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Periods for Seismic Design of Industrial Storage Racks

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PERIODS FOR SEISMIC DESIGN OF INDUSTRIAL STORAGE RACKS

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INTRODUCTION

Industrial steel storage racks have traditionally been designed mainly for vertical loads, and little attention has been given to earthquake loading. The need for considering seismic effects in the design of industrial steel storage racks has been recognized by two groups: the Rack Manufacturers Institute in its *Interim Specification for the Design, Testing and Utilization of Industrial Steel Storage Racks* (1972 edition),[6] and the International Conference of Building Officials in the *Uniform Building Code (UBC*, 1973, 1976, and 1979 editions).[5] Because the seismic criteria adopted by the two organizations are different in some respects, URS/John A. Blume & Associates, Engineers, conducted a comprehensive study[2] to reconcile those differences and to establish state-of-the-art seismic design requirements for industrial steel storage racks through correlation and evaluation of various testing results and analytical parameter variation studies. The study included static-cyclic tests of rack subassemblies and full-scale racks; structural performance full-scale shaking-table tests, including testing to determine the effects of loose merchandise on rack response to shaking; and engineering analysis reconciliations.

The study, which was summarized in a paper that was presented at the 1980 International Specialty Conference on Cold-Formed Steel Structures,[3] concluded that the lateral force provisions recommended in the *UBC* (Standard No. 27-11) appear generally to provide adequate seismic resistance in racks similar to those studied except that the load factor (modifier) of 1.25 recommended in the *UBC* for all members in braced frames might not be adequate. Either larger load factors or modifications to the rack fabrication are needed to preclude early nonductile damage during strong earthquake shaking. The *UBC*'s simple formulas ($T = 0.05h_n/\sqrt{D}$ and $T = 0.1N$) for estimating periods of vibration were found to be inappropriate for racks; designs using these formulas could be erroneous.

This paper summarizes the results of a further investigation that compares two alternative approaches (computer and manual) to estimating fundamental periods of vibration for industrial steel racks. The primary purpose of the paper is to provide formulas and procedures to allow manual calculation of periods to be done quickly and, for all practical purposes, accurately.

Three full-scale rack configurations were studied analytically: the standard pallet rack in both the longitudinal and transverse directions and the drive-in rack in the longitudinal direction. A computer analysis of two-dimensional mathematical models was carried out to compare calculated periods of vibration and mode shapes with those observed during the low-amplitude shaking-table tests and the pull-release free-vibration tests. The best-fit model for each rack configuration was used as a basis for manual calculation of the fundamental period of vibration. Both methods of calculation take into consideration the influence of semirigid beam-column connections in the unbraced-frame system and localized deformation at connections between the open-section column and open-section bracing members in the braced-frame system. The influence of column base fixity was considered in the development of mathematical models for computer analysis.

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DESCRIPTION OF RACKS

The standard pallet rack selected for investigation is currently the most common rack used for merchandise storage. Figure 1 shows the standard pallet rack assembly and its connection details. The rack is a modular assembly consisting of prefabricated uprights in the transverse direction (braced-frame system) and horizontal beams spanning between successive uprights in the longitudinal direction (unbraced-frame system). The upright columns have bearing plates at the base that have a single hole through which floor anchors are inserted. The beam end connections (shelf connectors) are of the clip-in type, and the upright columns are slotted along their full height to allow variations in the vertical spacing of beams.

In the drive-in storage rack (Figure 2), storage pallets are supported by rail members spanning between support arms that cantilever from the columns rather than by beams spanning the bay width, as in the standard pallet rack. The drive-in rack is accessible from one side, but forklifts cannot pass all the way through. The rack has two upright frames and two anchor frames in the longitudinal direction (perpendicular to the aisle). The upright frames are connected at the top by continuous tie members (overhead tie beams). For the anchor frames, ties (anchor beams) are provided at each story level. The horizontal-load-carrying system for the drive-in rack typically consists of bracing in the transverse direction and an unbraced-frame system in the longitudinal direction.

CALCULATION OF PERIODS BY COMPUTER

The period, T, of any structure can be calculated if its stiffness and mass characteristics are known. For very simple structures, the value of T can be obtained by hand calculation. For a complex structure, the number of calculations becomes so large that the use of a computer may be required. Even with a computer, the computed period values are only as accurate as the input data; engineering judgment must be exercised in this regard.

Many standard structural dynamics computer programs can do the frequency analysis of two-dimensional mathematical models with rigid beam-column connections. In this paper, the computer code SAP IV,[1] developed at the University of California, Berkeley, was used to calculate the periods and mode shapes of the rack structures.

Since the SAP IV computer code has no capability to handle the semirigid joint problem, the beam member properties must be modified. The beam rigidities are modified as follows:

$$\left(\frac{I_b}{L_b}\right)_{red} = \left[\frac{1}{1 + \frac{6EI_b}{K_\theta L_b}}\right]\frac{I_b}{L_b} \tag{1}$$

where:

I_b = moment of inertia of beam (in.^{4})

L_b = length of beam (in.)

E = Young's modulus of elasticity (lb/in.^{2})

K_θ = joint rotational spring stiffness (lb-in./rad)

Although this formulation was suggested by Driscoll[4] for determining the effective length of columns with semirigid connections, in this study, it was used to model the unbraced-frame system.

