The modified Newton-Raphson method is used to solve nonlinear equilibrium equations in most steps, but a Newton-Raphson method combined with restraining the length of displacement vector is applied to overcome the limit point. The load P_a at the limit point "A" is 0.4978kip(2.215kN). Bathe and Bolourichi(1980) only solve the loading path before the limit point, P_a=0.5034kip(2.24kN). Sabir and Lock(1972) also allow the limit point to be passed, P_a=0.4989kip(2.22kN), but existing small difference that the elastic modulous E=450.23ksi(3105N/mm^2), the length L=19.843in(504 mm). Their results are also given in Fig.5 and in close agreement with the solution of this paper.

Post-Buckling Behaviour of Rectangular Plate

The elastic-plastic post-buckling behaviour of a rectangular plate with initial deformation is investigated by the present method. The simply supported plate, with its side edges free to move under a uniform edge compression, has such assumed initial imperfection as $W_0=0.001b\sin(\pi x/b)\sin(\pi y/a)$ in the out-of-plane direction, referred to Fig.6. b/t=55,a/b=0.875, $E=2.99\times 10^4$ksi(2.062×10^5N/mm^2), μ=0.3, σ_Y=36.25ksi(250N/mm^2). Rerkshanandana, Usami and Karasudhi(1981) used a finite element method based on Lagrangian description and modified Ilyushin yield criterion to analyse the problem. In this paper, the 1/4 part of the plate is subdivided into 4 strips and 4 sections. The Mises yield criterion is used and the plastic development in the thick direction can be considered through Simpson integration. It is shown that the present method can be effectively used to study the post-buckling behaviour of plate through the comparison of results in Fig.6.

Lateral Buckling of Eccentrically Loaded Column

There are initial imperfections in an eccentrically loaded H-column. The eccentricity of loading e is 2.47in(62.7mm) in the web direction(Fig.7a). The slenderness ratio L/r is 60. B=H=8in(203mm), t_f=0.435in(11.05mm), t_w=0.287in(7.29mm). The initial curvature is assumed as very small, U_0/L=1/100000, so that an ultimate load of lateral instability can be obtained. The residual stress in the section is shown in Fig.7a. $E=2.958\times 10^4$ksi(2.04×10^5N/mm^2), μ=0.3, σ_Y=35.975ksi(248.1N/mm^2), the maximum compressive residual stress $\sigma_{rc}=-0.3\,\sigma_Y$. The computing results by a finite beam element method(Lu 1983) and the present method are both given in Fig.7b. It is shown that an excellent agreement exists between them.

Elastic-Plastic Buckling of Biaxially Loaded Member

A series of H-columns loaded eccentrically with respect to both principle axes of the end sections wee experimentally investigated by Birnstiel(Birnstiel 1968). The No.7 specimen of type B in the experimental program is taken as an example to be studied by the present method. The specimen was made by machining the edges of the flanges of rolled wide flange shapes which had been stress-relieved. The maximum values of initial deformation are U_0=0.0, W_0=0.03in(0.762mm), θ_0=0.001rad. The ends were simply supported and their warping were prevented. The cross-sectional dimensions are B=5in(127mm), H=6.3in (160mm), t_w=0.33in(8.4mm), t_f=0.472in(12mm). The length L is 96in(2438.4mm). The eccentricities of loading at top and bottom end are e_{xt}=-0.92in(-23.4mm), e_{zt}=2.78in (70.6mm) and e_{xb}=-0.85in(-21.6mm), e_{zb}=2.87in(72.9mm), respectively. The material is ASTM A36 steel with $E=2.958\times 10^4$ksi(2.04×10^5N/mm^2), μ=0.3, σ_Y=34.075ksi (235N/mm^2). The ultimate load of experiment is 77.978kip(347 kN). The present method is adopted to analyse this problem. The initial deformation and plastic development are considered. The theoretical ultimate load is 80.067kip(356.3kN). The curves of load versus strain at the midheight from computation are compared with that of experiment(Birnstiel 1968), as shown in Fig.8.