For the racks tested on the shaking table,[2] strong localized deformations were observed at the connections between the open-section columns and the open-section bracing members. This type of deformation should be considered in response analyses of the braced-frame system. The total deformation of the bracing members consists of two parts: the deformation due to the bracing member and the localized deformation of the column lip at the connection between the bracing and column members. No quantitative experimental data are available on the influence of local deformation. Because of this, it was assumed that the composite axial bracing member consisted of two parts; its stiffness was reduced as follows:

$$\left(\frac{EA}{l}\right)_{\text{red}} = \frac{1}{k}\left(\frac{EA}{l}\right) \quad (2)$$

where:

E = Young's modulus of elasticity (lb/in.^{2})

A = cross-sectional area of brace (in.^{2})

l = unbraced length of brace (in.)

k = a factor to account for local deformation at the diagonal-to-column connection

The influence of column base fixity can be incorporated into the models. Two alternative methods have usually been employed by structural engineers in connection with this problem. The first method uses rotational springs (assumed or experimentally determined) at the column base plates. The second method assumes the bases to be connected to a fictitious restraining beam. Because the first method requires special computer routines that may not be readily available for most frame-analysis programs and because the fictitious restraining beam elements of the second method are easily adaptable to SAP IV, the latter approach was used in the development of mathematical models.

MANUAL CALCULATION OF PERIODS

The Rayleigh method (Equation [12-3] in the 1979 *UBC*) is used for manually determining the period of vibration. The value of T can be expressed as follows:

$$T = 2\pi\sqrt{\frac{\sum m_i\,\delta_i^2}{\sum F_i\,\delta_i}} \quad (3)$$

where the values of F_i represent any lateral force distributed approximately in accordance with the formula in the *UBC* or any other rational distribution. The deflections, δ_i, can be calculated using the applied lateral forces, F_i. The values of m_i represent mass assigned to level i.

One of the most difficult tasks in manual calculation of periods is to estimate story deflections. Simple formulas for estimating story deflections are shown in Figures 3 and 4. The formulas in Figure 3a are for calculating interstory deflection due to joint rotation by considering the columns infinitely rigid, and those in Figure 3b are for calculating interstory drift due to column bending by considering the beams and joints infinitely rigid. For both cases, the effects due to beam and column shear stresses are generally insignificant and are not included in the formulas. The formulas in Figure 3 are for an unbraced-frame system; the values of P are estimated by dividing the story shear, V_i, by the number of columns at level i.

Figure 4a shows a formula for interstory deflection due to diagonal forces. The deflections shown in Figure 4b are caused by axial column deformations that result from bending of the frame as a whole because of overturning. Figure 4c shows the virtual work method, an alternative for calculating deflections for the braced-frame system.

Periods of racks can be calculated manually in five steps.

1. Assume an initial value of T, and calculate the base shear, V, in accordance with the *UBC* formula:

$$V = ZIKCSW \tag{4}$$

where:

Z = a seismic zone factor

I = an occupancy importance factor

K = a factor depending on the type of structure or structural system used

$C = \frac{1}{15\sqrt{T}}$

S = a factor designed to account for site-structure effects

W = the total dead load plus contents

2. Estimate story forces, F_x, in accordance with the *UBC* formula:

$$F_x = \frac{(V - F_t)m_x h_x}{\sum m_i h_i} \tag{5}$$

where:

F_t = $0.07TV$ when $T > 0.7$ sec

m_x, m_i = the story mass assigned to level x or i, respectively

h_x, h_i = the height above the base to level x or i, respectively

3. Estimate story deflections, δ_i, using story forces F_i and applying the appropriate formulas from Figures 3 and 4.

4. Calculate T using the Rayleigh method as shown in Equation (3).

5. Compare the calculated value of T with the initial value of T and repeat steps 1 through 4 using the calculated value of T, if necessary.

Although the period obtained from step 4 is theoretically correct, step 5 is required for obtaining accurate F_i for seismic design.

RESULTS

Standard Pallet Rack, Longitudinal (Unbraced-Frame System). Figure 5a shows the mathematical model developed for the standard pallet rack in the longitudinal direction. Because it assumes symmetric response for the two frames, an analytical model for a single frame was considered adequate. The centerline dimensions shown were used, and semirigid beam-column joints and partially fixed bases were assumed in evaluating stiffness. The mass of the dead load and the mass of the concrete blocks and wooden pallets were lumped at the nodes where the pallets were located. The mass per floor was estimated to be about 16.2 lb-sec^2/in. (2.9 kg-sec^2/cm). The minimum net section properties provided by the manufacturer are also shown in Figure 5a.

To account for semirigid connections, the beam properties were modified according to Equation (1). The value of $K_\theta = 10^6$ lb-in./rad (113 kN-m/rad) was adopted in the model, which was experimentally determined from the subassembly tests reported in Reference 2. To deal with the problem of column base fixity, fictitious restraining floor beams were added to simulate the actual column base condition. Because no experimental data on the column base fixity were available, various values of the parameter I_f (moment of inertia of fictitious floor beams) were tried. As shown in Table 1a, the results from the model that assumed $I_f = 0.2$ in.^{4} (8.3 cm^4) and $K_\theta = 10^6$ lb-in./rad (113 kN-m/rad) were in good agreement with the measured period and normalized mode shape.

The model for the manual calculation is shown in Figure 5b. The results presented in Table 1a show that the manually calculated periods fall within the range of those calculated with a computer, and good agreement is evident for the fixed-base case. Only fixed and pinned column bases were considered because the semifixed condition requires more calculation than can easily be done manually. Appendix C presents a sample calculation for a fixed column base. The period for the semifixed case can be approximated from the results of the fixed and pinned cases, but common practice is to assume a fixed base in order to obtain conservative values of $C \times S$.

Standard Pallet Rack, Transverse (Braced-Frame System). Figure 6a shows the mathematical model developed for the standard pallet rack in the transverse direction. Because it assumes symmetric response for the three upright frames, an analytical model for a single frame was considered adequate. Centerline dimensions were used, and fictitious restraining beams were used to account for the semifixed column bases. The localized deformation at the connections between the open-section bracing members and the open-section columns were also considered. The entire mass was lumped equally at the six nodal joints, and the mass per story level was estimated to be 10.8 lb-sec^2/in. (1.9 kg-sec^2/cm). The minimum net section properties provided by the manufacturer are also shown in Figure 6a.

To account for local deformations at the brace-column connections, the bracing member properties were modified according to Equations (2). Because no experimental data on the value of k were available, various combinations of k and I_f were tried. The computer model that assumed I_f = 0.2 in.^{4} (8.3 cm^4) and k = 12 was found to produce results closest to the experimental results (see Table 1b).

Two different methods of manual calculation were studied. The calculation for the fixed column base used the approximate formulas presented in Figures 4a and 4b (see Appendix D for a sample). The calculation for the pinned column base used the virtual work method presented in Figure 4c. As shown in Figure 6b, the model was simplified for the manual calculation. The periods derived manually are slightly shorter than those calculated by computer and would thus yield a slightly more conservative base shear coefficient.

Drive-In Rack, Longitudinal (Unbraced-Frame System). In the longitudinal direction, the drive-in rack assembly consists of two upright and two anchor frames, as shown in Figure 2. Although the structural systems and stiffnesses for these two types of frames are quite different, no torsion was detected from the experimentally obtained displacement time-history plots.[2] This negligible torsional effect makes possible two-dimensional modeling of this structure. Figure 7a shows the mathematical model developed for this rack assembly. It consists of one upright and one anchor frame connected by three fictitious rigid springs at the floor levels. Half of the total mass was included in the model. The masses at the first, second, and third levels were assumed to be 24.8 lb-sec^2/in. (4.4 kg-sec^2/cm). The mass at the fourth level was small enough to be neglected. Centerline dimensions and the minimum net section properties supplied by the manufacturer were used. Semirigid beam-column connections and partially fixed base conditions were assumed in the model.

The model that assumed I_f = 0.2 in.^{4} (8.3 cm^4) and K_θ = 10^6 lb-in./rad (113 kN-m/rad) was found to produce results closest to the measured period and mode shape (see Table 1c).

Because of the complexity of the drive-in rack assembly in the longitudinal direction, the first step in the manual calculation was to estimate the relative stiffness of the upright and anchor frames (i.e., the lateral force required to produce a unit deflection), which is the reciprocal of flexibility. The stiffness ratio of the upright and anchor frames was estimated to be approximately 1:3. Thus, about 75% of the lateral force would be resisted by the anchor frame and 25% by the upright frame. With this assumed distribution of lateral forces, the story deflections, δ_i, can be calculated with the formulas presented in Figure 3 and the procedures illustrated in Appendix E.

Table 1c compares the periods calculated by the computer and manual methods. The results for fixed column bases show good agreement.

CONCLUSIONS

Periods of vibration of industrial steel storage racks can be calculated by either of two methods. Use of a computer is more accurate because the base of the rack can be modeled as partially fixed. Manual calculation, although approximate, yields results that are accurate enough to be useful for seismic design.

APPENDIX A: REFERENCES

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3. Chen, C. K., R. E. Scholl, and J. A. Blume, "Seismic Response of Industrial Steel Storage Racks," *Proceedings*, Fifth International Specialty Conference on Cold-Formed Steel Structures, St. Louis, Missouri, November 1980.

4. Driscoll, G. C., "Effective Length of Columns with Semi-rigid Connections," *Engineering Journal*, American Institute of Steel Construction, Vol. 13, No. 4, 1976.

5. International Conference of Building Officials, *Uniform Building Code*, Whittier, California, 1973, 1976, 1979.

6. Rack Manufacturers Institute, *Interim Specification for the Design, Testing, and Utilization of Industrial Storage Racks*, Pittsburgh, Pennsylvania, 1972.

APPENDIX B: NOTATION

F_i, F_n, F_x = lateral force applied to level i, n, or x, respectively

F_t = portion of V considered concentrated at the top of the structure in addition to F_n

h_i, h_n, h_x = height above the base to level i, n, or x, respectively

I_f = moment of inertia of fictitious floor beam

k = parameter used to account for localized deformation at brace-column connection (see Equation [2])

K_θ = initial rotational spring of semirigid connection (see Equation [1])

m_i, m_x = mass located at level i or x, respectively

V = total lateral force or shear at the base (see Equation [4])

δ_i = deflection at level i relative to the base due to applied lateral forces, ΣF_i, for use in Equation (3)

APPENDIX C: EXAMPLE MANUAL CALCULATION, STANDARD PALLET RACK, LONGITUDINAL, COLUMN BASES FIXED

See Figure 3 to determine deflections and Figure 5 for the mathematical model.

Step 1: Assign T a value of 2.0 sec and assume that $S = K = I = Z = 1.0$. Then:

$$C = \frac{1}{15\sqrt{T}} = 0.047$$

$$W = 3 \times 16.2 \times 386 = 18{,}750 \text{ lb}$$

$$V = ZIKCSW = 887 \text{ lb}$$

$$F_t = 0.07TV = 124 \text{ lb}$$

Step 2: Calculate story forces and shears using Equation (5).