INFLUENCE OF COLD-BENDING RESIDUAL STRESS ON COLD-FORMED MEMBERS

The residual stress in a cold-formed thin-walled member are quite different from those in a hot-rolled or other member(Weng and Pekoz 1988; Fang et al. 1993). The magnitude and distribution of residual stress in a cold-formed lipped angle column were studied experimentally in Fang et al.(1993) through the electric discharge machining(EDM) technique, referred to Fig.9. Besides, the behaviour of short and long columns simply supported and centrically loaded was also studied as shown in Fig.10 and Fig. 11. The lengths are 250mm and 1200mm respectively. The widths of flange and lip are W_1=2.76in(70mm) and W_2=0.79in(20mm) respectively, plate thickness t=0.098in(2.5mm). Young's modules E=2.997$\times10^4$ksi(2.067$\times10^5$ N/mm^2), Poisson's ratio μ=0.3, yield stress σ_y=46.922ksi(323.6 N/mm^2), ultimate stress σ_u= 63.032ksi(434.7N/mm^2).

The two kinds of specimens are analysed by the present method with the subdivision of 12 strips(6 strips and 1 strip for each flange and lip) and 6 sections for 1/2 length. The 3$\times$3 Gauss formulation in plate plane and Simpson formulation with 7 points in the thick direction are used to integrate numerically. The actual residual stress measured is introduced into analysis. The initial curvature of the long specimen is assumed as V_0/L=1/1000. Fig.10 and Fig.11 show the results of average stress versus strain in short column and load versus lateral displacement in long column, respectively. The theoretic ultimate load of lateral buckling of long specimen is 20.647kip(91.88kN), which is clozed to the experimental value 20.0kip(89.0kN). It is shown that the method can analyse the influence of complicated residual stress effectively.

SUMMARY AND CONCLUSIONS

Based on the principle of virtual work and the total Lagrangian description, a nonlinear spline finite strip method for problems of large elastoplastic deflection is developed, and applied to analyze plates, shells and cold-formed thin-walled structures and members of geometric and material nonlinearity.

To deal with the material nonlinearity, the Prantl-Reuss plastic flow rule, von Mises yielding criterion with isotropic hardening and the incremental theory of plasticity are adopted. The strip elements are divided into layers to consider plastic zone spreading due to the yield of material at integral points. The Simpson formulation is used for the mathematical integration in the depth direction. The present method can deal with many kinds of complicated conditions such as arbitrary initial imperfections, different loading patterns, real plastic development as well as special support and displacement restraining conditions. It also provides a theoretical basis for further study on second type of interactive buckling in cold-formed thin-walled members and other complex nonlinear buckling problems.

The distribution of cold-bending residual stresses is complicated. The effect of the residual stresses on the local and overall buckling of cold-formed lip angle columns sometimes cannot be neglected, it should be taken into consideration. At inelastic stage, the local and overall behaviour of cold-formed thin-walled members are deteriorated by cold-bending residual stress. It is shown that the method can analyse the influence of complicated cold-bending residual stress effectively.

The numeric examples show that the nonlinear spline finite strip method possesses such advantages as less degrees of freedom, fine continuity, good boundary adaptation, quick computational speed and high accuracy etc. The efficiency of solving problems by the method is very satisfactory.

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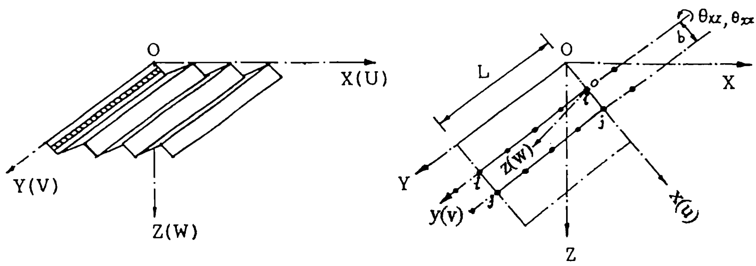
APPENDIX II.--NOTATION

The following symbols are used in this paper:

A	=	area;
$\{a\}$	=	general displacement vector of a spline strip;
$\{a_M\},\{a_F\}$	=	membrane and flexure parts of general displacement vector;
b	=	plate width or strip width;
$[B]$	=	strain matrix;
$[B_{L0}]$	=	part of strain matrix independent of displacement;
$[B_{L1}],[B_N]$	=	parts of strain matrix linearly depending on total; and incremental displacement, respectively;
$\{d\}$	=	real displacement vector at a point;
$[\overline{D}_{ep}],[D_T]$	=	real and general constructive matrices of material, respectively;
E	=	Young's modules;
$\{E\}$	=	real Green strain at a point, ε_x, ε_y, γ_{xy},
h	=	section length of spline interpolation
$[K_T]$	=	tangential stiffness matrix
$[K_0]$	=	linear component of tangential stiffness matrix
$[K_u],[K_\sigma]$	=	nonlinear components of tangential stiffness matrix
L	=	strip length or member length
m	=	number of spline sections
$[N]$	=	matrix of transverse shape functions
$[N_M],[N_F]$	=	matrices of transverse shape functions of strip corresponding to membrane and flexure parts, respectively
$\{p\},\{q\}$	=	volume, surface distributing force vectors
$\{\overline{R}\}$	=	equivalent knot force vector of external load at state $\Omega^{(n+1)}$
$\{R_S\}$	=	equivalent knot force vector of stress field at state $\Omega^{(n)}$
$\{S\}$	=	real Kirchhoff stress vector at a point, σ_x, σ_y, τ_{xy}
t	=	thickness
u,v,w,θ	=	strip displacements and rotation in local coordinate system
U,V,W,θ	=	strip displacements and rotation in global coordinate system
x,y,z	=	local coordinate axes of strip
X,Y,Z	=	global coordinate axes of structure
$\{\Delta\}$	=	real displacement vector at a point
$\{\varepsilon\}$	=	general strain vector of a spline strip
$[\Phi]$	=	matrix of B3-spline of a spline strip
$\phi, \tilde{\phi}$	=	standard and amended local B3-spline, respectively
μ	=	Poisson's ratio
$\{\sigma\}$	=	General stress vector of a spline strip
σ_r	=	residual stress
σ_Y, σ_U	=	yield stress and ultimate stress of material, respectively



(a) (b)

Fig.1 Spline Finite Strip

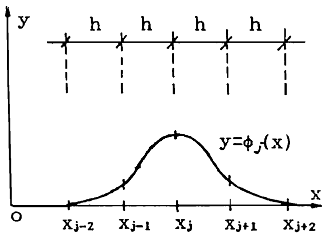


Fig.2 B3-Spline Function

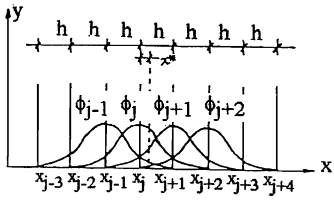


Fig.3 Related B3-Spline at $x=x_j+x^*$

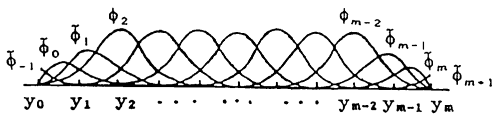


Fig.4 Longitudinal B3-Spline Interpolation

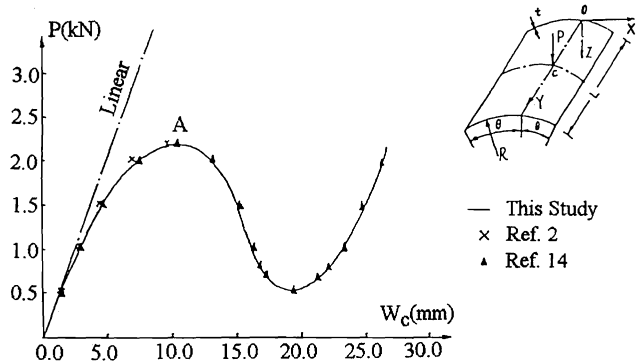


Fig.5 Elastic Large Deflection Analysis of Hinged Cylindric Shell(1in=25.4mm;1kip=4.45kN)