$F_3 = 508$ lb $\quad V_3 = 508$ lb

$F_2 = 254$ lb $\quad V_2 = 762$ lb

$F_1 = 125$ lb $\quad V_1 = 887$ lb

Step 3: Calculate δ_i using the formulas in Figure 3.

$$\delta_1 = \frac{P_1 h_1^3}{12EI_c} + \frac{P_1 h_1^2 L_b}{12EI_b'}$$

$$= \frac{887/3 \times 58^3}{12 \times 29.5 \times 10^6 \times 1.144} + \frac{887/3 \times 58^2 \times 99}{12 \times 29.5 \times 10^6 \times 0.47}$$

$$= 0.142 + 0.592 = 0.734 \text{ in.}$$

$$\delta_2 = \frac{P_2(h_2 - h_1)^3}{12EI_c} + \frac{P_2(h_2 - h_1)^2 L_b}{12EI_b'} + \delta_1$$

$$= 0.135 + 0.544 + 0.734 = 1.413 \text{ in.}$$

$$\delta_3 = \frac{P_3(h_3 - h_2)^3}{12EI_c} + \frac{P_3(h_3 - h_2)^2 L_b}{12EI_b'} + \delta_2$$

$$= 0.090 + 0.363 + 1.413 = 1.866 \text{ in.}$$

Step 4: Compute T using Equation (3).

$$\sum m_i \delta_i^2 = 97.4 \text{ and } \sum F_i \delta_i = 1{,}398.6$$

Therefore:

$$T = 1.66 \text{ sec}$$

Step 5: Assign T a value of 1.66 sec and repeat steps 1 through 4 to obtain new values of F_i for seismic design.

APPENDIX D: EXAMPLE MANUAL CALCULATION, STANDARD PALLET RACK, TRANSVERSE, COLUMN BASES FIXED

See Figure 4 to determine deflections and Figure 6 for the mathematical model.

Step 1: Assign T a value of 0.8 sec and assume that $S = 1.5$, $K = 1.33$, $I = Z = 1.0$, and $F_t = 0$. Then:

$$C = \frac{1}{15\sqrt{T}} = 0.075$$

$$W = 3 \times 10.8 \times 386 = 12{,}500 \text{ lb}$$

$$V = ZIKCSW = 1{,}850 \text{ lb}$$

Step 2: Calculate story forces and shears using Equation (5).

$$F_3 = 833 \text{ lb} \qquad V_3 = 833 \text{ lb}$$
$$F_2 = 666 \text{ lb} \qquad V_2 = 1{,}499 \text{ lb}$$
$$F_1 = 351 \text{ lb} \qquad V_1 = 1{,}850 \text{ lb}$$

Step 3: Calculate δ_i using the formulas in Figures 4a and 4b (assume a linear distribution of deflection due to column axial deformation).

$$\delta'_1 = \frac{P_1 l_1}{(EA)_{red} \cos^2\theta} + \frac{P_2 l_2}{(EA)_{red} \cos^2\theta}$$
$$= \frac{1{,}850 \times 52}{773{,}000 \times \left(\frac{39}{52}\right)^2} + \frac{1{,}850 \times 51}{773{,}000 \times \left(\frac{39}{51}\right)^2}$$
$$= 0.221 + 0.208 = 0.430 \text{ in.}$$

$$\delta'_2 = \frac{P_3 l_3}{(EA)_{red} \cos^2\theta} + \frac{P_4 l_4}{(EA)_{red} \cos^2\theta} + \delta'_1$$
$$= \frac{1{,}499 \times 51}{773{,}000 \times \left(\frac{39}{51}\right)^2} \times 2 + \delta'_1$$
$$= 0.338 + 0.430 = 0.768 \text{ in.}$$

$$\delta'_3 = \frac{P_5 l_5}{(EA)_{red} \cos^2\theta} + \delta'_2$$
$$= 0.111 + 0.768 = 0.880 \text{ in.}$$

$$\delta''_3 = \frac{11 V h_n^3}{15 E A_c D^2}$$
$$= \frac{11 \times 1{,}850 \times 178^3}{15 \times 29.5 \times 10^6 \times 0.69 \times 2 \times 39^2} = 0.124 \text{ in.}$$

$$\delta''_2 = \frac{140.5}{178} \times \delta''_3 = 0.098 \text{ in.}$$

$$\delta''_1 = \frac{75.5}{178} \times \delta''_3 = 0.053 \text{ in.}$$

Therefore:

$$\delta_3 = 1.00 \text{ in.}, \quad \delta_2 = 0.87 \text{ in.}, \quad \delta_1 = 0.48 \text{ in.}$$

Step 4: Compute T using Equation (3).

$$\sum m_i \delta_i^2 = 21.50 \text{ and } \sum F_i \delta_i = 1{,}580$$

Therefore:

$$T = 0.73 \text{ sec}$$

Step 5: Assign T a value of 0.73 sec and repeat steps 1 through 4 to obtain new values of F_i for seismic design.

APPENDIX E: EXAMPLE MANUAL CALCULATION, DRIVE-IN RACK, LONGITUDINAL, COLUMN BASES FIXED

See Figure 3 to determine deflections and Figure 7 for the mathematical model.

Step 1: Assign T a value of 2.4 sec and assume that $S = K = I = Z = 1.0$. Then:

$$C = \frac{1}{15\sqrt{T}} = 0.043$$

$$W = 74.6 \times 386 = 28{,}800 \text{ lb}$$

$$V = ZIKCSW = 1{,}238 \text{ lb}$$

$$F_t = 0.07TV = 208 \text{ lb}$$

Step 2: Calculate story forces and shears using Equation (5).

$$F_4 = 218 \text{ lb} \qquad V_4 = 218 \text{ lb}$$

$$F_3 = 515 \text{ lb} \qquad V_3 = 733 \text{ lb}$$

$$F_2 = 340 \text{ lb} \qquad V_2 = 1{,}073 \text{ lb}$$

$$F_1 = 165 \text{ lb} \qquad V_1 = 1{,}238 \text{ lb}$$

Step 3: Calculate δ_i assuming that 75% of the lateral loads are taken by the anchor frame.

$$\delta_1 = \frac{P_1 h_1^3}{12EI_c} + \frac{P_1 h_1^2 L_b}{12EI_b'}$$

$$= \frac{0.75 \times 1{,}238/3 \times 70^3}{12 \times 29.5 \times 10^6 \times 2.21} + \frac{0.75 \times 1{,}238/3 \times 70^2 \times 50}{12 \times 29.5 \times 10^6 \times 0.22}$$

$$= 0.136 + 0.974 = 1.109 \text{ in.}$$

$$\delta_2 = \frac{P_2(h_2 - h_1)^3}{12EI_c} + \frac{P_2(h_2 - h_1)^2 L_b}{12EI_b'} + \delta_1$$

$$= 0.128 + 0.893 + 1.109 = 2.130 \text{ in.}$$

$$\delta_3 = \frac{P_3(h_3 - h_2)^3}{12EI_c} + \frac{P_3(h_3 - h_2)^2 L_b}{12EI_b'} + \delta_2$$

$$= 0.092 + 0.609 + 2.130 = 2.832 \text{ in.}$$

$$\delta_4 = \frac{P_4(h_4 - h_3)^3}{12EI_c} + \frac{P_4(h_4 - h_3)^2 L_b}{12EI_b'} + \delta_3$$

$$= 0.020 + 0.380 + 2.832 = 3.232 \text{ in.}$$

Step 4: Compute T using Equation (3).

$$\sum m_i \delta_i^2 = 344 \text{ and } \sum F_i \delta_i = 3{,}070$$

Therefore:

$$T = 2.1 \text{ sec}$$

Step 5: Assign T a value of 2.10 sec and repeat steps 1 through 4 to obtain new values of F_i for seismic design.

TABLE 1

MEASURED AND COMPUTED RESULTS

a. Standard Pallet Rack, Longitudinal

	Measured Results	Computed Results				
		Computer Method			Manual Method	
		Partially Fixed Column Base*	Fixed Column Base	Pinned Column Base	Fixed Column Base	Pinned Column Base
Period (sec)	1.80-2.10	1.95	1.54	2.30	1.66	1.90
Normalized Mode Shape	1.00 0.79 0.44	1.00 0.77 0.42	1.00 0.67 0.25	1.00 0.82 0.51	--	--

b. Standard Pallet Rack, Transverse

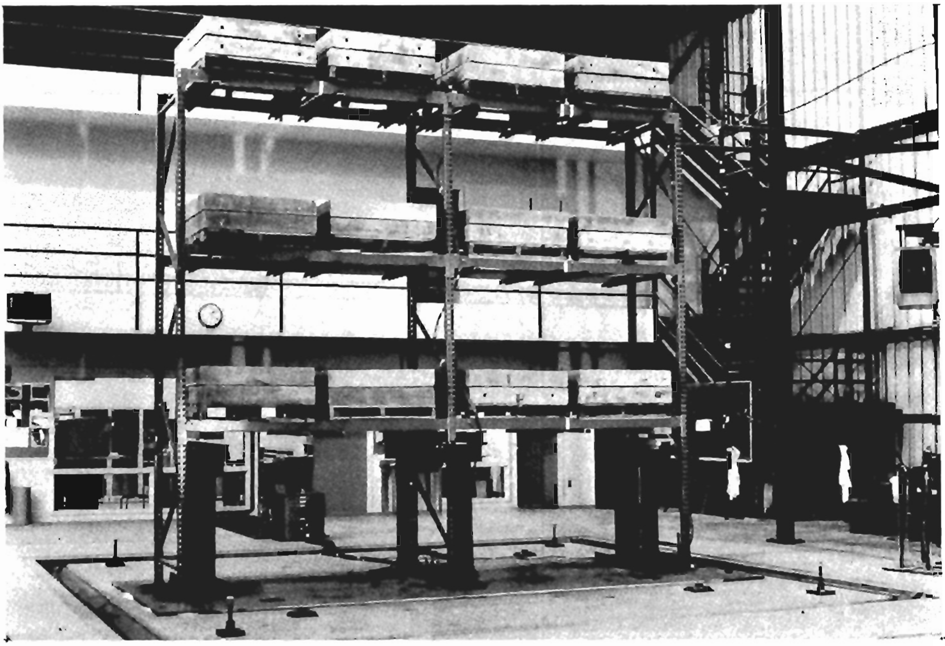
Period (sec)	0.83-0.85	0.82	0.75	0.87	0.73	0.80†
Normalized Mode Shape	1.00 0.75 0.43	1.00 0.72 0.41	1.00 0.67 0.30	1.00 0.75 0.48	--	--

c. Drive-In Rack, Longitudinal

Period (sec)	2.40-2.50	2.36	1.97	3.04	2.10	2.30
Normalized Mode Shape	1.00 0.80 0.59 0.31	1.00 0.82 0.57 0.28	1.00 0.79 0.51 0.21	1.00 0.84 0.63 0.34	--	--