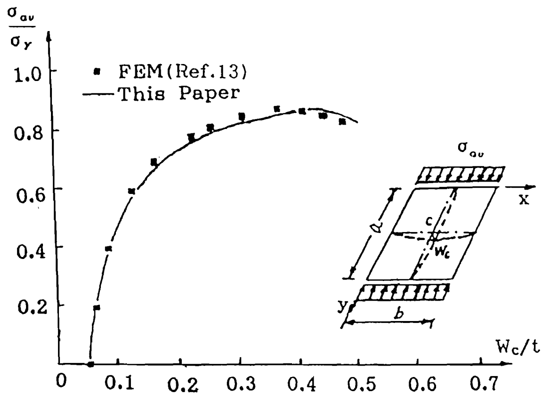


Fig.6 Post-Buckling Behaviour of Simply Supported Plate

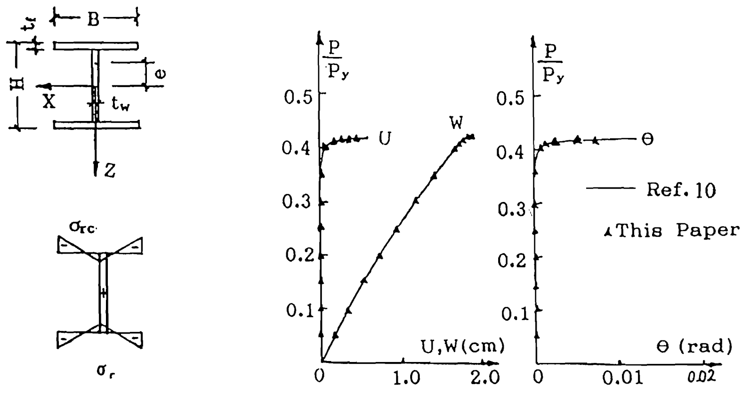


Fig.7 Lateral Buckling of Eccentrically Loaded Column(1in=25.4mm;1kip=4.45kN)

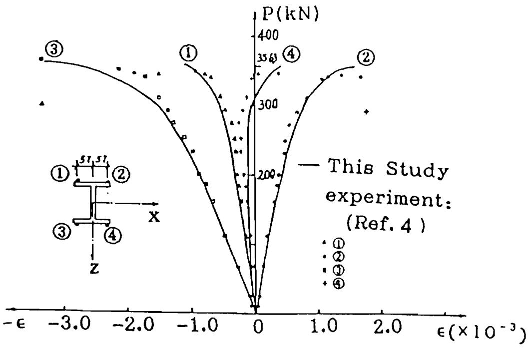


Fig.8 Load versus Strain at the Midheight of Biaxially Loaded Member (1 kip=4.45 kN)

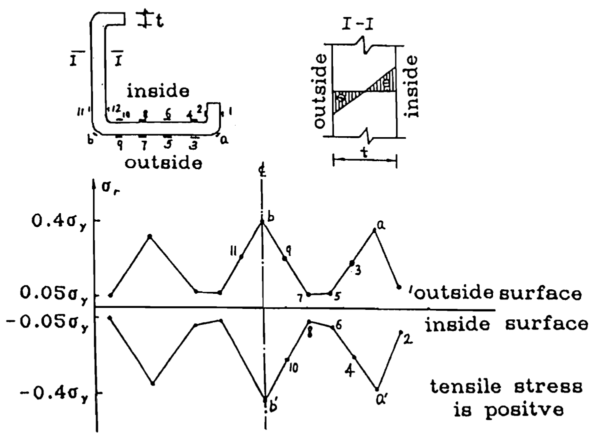


Fig.9. Residual Stress in Cold-Formed Lipped Angle Column

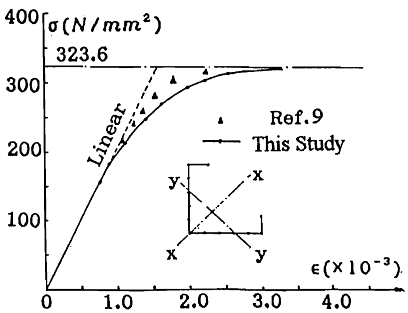


Fig.10. Average Stress-Strain Curve of Short Lipped Angle Column

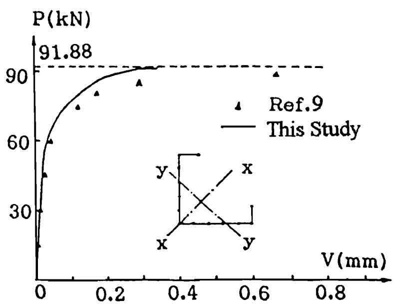


Fig.11. Load-Displacement Curve of Long Lipped Angle Column