*I_f = 0.2 in.^{4} (moment of inertia of fictitious floor beams)

†By the virtual work method



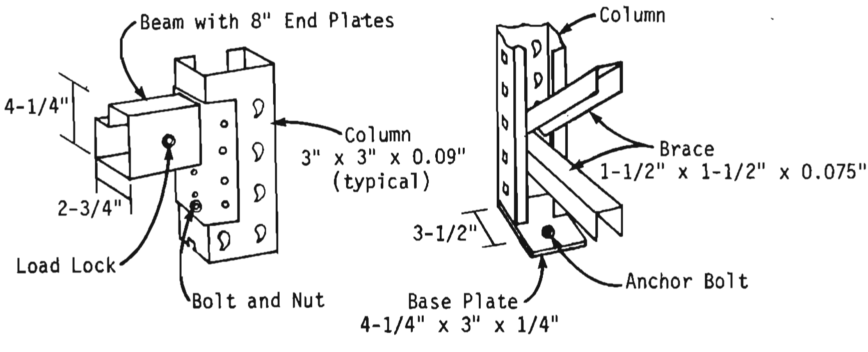
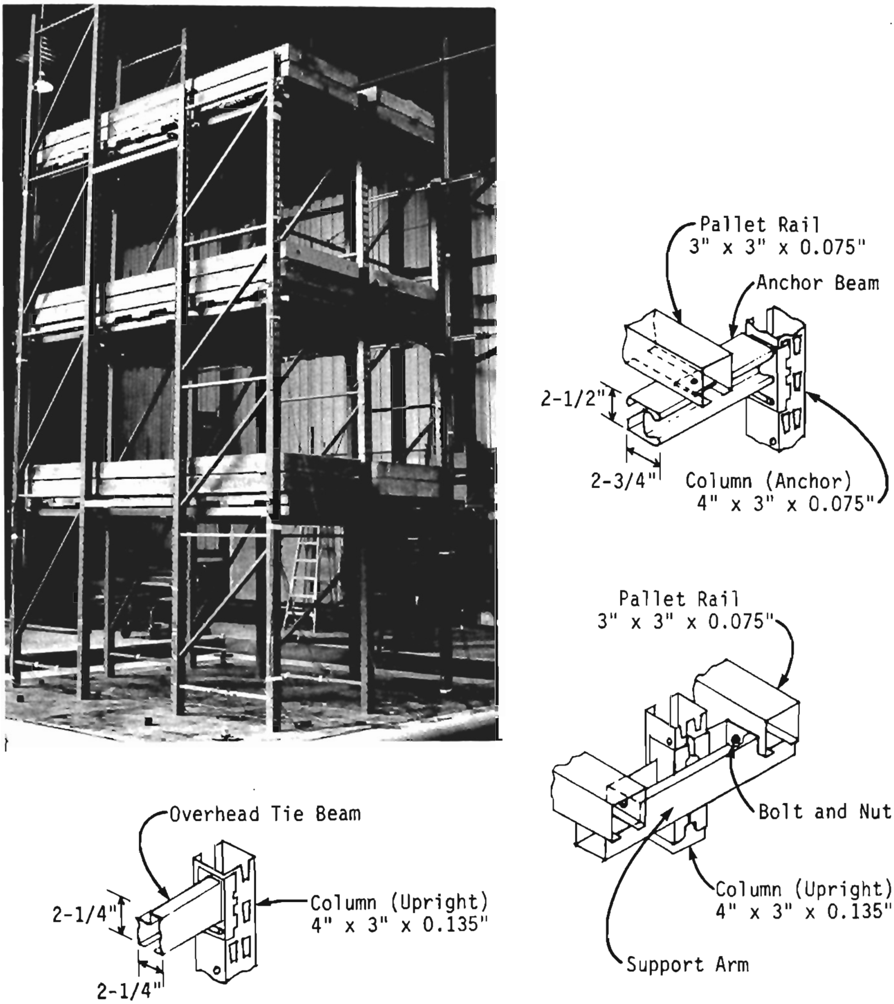
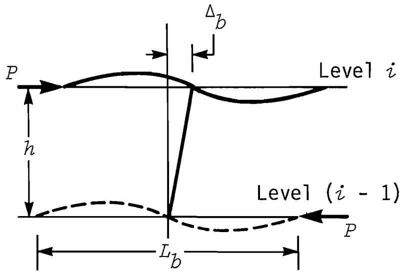


FIGURE 1 STANDARD PALLET RACK AND JOINT DETAILS



Note: 1 in. = 25.4 mm

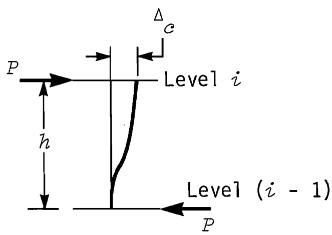
FIGURE 2 DRIVE-IN RACK AND JOINT DETAILS



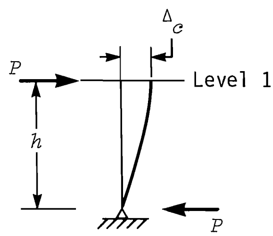
$$\Delta_b = \frac{Ph^2L_b}{12EI_b}$$

Note: $P = V_i/n$, n = number of columns, and I_b is reduced according to Equation (1).

a. Interstory Deflections Due to Beam Bending



Fixed-fixed column



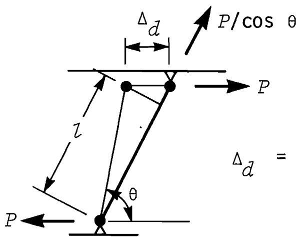
Fixed-pinned column

$$\Delta_c = \frac{Ph^3}{12EI_c} \qquad \Delta_c = \frac{Ph^3}{3EI_c}$$

Note: $P = V_i/n$, and n = number of columns.

b. Interstory Deflections Due to Column Bending

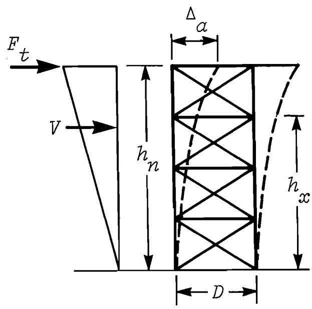
FIGURE 3 FORMULAS FOR CALCULATING INTERSTORY DEFLECTIONS IN UNBRACED-FRAME SYSTEMS



$$\Delta_d = \frac{Pl}{(AE)_{red}\cos^2\theta}$$

Note: for $(AE)_{red}$, see Equation (2).

a. Interstory Deflections Due to Diagonal Deformation



$F_t = 0$

$$\Delta_x = \frac{V}{15A_cD^2h_n^2}\left[-(h_n - h_x)^5 - 15(h_n - h_x)h_n^4 + 5(h_n - h_x)^4h_n + 11h_n^5\right]$$

$$\Delta_a = \frac{11h_n^3V}{15ED^2A_c} \quad \text{for } h_x = h_n$$

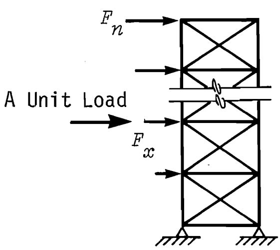
$V = 0$

$$\Delta_x = \frac{2F_t}{3ED^2A_c}\left[(h_n - h_x)^3 - 3(h_n - h_x)h_n^2 + 2h_n^3\right]$$

$$\Delta_a = \frac{4F_th_n^3}{3ED^2A_c} \quad \text{for } h_x = h_n$$

Note: A_c is the area of two columns.

b. Deflections Due to Column Deformation



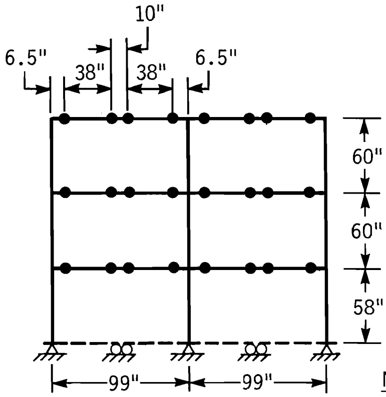
$$\Delta_x = \sum \frac{USl}{AE}$$

U = member force due to a unit load

S = member force due to lateral forces F_i

c. Virtual Work Method

FIGURE 4 FORMULAS FOR CALCULATING DEFLECTIONS AND INTERSTORY DEFLECTIONS IN BRACED-FRAME SYSTEMS



Column (net):
A = 0.69 in.^{2}
I = 1.15 in.^{4}

Shelf Beam:
A = 1.29 in.^{2}
I = 3.27 in.^{4}

Joint:
K_θ = 10^6 lb-in./rad

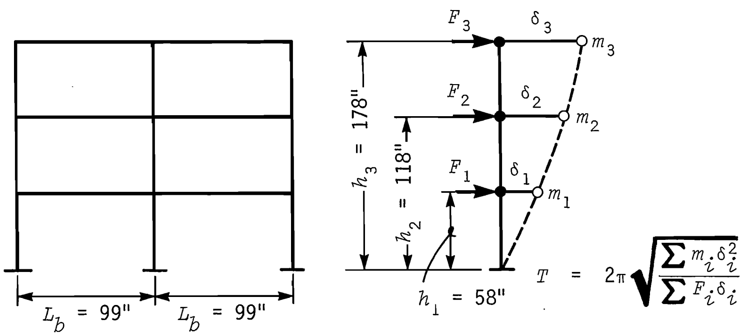
Floor Beam:
I_f = 0.20 in.^{4}

Note:

1 in. = 25.4 mm
1 in.^{2} = 6.5 cm^2
1 in.^{4} = 41.6 cm^4
1 lb-in./rad = 1.13 x 10^{-4} kN-m/rad

—— Beam-Column Element
- - - Beam-Column Element (Fictitious)
● Lumped Mass

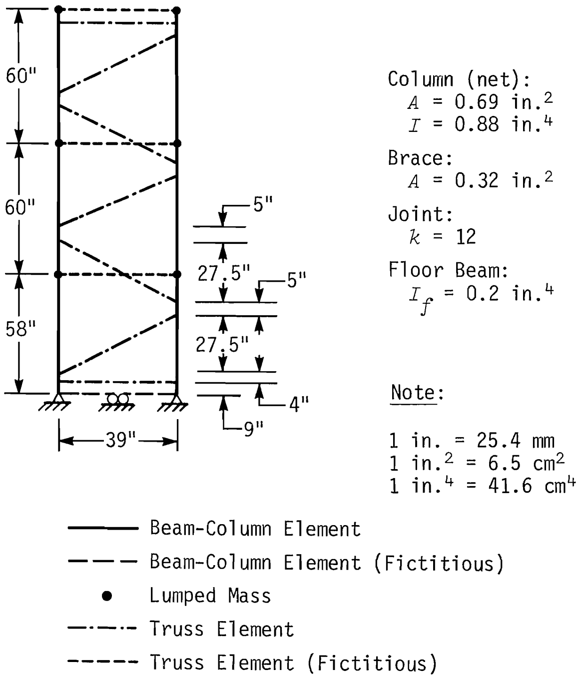
a. Computer Model



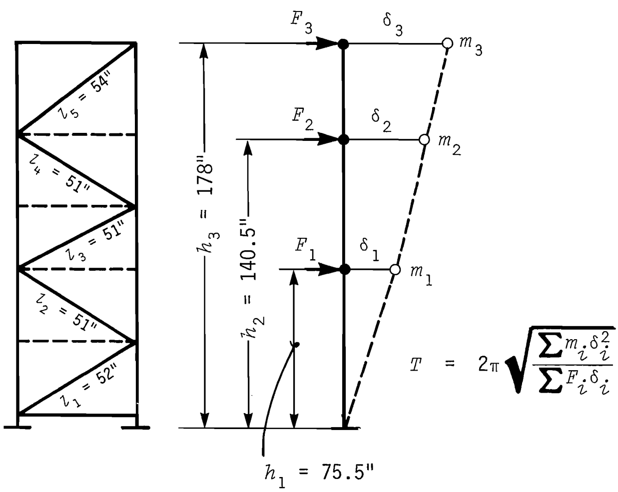
$m_3 = m_2 = m_1$ = 16.2 lb-sec^2/in. (2.9 kg-sec^2/cm)

b. Model for Manual Calculation

FIGURE 5 MATHEMATICAL MODEL - STANDARD PALLET RACK, LONGITUDINAL (UNBRACED-FRAME SYSTEM)



a. Computer Model

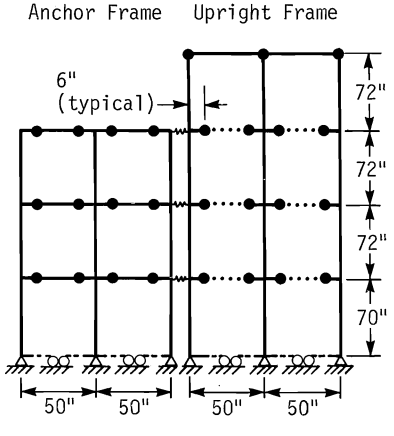


$$T = 2\pi\sqrt{\frac{\sum m_i\delta_i^2}{\sum F_i\delta_i}}$$

$m_3 = m_2 = m_1 = 10.8$ lb-sec^2/in. (1.9 kg-sec^2/cm)

b. Model for Manual Calculation

FIGURE 6 MATHEMATICAL MODEL - STANDARD PALLET RACK, TRANSVERSE



Anchor Column (net):
A = 0.75 in.^{2}
I = 2.21 in.^{4}

Upright Column (net):
A = 1.32 in.^{2}
I = 3.78 in.^{4}

Anchor Beam:
A = 1.09 in.^{2}
I = 1.18 in.^{4}

Upright Beam:
A = 0.46 in.^{2}
I = 0.33 in.^{4}

Floor Beam:
I_f = 0.2 in.^{4}

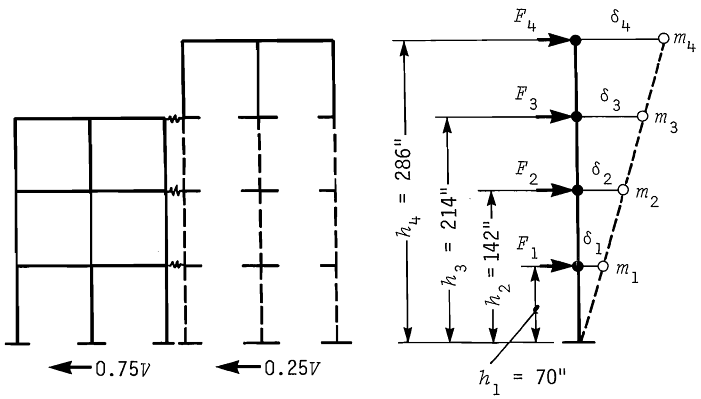
Joint:
K_θ = 10^6 lb-in./rad

········· Truss Element (Fictitious)
-^^^- Rigid Spring
—·— Beam-Column Element (Fictitious)
——— Beam-Column Element
● Lumped Mass

Note:

1 in. = 25.4 mm
1 in.^{2} = 6.5 cm^2
1 in.^{4} = 41.6 cm^4
1 lb-in./rad = 1.13 x 10^{-4} kN-m/rad

a. Computer Model



$m_3 = m_2 = m_1$ = 24.8 lb-sec^2/in. (4.4 kg-sec^2/cm).
m_4 = 0.2 lb-sec^2/in. (0.04 kg-sec^2/cm)

b. Model for Manual Calculation

FIGURE 7 MATHEMATICAL MODEL - DRIVE-IN RACK, LONGITUDINAL